

Holography for quark-gluon plasma formation in heavy-ions collisions

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3 keys points

Outlook

Quark-gluon plasma (QGP)

Holography

Heavy-ions collisions (HIC)

- New formula for multiplicity
- New phase transition

*14'th Ginzburg's
problem,
UFN, 2002*



On some advances in physics and astronomy over the past three years

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In 1999 the author published a paper "What problems of physics and astrophysics seem now to be especially important and interesting (thirty years later, already on the verge of XXI century?" [1]. By its very nature and intention, the content of this paper should be modified on a continuous basis to keep up with advances in science. In the last three years important results of a fundamental nature have been obtained which the author finds appropriate to summarize briefly in this article — not least because of the great readers' interest generated by Ref. [1].

PACS numbers: **01.55. + b**, **01.90. + g**

Bibliography 69 references

Received 16 January 2002

Ginzburg V L About Science, Myself, and Others

12. Разеры, газеры, сверхмощные лазеры.
13. Сверхтяжелые элементы. Экзотические ядра.
14. ~~Спектр масс. Кварки и глюоны. Квантовая хромодинамика.~~
Кварк-глюонная плазма.
15. Единая теория слабого и электромагнитного взаимодействия. W^{\pm} - и Z^0 -бозоны. Лептоны.
16. Стандартная модель. Великое объединение. Суперобъединение. Распад протона. Масса нейтрино. Магнитные монополи.

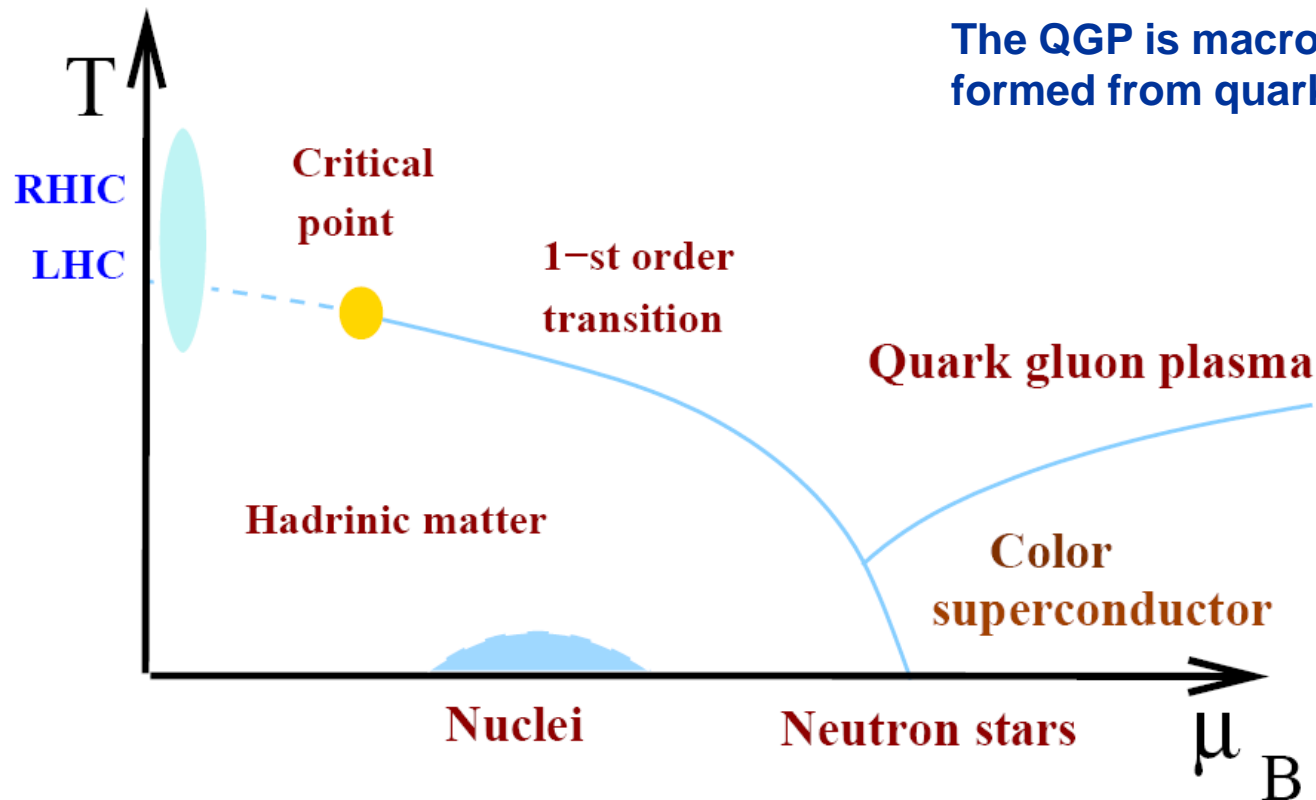
стве металлов [12]. В этой связи особенно интересны результаты [13], опубликованные в самом конце 2001 г., и быть может свидетельствующие о том, что в купратах "элементарные возбуждения", переносящие ток и тепло, довольно существенно отличны от электронов и дырок. Мне, однако, осталась неясной роль присутствующего в этих экспериментах сильного магнитного поля. Несомненно, опыты [13] должны быть (и будут!) повторены и

Quark-Gluon Plasma (QGP): a new state of matter

Asymptotic freedom

T increases, or
density increases

nuclear matter \longrightarrow Deconfined phase



Heavy ions collisions

Experiments

- started in the 1990's at the Brookhaven Alternating Gradient Synchrotron (AGS),
- the CERN Super Proton Synchrotron (SPS)
- the Brookhaven Relativistic Heavy-Ion Collider (RHIC)
- the LHC collider at CERN.

$$\sqrt{s_{NN}} = 4.75 \text{ GeV}$$

$$\sqrt{s_{NN}} = 17.2 \text{ GeV}$$

$$\sqrt{s_{NN}} = 200 \text{ GeV}$$

$$\sqrt{s_{NN}} = 2.76 \text{ TeV}$$

Theory (macroscopic: thermodynamics, hydrodynamics, ... microscopic - QCD, holographic)

Landau(1953); Fermi(1950); Pomeranchuk; Rozentel, Cernavskij (UFN,1954);

Landau, Bilenkij (UFN,1955); Cernavskij, Feiberg (1972);

