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A NEW CLASS OF INFINITE-DIMENSIONAL LIE ALGEBRAS: AN ANALYTICAL CONTINUATION OF THE ARBITRARY FINITE-DIMENSIONAL SEMISIMPLE LIE ALGEBRA

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With any semisimple Lie algebra g we can associate an infinite-dimensional Lie algebra AC(g) which is an analytic continuation of g from its root system to its root lattice. The manifest expressions for the structure constants of analytic continuations of the symplectic Lie algebras sp_{2n} are obtained by the Poisson-bracket realizations method and AC(g) for $g = sl_n$ and so_n are discussed. The representations, central extension, supersymmetric and higher spin generalizations are considered. The Virasoro theory is a particular case where $g = sp_2$.

1. Introduction

Infinite-dimensional Lie algebras play an increasingly important role in modern quantum field theory and quantum statistics. The most important known algebras are the affine Kac-Moody algebras,[1] the higher spin algebras,[2] and the Virasoro algebra.[3] It is of curiosity that the Virasoro algebra containing sl_2 as a finite-dimensional subalgebra is usually considered separately, but not as a member of some class of the infinite algebras. Physically the cause is that only in one and two dimensions, the conformal group is infinite-dimensional and, on the other hand, in more than two dimensions, the conformal algebra is finite-dimensional. The theory of the Virasoro algebra and its representations is a corner stone of the algebraic theory of exactly solvable two-dimensional conformal models. At the same time, the functional theory of D-dimensional models ($D \geq 2$) has been rather elaborated.[4] However, the algebraic theory of D-dimensional exactly solvable models has not been constructed yet. What is the analog of the Virasoro algebra in D-dimensions? This paper gives a mathematical answer. In the first part of the paper general theorems are given, and in the second part the concrete algebras are considered.

2. The General Theorems

Let g be some arbitrary semisimple complex[a] range-n Lie algebra, h its Cartan subalgebra with the basis $\{H_i, i = 1, \ldots, n\}$, Δ its root system, and E_α the step

[a] All algebras are supposed to be complex, however, all the results can be formulated also over R.

generators of g ($\boldsymbol{\alpha} \in \Delta$, we denote the roots by small Greek letters $\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \ldots$). Then commutation relations in the Cartan-Weyl basis $\{H_i, E_{\boldsymbol{\alpha}}\}$ have the familiar forms

$$\begin{aligned} &[H_i, H_j] = 0,\ [H_i, E_{\boldsymbol{\alpha}}] = \alpha_i E_{\boldsymbol{\alpha}}\,, \\ &[E_{\boldsymbol{\alpha}}, E_{-\boldsymbol{\alpha}}] = \boldsymbol{\alpha}\cdot\mathbf{H}(\boldsymbol{\alpha}\cdot\mathbf{H} = \alpha^i H_i)\,, \\ &[E_{\boldsymbol{\alpha}}, E_{\boldsymbol{\beta}}] = N(\boldsymbol{\alpha}, \boldsymbol{\beta})\, E_{\boldsymbol{\alpha}+\boldsymbol{\beta}},\ \boldsymbol{\alpha} \neq -\boldsymbol{\beta}\,, \end{aligned} \tag{1}$$

where $N(\boldsymbol{\alpha}, \boldsymbol{\beta}) = -N(\boldsymbol{\beta}, \boldsymbol{\alpha})$ are the structure constants of g defined by the root system up to a redefinition of $E_{\boldsymbol{\alpha}}$.

