Modern Physics Letters A Vol. 4, No. 27 (1989) 2635–2647

HIGHER-SPIN SYMMETRY IN ONE AND TWO DIMENSIONS (I)

E. S. FRADKIN and V. Ya. LINETSKY

P. N. Lebedev Physical Institute, USSR Academy of Sciences, Leninsky Prospect 53, Moscow 117924, USSR

and

Institute for Nuclear Study, University of Tokyo, Midori-cho, Tanashi, Tokyo 188, Japan

Received 28 September 1989

Higher spin theories in one and two dimensions are considered. The analysis of the ghost sector is carried out and a possible analogy with the ghost sector of the superparticle and the Green-Schwarz superstring in the covariant gauge is discussed.

1. Introduction

Considerable progress in the construction of theories of massless higher spin fields which interact among themselves and with gravity has recently become possible due to the introduction of a new class of infinite dimensional higher spin superalgebras. The algebras $\mathrm{shs}(N|2M)$, which generalize $\mathrm{osp}(N|2M)$, were constructed in Refs. 1–3. The superalgebra of higher spins and auxiliary fields, shsa(1), was constructed in Ref. 4. Based on these superalgebras, the theory of massless higher spin fields in adS_4 was developed in Refs. 1–10. In particular, in Refs. 5 and 6 the cubic self-coupling and coupling with gravity of massless higher spin fields were constructed which turned out to be non-analytical in the cosmological constant. The theory beyond the cubic approximation was considered in Ref. 7. Conformally invariant higher spin theory has been developed in Refs. 11–16. The Chern-Simons conformal higher spin theory in $D = 2 + 1$, which generalizes the $D = 2 + 1$ conformal supergravity, was considered in Refs. 11–13. It is based on the conformal superalgebra $\mathrm{shsc}(N|3)$ (c for conformal) which by construction is isomorphic to $\mathrm{shs}^E(4|N)$. New superalgebras $\mathrm{shsc}^\infty(N|4)$ and $\mathrm{shsc}^{(n)}(4|N)$ which generalize the $D = 3 + 1$ conformal superalgebra $\mathrm{SU}(2, 2|N)$ were constructed in Refs. 13–15. In Ref. 16, $\mathrm{hsc}^\infty(4)$ (a generalization of SU(2, 2)) was used to build the cubic interaction of conformal higher spins among themselves and with conformal gravity. The corresponding lagrangian generalizes the Weyl lagrangian C^2. The supersymmetric version of Ref. 16 will soon be published.

The present paper is devoted to higher spin theories in one and two dimensions, which generalize one- and two-dimensional supergravity. Though these theories by themselves, similarly to the $D = 2 + 1$ theory, do not possess physical degrees of freedom on the mass shell, yet they may play an important role in understanding the theories involving matter fields[a] in $D = 1, 2, 3$. Representations of the corresponding higher spin superalgebras can be realized as symmetries of the space of states in such theories.

The conformal algebra is infinite dimensional in one and two dimensions. However, it contains a finite-dimensional subalgebra, the little conformal algebra. In this paper we consider an extension of the little conformal algebra (and also of the adS_2 and P_2 superalgebras) which involves higher spins in $D = 1$ and $D = 1 + 1$. The higher spin extensions of the Virasoro algebra (and the superconformal algebras) will be considered in a subsequent paper.

2. Conformal Higher Spins in One Dimension

The little conformal algebra in one dimension,

$$\mathrm{so}(2, 1) \simeq \mathrm{SU}(1, 1) \simeq \mathrm{sl}(2; \mathbb{R}) \simeq \mathrm{sp}(2; \mathbb{R})$$

admits a simple oscillator representation in terms of Heisenberg operators

$$[\hat{a}, \hat{a}^\dagger] = 2, \tag{1}$$

$$\hat{T}_+ = \frac{1}{4}\hat{a}^\dagger\hat{a}^\dagger,\ \hat{T}_0 = \frac{1}{8}(\hat{a}^\dagger\hat{a} + \hat{a}\hat{a}^\dagger),\ \hat{T}_- = \frac{1}{4}\hat{a}\hat{a}, \tag{2}$$

$$[\hat{T}_0, \hat{T}_\pm] = \pm\hat{T}_\pm, [\hat{T}_+, \hat{T}_-] = -2\hat{T}_0, \tag{3}$$

where $T_0 = D$, $T_+ = K$, $T_- = P$ are dilatation, conformal boosts and translation generators. The so(2, 1) algebra admits a number of extensions which were classified in Ref. 17. We shall be concerned only with the superalgebra $\mathrm{osp}(N|2)$. The $\mathrm{osp}(1|2)$ superalgebra follows by adding the following generators

$$\hat{T}_{+1/2} = \frac{1}{2}\hat{a}^\dagger,\quad \hat{T}_{-1/2} = \frac{1}{2}\hat{a} \tag{4}$$

to (2).

