Superconductor-insulator transition, energy localization and level statistics.

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In collaboration with:

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Marc Mezard — Orsay University
Emilio Cuevas — University of Murcia

Previous publications on the related subject:

Phys Rev Lett. 98, 027001(2007) (M.F.,L. Ioffe,V. Kravtsov, E.Yuzbashyan)
Annals of Physics 325, 1368 (2010) (M.F., L.Ioffe, V.Kravtsov, E.Cuevas)
Phys.Rev. B 82, 184534 (2010) (M.F.,L. Ioffe, M. Mezard)
Nature Physics, 7, 239 (2011) (B.Sacepe,T.Doubochet,C.Chapelier,M.Sanquer, M.Ovadia,D.Shahar, M. F., L.Ioffe)

Plan of the talk

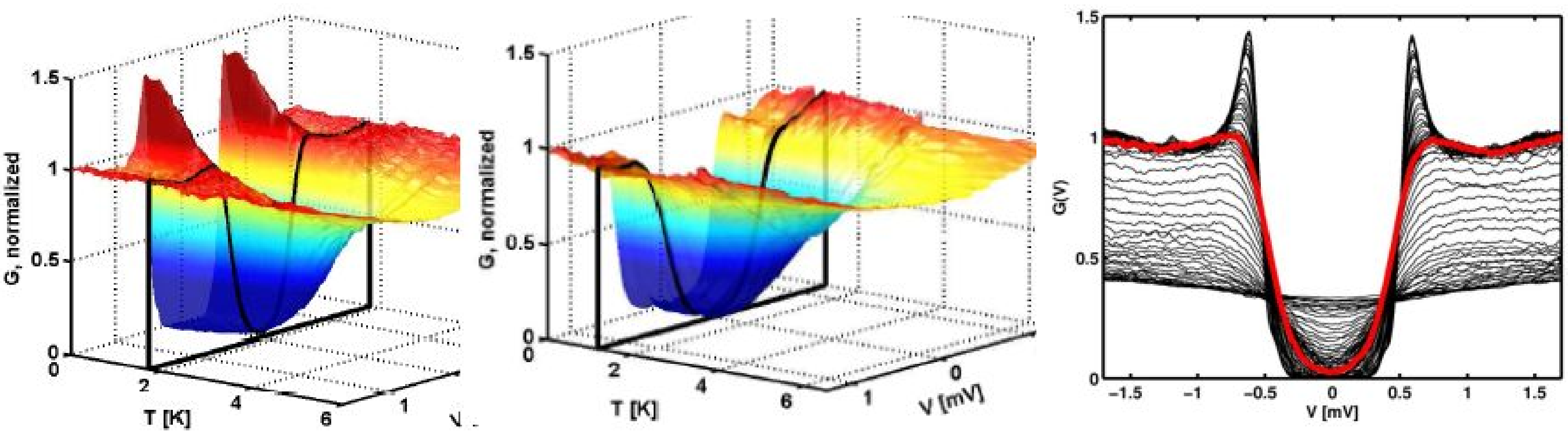
1. Superconductivity with pseudogap and effective spin-1/2 model
2. Bethe lattice model of quantum phase transition. Critical lines from the analitical solution
3. Level statistics on small random graph: exact numerical diagonalization.
4. Summary of results

Nature Physics, 7, 239 (2011)

SC side: local tunneling conductance

Spectral signature of localized Cooper pairs in disordered superconductors.

Benjamin Sacépé,[1,*] Thomas Dubouchet,[1] Claude Chapelier,[1] Marc Sanquer,[1] Maoz Ovadia,[2] Dan Shahar,[2] Mikhail Feigel'man,[3] and Lev Ioffe[4,3]



Superconductive state with a pseudogap: Fermi-level in the localized band

Superconductive state near SIT is very unusual:

The spectral gap appears much before (with T decrease) than superconductive coherence does

Coherence peaks in the DoS appear together with resistance vanishing

Distribution of coherence peaks heights is very broad near SIT

Single-electron states suppressed by pseudogap $\Delta_P \gg T_c$

$Z \sim \nu_0 T_c L^d_{loc}$

„Pseudo spin" representation:

$S^+_\mu = a^+_{\mu\uparrow} a^+_{\mu\downarrow}$ $S^-_\mu = a_{\mu\uparrow} a_{\mu\downarrow}$

$2S^z_\mu = a^+_{\mu\uparrow} a_{\mu\uparrow} + a^+_{\mu\downarrow} a_{\mu\downarrow}$

H_{BCS} acts on Even sector: all states which are 2-filled or empty

$$\hat{H} = \sum_\mu 2\xi_\mu S^z_\mu - g \sum_{\mu,\nu} M_{\mu\nu} S^+_\mu S^-_\nu + \sum_{B_\mu} \left(\xi_\mu + \frac{G_\mu}{2}\right)$$

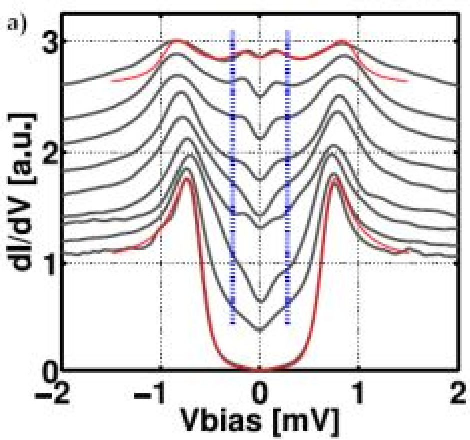
B: „blocked" states

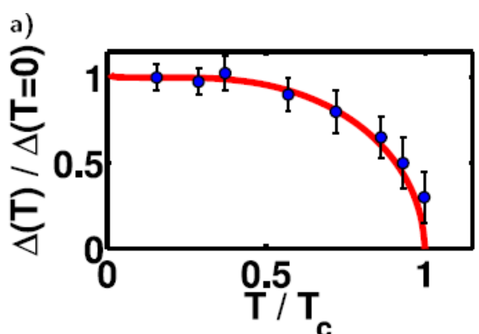
"Pseudospin" approximation

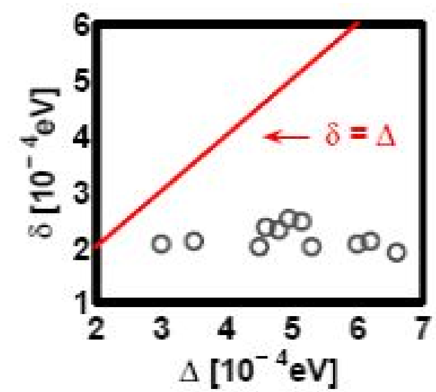
$$\bar{M}_{\mu\nu} = \frac{1}{\nu V} M(\xi_\mu - \xi_\nu)$$