Let $\Lambda(g)$ be a root lattice of g defined by the root system Δ, i.e., $\Lambda(g)$ is an infinite set of vectors $\mathbf{A} = A^i\boldsymbol{\alpha}_{(i)}$, $A^i \in \mathbb{Z}$ (we denote vectors from $\Lambda(g)$ by capital Greek letters $\mathbf{A}, \mathbf{B}, \boldsymbol{\Gamma}, \ldots$ and $\boldsymbol{\alpha}_{(i)}$, $i = 1, \ldots, n$ are the simple roots). For convenience, let us denote all the generators of g as $E^i_{\boldsymbol{\alpha}}$ so that for $i = 1$ $\boldsymbol{\alpha} \neq \mathbf{0}$ and $E^1_{\boldsymbol{\alpha}} = E_{\boldsymbol{\alpha}}$, and for $i = 1, \ldots, n$, $\boldsymbol{\alpha} = \mathbf{0}$ and $E^i_{\mathbf{0}} = H_i$ (we have included the zero-root $\mathbf{0}$ in the lattice $\Lambda(g)$). The commutation relations in these notations are

$$\left[E^i_{\boldsymbol{\alpha}}, E^j_{\boldsymbol{\beta}}\right] = N^{ij}_k(\boldsymbol{\alpha}, \boldsymbol{\beta}) E^k_{\boldsymbol{\alpha}+\boldsymbol{\beta}}\,. \tag{2}$$

Lemma 1.

There exists a unique (up to the equivalency) non-decomposable infinite-dimensional representation ρ of the complex semisimple Lie algébra g containing a given irreducible finite-dimensional representation of g in its invariant subspace, such that its weight diagram under $h \subset g$ is a n-dimensional lattice generated by the weight system of ρ.

This lemma follows from the results of Ref. 5. Such infinite-dimensional representations are members of the basic non-unitary series of representations of the group G (exactly, they are their infinitesimal forms). They are often referred to as elementary g-modules or Harish-Chandra modules. The matrix elements of these infinite-dimensional representations are obtained as a straightforward analytic continuation of the matrix elements of the finite-dimensional g-representations in some suitable basis. Let *qadg* (q-quasi) denote such a representation which contains the adjoint representation *adg* in its invariant subspace. We call it a quasi-adjoint representation or an analytic continuation of *adg*. The basis in *qadg* can be chosen in the form $E^I_{\mathbf{A}}$, where $\mathbf{A} \in \Lambda(g)$ is a weight under h and I is an index in the weight subspace $V(\mathbf{A})$. In these notations, the finite-dimensional subalgebra g generators are $E^i_{\boldsymbol{\alpha}}$ when $\mathbf{A} = \boldsymbol{\alpha} \in \Delta$ and n first generators $H_i = E^I_{\boldsymbol{\alpha}}$ when $I = i = 1, \ldots, n$.

We define an infinite-dimensional Lie algebra associated with *qadg* as follows,

$$\left[E^I_{\mathbf{A}}, E^J_{\mathbf{B}}\right] = N^{IJ}_K(\mathbf{A}, \mathbf{B}) E^K_{\mathbf{A}+\mathbf{B}}\,, \tag{3}$$

where it is supposed that when both $E^I_{\mathbf{A}}$ and $E^J_{\mathbf{B}}$ belong to g, the structure constants coincide with the Cartan-Weyl ones (1, 2), and when only one of them belongs to g the structure constants coincide with the matrix elements of *qadg* (these are the "initial conditions"). We call such an algebra an analytic continuation of g and denote it by AC(g).

Theorem 1.

For any semisimple finite-dimensional Lie algebra g there exists an unique analytic continuation AC(g).

In order to prove this, it should be shown that, from the Jacobi identities for (3) and the "initial conditions" for the structure constants, all the structure constants are obtained in a unique way (up to a redefinition of the generators' normalization). However from the Wigner-Eckart theorem for the basic non-unitary series it follows that the structure constants are the Clebsh-Gordan coefficients[6] for the tensor product of two quasi-adjoint representations ($qadg \otimes qadg = qadg$ + other terms). Hence they may be found in a unique way up to a generators' renormalization. Later the manifest expressions for the structure constants of AC(g) for the symplectic Lie algebras will be obtained from the Poisson-bracket realization.

Conjecture 1.

The algebra AC(g) for any semisimple g admits a unique non-trivial central extension $\widetilde{AC}(g)$.