The infinite-dimensional extension of the $\mathrm{osp}(1|2)$ superalgebra follows by considering polynomials of arbitrary degree in the generating elements a and $a^\dagger$.[3]

[a] Interaction of the scalar field with higher spins in $D = 1$ and 2 was considered in Ref. 37.

For calculational purposes it is very convenient to go over to Weyl symbols A of the operators $\hat{A}$ and work with the associative $*$-multiplication of symbols (see Refs. 3, 4, and 18–20)

$$A * B = A \exp\left(\frac{\overleftarrow{\partial}}{\partial a}\frac{\overrightarrow{\partial}}{\partial a^\dagger} - \frac{\overleftarrow{\partial}}{\partial a^\dagger}\frac{\overrightarrow{\partial}}{\partial a}\right) B. \tag{5}$$

The basis in shs(1|2) is naturally chosen as

$$T^s_m = \frac{(a^\dagger)^{s+m}\,(a)^{s-m}}{2\sqrt{(s+m)!(s-m)!}}, \tag{6}$$

where $s = 0, 1/2, 1, \ldots$ is an so(2, 1)-spin, and $m = -s, -s+1, \ldots, s$ can be interpreted as conformal weight ($[D, T^s_m] = m T^s_m$).

The associative multiplication takes in this basis the form

$$T^s_m * T^{s'}_{m'} = \sum_{s''} a(s, s', s'')\, C^{ss's''}_{mm'm+m'}\, T^{s''}_{m+m'}, \tag{7}$$

where the reduced matrix element is defined by

$$a(s, s', s'') = \left[\frac{(s+s'+s''+1)!}{(s+s'-s'')!(s-s'+s'')!(s'+s''-s)!}\right]^{1/2} \tag{8}$$

(2s″+1)

and C are Clebsch-Gordan coefficients.[21] The super-commutator is defined in shs(1|2) by

$$[A,B\}_* = A * B - (-1)^{\pi(A)\pi(B)}\, B * A, \tag{9}$$

where the Grassmann parity of generators is o(1) for s (half) integer:

$$\pi(T^s_m) = |2s|_2 = \pi_s. \tag{10}$$

The super-commutator of two basis elements assumes then the form[11,12]

$$[T^s_m, T^{s'}_{m'}\}_* = \sum_{s''} \delta(|4ss'+s+s'-s''+1|_2)\, a(s,s',s'')\, C^{ss's''}_{mm'm+m'} \times T^{s''}_{m+m'}, \tag{11}$$

where $\delta(|n|_2) = 1(0)$ for n even (odd).

The superalgebra shs(1|2) admits invariant multilinear forms:

$$\operatorname{Tr}(A * B * \ldots * C), \tag{12}$$

$$\operatorname{Tr}(A(a, a^\dagger)) = A(0,0), \tag{13}$$

$$\operatorname{Tr}(T^s_m * T^{s'}_{m'}) = (-1)^{s+m}\,\delta^{s,s'}\,\delta_{m,-m'} \tag{14}$$

for $A, B, \ldots, C$ being the elements of the Grassmann shell of the first or second class.[22,3]

Gauge fields and parameters of shs(1|2) in $D = 1$ are of the form

$$\omega_0 = \sum_{s,m} \omega^s_{0,m}\, T^s_m\,(i^{-\pi_s}),\ \varepsilon = \sum_{s,m} \varepsilon^s_m\, T^s_m\,(i^{-\pi_s}). \tag{15}$$

Gauge transformations are defined in the usual way, as ($x^0 = t$)

$$\delta\omega_0 = \mathscr{D}_0\varepsilon = \dot{\varepsilon} + [\omega_0, \varepsilon]_*, \tag{16}$$

where ω_0 and ε are elements of the Grassmann shell of the second-class by definition, i.e.,

$$(-1)^{4ss'}\,\omega^s_{0,m}\,T^{s'}_{m'} = T^{s'}_{m'}\,\omega^s_{0,m},\ \omega^s_{0,m}\omega^{s'}_{0,m'} = (-1)^{4ss'}\,\omega^{s'}_{0,m'}\,\omega^s_{0,m}.$$

With the field transformation law (16) at hand, we can write down the Lagrangian for ghosts which correspond to the shs(1|2)-symmetry.