D.o.S ↑ ↖ total volume

T. Dubouchet,[1,*] C. Chapelier,[1] M. Sanquer,[1] B. Sacépé,[2,3] Maoz Ovadia,[3] and Dan Shahar[3]







$2eV_1 = 2\Delta$

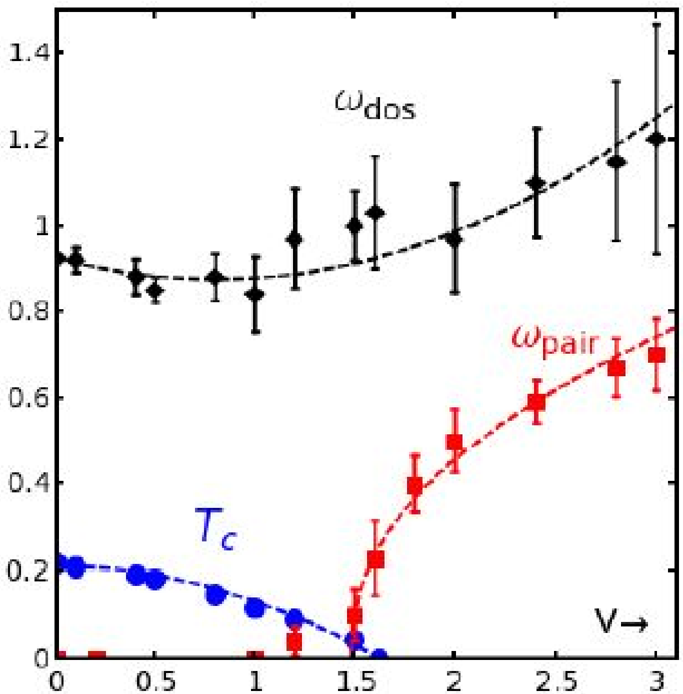
$eV_2 = \Delta + \Delta_P$

Andreev point-contact spectroscopy

Single and two-particle energy gaps across the disorder-driven superconductor-insulator transition

Karim Bouadim, Yen Lee Loh, Mohit Randeria, and Nandini Trivedi

arXiv:1011.3275
Nature Physics 2011



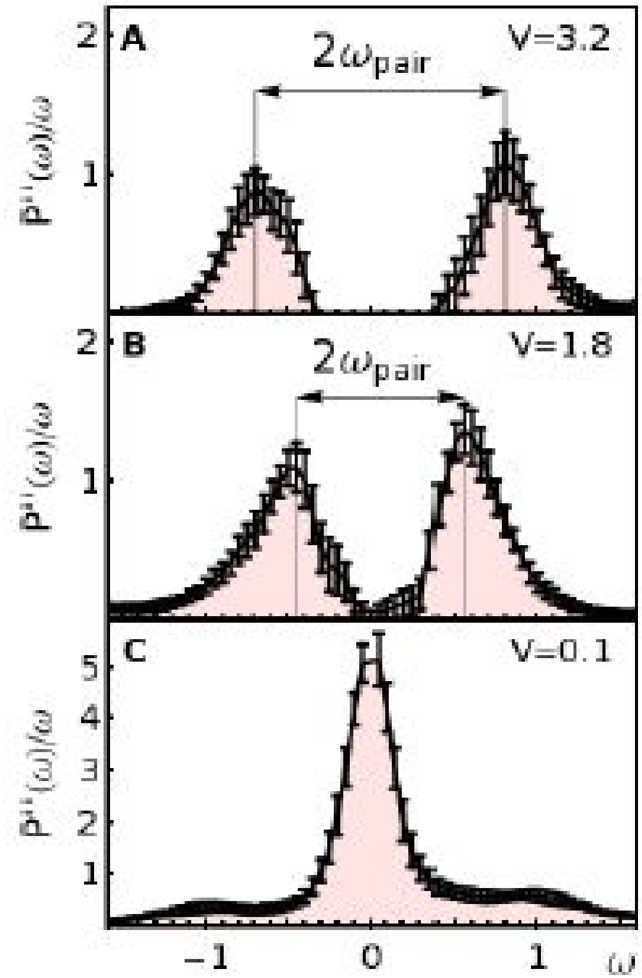


FIG. 3: Imaginary part of the dynamical pair susceptibility $P''(\omega)/\omega$ at $T = 0.1t$, averaged over 10 disorder realizations at three disorder strengths. Error bars represent

S-I-T: Third Scenario

- Bosonic mechanism: preformed Cooper pairs + competition Josephson v/s Coulomb – S I T in arrays
- Fermionic mechanism: suppressed Cooper attraction, no pairing – S M T
- Pseudospin mechanism: individually localized pairs

- S I T in amorphous media

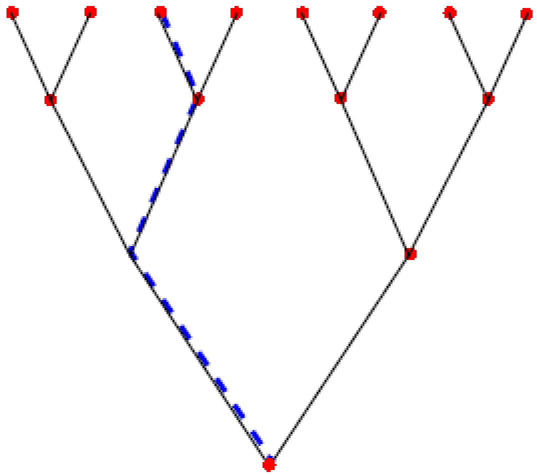
SIT occurs at small Z and lead to paired insulator

How to describe this quantum phase transition ?