We expect that the corresponding cocycle is given by the general Batalin-Fradkin formula[7] (see Ref. 6, pp. 16–19)

$$C^{IJ}(\mathbf{A}, \mathbf{B}) = \delta_{\mathbf{A}+\mathbf{B},\mathbf{0}}$$

$$\times \sum_{K,L} \sum_{\Gamma \in \Lambda} \theta(\Gamma)\theta(\mathbf{A}-\Gamma) N^{IL}_{K}(\mathbf{A}, -\Gamma) N^{JK}_{L}(-\mathbf{B}, \Gamma-\mathbf{A}), \tag{4}$$

where $\theta(\Gamma) = 1(\Gamma > \mathbf{0}), 0(\Gamma < \mathbf{0}), 1/2(\Gamma = \mathbf{0})$, (we have introduced usual ordering in $\Lambda(g)$).

3. Representations

Theorem 2.

There exists a unique analytic continuation of each of the finite-dimensional irreducible representations of g to the infinite-dimensional representation of AC(g).

The proof is based on Lemma 1 and the Wigner-Eckart theorem for the tensor product of $qadg$ and the non-decomposable representation from Lemma 1 containing the given irreducible finite-dimensional representation in its invariant subspace.

4. A Manifest Construction for the Simple Finite-Dimensional Lie Algebras of the Classical Series

Here we give the Poisson-bracket realizations of AC(g) for $g = \mathrm{sp}_{2n}$.

Let $\{a_k, a^{\dagger}_k, k = 1, \ldots, n\}$ be a set of $2n$ variables and V^2 be a linear space of quadratic polynomials on these variables. Then V^2 becomes a Lie algebra isomorphic to sp_{2n} after introducing the Poisson brackets

$$[f, g] = \sum_{k=1}^{n}\left(\frac{\partial f}{\partial a_k}\frac{\partial g}{\partial a_k^\dagger} - \frac{\partial f}{\partial a_k^\dagger}\frac{\partial g}{\partial a_k}\right). \tag{5}$$

To perform an analytic continuation to AC(sp_{2n}) it is convenient to choose the following basis in sp_{2n} ($\dim V^2 = \dim(sp_{2n}) = n(2n+1)$),

$$E^{\mathbf{l}}_{\mathbf{m}} = \prod_{k=1}^{n}(a_k^\dagger)^{\frac{l_k+m_k}{2}}(a_k)^{\frac{l_k-m_k}{2}}, \tag{6}$$

where $\mathbf{m} = (m_1, \ldots, m_n)$, $\boldsymbol{l} = (l_1, \ldots, l_n)$, $m_K \in \mathbb{Z}$, $l_K \in \mathbb{Z}$ and the following conditions are satisfied,

$$\text{i)} \sum_{k=1}^{n} l_k = 2\,, \tag{7a}$$

$$\text{ii)}\; l_k \geq 0,\; k = 1, \ldots, n\,, \tag{7b}$$

$$\text{iii)}\; (-1)^{l_k+m_k} = 1,\; k = 1, \ldots, n\,, \tag{7c}$$

$$\text{iv)}\; |m_k| \leq l_k,\; k = 1, \ldots, n\,. \tag{7d}$$

The basis (6) is related with the reduction $sp_{2n} \to sp_2 \oplus \ldots \oplus sp_2$ (n times). The vector $\mathbf{l}$ defines irreps of $sp_2 \oplus \ldots \oplus sp_2$ and $\mathbf{m}$ labels basis elements in each irrep defined by $\mathbf{l}$. Evidently only the following vectors $\mathbf{l}$ satisfy conditions i and ii,[b]

$$\mathbf{l}_{(k)} = (0 \ldots \underset{k\text{th}}{2} \ldots 0)\,(n \text{ vectors})\,, \tag{8a}$$

$$\mathbf{l}_{(k,\,p)} = (0 \ldots \underset{k\text{th}}{1} \ldots \underset{p\text{th}}{1} \ldots 0),\; k < p\left(\frac{n(n-1)}{2} \text{ vectors}\right). \tag{8b}$$