We introduce ghost fields

$$C = \sum_{s,m} C^s_m\,T^s_m,\ \pi(C^s_m) = \pi(\omega^s_{0,m}) + 1. \tag{17}$$

In accordance with the general theory,[23–26] the ghost lagrangian is of the form

$$\mathscr{L}_{\text{gh}} = \operatorname{Tr}\left(\frac{\delta_r\Psi}{\delta\omega_0} * \mathscr{D}_0 C + \frac{\delta_r\Psi}{\delta c} * C * C\right), \tag{18}$$

where the gauge fermion is equal to

$$\Psi = \sum_{s,m} \int dt\,\bar{C}^s_m \Psi^s_m,\ \pi(\Psi) = 1,\ \pi(\bar{C}^s_m) = \pi(C^s_m) \tag{19}$$

and

$$\frac{\delta_r \Psi}{\delta \varphi} = \sum_{s,m} \frac{\delta_r \Psi}{\delta \varphi^s_m} T^s_{-m}(-1)^{s+m}, \varphi^s_m = (\omega^s_{0,m}, C^s_m). \tag{20}$$

Choose the gauge in which all fields except for $\omega^1_{0,-1} = 1/\sqrt{2}$ are set to zero ($e = \sqrt{2}\,\omega^1_{0,-1}$ is the einbein, and $T^1_{-1} = \sqrt{2}P$ is the translation generator). Then

$$\Psi = \sum_{s,m} \int dt\, \bar{C}^s_m \left(\omega^s_{0,m} - \frac{1}{\sqrt{2}}\delta(s-1)\,\delta(m+1)\right). \tag{21}$$

The ghost lagrangian becomes in this gauge a sum of quadratic terms

$$\mathscr{L}_{\text{gh}} = \sum_{s,m} \bar{C}^s_m (\dot{C}^s_m + a^s_m C^s_{m+1}), \tag{22}$$

where $a^s_m = \sqrt{(s-m)(s+m+1)}$.

The lagrangian (2.2) can be brought to the form

$$\mathscr{L}_{\text{gh}} = \sum_{s,m} \bar{C}^s_m \dot{\tilde{C}}^s_m, \tag{23a}$$

where $\tilde{C}^s_m = C^s_m + a^s_m \int dt\, C^s_{m+1}$.

Consider now an extended version based on the superalgebra shs$(N+1|2)$.[3] It follows by adding the Clifford generating elements

$$\{\hat{\psi}_i, \hat{\psi}_j\} = 2\delta_{ij}, i, j = 1, \dots, N. \tag{24}$$

The gauge and ghost fields which correspond to shs$(N+1|2)$, are of the form

$$\omega_0 = \sum_{k=0}^{N} \omega_0^{i(k)}, \psi_{i_1}\dots\psi_{i_k}, C = \sum_{k=0}^{N} C^{i(k)}\psi_{i_1}\dots\psi_{i_k}, \tag{25}$$

where (i) $\omega_0^{i(k)}$, and $C^{i(k)}$ have the structure given by Eqs. (15) and (17) and are antisymmetric tensors with respect to the internal indices; (ii) ψ_i are the symbols of $\hat{\psi}_i$.

The structure of the ghost Lagrangian $\mathscr{L}^N$ is the same as that in Eq. (18) with the ∗-multiplication and the trace now being defined with the ψ_i-s accounted for. The ghost Lagrangian (23a) in this case takes the form

$$\mathscr{L}^N_{\text{gh}} = \sum_{s,m} \bar{C}^{s,\alpha}_m \dot{C}^s_{m,\alpha}, \tag{23b}$$

where α is an index ranging from 1 to 2^N ($\alpha = \{i(k), k=1,\dots,N\}$).

3. Conformal Higher Spins in Two Dimensions

The little conformal algebra in $D = 1 + 1$ is given by a sum of two conformal algebras for one dimension

$$\text{so}(2, 2) \simeq \text{so}(2, 1)_+ \oplus \text{so}(2, 1)_-$$

which act, respectively, on the coordinates x^+ and x^-. The super-extension involving conformal higher spins is of the form

$$\text{shs}\,(1|2)_+ \oplus \text{shs}(1|2)_- . \tag{26}$$

The corresponding gauge fields take the form

$$\omega_\mu = {}^+\omega_\mu + {}^-\omega_\mu, \; {}^\pm\omega_\mu = \sum_{s,m} {}^\pm\omega^s_{\mu,m}\, {}^\pm T^s_m (i^{-\pi_s}), \tag{27}$$

where ${}^\pm T^s_m$ are the generators (6) with the substitution

$$(a, a^\dagger) \to ({}^\pm a, {}^\pm a^\dagger).$$

The gauge fields $\omega^s_{\mu,m}$ represent tensor and spin-tensor (for half-integers) components in the light-cone coordinates.