Bethe lattice model is solved

Phys. Rev.Lett. 105, 037001 (2010)
L.Ioffe, M. Mezard
Phys.Rev. B 82, 184534 (2010)
M. Feigelman, L.Ioffe, M. Mezard

$$H = 2\sum_i \xi_i s_i^z - \sum_{ij} M_{ij}(s_i^x s_j^x + s_i^y s_j^y)$$



Distribution function for the order parameter

General recursion:

$$B_j = \frac{g}{K}\sum_{k=1}^{K}\frac{B_k}{\sqrt{B_k^2+\xi_k^2}}\tanh\beta\sqrt{B_k^2+\xi_k^2}\ .$$

Linear recursion ($T=T_c$)

$$B_i = (g/K)\sum_k (B_k/\xi_k)\tanh(\beta\xi_k)\ ,$$

$$P(B) = \frac{B_0^m}{B^{1+m}}$$

Laplace transform satisfies the equation:

$$\mathcal{P}(s) = \left[\int_0^1 d\xi\, \mathcal{P}\left(s\frac{g}{K}\frac{\tanh\beta\xi}{\xi}\right)\right]^K$$

Diverging 1st moment

Solution in the RSB phase: $\mathcal{P}(s) = 1 - As^x$ with $x<1$

$$1 = K\int\frac{d\xi}{\xi}\left(\frac{g}{K}\frac{\tanh(\beta\xi)}{\xi}\right)^x$$

$$\int_{-1}^{1}\frac{d\xi}{2}\frac{\tanh^x\beta\xi}{\xi^x}\ln\left(\frac{g}{\xi K}\tanh\beta\xi\right) = 0$$

T=0

$$g_c e^{1/(eg_c)} = K$$

$$m = 1 - eg_c$$

Vicinity of the Quantum Critical Point

$$T_c(K) = \vartheta(y_c)\left(\frac{K}{K_c} - 1\right)^{1/y_c}$$

$$y_c = eg. \quad \mathbf{<< 1}$$

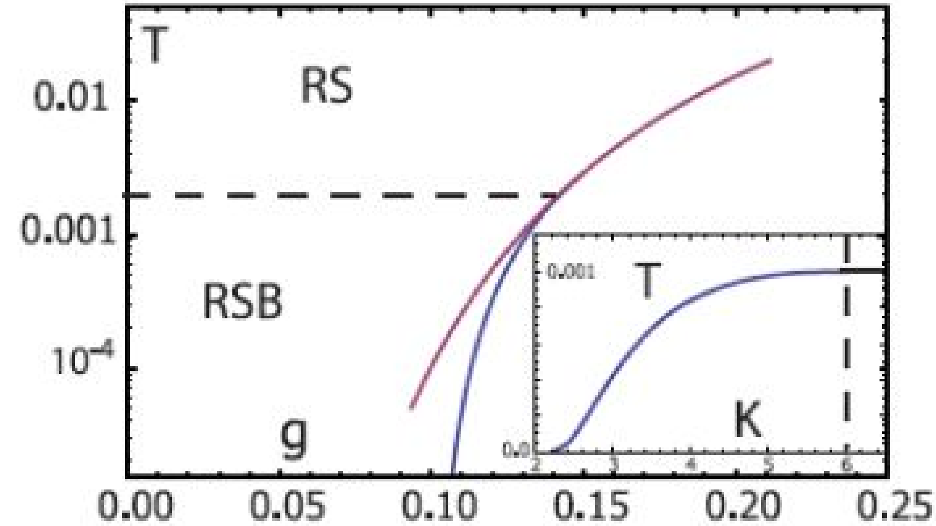


FIG. 2: Main panel: phase diagram in plane (g, T) for $K = 4$. Full lines show the critical temperature as function of g. The low temperature phase is superconducting, the high temperature phase is insulating. The top curve is the naive mean-field prediction which gives the correct result above $T_{RSB} = 0.0207$. The bottom curve is the result of the correct analysis on the Bethe lattice, including the RSB effects in the DP problem, which occur at temperatures $T < T_{RSB}$. The insert shows the phase diagram as function of K for $g = 0.129$. For this value of g the replica symmetric solution gives K-independent transition temperature $T_c = 0.001$; this value roughly correspond to the experimental situation in disordered InO films (see section VI). The prediction of replica symmetric theory is correct for $K > K^{RSB} \simeq 6$. For smaller K the transition temperature starts to drop, the quantum critical point corresponds to $K_c \simeq 2.2$. Notice that in a numerically wide regime the replica symmetry is broken but the effect on transition temperature is small.

Insulating phase: continuous v/s discrete spectrum ?

Consider perturbation expansion over M_{ij} in H below:

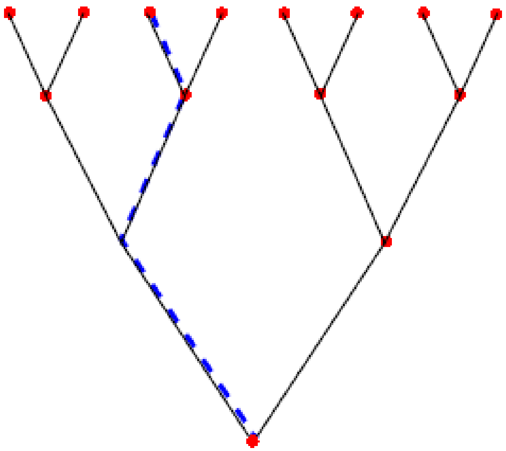
$$H = 2\sum_i \xi_i s_i^z - \sum_{ij} M_{ij}(s_i^x s_j^x + s_i^y s_j^y)$$

Within convergence region the many-body spectrum is qualitatively similar to the spectrum of independent spins

⇓

No thermal distribution, no energy transport, distant regions "do not talk to each other"

Recursion relations for level widths



$$\Gamma_i = (2g/K)^2 \sum_{k(i)} \frac{\Gamma_k}{(\omega - 2\xi_k)^2 + \Gamma_k^2}$$

$$\Gamma_0 = \sum_P \prod_{k \in P} \left[\frac{2g/K}{\omega - 2|\xi_k|} \right]^2 S(\omega)$$

$S(\omega)$. Spectral function of external noise

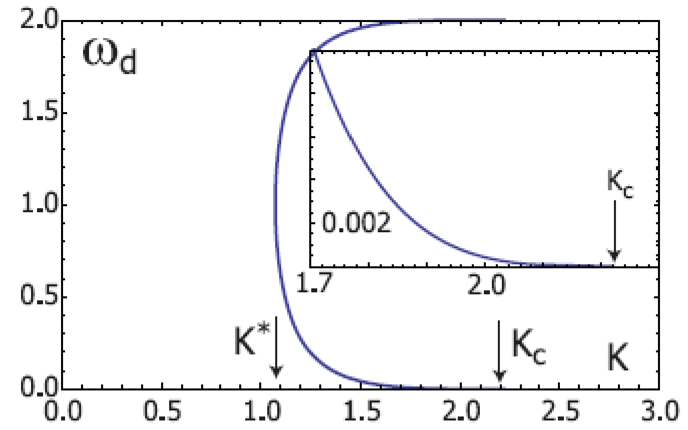
We look for the distribution function of the form

$$W(\Gamma) = \frac{\Gamma_0^b}{\Gamma^{b+1}} \text{ where } \Gamma_0 \le \Gamma \le \Gamma_{\max}$$