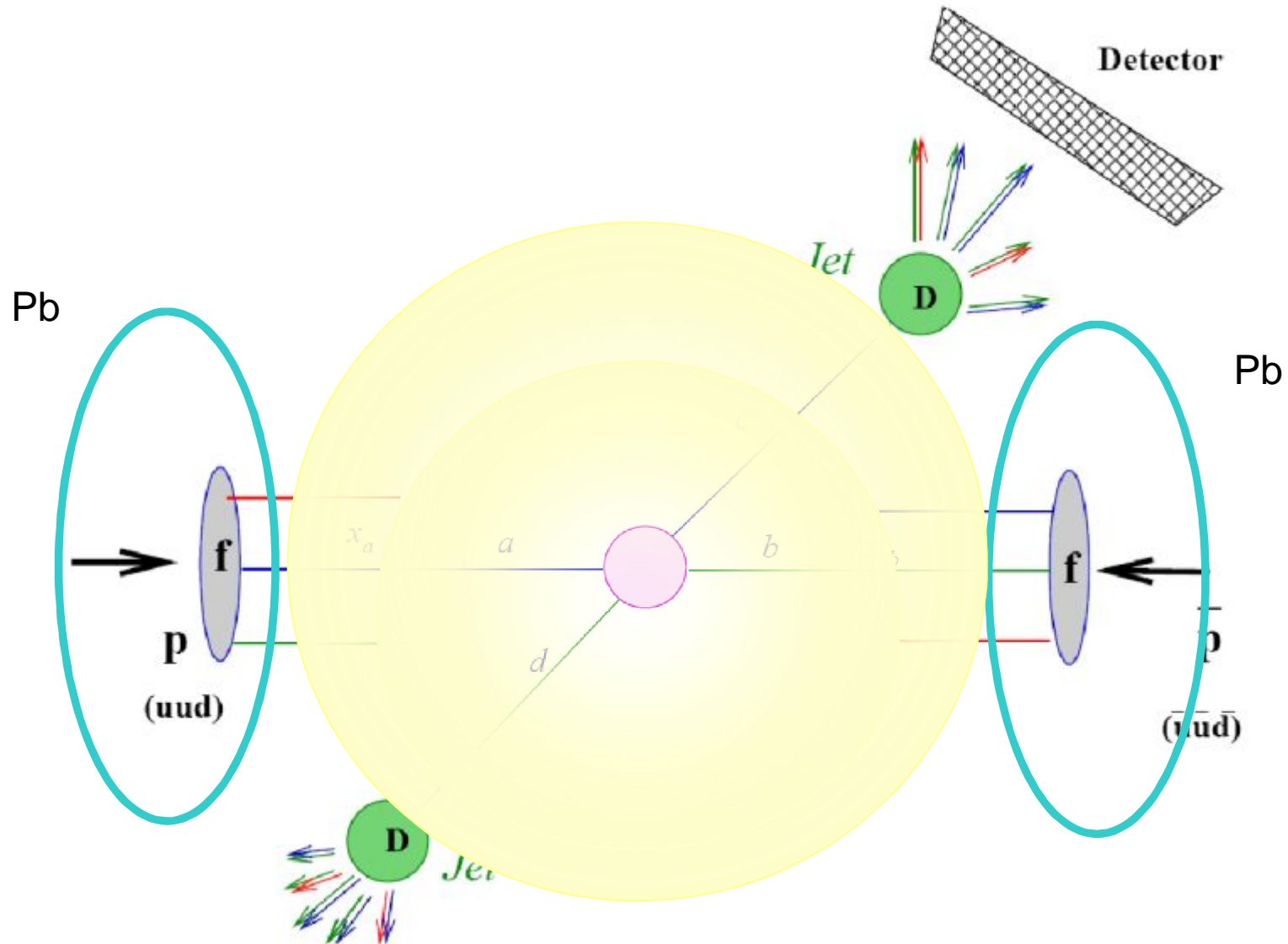
Cooper,...(1975); Bjorken(1983);

Kolb, Heinz (2003); Janik, Peschansky (2006); Shuryak(2009); Peigne, Smilga (UFN, 2009)

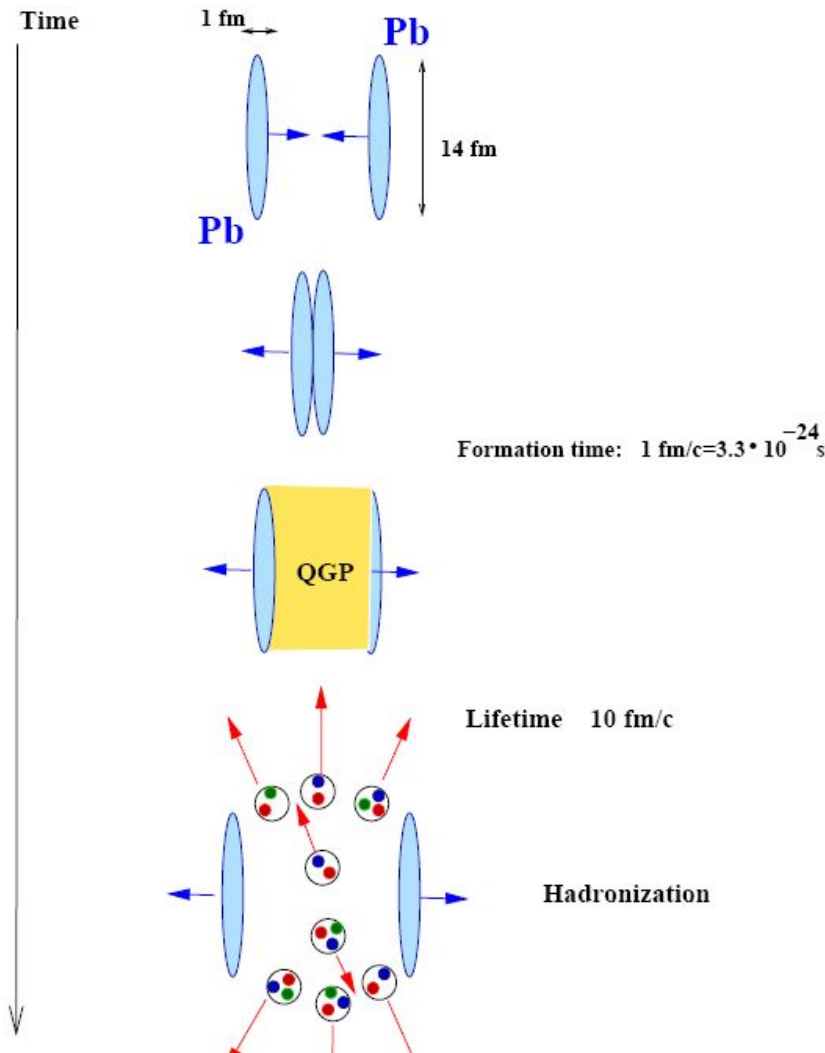
Dremin, Leonidov (UFN, 2010); Muller, Schafer(2011),....