The curvatures of the superalgebra (26) have the form

$$R_{\mu\nu} = {}^+R_{\mu\nu} + {}^-R_{\mu\nu}, \; {}^\pm R_{\mu\nu}, = \sum_{s,m} {}^\pm R^s_{\mu\nu,m}\, {}^\pm T^s_m (i^{-\pi_s}), \tag{28}$$

$$\begin{aligned} {}^+R^s_{\mu\nu,m} = {} & \partial^+_\mu \omega^s_{\nu,m} - \partial_\nu {}^+\omega^s_{\mu,m} \\ & + \sum \delta(|4s' s'' + s' + s'' - s + 1|_2)\, a(s', s'', s) \\ & \times C^{s's''s}_{m'm''m}\, {}^+\omega^{s'}_{\mu,m'}\, {}^+\omega^{s''}_{\nu,m''} \end{aligned} \tag{29}$$

and similarly for ${}^-R$.

Now, choose the invariant action as[b]

$$S_0 = \int \mathrm{Tr}\,(\lambda * R), \tag{30}$$

where $\lambda = {}^+\lambda + {}^-\lambda$, ${}^\pm\lambda = \Sigma\,(i^{-\kappa_s})\,{}^\pm\lambda^s_m\,{}^\pm T^s_{-m}\,(-1)^{s+m}$ are the Lagrangian multipliers, and $R = 1/2\; R_{\mu\nu}\, dx^\mu \wedge dx^\nu$.

By the invariance of the bilinear form, the action (30) is invariant under the gauge transformations

$$\begin{aligned} \delta\omega_\mu &= \partial_\mu\,\varepsilon + [\omega_\mu, \varepsilon]_* = \mathscr{D}_\mu \varepsilon\,, \\ \delta\lambda &= [\lambda, \varepsilon]_*\,. \end{aligned} \tag{31}$$

The ghost lagrangian in this case assumes the form

$$\mathscr{L}_{\mathrm{gh}} = \mathrm{Tr}\left(\frac{\delta_r\Psi}{\delta\omega_\mu} * \mathscr{D}_\mu C + \frac{\delta_r\Psi}{\delta\lambda} * [\lambda, C]_* + \frac{\delta_r\Psi}{\delta C} * C * C\right), \tag{32}$$

$$\Psi = {}^+\Psi + {}^-\Psi,\ {}^\pm\Psi = \sum_{s,m} \int d^2x\, {}^\pm\bar{C}^s_m\, {}^\pm\Psi^s_m. \tag{33}$$

Now, choose the light-cone gauge

$$\begin{aligned} \Psi = \sum \int d^2x \Big[&{}^+\bar{C}^s_m \left({}^+\omega^s_{+,m} - \frac{1}{\sqrt{2}}\,\delta(s-1)\,\delta(m+1) \right) \\ &+ {}^-\bar{C}^s_m \left({}^-\omega^s_{-,m} - \frac{1}{\sqrt{2}}\,\delta(s-1)\,\delta(m+1) \right) \Big] \end{aligned} \tag{34}$$

(with ${}^+\omega^1_{+,-1}$ and ${}^-\omega^1_{-,-1}$ being the nonzero components of the flat zweibein, see the Appendix). In this gauge the total action can be brought to the form (which follows upon integrating out ${}^+\omega^s_{+,m}$ and ${}^-\omega^s_{-,m}$):

[b] Note that our equations for $S = 1/2, 1$ ${}^+R^s_{\mu\nu,m} = 0$, ${}^-R^s_{\mu\nu,m} = 0$ (${}^\pm\omega^s_{\mu,m} = 0$ for $s > 1$) differ from the equations of Ref. 29. Namely, instead of the constraints $f_{((\mu\nu))} = \phi_{(\mu)} = 0$ we have imposed the constraints $R(K) = R(S) = 0$.

$$S = \sum_{s,m} \int d^2x [{}^+\lambda^s_m(\partial_+ {}^+\omega^s_{-,m} + a^s_m {}^+\omega^s_{-,m+1})$$

$$+ {}^-\lambda^s_m(\partial_- {}^-\omega^s_{+,m} + a^s_m {}^-\omega^s_{+,m+1})$$

$$+ {}^+\bar{C}^s_m(\partial_+ {}^+C^s_m + a^s_m {}^+C^s_{m+1}) + {}^-\bar{C}^s_m(\partial_- {}^-C^s_m + a^s_m {}^-C^s_{m+1}). \quad (35)$$

