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On space–time interpretation of the coset models in $D<26$ critical string theory

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Manifest expressions for the 3D string metric–dilaton backgrounds corresponding to the (anti-) de Sitter coset models introduced previously are obtained in the leading order approximation. They may be interpreted as non-static cosmological solutions in $D=3$ critical string theory. A generalization to $D>3$ space–time dimension is discussed.

To investigate the possibilities to achieve critical strings in space–time dimensions less than 26 (or 10 for superstrings) is an interesting direction that might be an alternative for the Kaluza–Klein compactification. So a family of critical strings in $D<26$ ($D<10$) described by the anti-de Sitter non-compact coset models $SO(D-1, 2)/SO(D-1, 1)$ has been recently introduced in refs. [1,2] [#1]. The possibility to have the critical dimension less than 26 (10) is provided by the presence of the background curvature with a scale parameter of the Planck order. The unitarity problem is discussed in refs. [1,3].

However, a manifest space–time interpretation of the coset models turns out to be rather non-trivial. So the background metric, where strings described by the G/H coset model propagate in, represents a non-static Universe and has quite a little to do with the metric on the very coset G/H, partially due to the presence of a non-constant background dilaton.

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[#1] After the paper [2] was published we became aware of the earlier paper by Bars and Nemeshansky [1] where the critical strings based on the AdS_D coset models were introduced.

For the two-dimensional coset models $SO(2, 1)/SO(1, 1)$ and $SO(2, 1)/SO(2)$ (the latter model has been proved to be unitary in ref. [4]) a manifest space–time interpretation has been recently given in ref. [5] where it is suggested to regard them as describing 2D black holes. The corresponding metric–dilaton background was earlier found as a solution to the effective equations of motion (or, equivalently, Weyl invariance conditions) in the leading order approximation in ref. [6]. The exact solution has been conjectured in refs. [7,8].

The goal of the present letter is to obtain a manifest solution for the metric–dilaton background for the three-dimensional coset models $SO(2, 2)/SO(2, 1)_{diag}$, $SO(3, 1)/SO(3)$ and $SO(3, 1)/SO(2, 1)$ in the leading order approximation and to discuss corresponding 3D critical strings. It should be stressed we shall consider coset (gauged WZW) models, rather than the $SO(2, 1)$ WZW model [9] which is drastically different in its space–time interpretation, describing a true group manifold with a torsion.

The 3D critical string in question can be defined as a GKO coset model for the non-compact coset $SO(2, 2)/SO(2, 1)_{diag}$ subject to the Virasoro constraints on its physical states (energy–momentum tensor being the GKO one) with the Virasoro central charge $c(k)=6k/(k-2)-3k/(k-1)=26$, where k is the underlying SO(2, 2) Kac–Moody algebra level [1,2]. Note that for $D=3$ there is a family of the possible coset models $SO(2,1)_{k_1}\times SO(2,1)_{k_2}/SO(2,1)_{k_1+k_2}$. We shall consider just the case $k_1=k_2=k$. Also we will not fix k equal to one of its two critical values, since the anomaly cancellation condition $c(k)=26$ may be relaxed to $c(k)+c_{int}=26$ by adding some internal unitary conformal theory to describe extra dimensions.

To find the metric–dilaton background in the leading order approximation corresponding to the G/H coset model one can start with the gauged WZW model action (see e.g. ref. [10]), choose an appropriate parametrization of the group manifold G, fix a gauge, and then gauge fields A_α ($\alpha=1,2$) localizing the symmetry $g\to hgh^{-1}$, $g\in G$, $h\in H$ (or certain of its modifications, see (36) below) can be integrated out as auxiliary fields. As a result one finds the σ-model metric corresponding to the G/H GKO coset construction. The dilaton then can be found by solving the leading order effective equations of motion [11] for a given $G_{\mu\nu}$ (the anti-symmetric tensor when G/H is a symmetric space vanishes identically in a suitable gauge). For $G=SL(2;\mathbb{R})$, $H=SO(2)$ and $SO(1,1)$ it was explicitly done in ref. [5].

For the sake of simplicity we will first consider the compact euclidean case $G=SO(4)$ with $H=SO(3)$ being its diagonal subgroup. Then we shall pass to all the non-compact non-euclidean cases via various possible analytic continuations.