pp collisions vs heavy ions collisions



Heavy Ions collisions

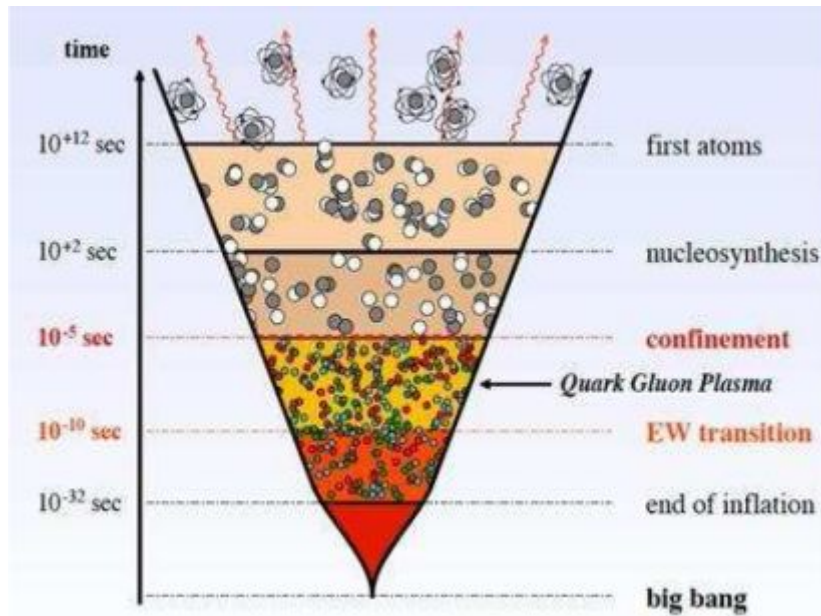


- Due to multiple interactions the system will thermalize and form the QGP
- QGP undergoes a collective expansion and eventually becomes so dilute that it hadronizes.

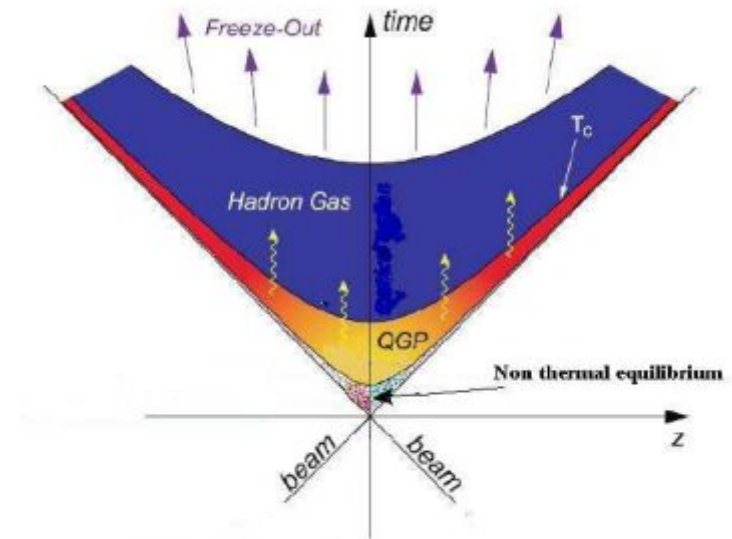
We can “see” this collective expansion (“elliptic flow”)

QGP in Heavy Ion Collision and Early Universe

- One of the fundamental questions in physics is: what happens to matter at extreme densities and temperatures as may have existed in the first microseconds after the Big Bang
- The aim of heavy-ion physics is to collide nuclei at very high energies and create such a state of matter **in the laboratory**.



Evolution of the Early Universe

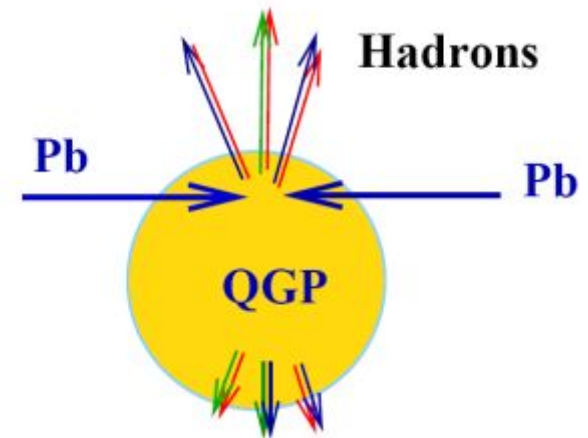


Evolution of a Heavy Ion Collision

QGP in heavy-ions collisions

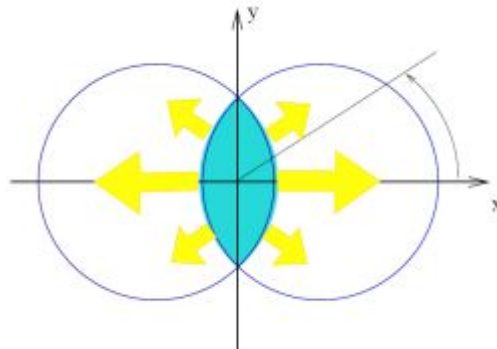
There are strong experimental evidences that RHIC or LHC have created some medium which behaves collectively:

suppression of back-to-back jet correlations
(signals that some jets are “lost” going through the medium)



modification of particle spectra (compared to p+p)

Elliptic flow



Hydrodynamics simulations can give estimations for V_2 .

more pressure along the small axis

QGP and Relativistic Hydrodynamics

Hydrodynamics appears as an effective description, valid on

length scales \gg mean free path

Fermi,
L.D.Landau (1953),
J.D.Bjorken (1982)

Energy-Momentum tensor (**perfect fluid**)

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - p\eta^{\mu\nu} ; u^\mu u^\mu = -1$$

Hydrodynamics

$$\boxed{\partial_\mu T^{\mu\nu} = 0} ; \frac{\partial p}{\partial \epsilon} = c_s^2 (= 1/g) ; \boxed{T^{\mu\mu} = 0} \Rightarrow g = 3$$

Fluid with viscosity

η - **shear viscosity**

ζ - **bulk viscosity**

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - p\eta^{\mu\nu} + P^{\mu\alpha} P^{\nu\beta} [\eta(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{3}g_{\alpha\beta} \partial \cdot u) + \zeta g_{\alpha\beta} \partial \cdot u]$$

$$P^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

QCD as a strongly coupled fluid

- A remarkable conclusion from the RHIC and LHC experiments is that the QGP does not behave as a weakly coupled gas of quarks and gluons, but rather as a **strongly coupled fluid**. Shuryak, 04
- This makes perturbative methods inapplicable
- The lattice formulation of QCD does not work, since we have to study real-time phenomena.
- This has provided a strong motivation for understanding the dynamics of strongly coupled QGP through the gauge/string duality



Dual description of QGP (Quark Gluon Plasma) as a part of Gauge/string duality

(Maldacena '97):

- The Gauge/Gravity duality gives an correspondence between the 4-dim physical space where the **gauge theory** lives and the 5-dimensional space where the **supergravity** (weak curvature) approximation of the 10-dimensional string theory is valid.
- Or in others words, the properties of the gauge theory in (physical) Minkowski space in $3+1$ dimensions are in one-to-one relation with properties of the bulk theory.
- The best known example of such theories is $N = 4$ super Yang-Mills, a superconformal field theory with matter in the adjoint representation of the gauge group $SU(N_c)$.