Threshold energy at T=0

$$K\left(\frac{g}{K}\right)^{2b}\int_0^1 d\xi \frac{1}{|\xi-\omega/2|^{2b}} = 1$$

$$K\left(\frac{g}{K}\right)^{2b}\int_0^1 \frac{d\xi}{|\xi-\omega/2|^{2b}}\ln\frac{1}{|\xi-\omega/2|} = \ln\frac{K}{g}$$



Low-energy limit
$\omega << 1$

$$\omega_d(g,K) = 2\left[\left(\frac{K_c(g)}{K}\right)^{eg} - 1\right]^{1/(eg)}$$

Full band localization
$\omega = 1$

$$K^*(g) = 2ge^{1/(2eg)}$$

$$z^*(g) = 2eg$$

Now set T>0. What happens to level width at low excitation energies ?

Threshold for activated transport

Nonzero line-width appears above threshold frequency only:

$$\omega(g,K) = 2(eg)^{1/eg}\left(1-\frac{K}{K_c}\right)^{1/eg}$$

$$\Gamma^{typ}(\omega) \simeq \Gamma_0(\omega) \simeq e^{-1/eg} exp\left(-\frac{\omega_1}{\omega-\omega_c}\right)$$

$$\omega_1 = \frac{C}{e(eg)^3}\frac{\omega_c}{1-K/K_c} \gg \omega_c$$

This is $T = 0$ result !

Nonzero activation energy for transport of pairs is due to the absence of thermal bath at low ω

Nonzero but low temperatures:

$$\Gamma^{typ}(\omega,T) \approx \max\left(\eta(T),\Gamma_0(\omega)\right)$$

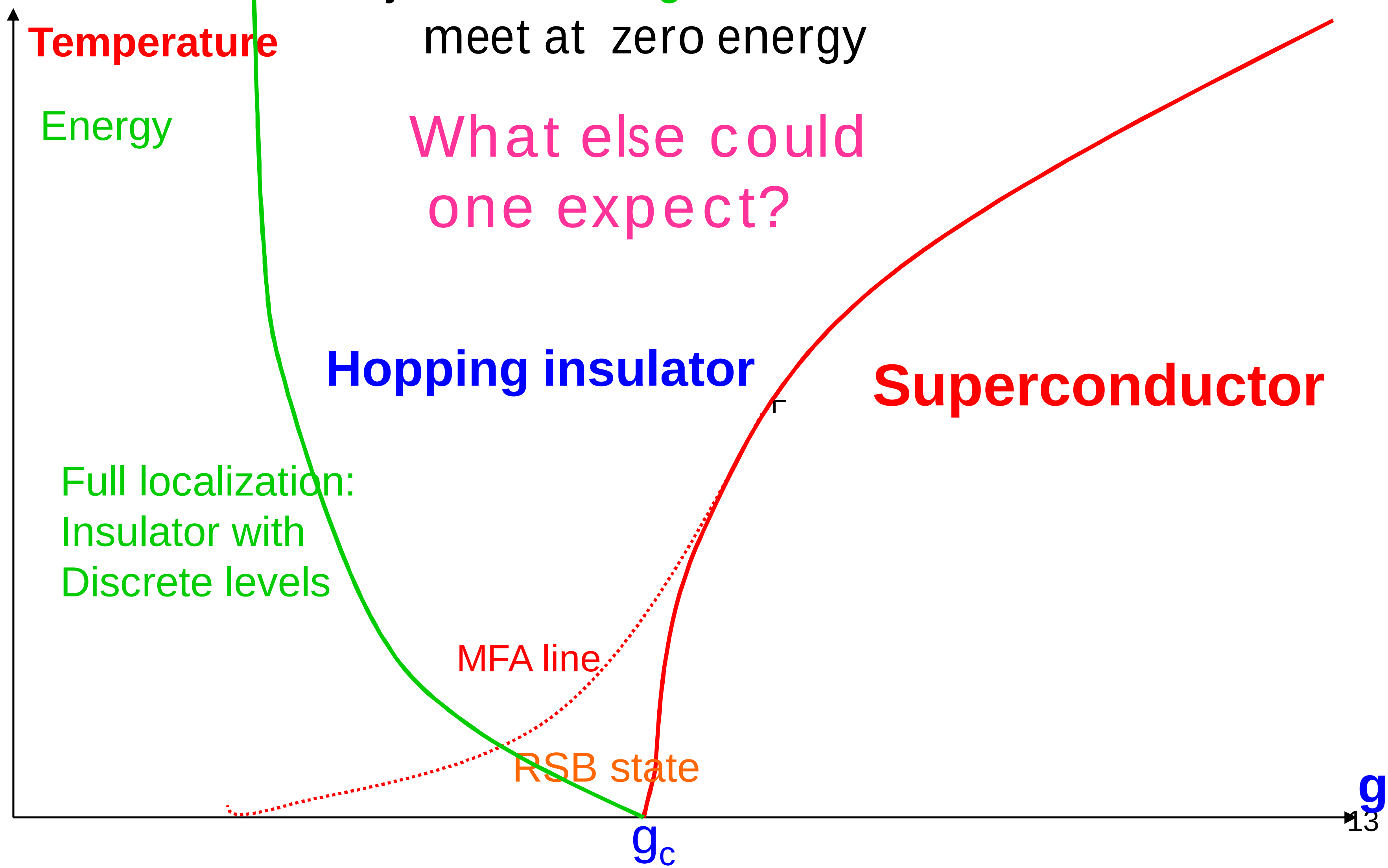
$$\eta(T) = \left(\frac{2g}{K}\right)^2 \frac{4\sqrt{\pi}\omega_1 e^{-1/eg}}{(\omega_c+\sqrt{\omega_1 T})^2}\left(\frac{T}{\omega_1}\right)^{3/4} \exp\left[-\frac{\omega_c}{T} - 2\sqrt{\frac{\omega_1}{T}}\right]$$