Only the following vectors $\mathbf{m}$ (root vectors) satisfy conditions iii and iv for a given fixed $\mathbf{l}$,

$$\mathbf{l}_{(k)}\!: \mathbf{m}^{\pm}_{(k)} = (0 \ldots \pm\underset{k\text{th}}{2} \ldots 0),\; \mathbf{0}\;(3 \text{ vectors})\,, \tag{9a}$$

$$\mathbf{l}_{(k,\,p)}\!: \mathbf{m}^{\pm\pm}_{(k,p)} = (0 \ldots \pm\underset{k\text{th}}{1} \ldots \pm\underset{p\text{th}}{1} \ldots 0)\;(4 \text{ vectors}). \tag{9b}$$

The commutation relations for sp_{2n} in this basis have a form

$$\left[E^{\mathbf{l}}_{\mathbf{m}}, E^{\mathbf{l}'}_{\mathbf{m}'}\right] = \frac{1}{2}\sum_{k=1}^{n}(l_k m'_k - l'_k m_k)E^{\mathbf{l}+\mathbf{l}'-\mathbf{l}_{(k)}}_{\mathbf{m}+\mathbf{m}'}\,. \tag{10}$$

Here $E^{\mathbf{l}_{(k)}}_{\mathbf{0}} = H_k$ are the generators of the Cartan subalgebra and all the other generators are step operators:

$$\left[H_k, E^{\mathbf{l}}_{\mathbf{m}}\right] = m_k E^{\mathbf{l}}_{\mathbf{m}}\,. \tag{11}$$

[b] To prevent misunderstanding, note that l_k are components of the arbitrary vector $\mathbf{l} = (l_1, \ldots, l_n)$, while $\mathbf{l}_{(k)}$ are the vectors with 2 on the kth position, i.e., $l_{(k)p} = 2\delta_{k,p}$.

The generators $E^{\mathbf{l}(k)}_{\mathbf{m}}$ form a subalgebra in sp_{2n} which is isomorphic to $sp_2 \oplus \ldots \oplus sp_2$ (n times):

$$\left[E^{\mathbf{l}(k)}_{\mathbf{m}}, E^{\mathbf{l}(k')}_{\mathbf{m}'}\right] = m'_k\, E^{\mathbf{l}(k')}_{\mathbf{m}+\mathbf{m}'} - m_{k'}\, E^{\mathbf{l}(k)}_{\mathbf{m}+\mathbf{m}'}. \tag{12}$$

Now let us perform an analytic continuation from Δ to Λ (sp_{2n}), i.e., revoke the condition (7d). Now $m_k \in \mathbb{Z}$, $(-1)^{l_k+m_k} = 1$.

It means that the monomials (6) now contain both the positive and negative powers of their variables. Along with the revocation of (7d), the condition (7b) should also be revoked because in the r.h.s. of (10) both the positive and negative values of $l_p + l'_p - 2\delta_{p,k}$ (components of the vector $\mathbf{l} + \mathbf{l}' - \mathbf{l}_{(k)}$) may appear. The algebra obtained in this way is isomorphic to $AC(sp_{2n})$ owing to the non-decomposibility of representation of g and the uniqueness Theorem 1. The generators of $AC(sp_{2n})$ $E^{\mathbf{l}}_{\mathbf{m}}$ are defined by the two vectors $\mathbf{l}$ and $\mathbf{m}$, so that $\sum_{k=1}^{n} l_k = 2$, $(-1)^{l_k+m_k} = 1$, and the structure constants in (10) are straightforward analytic continuation of the sp_{2n} ones.

The commutation relations (12), when $m_k \in 2\mathbb{Z}$ (even numbers), give an algebra $AC(sp_2 \oplus \ldots \oplus sp_2)$.

The manifest expression for the cocycle (4) over the algebra $AC(g)$ will be given separately.