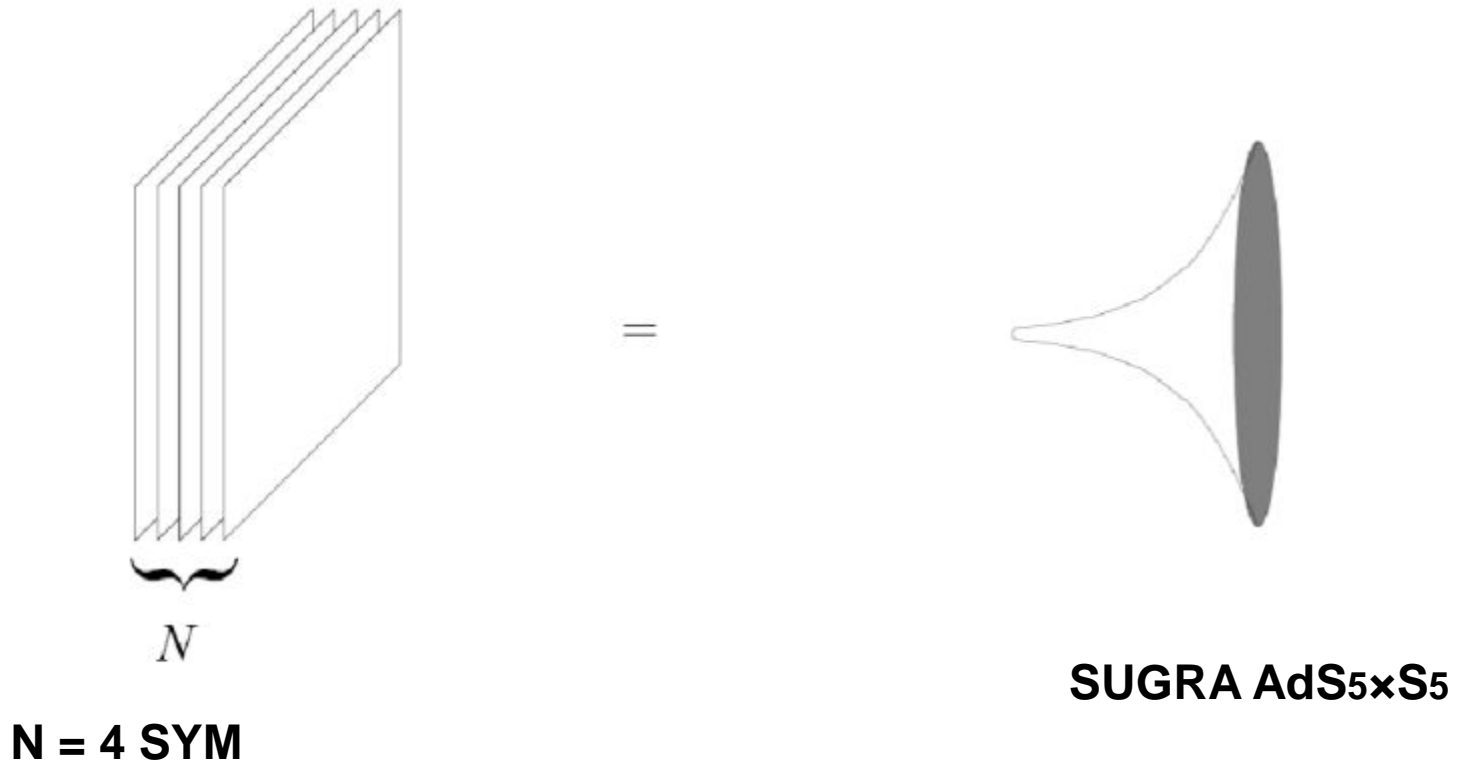


Dual description of QGP (Quark Gluon Plasma) as a part of Gauge/string duality

- However, **there is not yet exist a gravity dual construction for QCD.**
- Differences between $N = 4$ SYM and QCD are less significant, when quarks and gluons are in the deconfined phase (because of the conformal symmetry at the quantum level $N = 4$ SYM theory does not exhibit confinement.)
- Lattice calculations show that QCD exhibits a quasi-conformal behavior at temperatures $T > 300$ MeV and the equation of state can be approximated by $\epsilon = 3 p$ (a traceless conformal energy-momentum tensor).
- The above observations, have motivated to use the AdS/CFT correspondence as a tool to get non-perturbative dynamics of QGP.
- There is the considerable success in description of the static quark-gluon plasma, in particular in the evaluation of η/s [Policastro, Son, Starinets, 01]



The Gauge/Gravity Duality



Relation between parameters:

$$g^2 = 4\pi g_{\text{st}}$$
$$g^2 N_c = \frac{R^4}{\ell_s^4}$$

Dual description of QGP (Quark Gluon Plasma) as a part of Gauge/string duality

In the phenomenological hydrodynamical (Landau or Bjorken models of QGP),
the plasma is characterized by a space-time profile of the energy-momentum tensor

$$T_{\mu\nu}, \mu, \nu = 0, \dots, 3$$

$$T_{\mu\nu} = \langle \hat{T}_{\mu\nu} \rangle$$

ADS/CFT: operators in the gauge theory correspond to fields in SUGRA

$$\langle \mathcal{O} \rangle \Leftrightarrow \mathcal{A}_i$$

If $\hat{T}_{\mu\nu}$ then g_{MN}

In the case of the energy-momentum tensor, the corresponding field is the 5D metric.

Dual description of QGP (Quark Gluon Plasma) as a part of Gauge/string duality

Janik, 05

Static uniform plasma

$$\langle T_{\mu\nu} \rangle \propto g_{\mu\nu}^{(4)} = \begin{pmatrix} 3/z_0^4 & 0 & 0 & 0 \\ 0 & 1/z_0^4 & 0 & 0 \\ 0 & 0 & 1/z_0^4 & 0 \\ 0 & 0 & 0 & 1/z_0^4 \end{pmatrix}.$$

$$G_{MN} = \Lambda g_{MN}, \quad g_{\mu\nu} \big|_{\text{boundary}} = g_{\mu\nu}^{(4)}$$