Activation law

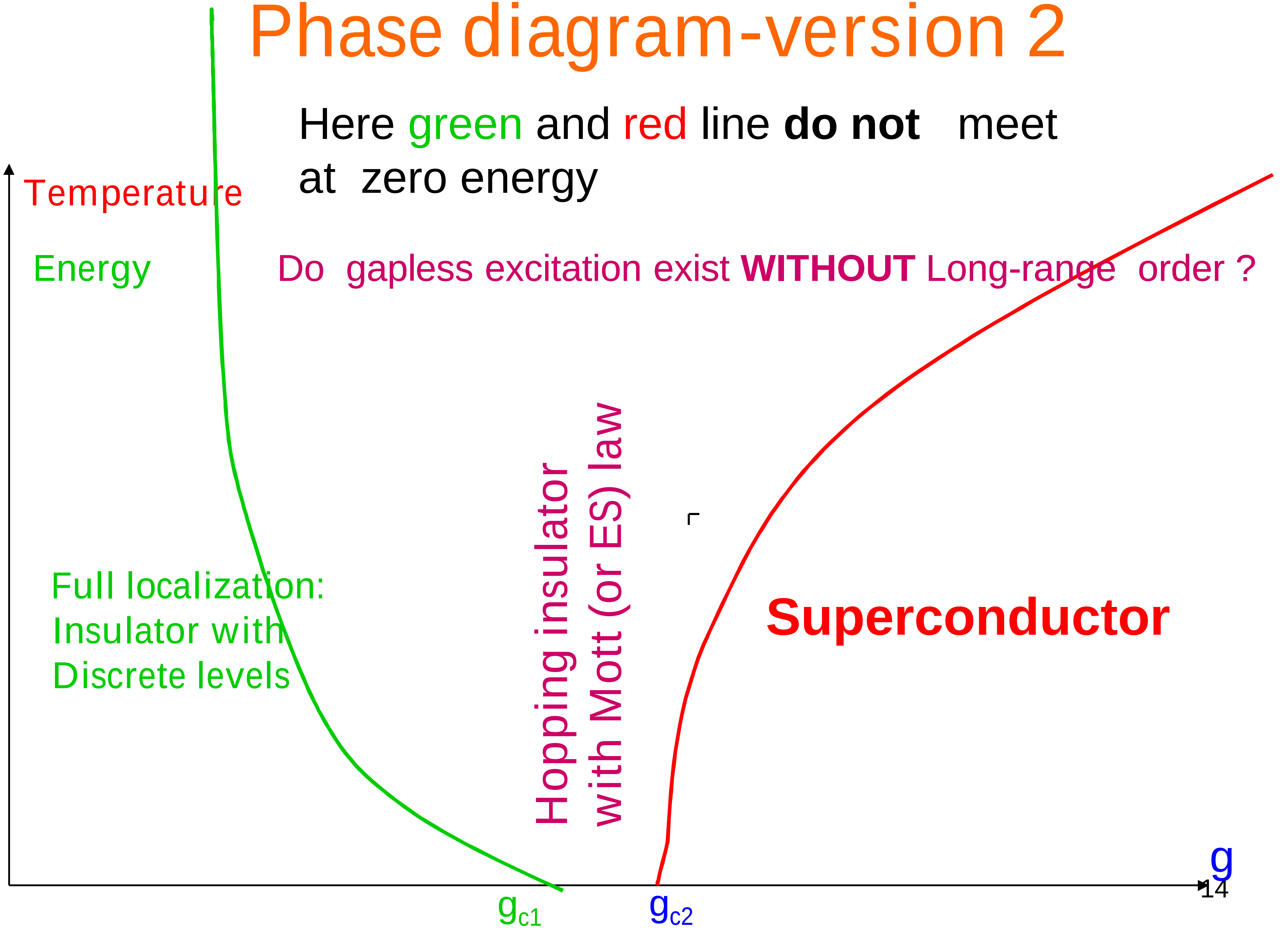
Phase diagram

Major feature: green and red line meet at zero energy

What else could one expect?