An arbitrary group element $g\in SO(4)$ can be conveniently parametrized by the generalized Euler angles [12]

$$g=g_1(\theta_1^3)g_2(\theta_2^3)g_3(\theta_3^3)h\,, \tag{1}$$

where $h\in SO(3)\subset SO(4)$ and

$$h=g_1(\theta_1^2)g_2(\theta_2^2)g_1(\theta_1^1)\,. \tag{2}$$

Here $g_k(\alpha)=\exp(\alpha T_{k+1,k})$ are one-parameter subgroups corresponding to the SO(4) generators $T_{k+1,k}$ ($k=1,2,3$) with the matrices $(T_{k+1,k})^j_i=\delta_{k,i}\delta^j_{k+1}-\delta_{k+1,i}\delta^j_k$.

The SO(4)/SO(3) gauged WZW model is invariant under the gauge transformations

$$g\to aga^{-1},\quad g\in SO(4),\quad a\in SO(3)\subset SO(4)\,. \tag{3}$$

A gauge can be conveniently fixed by the following choice for the gauge slice:

$$g=g_1(\varphi)g_2(\theta)g_3(2t)g_2(\theta)g_1(\varphi)\,, \tag{4}$$

where

$$0\leqslant\varphi<2\pi,\quad 0\leqslant\theta<\pi,\quad 0\leqslant t<\tfrac{1}{2}\pi \tag{5}$$

[$t=t(\sigma,\tau)$, $\theta=\theta(\sigma,\tau)$, $\varphi=\varphi(\sigma,\tau)$ are functions of the world-sheet coordinates σ and τ].

The SO(4) currents $J_-=g^{-1}\partial_-g$ in this gauge take the form

$$J_-^{21}=(1+\cos^2\theta-\sin^2\theta\cos 2t)\,\partial_-\varphi\,,$$
$$J_-^{32}=(1+\cos 2t)\times(\cos\varphi\,\partial_-\theta+\sin\theta\cos\theta\sin\varphi\,\partial_-\varphi)\,,$$
$$J_-^{31}=(1+\cos 2t)\times(-\sin\varphi\,\partial_-\theta+\sin\theta\cos\theta\cos\varphi\,\partial_-\varphi)\,,$$
$$J_-^{43}=2\cos\theta\,\partial_-t+\sin 2t\sin\theta\,\partial_-\theta\,,$$
$$J_-^{42}=-2\sin\theta\cos\varphi\,\partial_-t+\sin 2t\times(\cos\theta\cos\varphi\,\partial_-\theta+\sin\theta\sin\varphi\,\partial_-\varphi)\,,$$
$$J_-^{41}=2\sin\theta\sin\varphi\,\partial_-t+\sin 2t\times(-\cos\theta\sin\varphi\,\partial_-\theta+\sin\theta\cos\varphi\,\partial_-\varphi)\,, \tag{6}$$

as can be found by straightforward calculation. Currents $J_+=\partial_+g\,g^{-1}$ are given by similar expressions with the only substitution of ∂_+ for ∂_- and additional overall minus signs in the expressions for J_+^{42} and J_+^{31}.

The WZ term identically vanishes in this gauge (since the corresponding closed three-form vanishes) and the WZW lagrangian reduces to

$$L_{WZW}=\frac{k}{4\pi}\,[\partial_+t\,\partial_-t+\tfrac{1}{2}(1+\cos 2t)\,\partial_+\theta\,\partial_-\theta+\tfrac{1}{2}(1+\cos^2\theta-\sin^2\theta\cos 2t)\,\partial_+\varphi\,\partial_-\varphi]\,. \tag{7}$$

The gauged WZW lagrangian is then given by

$$L_{gauged}=L_{WZW}-\frac{k}{16\pi}\mathrm{Tr}(A_+J_-+A_-J_+-A_+A_-+A_+g^{-1}A_-g)\,, \tag{8}$$

where $A_\pm\in so(3)$ ($A_\pm=A_\pm^{21}T_{21}+A_\pm^{32}T_{32}+A_\pm^{31}T_{31}$) and Tr is the usual matrix trace.