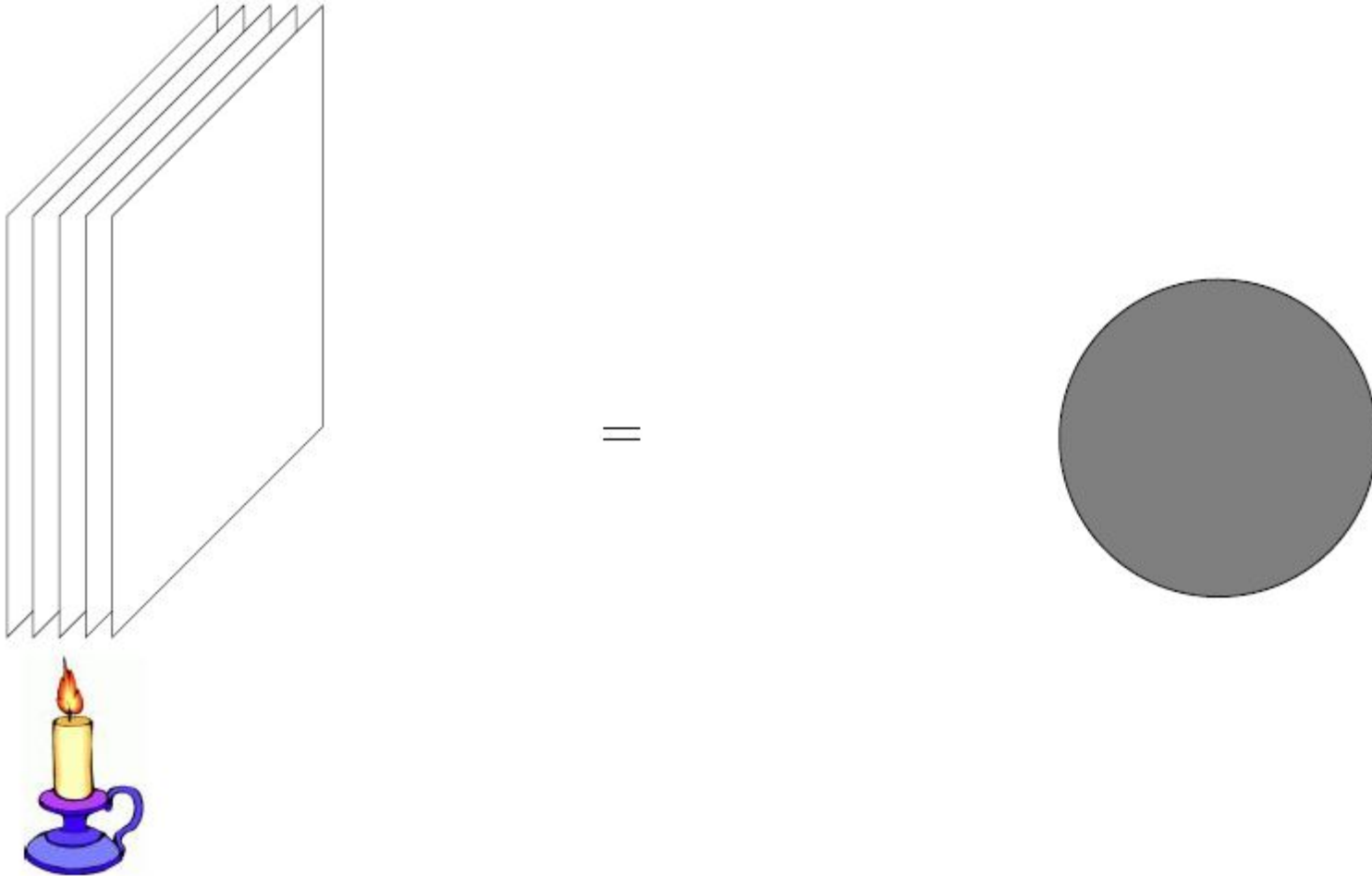
Solution:
$$ds^2 = -\frac{(1 - z^4/z_0^4)^2}{(1 + z^4/z_0^4)z^2} dt^2 + (1 + z^4/z_0^4) \frac{dx^2}{z^2} + \frac{dz^2}{z^2}$$

Performing a change of coordinates
$$\tilde{z} = \frac{z}{\sqrt{1 + \frac{z^4}{z_0^4}}}$$

The standard AdS static black hole
$$ds^2 = -\frac{1 - \tilde{z}^4/\tilde{z}_0^4}{\tilde{z}^2} dt^2 + \frac{dx^2}{\tilde{z}^2} + \frac{1}{1 - \tilde{z}^4/\tilde{z}_0^4} \frac{d\tilde{z}^2}{\tilde{z}^2}$$

with $\tilde{z}_0 = z_0/\sqrt{2}$.

Dual description of QGP (Quark Gluon Plasma)



“Quark gluon plasma” = black hole (in anti de-Sitter space)

Dual description of formation of QGP

CONJECTURE:

Black Hole formation in AdS₅

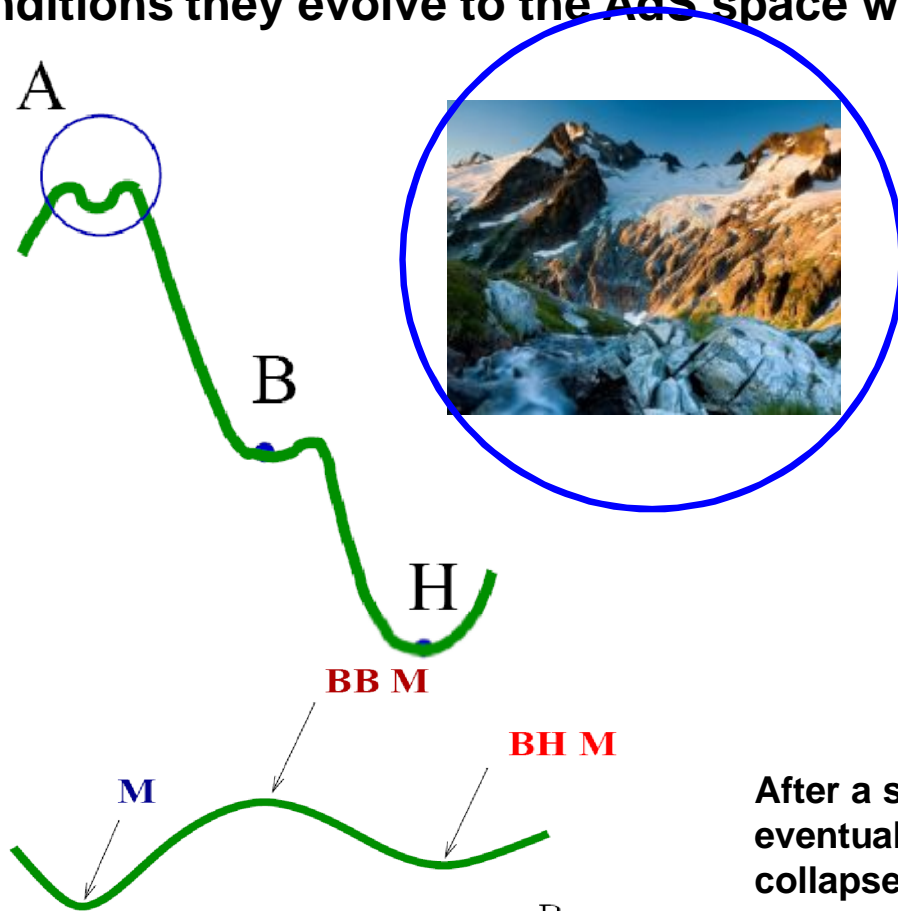
= thermalization (formation of QGP) of 4-dim QCD



BH formation in AdS (or holographic thermalization)

Thermalization of some class of space-time geometries = space-time geometries without an event horizon (EH) evolve to space-times with EH.

Take deformed AdS space-times (asymptotical AdS) and ask under which conditions they evolve to the AdS space with a BH, or black branes.