The case $g = sl_n$ ($n > 2$ as $sl_2 \simeq sp_2$) can in principle be obtained from the case $g = sp_{2n}$. Note that the embedding $sl_n \subset sp_{2n}$ takes place and the subalgebra sl_n is formed by the generators commutative with the "particle number" operator

$$N = \sum_{k=1}^{n} H_k = \sum_{k=1}^{n} E^{\mathbf{l}(k)}_{\mathbf{0}}, \tag{13}$$

except for N itself. In this way, the generators

$$E^{\mathbf{l}}_{\mathbf{m}} - \frac{1}{n}\delta_{\mathbf{m},0}\, N$$

with

$$\sum_{k=1}^{n} m_k = 0 \tag{14}$$

form the subalgebra. The analytic continuation to $AC(sl_n)$ and $\widetilde{AC}(sl_n)$ again consists in revoking the conditions (7b), (7d), conserving the conditions (7a), (7c) and (15). However, the algebra obtained in this way does not coincide directly with $AC(sl_n)$, but contains it as a subalgebra. The case $g = so_n$ ($n > 5$) is obtained by the embedding $so_n \subset sl_n$. The so_n subalgebra in sl_n is formed by the generators

$$L^{\mathbf{l}}_{\mathbf{m}} = E^{\mathbf{l}}_{\mathbf{m}} - E^{\mathbf{l}}_{-\mathbf{m}},\ \mathbf{m} > \mathbf{0},\ \sum_{k=1}^{n} m_k = 0, \tag{15}$$

and the so_n commutation relations are

$$\left[L^{\mathbf{l}}_{\mathbf{m}}, L^{\mathbf{l}'}_{\mathbf{m}'}\right] = \frac{1}{2}\sum_{k=1}^{n}\left[(l_k m'_k - l'_k m_k)\, L^{\mathbf{l}+\mathbf{l}'-\mathbf{l}(k)}_{\mathbf{m}+\mathbf{m}'} + (l_k m'_k + l'_k m_k)\, L^{\mathbf{l}+\mathbf{l}'-\mathbf{l}(k)}_{\mathbf{m}-\mathbf{m}'}\right]. \tag{16}$$

The analytic continuation of this formula, as in the sl_n case, does not coincide with $AC(so_n)$ but contains it as a subalgebra (this is due to the basis (15) which is not the Cartan-Weyl basis for so_n). The basis in which the analytic continuation can be performed directly for sl_n and so_n will be considered separately.

Thus we have obtained the manifest construction for analytic continuations of the classical simple Lie algebras. The exceptional algebras can be embedded in sl_n (for some suitable n) and we expect that their $AC \subset AC(sl_n)$.

It should also be mentioned that one can make partial analytic continuations only for some of the parameters, i.e., revoke the conditions (7b), (7d) only for some values of the index k. Then we obtain a series of partial analytic continuations of g $PAC_K(g)$, where K is a set of the values of k for which an analytic continuation in the formula (10) has been performed. Evidently all the partial analytic continuations are contained in $AC(g)$ as subalgebras. The weight diagrams of PAC are subsets of the lattice $\Lambda(g)$. The PAC form a composition (or Jordan-Gelder) series of *qadg*.

To conclude this section, we note that the realization presented here was proposed in Ref. 8.

5. An Analytic Continuation of the Lie Superalgebras

Here we present an analytic continuation of $osp(1|2n)$. In addition to (10), we introduce Fermi generators

$$Q^{\mathbf{l}}_{\mathbf{m}} = \prod_{k=1}^{n} (a_k^\dagger)^{(l_k+m_k)/2} (a_k)^{(l_k-m_k)/2} , \tag{17}$$

where instead of condition (7a), the following condition is now satisfied,

$$\sum_{k=1}^{n} l_k = 1 , \tag{18}$$

and the other conditions remain in force. The $osp(1|2n)$ is then defined by the commutation relations (10) and

$$\left\{Q^{\mathbf{l}}_{\mathbf{m}}, Q^{\mathbf{l}'}_{\mathbf{m}'}\right\} = 2\, E^{\mathbf{l}+\mathbf{l}'}_{\mathbf{m}+\mathbf{m}'} , \tag{19}$$

$$\left\{E^{\mathbf{l}}_{\mathbf{m}}, Q^{\mathbf{l}'}_{\mathbf{m}'}\right\} = \frac{1}{2} \sum_{k=1}^{n} (l_k m'_k - l'_k m_k)\, Q^{\mathbf{l}+\mathbf{l}'-\mathbf{1}_{(k)}}_{\mathbf{m}+\mathbf{m}'} \tag{20}$$

(here $\{Q, Q\} = 2Q$ is an anticommutator, i.e., simply the product of two functions).