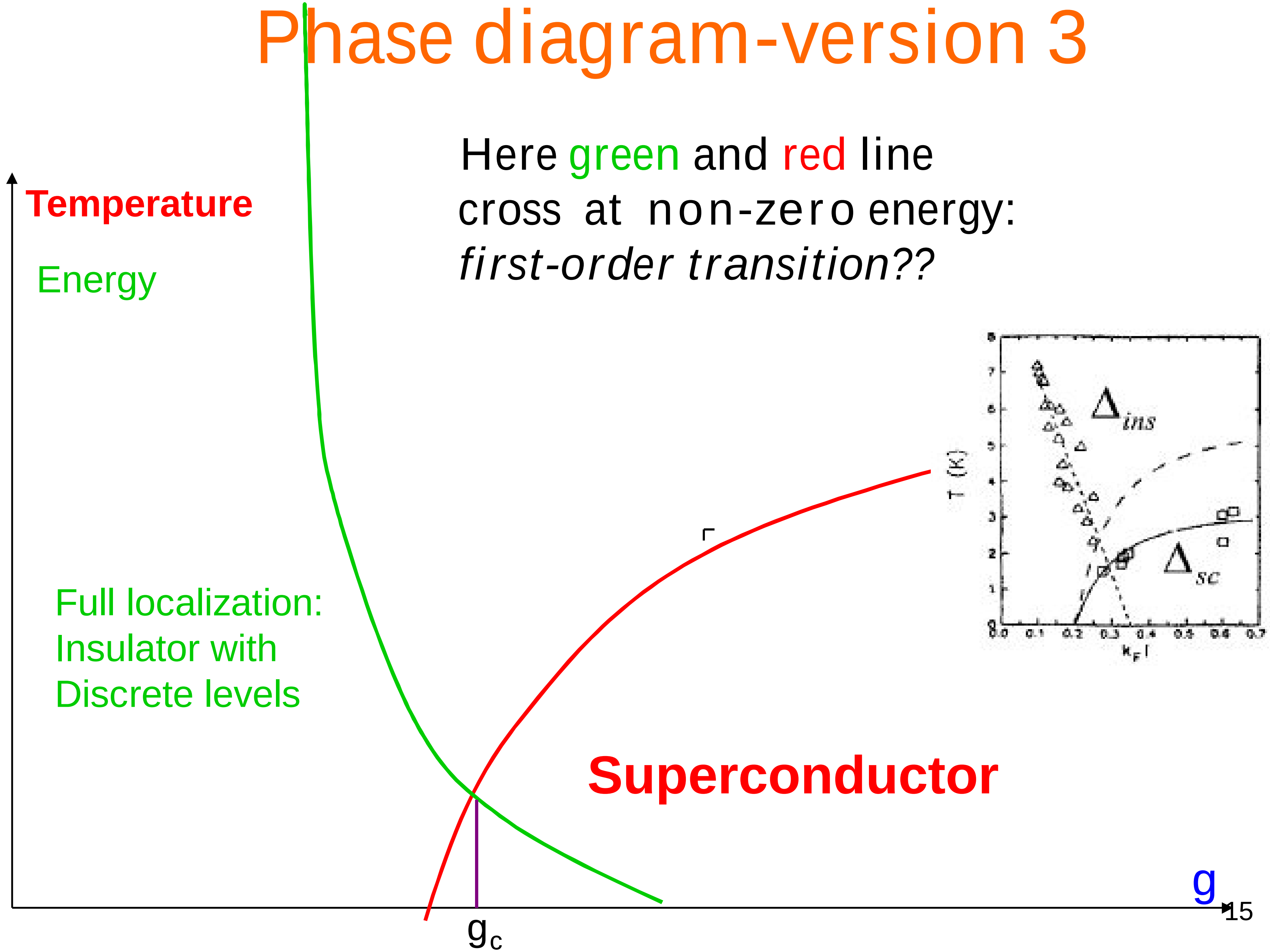


Phase diagram-version 2



Phase diagram-version 3

Here green and red line
cross at non-zero energy:
first-order transition??



Major results from Bethe lattice study

- Full localization of eigenstates with
  E ~ W at weakest coupling between spins, $g < g^*$ (or $K < K^*(g)$)
- No intermediate phase *without both* order parameter *and* localization of low-energy modes

Questions:

1) what about highly excited states with E >> W
2) how universal is the absence of intermediate phase ?
3) How to avoid the use of Bethe lattice ?

Different definitions for the fully many-body localized state

- 1. No level repulsion (Poisson statistics of the full system spectrum)
- 2. Local excitations do not decay completely
- 3. Global time inversion symmetry is not broken (no dephasing, no irreversibility)
- 4. No energy transport (zero thermal conductivity)
- 5. Invariance of the action w.r.t. local time transformations $t \rightarrow t + \varphi(t,r)$:
  $d\,\varphi(t,r)\,/\,dt = \xi\,(t,r)$ – Luttinger's gravitational potential

Different physical properties of the excitations at low and high energies in the infinite system are reflected in different statististical properties of the spectra of finite systems at $E < \mathcal{E}_c$ and at $E > \mathcal{E}_c$. Intuitively, if the eigenvectors are extended as expected for the state where local excitations decay, they are subject to inter-level repulsion. Conversely, if the eigenvectors are localized, eigenvalues corresponding to excitations localized in different parts of the system are independent and Poisson distribution of levels is expected. To show it, consider a small perturbation of the Hamiltonian that controls the dynamics of a generic quantum system in thermodynamic limit:

$$H \to H(1 + \dot{\phi}(t, x)) \tag{1}$$

where $\phi(t, x)$ is arbitrary slow function of coordinates and time. The small perturbation (1) results in the slow (adiabatic) motion of energy levels $E_n(t)$. In the absence of level repulsion, different levels cross without affecting each other, so that this motion leads only to the total phase of the wave function. Because the field $\dot{\phi}(t, x)$ (which is similar to the gravitational potential[11]) is conjugated to the energy density, the absence of response to it implies absence of the energy flux. An excitation with energy ΔE localized around point x acquires phase $\exp -i\Delta E\phi(t, x)$ due to perturbation (1); in contrast, a delocalized excitation becomes a superposition of the excitations. Thus, the absence of the effect of this perturbation implies that excitations are localized and do not decay. We conclude that the absence of level repulsion implies the localization

Level statistics: Poisson v/s WD

- Discrete many-body spectrum with zero level width: Poisson statistics
- Continuous spectrum (extended states) : Wigner-Dyson ensemble with level repulsion

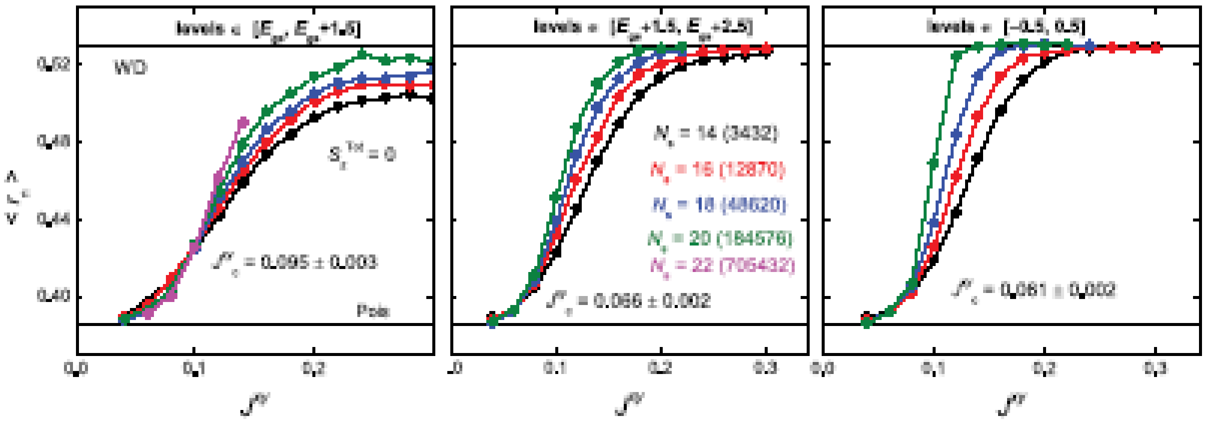
V.Oganesyan & D.Huse

Phys. Rev. B **75**, 155111 (2007)

Model of interacting fermions (no-conclusive concerning sharp phase transition)