By the transformation

$$\begin{aligned} {}^+\tilde{\omega}^s_{-,m} &= {}^+\omega^s_{-,m} + a^s_m \int dx^{+} {}^+\omega^s_{-,m+1}, \\ {}^-\tilde{\omega}^s_{+,m} &= {}^-\omega^s_{+,m} + a^s_m \int dx^{-} {}^-\omega^s_{+,m+1}, \\ {}^+\tilde{C}^s_m &= {}^+C^s_m + a^s_m \int dx^{+} {}^+C^s_{m+1}, \\ {}^-\tilde{C}^s_m &= {}^-C^s_m + a^s_m \int dx^{-} {}^-C^s_{m+1} \end{aligned} \quad (36)$$

the action (35) can be brought to the form

$$S = \sum \int d^2x [{}^+\lambda^s_m \partial_+ {}^+\tilde{\omega}^s_{-,m} + {}^-\lambda^s_m \partial_- {}^-\tilde{\omega}^s_{+,m} + {}^+\bar{C}^s_m \partial_+ {}^+\tilde{C}^s_m$$

$$+ {}^-\bar{C}^s_m \partial_- {}^-\tilde{C}^s_m] \quad (37)$$

which is similar to Eq. (23).

The above gauge, similar to any axial gauge, leaves a residual symmetry with parameters obeying certain differential relations such as

$$\partial_+ {}^+\varepsilon^s_m = 0, \partial_- {}^-\varepsilon^s_m = 0.$$

The (N, M)-extended conformal superalgebra in two dimensions is a sum of two superalgebras for one dimension

$$\text{shs}(N+1|2)_+ \oplus \text{shs}(M+1|2)_-.$$

The most interesting case is $N = M$. When $N = M = 2k$,

$$\text{shs}(2k+1|2)_+ \oplus \text{shs}(2k+1|2)_- \simeq \text{shs}(2k+2|2).$$

The Clifford generating elements $\hat{\psi}_i\,(i=1,\ldots,2k+1)$ are split into two sets[3]

$$ {}^{\pm}\hat{\varphi}_i = \hat{\Pi}_{\pm}\,\hat{\psi}_i,\ \hat{\Pi}_{\pm} = \frac{1}{2}(1\pm\hat{\psi}),\ \hat{\psi} = i^k\psi_1\ldots\psi_{2k+1} \tag{38} $$

with $\hat{\Pi}_{\pm}$ projecting into the 'right' and the 'left' subalgebra.

Let us write down the expression for curvatures in this case

$$ \begin{aligned} {}^{+}R^{s}_{\mu\nu,m,i(k)} = {} & \partial_\mu\,{}^{+}\omega^{s}_{\nu,m,i(k)} - \partial_\nu\,{}^{+}\omega^{s}_{\mu,m,i(k)} \\ & + \sum_{(u,v,r)}\ \sum_{(s',s'',m',m'')} \delta(|4s's''+s'+s''-s+r(u+v)+uv+1|_2)\,\frac{k!}{u!v!r!} \\ & \times i^{r-|r|_2}\,a(s',s'',s)\,C^{s's''s}_{m'm''m}\,\delta(k-u-v)\times{}^{+}\omega^{s'}_{\mu,m',i(u)j(r)}\,{}^{+}\omega^{s''}_{\nu,m'',i(v)j(r)} \end{aligned} \tag{39} $$

and similarly for ${}^{-}R$.

4. Higher Spins in adS$_2$

The adS$_2$-superalgebra osp$(1|2)$ is defined by the commutation relations

$$ \begin{aligned} [P_+,P_-] &= -2\lambda^2 M,\ [M,P_\pm] = \pm\,P_\pm, \\ \{Q_{+1/2},Q_{-1/2}\} &= \lambda M,\ \{Q_{\pm1/2},Q_{\pm1/2}\} = P_\pm, \\ [M,Q_{\pm1/2}] &= \pm\frac{1}{2}Q_{\pm1/2},\ [P_\pm,Q_{\mp1/2}] = \mp\,Q_{\pm1/2}, \end{aligned} \tag{40} $$

where λ is the inverse adS$_2$ radius. Gauge fields and curvatures of the corresponding infinite-dimensional higher spin extension, shs$(1|2)$, are of the form

$$ \omega_\mu = \sum_{s,m}\omega^{s}_{\mu,m}\,T^{s}_{m}\,(i^{-\pi_s}), \tag{41} $$

$$ \begin{aligned} R^{s}_{\mu\nu,m} = {} & \partial_\mu\omega^{s}_{\nu,m} - \partial_\nu\omega^{s}_{\mu,m} + \sum \delta(|4s's''+s'+s''-s+1|_2)\,\lambda^{|m'|+|m''|-|m'+m''|} \\ & \times a(s',s'',s)\,C^{s's''s}_{m'm''m}\,\omega^{s'}_{\mu,m'}\,\omega^{s''}_{\nu,m''}. \end{aligned} \tag{42} $$

A flat space limit $\lambda\to0$, which gives rise to $\delta(|m'|+|m''|-|m'+m''|)$, is available in Eq. (42). The corresponding curvatures generalize the $D=1+1$

Poincaré supergravity curvatures, and the superalgebra $shs^f(1|2)$ (f for flat) extends the $D = 1 + 1$ Golfand-Lichtman superalgebra.