Gauge fields $A_\pm$ now can be expressed through the physical σ-model fields t, θ and φ by solving the classical equations of motion

$$(J_-)_{so(3)} = A_- - (g^{-1}A_-g)_{so(3)}, \tag{9a}$$

$$(J_+)_{so(3)} = A_+ - (gA_+g^{-1})_{so(3)}, \tag{9b}$$

where $(\ )_{so(3)}$ stands for the so(3) projection.

To simplify the above equations it is convenient to perform first an SO(2) rotation

$$A'_- = g_1(-\varphi)A_-g_1(\varphi),$$
$$A'_+ = g_1(\varphi)A_+g_1(-\varphi), \tag{10a}$$

$$J'_- = g_1(\varphi)J_-g_1(-\varphi),$$
$$J'_+ = g_1(-\varphi)J_+g_1(\varphi). \tag{10b}$$

Then the equations we are to solve become in components

$$(1+\cos 2t)\sin\theta(A^{21}_- \sin\theta + A^{31}_- \cos\theta)$$
$$= (1+\cos^2\theta - \sin^2\theta\cos 2t)\,\partial_-\varphi, \tag{11a}$$

$$(\cos 2\varphi - \cos 2t)A^{32}_- - A^{31}_- \sin 2\varphi$$
$$= (1+\cos 2t)\,\partial_-\theta, \tag{11b}$$

$$(\cos 2\varphi + \sin^2\theta - \cos^2\theta\cos 2t)A^{31}_- + A^{32}_- \sin 2\varphi$$
$$- \cos\theta\sin\theta(1+\cos 2t)A^{21}_-$$
$$= \sin\theta\cos\theta(1+\cos 2t)\,\partial_-\varphi, \tag{11c}$$

where we have omitted primes.

A solution with respect to A_- can be written as follows:

$$A^{21}_- = \tan\varphi\cot\theta\cot^2 t\,\partial_-\theta$$
$$+ (\tan^2 t - \cos^2\theta\cot^2 t)\frac{\partial_-\varphi}{\sin^2\theta}, \tag{12a}$$

$$A^{32}_- = \cot^2 t\,\partial_-\theta + \frac{2\cot\theta\tan\varphi\,\partial_-\varphi}{1-\cos 2t}, \tag{12b}$$

$$A^{31}_- = -\tan\varphi\cot^2 t\,\partial_-\theta$$
$$+ \frac{(\cos 2\varphi - \cos 2t)\cot\theta\,\partial_-\varphi}{(1-\cos 2t)\cos^2\varphi}. \tag{12c}$$

A_+ are given by similar expressions. Substituting it into the gauged WZW lagrangian (8), we finally obtain a σ-model lagrangian expressed only through the physical fields

$$L = \frac{k}{4\pi}\Big(\partial_+ t\,\partial_- t + \cot^2 t\,(\partial_+\theta + \tan\varphi\cot\theta\,\partial_+\varphi)$$
$$\times(\partial_-\theta + \tan\varphi\cot\theta\,\partial_-\varphi) + \frac{\tan^2 t}{\sin^2\theta}\partial_+\varphi\,\partial_-\varphi\Big), \tag{13}$$

which corresponds to the string σ-model metric

$$ds^2 = \alpha' k\Big(dt^2 + \cot^2 t(d\theta + \tan\varphi\cot\theta\,d\varphi)^2$$
$$+ \frac{\tan^2 t}{\sin^2\theta}d\varphi^2\Big). \tag{14}$$

By means of introducing new variables

$$x = \sin\varphi, \quad y = \cos\theta\cos\varphi, \tag{15}$$

such that

$$0 \leqslant x^2 + y^2 \leqslant 1, \tag{16}$$

the metric can be brought to the simpler form

$$ds^2 = \alpha' k\Big(dt^2 + \frac{\tan^2 t\,dx^2 + \cot^2 t\,dy^2}{1-x^2-y^2}\Big). \tag{17}$$