Numerical results for random Z=3 graph, E.Cuevas

$$0 < r_n = \min(\delta_n, \delta_{n-1}) / \max(\delta_n, \delta_{n-1}) < 1$$



E

J_c = 0.061

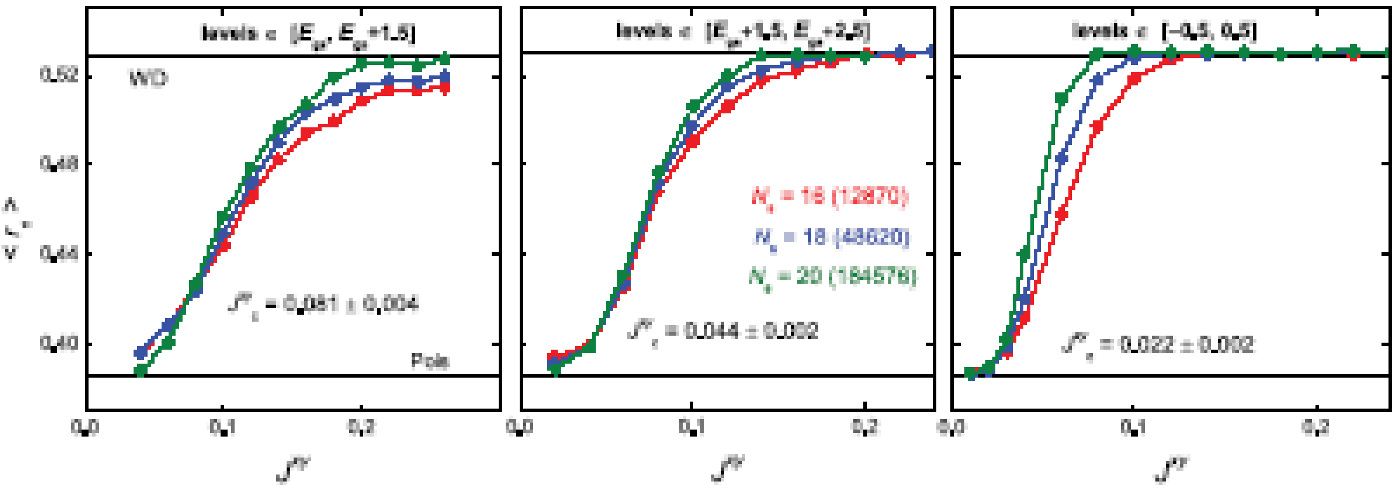
J_c = 0.066

J_c = 0.095

Fig. 2: The average $\langle r_n \rangle$ that distinguishes Wigner-Dyson and Poisson distributions (values of $\langle r_n \rangle$ corresponding to these distributions are shown by dashed lines). The left panel shows the statistics of the low-energy excitations in the energy interval $(E_{gs}, E_{gs} + 1.5)$ as a function of the transverse interaction constant, J^{xy}, for the $Z = 3$ random graph with bandwidth $W = 1$. The middle panel shows similar results for intermediate energies, and the right panel corresponds to high energies, close to the center of the many-body spectrum. Each data point represents the average over $N_r = 2000$, 200, 100 and 60 disorder realizations for $N_s = 14$, 16, 18 and 20 respectively. A large (exponential) increase in the number of states implies that larger samples require less averaging over realization to achieve the same accuracy.

Role of $J_z S_i^z S_j^z$ interaction

$$\tilde{H}_{XY} = -2\sum_i \xi_i s_i^z - \sum_{(ij)} J s_i^z s_j^z - \sum_{(ij)} J_{ij}^{XY}\left(s_i^+ s_j^- + s_i^- s_j^+\right)$$



The average $\langle r_n \rangle$ in the presence of weak longitudinal spin coupling J^{zz} for low energy levels (left panel), intermediate energies (middle panel) and center of the many body band (right panel). Even a small coupling $J^{zz} = 0.1$ has a large effect, it shifts the transition to much smaller values of the transverse coupling g. These results were obtained by averaging over the same number of realizations as in Fig. 2

0.061 → 0.022 (midband)

0.066 → 0.044 (intermeadiate)

0.095 → 0.081 (low E)

Temperature-controlled transition to the state with zero level widths and zero conductivity (Basko, Aleiner & Altshuler 2006)

Original model: XY exchange + transverse field

$$\Gamma_i = (2g/K)^2 \sum_{k(i)} \frac{\Gamma_k}{(\omega - 2\xi_k)^2 + \Gamma_k^2}$$

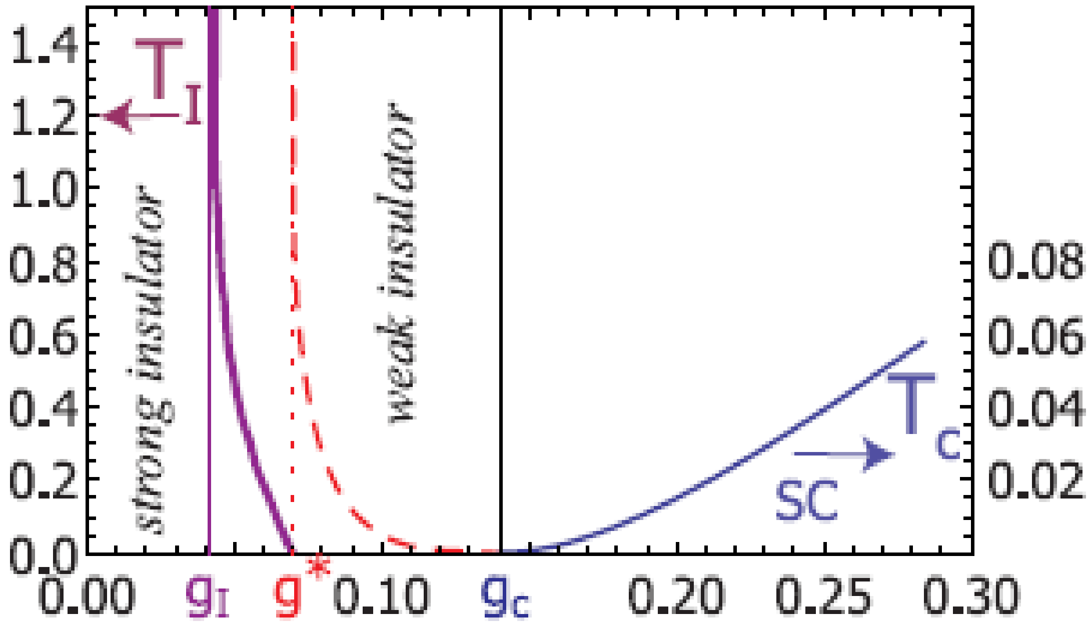
Full model with S_z-S_z coupling

$$\Gamma_i = (2g/K)^2 \sum_{k(i)} \sum_{s^z_{n(k)}} \frac{e^{\beta \xi_n s^z_{n(k)}}}{Z_k} \frac{\Gamma_k}{\left(\omega - 2\tilde{\xi}_k\right)^2 + \Gamma_k^2}$$

$$\xi_j \rightarrow \tilde{\xi}_j = \xi_j + \frac{1}{2} \sum_{k_j = 1..Z-1} J s_k$$