The action for these theories has the form (30) and the ghost Lagrangian coincides with that in Eq. (32), with all quantities now taking values in $shs(1|2)$.

5. Conclusion

Let us note that the conformal higher-spin infinite-dimensional symmetries in one and two dimensions considered above generate the ghost sector (23), (37) analogous to the ghost sector which emerges when covariantly quantizing the superparticle[27,28] and the closed Green-Schwarz superstring.[29,30,31] Note at the same time that while the superparticle and superstring theories involve reducible generators and, accordingly, ghosts of each generation are ghosts for ghost of the proceeding generation, in the higher spin theory, on the other hand, the infinite dimensional symmetry with irreducible generators is given as a whole from the very beginning and all the ghosts arise just as ghosts for this symmetry.

This supports the idea that a formulation of superstring and superparticle exists in which gauge fields of all spins are present and the gauge algebra described in the configuration space by an infinite number of irreducible generators.

It may be hoped that in the open string theory the adS_2-higher spin symmetry plays an important role. Both in the open string theory and in the adS_2-higher spin theory, the little conformal group in two dimensions, so(2,2), gets broken down to so(2,1). In that case the dimensional constant of the open string theory, the strings' length parameter, is related to the cosmological constant of the adS_2-higher spin theory.

Extending to three dimensions the analogy between the ghost sector of higher spin theories in $D = 1,2$ and superparticle and superstring theories, it may be conjectured that the ghost lagrangian of the supermembrane theory in the covariant quantization has the structure of the type of $\mathrm{Tr}(\bar{C} * \partial C)$, similar to the higher spin theory in $D = 2 + 1$ in the axial gauge.[12,13]

It may be conjectured that the adS_3-higher spin theory is related to the supermembrane without Weyl invariance[32,33] whereas the higher spin superconformal theory corresponds to the Weyl-invariant supermembrane.[34–36]

We hope to return to these problems in the future.

Acknowledgments

One of the authors (E.S.F.) would like to thank Professor H. Terazawa for his warm hospitality extended to him during his visit to Institute for Nuclear Study, University of Tokyo.

Appendix

Spin $s+1$ in $D \geqslant 4$ was described in Refs. 10, 38, and 39 by the following set of gauge fields:

(1) for integer s,

$$\omega_{\mu,b(t),a(s)},\ t=0,1,\ldots,S, \tag{A.1}$$

$$t\,\omega_{\mu,b(t-1)a,a(s)}=0,\ s(s-1)\,\omega_{\mu,b(t),}{}^{c}{}_{c\,a(s-2)}=0 \tag{A.2}$$

(2) for half-integer s

$$\omega_{\mu,b(t),a(s-\frac{1}{2});\alpha},\ t=0,1,\ldots,s-\frac{1}{2}, \tag{A.3}$$

$$t\,\omega_{\mu,b(t-1)a,a(s-\frac{1}{2});\alpha}=0,\ \gamma^{c\ \beta}_{\alpha}\omega_{\mu,b(t),a(s-3/2)c;\beta}=0, \tag{A.4}$$

where α and β are spinor indices and $\{\gamma^a,\gamma^b\}=2\eta^{ab}$. Consider this set of fields in $D=2$.[c]

In the light-coordinates the only nonzero components are

(1) integer s

$$\omega^{s}_{\mu,m}=\begin{cases}\omega_{\mu,\underbrace{-\ldots-}_{s-m},\underbrace{+\ldots+}_{s}},\ s\geqslant m>0\\[4pt] \omega_{\mu,\underbrace{+\ldots+}_{s+m},\underbrace{-\ldots-}_{s}},\ -s\leqslant m<0\\[4pt] \omega_{\mu,\underbrace{-\ldots-}_{s},\underbrace{+\ldots+}_{s}}=\omega_{\mu,\underbrace{+\ldots+}_{s},\underbrace{-\ldots-}_{s}}(-1)^s,\ m=0\end{cases} \tag{A.5}$$