Similarly $AC(sp_{2n})$, $AC(osp(1|2n))$ are defined by revocation of the conditions (7b), (7d) both for E and Q generators. Thus the theory proposed here may be expanded with slight modifications on the superalgebras.

6. Analytic Continuations of the Higher Spin Algebras

The infinite-dimensional higher spin algebras $hs^*(2n)$ (* is for Poisson-bracket version)[9] containing sp_{2n} as a subalgebra can be defined as a Poisson-bracket algebra with the generators (6) in which the conditions (7b – d) take place, but instead of the condition (7a), we require only

$$\sum_{k=1}^{n} l_k = 2s, \quad s = 1, 2, \ldots \tag{21}$$

(in the higher spin algebra s takes all the integer values ≥ 1). This algebra under the representation of sp_{2n} (generators with $\sum l_k = 2$) can be decomposed into a direct sum of the "higher spin" representations $\bigoplus_{s=1}^{\infty} (adg)_s$ ($(adg)_1 = adg$), where the irreducible sp_{2n} representations with maximal dimensions in the decomposition $ad(sp_{2n}) \oplus \ldots \oplus ad(sp_{2n})$ (n times) $= (ad(sp_{2n}))_s$ + other terms. Revoking the conditions (7b) and (7d), we obtain an analytic continuation of $hs^*(2n)$ – $AC(hs^*(2n))$, the "higher spin" representation $(adg)_s$ being continued to the infinite-dimensional representations of $AC(sp_{2n})$ (see Theorem 2).

7. Analytic Continuation of sp_2: The Virasoro Algebra

When $g = sp_2$, formula (10) becomes the usual formula $[E_n^2, E_m^2] = (m - n) E_{n+m}^2$ ($l = 2$ in this case) for the algebra diff(S^1) and the central extension is defined by the Gel' fand-Fuchs cocycle (which is given by Eq. (4)). Thus $AC(sp_2) \simeq$ diff(S^1) and $\widetilde{AC}(sp_2) \simeq$ Vir. The representations from Theorem 2 become the spin-s representations of Vir and the Verma modules become the Verma modules for Vir. In this way the theory of the Virasoro algebra and its representations is an analytic continuation of sl_2 theory; the general theory expounded here is an analytic continuation of the theory of the semisimple algebras and their representations. An analytic continuation $AC(hs^*(2))$ is a higher spin generalization of the Virasoro algebra[8] (the Poisson bracket version of W_∞ algebra). In the supersymmetric case the algebra $AC(osp(1|2))$ is isomorphic to the Neveu-Schwarz superalgebra. Note that our algebra $AC(sp_2 \oplus sp_2)$ (see (12)) does not coincide with the algebra Vir $\oplus$ Vir but contains it as a subalgebra. It may be possible that in two dimensions $AC(sp_2 \oplus sp_2)$ plays the role of some "extended" conformal algebra.

8. Applications

In our opinion, the theory proposed may play the role of algebraic basis for exactly solvable D-dimensional quantum models, similar to the Virasoro theory in two dimensions. In particular, the algebra $\overset{(\sim)}{AC}(so(D, 2))$ might be considered as a hidden extended conformal algebra in D-dimensions similar to the Virasoro algebra. The algebra $\overset{(\sim)}{AC}(sp_4)$ ($sp_4 \simeq SO(3, 2)$) (sll(10), $n = 2$) may be of importance in the $D = 3$ phase transitions theory.

Among other possible applications, we may mention the relation between these algebras and the p-branes.

An important mathematical question that remains consists of constructing Lie groups corresponding to $AC(g)$ and making clear their geometrical meaning (some subalgebras in the diffeomorphism algebras on the manifolds).

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