- H Sch AdS is an equilibrium "point"
- B Black branes AdS
- A AdS is stable under small fluctuations and is unstable under large nonlinear fluctuations

Bizon and Rostworowski, PRL, 2011;
Dias, Horowitz and Santos, 1109.1825

Circle A - large chaotic deformations

After a sufficiently long time, any finite excitation of AdS eventually finds itself inside its Schw. radius and collapses to a BH

Deformations of AdS metric

- colliding gravitational shock waves
- drop of a shell of matter with vanishing rest mass ("null dust"),
infalling shell geometry = Vaidya metric Balasubramanian et al, PRL '11
- sudden perturbations of the metric near the boundary that propagate into the bulk Chesler, Yaffe, PRL '09



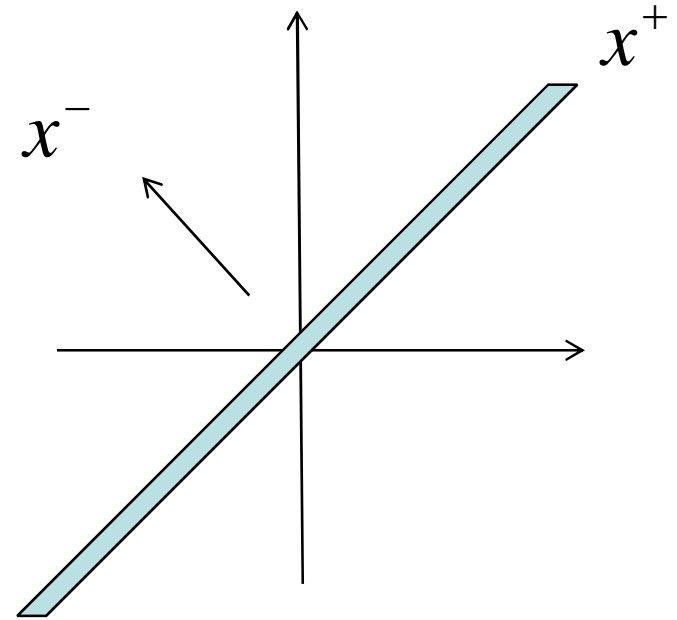
Single Nucleus in AdS/CFT

An ultrarelativistic nucleus is a shock wave in 4d with the energy-momentum tensor

$$\langle T_{--} \rangle \sim \mu \delta(x^-)$$

The metric of a shock wave in AdS corresponding to the ultrarelativistic nucleus in 4d is

$$ds^2 = \frac{L^2}{z^2} \left[-2 dx^+ dx^- + \frac{2\pi^2}{N_C^2} \langle T_{--}(x^-) \rangle z^4 dx^{-2} + dx_{\perp}^2 + dz^2 \right]$$



Janik, Peschanski '05

Ultrarelativistic particle = shock wave

- Aichelburg-Sexl shock wave

$$ds^2 = -dUdV + dx^{i2} + F(x^i)\delta(U) dU^2,$$

$$F(x^i) = \frac{c_k}{\left| \sum (x^i - x_{k0}^i)^2 \right|^{(D-4)/2}}, \quad U = x$$

**Smooth coordinates: P.D'Eath coordinates,
Dray and 't Hooft**

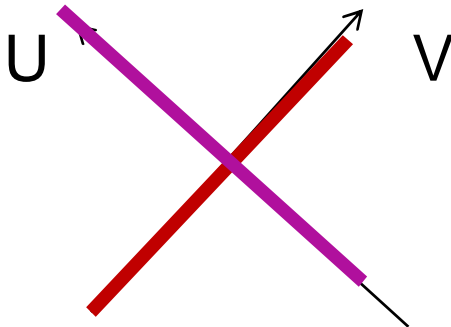


2 Ultrarelativistic particles = 2 shock waves

- 2 Aichelburg-Sexl shock waves

$$ds^2 = -dUdV + dX^{i2} + \underline{F_1(X^i)\delta(U)} dU^2 + \underline{F_2(V^i)\delta(V)} dV^2,$$

$$F_k(X^i) = \frac{c_k}{\left| \sum (X^i - X_{k0}^i)^2 \right|^{(D-4)/2}}$$



BLACK HOLE FORMATION = \exists Trapped Surface(TS)

- Theorem (Penrose): **BH Formation = \exists TS**

- **Theorem.** \exists **TS for two shock waves =**
• \exists **solution to the following Dirichlet problem**

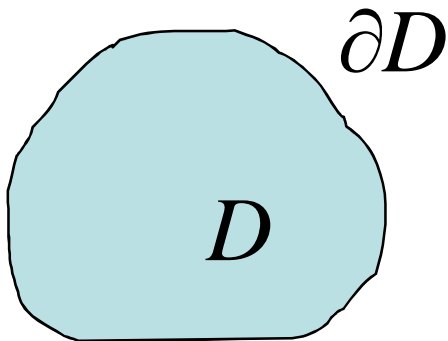
- $\Psi_{1,2} > 0, X \in D, \Psi_{1,2} = 0, X \in \partial D$

- $\nabla^2 \Psi_{1,2} = \delta^{(D-2)}(X - X_{(1,2)}), X \in D,$

the outer null normals have zero convergence

- $\nabla \Psi_1 \cdot \nabla \Psi_2 = 4, X \in \partial D$

no δ - function in convergence



Different profiles ----- different multiplicities

An arbitrary gravitational shock wave in *AdS₅*

$$ds^2 = \frac{L^2}{z^2} \left(-dx^+ dx^- + dx_\perp^2 + \phi(x_\perp, z) \delta(x^+) dx^{+2} + dz^2 \right)$$

$$\Phi(z, x_\perp) \equiv \frac{L}{z} \phi(z, x_\perp) \quad \left(\square_{H_3} - \frac{3}{L^2} \right) \Phi(z, x_\perp) = -16\pi G_5 \frac{z}{L} J_{uu}(z, x_\perp)$$

- Point sourced shock waves

$$J_{uu} = E \delta(u) \delta(z - L) \delta(x^1) \delta(x^2)$$

$$\Phi(z, x_\perp) = \Phi^p(q)$$

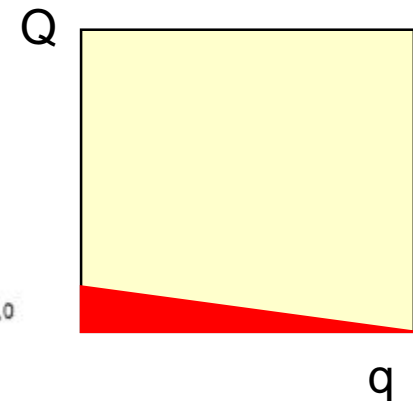
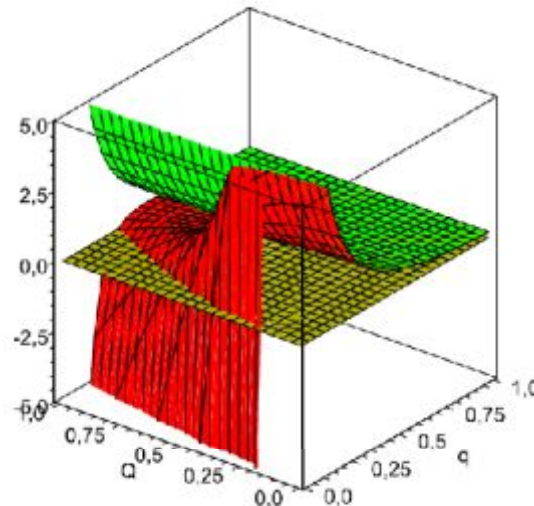
$$q = \frac{x_\perp^2 + (z - L)^2}{4zL}$$