(2) half-integer s

$$\omega^{s}_{\mu,m}=\begin{cases}\omega_{\mu,\underbrace{-\ldots-}_{s-m},\underbrace{+\ldots+}_{s-1/2},+1/2},\ s\geqslant m\geqslant\frac{3}{2}\\[4pt] \omega_{\mu,\underbrace{+\ldots+}_{s+m},\underbrace{-\ldots-}_{s-1/2},-1/2},\ -s\leqslant m\leqslant-\frac{3}{2}\\[4pt] \omega_{\mu,\underbrace{-\ldots-}_{s-\frac{1}{2}},\underbrace{+\ldots+}_{s-\frac{1}{2}},+1/2},\ m=\frac{1}{2}\\[4pt] \omega_{\mu,\underbrace{-\ldots-}_{s-\frac{1}{2}},\underbrace{+\ldots+}_{s-\frac{1}{2}},-1/2},\ m=-\frac{1}{2}\end{cases} \tag{A.6}$$

[c] Our notations for two dimensions are: $a,b,c=0,1$ (tangent space indices), $\mu,\nu=0,1$ (world indices), $\eta^{ab}=(+,-)$, $x^{\pm}=(x^0\pm x^1)/\sqrt{2}$, $\eta^{++}=\eta^{--}=0$, $\eta^{+-}=1$, $\varepsilon_{01}=-\varepsilon_{+-}=1$ and the two dimensional Majorana Spinor is $\psi=\begin{pmatrix}\psi_{+1/2}\\ \psi_{-1/2}\end{pmatrix}$.

Thus, in the spin-tensor formalism the shs(1|2)- (and its flat contraction shs^f(1|2)) gauge fields have the form (A.1–4) with $s = 0, 1/2, 1, \ldots$. Further, the two-dimensional supergravity fields are, in our notations,

$$(e^a_\mu, \omega_{\mu ab}, \psi_{\mu a}) \leftrightarrow (\omega^1_{\mu,\pm 1}, \omega^1_{\mu,0}, \omega^{1/2}_{\mu,\pm 1/2}). \tag{A.7}$$

Gauge fields of $D = 1 + 1$ conformal higher spin superalgebra are given by a double set of fields (A.1–4).

The conformal algebra in $D = 1 + 1$ reads

$$\begin{aligned} [P_+, K_-] &= D - M, [P_-, K_+] = D + M, \\ [M, K_\pm] &= \pm K_\pm, [M, P_\pm] = \pm P_\pm, \\ [D, P_\pm] &= -P_\pm, [D, K_\pm] = K_\pm, \end{aligned} \tag{A.8}$$

where, P, K are light-cone components of the translation and conformal boosts generators, and D and M are generators of dilations and pseudorotations.

The ^+T and ^-T generators, which are given by

$$\begin{aligned} {}^+T_+ &= K_+, {}^+T_- = P_-, {}^+T_0 = \frac{1}{2}(M + D) \\ {}^-T_+ &= K_-, {}^-T_- = P_+, {}^-T_0 = \frac{1}{2}(D - M). \end{aligned} \tag{A.9}$$

Span the so(2, 1)$_+$ and so(2, 1)$_-$ algebras respectively, the fields of the conformal gravity are, in our notations,

$$\begin{aligned} (e^a_\mu; \omega_{\mu ab}; b_\mu; f^a_\mu) \leftrightarrow ({}^+\omega^1_{\mu,-1}, {}^-\omega^1_{\mu,-1}; {}^+\omega^1_{\mu,0} - {}^-\omega^1_{\mu,0}; {}^+\omega^1_{\mu,0} + {}^-\omega^1_{\mu,0}; \\ {}^+\omega^1_{\mu,1}, {}^-\omega^1_{\mu,1}) \end{aligned} \tag{A.10}$$

because $\sqrt{2}\, T_+ = T^1_1, \sqrt{2}\, T_- = T^1_{-1}, 2T_0 = T^1_0$ (cf. Eqs. (2) and (6)).

The flat zweibein is $e^a_\mu = \delta^a_\mu$, or,

$$\begin{aligned} {}^+\omega^1_{+,-1} &= {}^-\omega^1_{-,-1} = \frac{1}{\sqrt{2}}, \\ {}^+\omega^1_{-,-1} &= {}^-\omega^1_{+,-1} = 0 \end{aligned} \tag{A.11}$$

($1/\sqrt{2}$ comes from the normalization $\sqrt{2} P_\pm = {}^\pm T^1_{-1}$).