Now we are to find a background dilaton which satisfies the leading order effective equations of motion [11]

$$R_{\mu\nu} = \mathscr{D}_\mu\mathscr{D}_\nu\phi. \tag{18}$$

Non-zero components of the Christoffel connection and Ricci curvature for the above metric are

$$\Gamma^t_{xx} = -\frac{\sin t}{(1-r^2)\cos^3 t}, \quad \Gamma^x_{tx} = -\frac{1}{\sin t\cos t}, \tag{19a}$$

$$\Gamma^t_{yy} = \frac{\cos t}{(1-r^2)\sin^3 t}, \quad \Gamma^y_{ty} = \frac{1}{\sin t\cos t}, \tag{19b}$$

$$\Gamma^x_{xy} = \Gamma^y_{yy} = \frac{y}{1-r^2}, \quad \Gamma^y_{yx} = \Gamma^x_{xx} = \frac{x}{1-r^2}, \tag{19c}$$

$$\Gamma^x_{yy} = -\frac{x\cot^4 t}{1-r^2}, \quad \Gamma^y_{xx} = -\frac{y\tan^4 t}{1-r^2}, \tag{19d}$$

$$R_{tt} = -\frac{2}{\sin^2 t\cos^2 t}, \tag{20a}$$

$$R_{xx} = -\frac{2\tan^4 t}{1-r^2} - \frac{2(x^2+y^2\tan^4 t)}{(1-r^2)^2}, \tag{20b}$$

$$R_{yy}=-\frac{2\cot^4 t}{1-r^2}-\frac{2(y^2+x^2\cot^4 t)}{(1-r^2)^2}, \tag{20c}$$

where $r^2=x^2+y^2$. Then the equations of motion (18) reduce to differential equations for $\phi=\phi(t, x, y)$ which admit a general solution

$$\phi=2\ln(\sin t\cos t)+\ln(1-x^2-y^2)+\phi_0, \tag{21}$$

where ϕ_0 is the integration constant.

In this way, we have obtained the metric–dilaton background (17), (21) corresponding to the SO(4)/SO(3) coset model in the leading order approximation. It defines the string σ-model [11]

$$S=\frac{1}{4\pi\alpha'}\int d\sigma\, d\tau\sqrt{-g}\,[g^{\alpha\beta}\,\partial_\alpha x^\mu\,\partial_\beta x^\nu\, G_{\mu\nu} -\tfrac{1}{2}\alpha' R^{(2)}\phi]. \tag{22}$$

Now let us take a look at the dilaton β-function

$$\beta^\phi=\tfrac{1}{6}(D-26)+\tfrac{1}{4}\alpha'[(\partial\phi)^2+\mathscr{D}^2\phi]+O(\alpha'^2). \tag{23}$$

In the background (17), (21) it reduces to a constant

$$\beta^\phi=\tfrac{1}{6}(D-26)-3/k+O(1/k^2) \tag{24}$$

($D=3$ in our case). One can see that it is nothing but the $1/k$ expansion of the SO(4)/SO(3) central charge

$$\beta^\phi=\tfrac{1}{6}[c(k)-26]=\frac{1}{6}\left(\frac{6k}{k+4}-\frac{3k}{k+2}-26\right) =\tfrac{1}{6}(3-26)-1/k+O(1/k^2). \tag{25}$$

However, the compact coset model SO(4)/SO(3) has the Virasoro central charge $c\leqslant 3$ (k is a positive integer). To get $c>3$ (and, in particular, $c=26$) one has to pass to the non-compact algebras so(2, 2) and so(3, 1). It can be done by taking some of the generators T_{43}, T_{32}, T_{21} in (4) to be non-compact. Equivalently, one can Wick rotate ($\alpha\to i\alpha$) the Euler angles corresponding to the non-compact generators directly in the final expression (14) for the σ-model metric. Simultaneously one is to change the sign of k, $k\to -k$, to obtain unitary representations of the non-compact algebra [13] [and to obtain $\beta^\phi=0$ ($c=26$)]. As a result, there are seven cases.

First, by taking $t\to it$, $k\to -k$ (T_{43} is non-compact) we obtain the non-compact euclidean so(3, 1)/so(3) coset model and a corresponding one-loop background is

$$ds^2=\alpha' k\left(dt^2+\frac{\tanh^2 t\, dx^2+\coth^2 t\, dy^2}{1-x^2-y^2}\right), \tag{26}$$

$$\phi=2\ln(\sinh t\cosh t)+\ln(1-x^2-y^2)+\phi_0. \tag{27}$$

It describes a unitary [#2] euclidean cosmological solution of 3D critical string theory which incorporates both Witten's euclidean SO(2, 1)/SO(2) black hole [5]

$$ds^2=dt^2+\tanh^2 t\, d\theta^2 \tag{28a}$$

and its dual

$$ds^2=dt^2+\coth^2 t\, d\theta^2. \tag{28b}$$

Thus this model is "self-dual" (see also below).