The chordal coordinate

Hotta, Tanaka, 93

- Point charged shock waves

$$\Phi(z, x_\perp) = \Phi^p(q) + \Phi^q(q) \Theta(q - q_{Th})$$



Dual description of QGP (Quark Gluon Plasma)

- **QGP formation for heavy ions collisions in M_4
in dual description:**

BH formation in shock waves collisions in AdS_5

- **Trapped surface area ----- multiplicity**
- **BH charge ----- chemical potential**



Multiplicity=Entropy=area of trapped surface

$$S_{\text{trapped}} \approx \pi \left(\frac{L^3}{G_5} \right)^{1/3} (2EL)^{2/3}$$

Gubser, Pufu, Yarom, 0805.1551,
 Alvarez-Gaume, C. Gomez, Vera,
 Tavanfar, Vazquez-Mozo, 0811.3969
 IA, Bagrov, Guseva, 0905.1087

Lattice calculations $ET^4 \approx 11$

Hawking-Page relation

$$\frac{L^3}{G_5} = \frac{16ET^4}{3\pi^3}$$

$$\underbrace{\hspace{15em}}_{\frac{L^3}{G_5} \approx 1.9}$$

From a Woods-Saxon profile for the nuclear density

$$Au: \quad L \approx 4.3 \text{ fm} = 4.3 \cdot 5 \frac{1}{\text{GeV}}; \quad Pb: \quad L \approx 4.4 \text{ fm}$$

$$EL \Big|_{Au-Au, \sqrt{s_{NN}}=200 \text{ GeV}} \approx 4.3 \times 10^5 \quad EL \Big|_{Pb-Pb, \sqrt{s_{NN}}=5.5 \text{ TeV}} \approx 1.27 \times 10^7$$

$$S \geq S_{\text{trapped}} \approx 35000 \left(\frac{\sqrt{s_{NN}}}{200 \text{ GeV}} \right)^{2/3}$$

Multiplicity: Experimental data, Landau, AdS-estimation

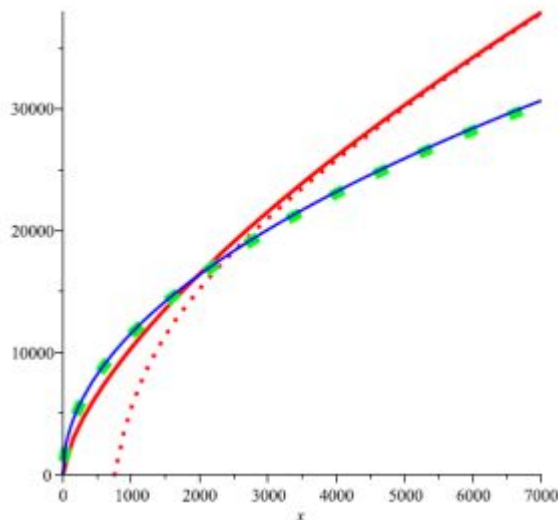
Phenomenological estimation: total multiplicity and the number of charged particle

$$S_{\text{trapped}} \leq S_{\text{AdS}} = S_{\text{QGP}} \approx 7.5 N_{\text{ch}}$$

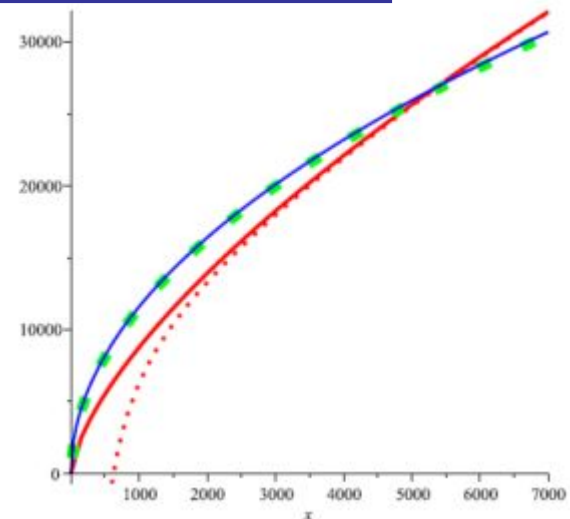
$$\mathcal{M} \cdot S_{\text{trapped}} < N_{\text{ch}}$$

$$S_{\text{trapped}} \propto S_{NN}^{1/3}$$

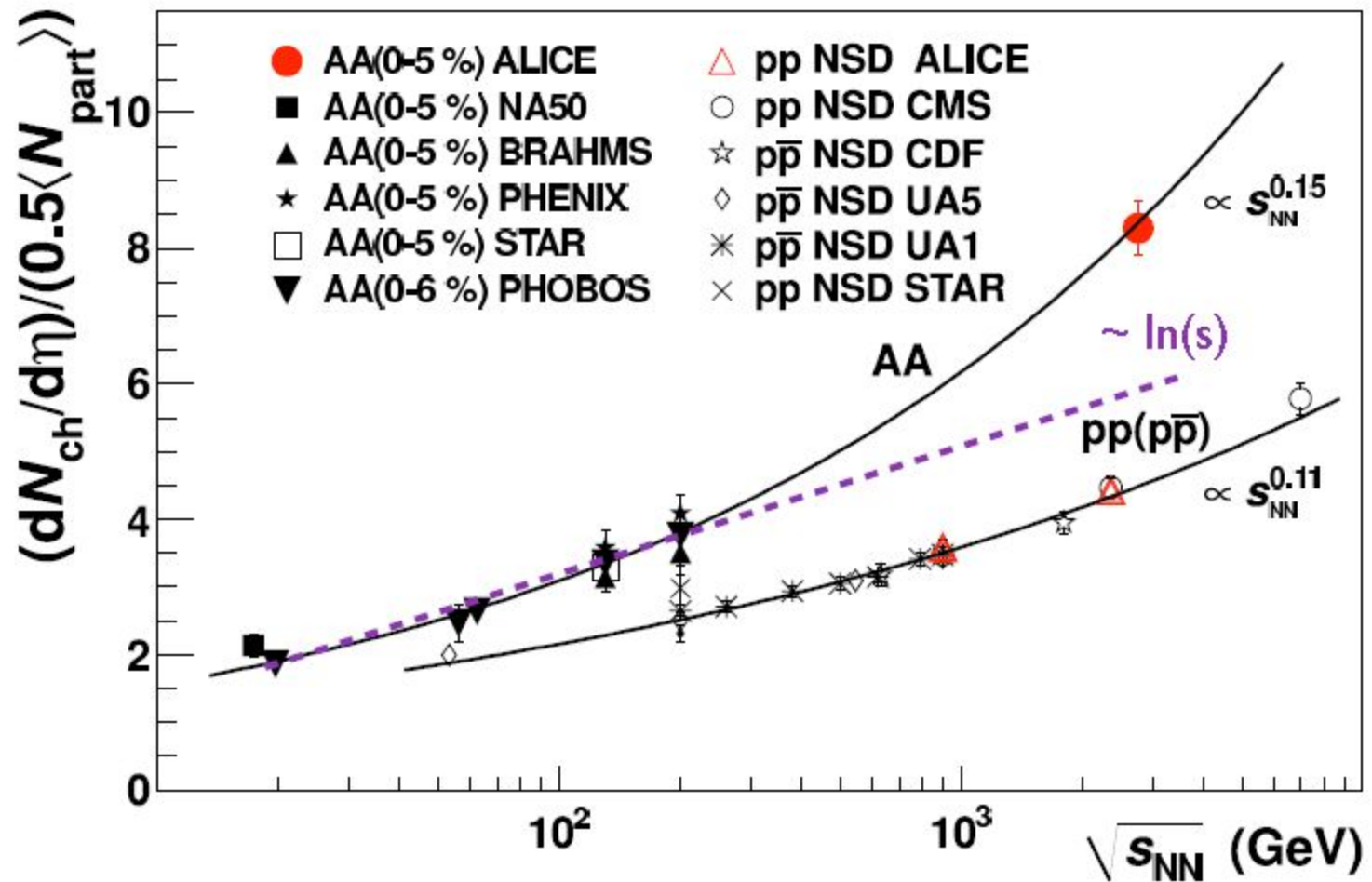
$$S_{\text{Landau}} \propto S_{NN}^{1/4}$$