References

1. M. A. Vasiliev and E. S. Fradkin, *Dok. Akad. Nauk.* **29** (1986) 1100, *Zh. Eksp. Teor. Fiz.* **44** (1986) 484.
2. E. S. Fradkin and M. A. Vasiliev, *Ann. Phys.* **177** (1987) 63.
3. M. A. Vasiliev, *Fortschr. Phys.* **36** (1988) 33.
4. E. S. Fradkin and M. A. Vasiliev, *Int. J. Mod. Phys.* **A3** (1988) 2983.
5. E. S. Fradkin and M. A. Vasiliev, *Phys. Lett.* **B189** (1987) 89.
6. E. S. Fradkin and M. A. Vasiliev, *Nucl. Phys.* **B291** (1987) 141.
7. M. A. Vasiliev, *Ann. Phys.* **190** (1989) 59.
8. M. A. Vasiliev, *Fortschr. Phys.* **35** (1987) 741; *Nucl. Phys.* **B307** (1988) 319.
9. M. A. Vasiliev, *Phys. Lett.* **B209** (1988) 491.
10. M. A. Vasiliev, *Sov. J. Nucl. Phys.* **32** (1980) 439.
11. E. S. Fradkin and V. Ya. Linetsky, *Mod. Phys. Lett.* **A4** (1989) 731.
12. E. S. Fradkin and V. Ya. Linetsky, preprint Lab. de Phys. Theor. et Hautes Energies PAR-LPTHE 89-06.
13. E. S. Fradkin and V. Ya. Linetsky, *Mod. Phys. Lett.* **A**, in press.
14. E. S. Fradkin and V. Ya. Linetsky, preprint Lab. de Phys. Theor. et Hautes Energies PAR-LPTHE 89-22.
15. E. S. Fradkin and V. Ya. Linetsky, DAN USSR (1989).
16. E. S. Fradkin and V. Ya. Linetsky, preprint Trieste ICTP/89/173.
17. M. Günaydin, G. Sierra, and P. K. Townsend, *Nucl. Phys.* **B274** (1986) 429.
18. F. A. Berezin, in *The Method of Second Quantization* (Academic Press, 1966).
19. F. A. Berezin and M. S. Marinov, *Ann. Phys.* **104** (1977) 336.
20. I. A. Batalin and E. S. Fradkin, *Ann. Inst. Henri Poincaré* **49** (1988) 145.
21. L. C. Biedenharn and J. D. Louck, in *Angular Momentum in Quantum Physics* (Addison-Wesley, 1981), Vol. 1.
22. F. A. Berezin, in *Introduction into the Algebra and Analysis with* Anticommuting Variables (Moscow State University press, 1983), in Russian.
23. E. S. Fradkin and G. A. Vilkovisky, *Phys. Lett.* **B55** (1975) 224.
24. I. A. Batalin and G. A. Vilkovisky, *Phys. Lett.* **B69** (1977) 309; E. S. Fradkin and T. E. Fradkina, *Phys. Lett.* **B72** (1978) 343.
25. I. A. Batalin and G. A. Vilkovisky, *Phys. Lett.* **B102** (1981) 27.
26. I. A. Batalin and E. S. Fradkin, *Phys. Lett.* **B128** (1983) 303; *Rivista del Nuovo Cim.* **9** (1986) 1.
27. M. B. Green and C. Hull, *Phys. Lett.* **B225** (1989) 57.
28. U. Lindström, M. Rocek, W. Siegel, P. van Nieuwenhuizen, and A. E. van de Ven, preprint ITP-SB-89-17.
29. S. J. Gates, Y. Hanssoun, and P. van Nieuwenhuizen, preprint ITP-SB-88-43 (UNDEPP-89-46).
30. R. E. Kallosh, *Phys. Lett.* **B225** (1989) 49.
31. S. J. Gates, Jr., M. T. Grisary, U. Lindström, M. Rocek, W. Siegel, P. van Nieuwenhuizen, and A. E. van de Ven, *Phys. Lett.* **B225** (1989) 44.
32. M. Duff, *Lectures at Superworld III*, CTP-TAMU-04/89.
33. P. K. Townsend, *Three lectures on Supermembranes*, DAMTP.
34. U. Lindström and G. Theororidis, *Phys. Lett.* **B208** (1988) 407.
35. A. Karlhede and U. Lindström, *Phys. Lett.* **B209** (1988) 441.
36. U. Lindström and M. Rocek, *Phys. Lett.* **B218** (1989) 207.
37. A. K. H. Bengtsson and I. Bengtsson, *Phys. Lett.* **B174** (1986) 294.
38. V. E. Lopatin and M. A. Vasiliev, *Mod. Phys. Lett.* **A3** (1988) 257.
39. M. A. Vasiliev, *Nucl. Phys.* **B301** (1988) 26.