Second, by taking $\theta\to i\theta$, $k\to -k$ (T_{32} is non-compact) we obtain the anti-de Sitter SO(2, 2)/SO(2, 1)$_{\rm diag}$ coset model with

$$ds^2=\alpha' k\left(-dt^2+\frac{\tan^2 t\, dx^2+\cot^2 t\, dy^2}{x^2+y^2-1}\right), \tag{29}$$

$$\phi=2\ln(\sin t\cos t)+\ln(x^2+y^2-1)+\phi_0, \quad x^2+y^2\geqslant 1. \tag{30}$$

The time t here is a periodic coordinate [similar to the true $AdS_3=SO(2, 2)/SO(2, 1)$ manifold which is topologically $S^1\times\mathbb{R}^2$].

Third, there are also two other analytic continuations corresponding to SO(2, 2)/SO(2, 1)$_{\rm diag}$. For $t\to it$, $\varphi\to i\varphi$, $k\to -k$ (T_{21}, T_{43} are non-compact); and $t\to it$, $\theta\to i\theta$, $\varphi\to i\varphi$, $k\to -k$ (T_{21}, T_{32} are non-compact), we have

$$ds^2=\alpha' k\left(dt^2+\frac{-\tanh^2 t\, dx^2+\coth^2 t\, dy^2}{1+x^2-y^2}\right), \tag{31a}$$

$$\phi=2\ln(\sinh t\cosh t)+\ln(1+x^2-y^2)+\phi_0, \quad 0\leqslant y^2-x^2\leqslant 1 \tag{31b}$$

and

$$ds^2=\alpha' k\left(dt^2+\frac{\tanh^2 t\, dx^2-\coth^2 t\, dy^2}{y^2-x^2-1}\right), \tag{32a}$$

[#2] G/H models, where H is a maximal compact subgroup of G are shown to be unitary in general [13].

$$\phi = 2\ln(\sinh t\cosh t) + \ln(y^2 - x^2 - 1) + \phi_0 ,$$

$$y^2 - x^2 \geqslant 1 , \qquad (32b)$$

correspondingly. These solutions are 3D generalizations of the 2D minkowskian black holes [5].

Finally, the analytic continuations $t \to it$, $\theta \to i\theta$ (T_{43}, T_{32} are non-compact); $\varphi \to i\varphi$ (T_{21} is non-compact); and $\varphi \to i\varphi$, $\theta \to i\theta$ (T_{21}, T_{32} are non-compact) lead to the SO(3, 1)/SO(2, 1) coset model with background metrics having two time directions:

$$ds^2 = \alpha' k\left(dt^2 - \frac{\tanh^2 t\, dx^2 + \coth^2 t\, dy^2}{x^2 + y^2 - 1}\right),$$

$$x^2 + y^2 \geqslant 1 , \qquad (33a)$$

$$ds^2 = \alpha' k\left(-dt^2 + \frac{\tan^2 t\, dx^2 - \cot^2 t\, dy^2}{1 - y^2 + x^2}\right),$$

$$0 \leqslant y^2 - x^2 \leqslant 1 , \qquad (33b)$$

and

$$ds^2 = \alpha' k\left(-dt^2 + \frac{\cot^2 t\, dy^2 - \tan^2 t\, dx^2}{y^2 - x^2 - 1}\right),$$

$$y^2 - x^2 \geqslant 1 , \qquad (33c)$$

respectively. Three different metrics correspond to three different embeddings $SO(2, 1) \subset SO(3, 1)$.

One can see that all the above metrics have various singularities [e.g. (26) – for $t=0$ and $x^2 + y^2 = 1$]. However, these singularities are unphysical in the following sense. Recall that the "observable" physical metric $G^{\text{phys}}_{\mu\nu}$ satisfying the standard Einstein equations

$$R^{\text{phys}}_{\mu\nu} - \tfrac{1}{2} G^{\text{phys}}_{\mu\nu} R^{\text{phys}} = T_{\mu\nu} , \qquad (34)$$

where $T_{\mu\nu}$ is the dilaton energy–momentum tensor, is related to the σ-model metric by

$$G^{\text{phys}}_{\mu\nu} = \exp[2\phi/(D-2)]\, G^{\sigma\text{-model}}_{\mu\nu} . \qquad (35)$$

Then, as is easy to see, such a rescaling removes all the singularities and the physical metric $G^{\text{phys}}_{\mu\nu}$ is regular.