red line - TS estimation
blue line - Landau model
dotted green - the experimental data
dashed line red - TS estimation with a chemical potential

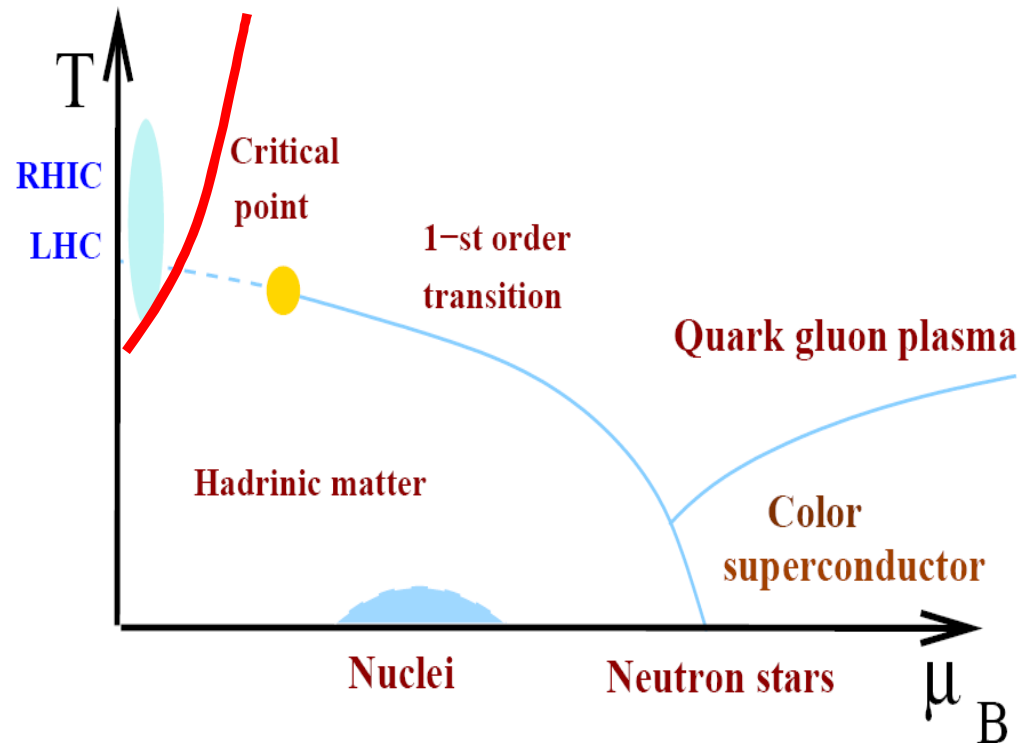
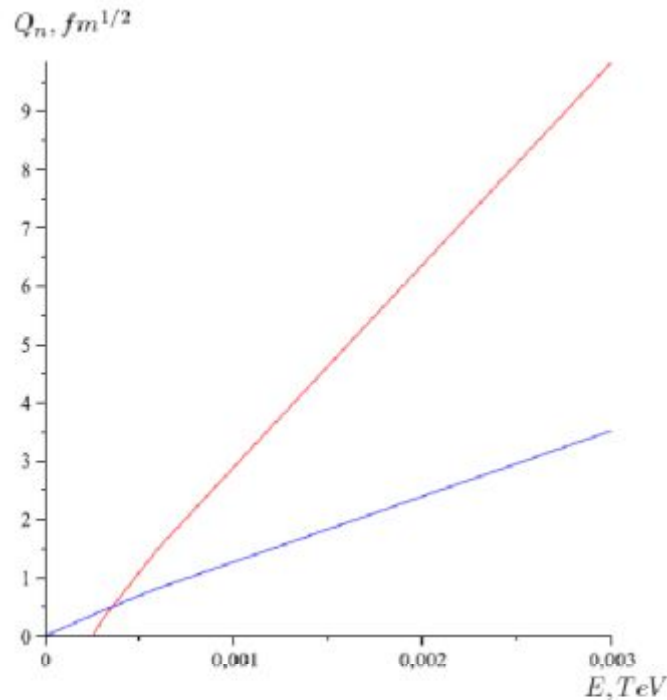


Multiplicity: Experimental data



Phase diagram from dual approach


Formation of trapped surfaces is only possible when $Q < Q_{cr}$



Red for smeared matter
Blue for point-like

I.A., A.Bagrov, Joukovskaya, 0909.1294
I.A., A.Bagrov, E.Pozdeeva, 1201.6542

Different profiles ----- different multiplicities



Goal: try to find a profile to fit experimental data

- Dilaton shock waves

Cai, Ji, Soh, gr-qc/9801097

IA, 0912.5481

Multiplicity very closed to LHC data

Kiritsis, Taliotis, 1111.1931

Further directions

Janik, Witaszczyk, 08

- Anisotropic thermalisation
- Isotropisation time
(isotropisation due to instability
in anisotropic plasma)

$$\tau_{isotr} \sim 0.1 fm$$

Mateos, Trancanelli, PRL, '11
PRL, '12

$$\langle T^{\mu\nu} \rangle = \text{diag}(\varepsilon, P_{\perp}, P_{\perp}, P_L)$$

a conformal anomaly $\langle T_{\mu}^{\mu} \rangle \propto a^4$

5-dimensional axion-dilaton gravity

$$\chi = a x_L$$



Conclusion

Using conjecture that BH production in AdS5 gives QGP formation in 4-dim QCD we have got:

- Multiplicity
- New phase transition (T vs μ)