Above we have considered the vector gauging $g \to hgh^{-1}$. However, it is possible to consider a more general situation

$$g \to hg(*h)^{-1} , \qquad (36)$$

where $*$ is a certain automorphism of the subgroup H. For involutive $*$ ($*^2 = 1$) such a construction gives a non-abelian generalization of the axial gauging $g \to hgh$ [$h \in SO(2)$] of ref. [5], where $*$ is simply $*T_{21} = -T_{21}$. Generally, for non-abelian H, transformations $g \to hgh$ obviously do not constitute a group, while the transformations (36) do. A corresponding group of involutive automorphisms of H may play the role of the generalized duality in the multidimensional non-abelian case (for the rôle of duality in string cosmology see ref. [14]). We are going to discuss it elsewhere, but here we shall only explain in what sense the solution (26) is "self-dual".

Suppose we are given an involutive automorphism $*$ of SO(3) defined by

$$*T_{21} = T_{21} , \quad *T_{32} = -T_{32} , \quad *T_{31} = -T_{31} . \qquad (37)$$

Then we can start with the gauging (36). An appropriate gauge now is

$$g = g_1(\varphi) g_2(-\theta) g_3(2t) g_2(\theta) g_1(\varphi) . \qquad (38)$$

Performing again all the necessary calculations we arrive at the σ-model metric

$$ds^2 = \alpha' k\left(dt^2 + \tan^2 t(d\theta + \tan\varphi \cot\theta\, d\varphi)^2 + \frac{\cot^2 t}{\sin^2\theta} d\varphi^2\right). \qquad (39)$$

Introducing new variables (15) we get

$$ds^2 = \alpha' k\left(dt^2 + \frac{\cot^2 t\, dx^2 + \tan^2 t\, dy^2}{1 - x^2 - y^2}\right), \qquad (40)$$

so that the x and y have only exchanged their places. On the other hand, putting $\varphi = 0$ one sees the metric (14) equals (28b), while (39) equals (28a) (when $t \to it$, $k \to -k$).

All the consideration presented here can be expanded to the AdS_D coset models $SO(D-1, 2)/SO(D-1, 1)$ [as well as to the euclidean models $SO(D, 1)/SO(D)$] for arbitrary D (and, generally speaking, to any G/H coset models), though to evaluate manifest expressions for the metric–dilaton background becomes more complicated. One again may start with the generalized Euler parametrization of $SO(N)$, fix a gauge

$$g = g_1(\theta_1) \ldots g_{N-1}(\theta_{N-1}) g_N(2t) \times g_{N-1}(\theta_{N-1}) \ldots g_1(\theta_1) , \qquad (41)$$

solve the equations for $A_{\pm}$, calculate the σ-model metric by substituting the solution to the gauged WZW lagrangian, and find a dilaton by solving the β-function equations. We are going to return to it and especially to the $D=4$ case, as well as to physical interpretations of these cosmological solutions, in a more detailed publication.

To conclude, we would like to emphasize once again that in string theory there are possible exact non-perturbative vacua with $D<26$ ($D<10$) and with curved background space–time. A class of such exact vacua is described by the (anti-)de Sitter coset models [euclidean versions $SO(D,1)/SO(D)$ provide solutions in D-dimensional critical euclidean string theories], as introduced in refs. [1,2]. However, the manifest space–time interpretation of these models is highly non-trivial since they describe complicated non-static cosmological solutions for the background metric with a non-constant dilaton. The true (anti-)de Sitter regime could probably be realized only at very small t, but then non-static regimes are realized. This non-staticity and a non-constant dilaton were missed in our original discussion of the AdS coset models in ref. [2]. It also should be mentioned there may appear a dilaton mass term (and a more general dilaton potential) as a result of string loop and non-perturbative effects. It would give an additional contribution to the curvature and change essentially the metric.

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