Summation over large number of configurations with different $\tilde{\xi}_j$ makes it easier to meet resonant conditions

Phase diagram



Phase diagram in the temperature - coupling constant plane for the model (2) with $Z = 3$ ($K = 2$, obtained from the solution of cavity equations and confirmed by numerical simulations. The strength of the $s^z s^z$ interaction is $J^{zz} = 0.1$, in the absence of this interaction the line separating weak and strong insulators becomes vertical. In the weak insulator, excitations at sufficiently high energies can decay even at zero temperature. A non-zero temperature results in non-zero relaxation of all excitations, even the ones of lowest energy. In contrast, in the strong insulator, no excitation with intensive energy can decay. As the interaction constant is decreased, the temperature separating these phases goes to infinity at $g = g_I$. At smaller coupling $g < g_I$, all excitations, even those with *extensive* energy remain localized. The value of $g_I \approx 0.042$ is approximately equal to $0.30 g_c$. The ratio $g_I/g_c = 0.3$ is in good agreement with the results of the direct diagonalization on small graphs, as

Conclusions

New type of S-I phase transition is described

On insulating side activation of pair transport is due to ManyBodyLocalization threshold

Results from level statistics studies support general shape of the phase diagram, but the possibility of intermediate phase cannot be excluded in this way

Interaction in the “density channel” is crucially important for the shape of the phase diagram

Open problems

- Analitical study of energy localization in Euclidean space or RGM: order parameter ?

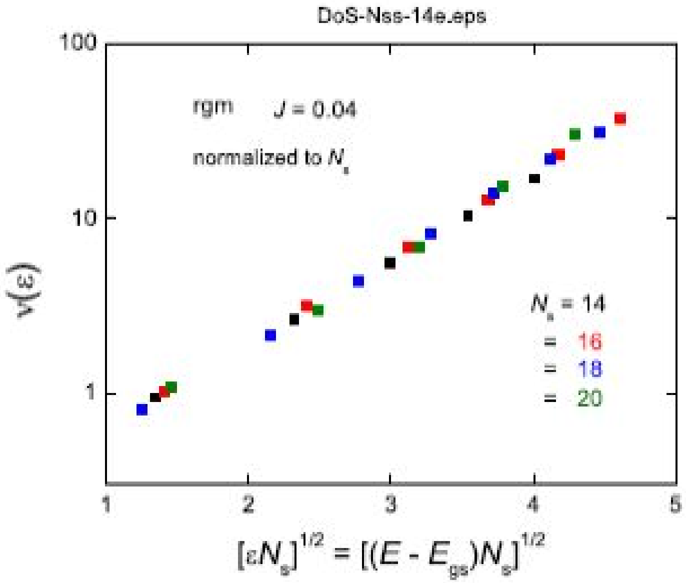
  anything to do with compactification of space and black holes ?

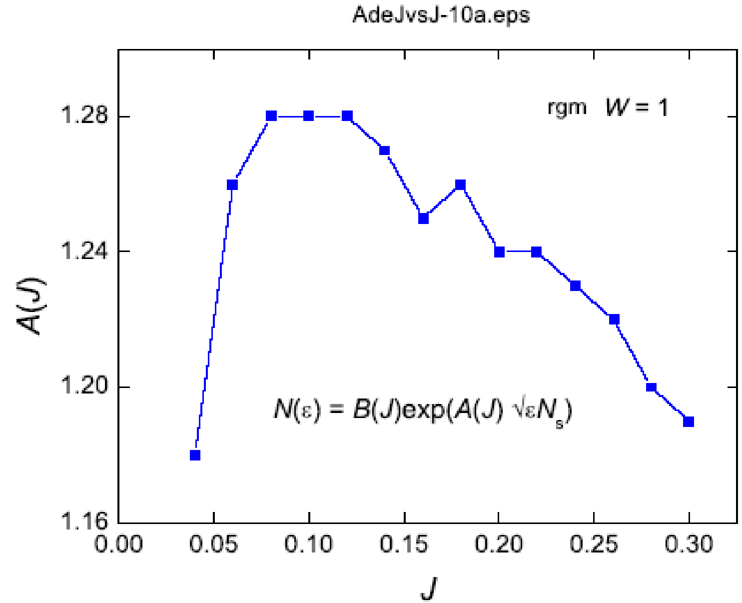
- Is it possible to modify the model in a way to find an intermediate phase or 1st order?
- How to calculate electric and thermal conductivities directly within recursion relations approach?
- rf- stimulated conductivity: search for threshold effect

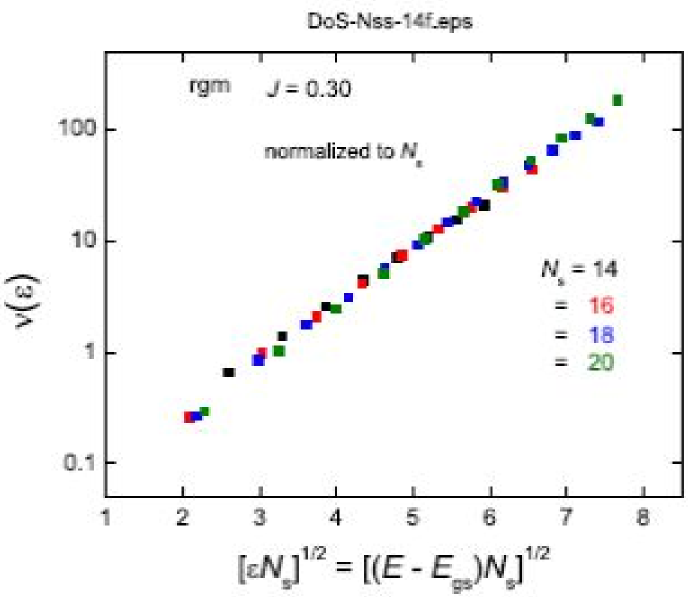
The End

Density of States

$$\nu(\epsilon) = C N_s \exp(\alpha\sqrt{\epsilon N_s})$$







$T = (dS / dE)^{-1} = \# \, (E / N_s)^{1/2}$

$E \sim N_s T^2$ (at $T \ll 1$)

Fractal superconductivity near localization threshold

Annals of Physics 325 (2010) 1390–1478

M.V. Feigel'man [a,b], L.B. Ioffe [a,c,d,*], V.E. Kravtsov [a,e], E. Cuevas [f]

We develop a semi-quantitative theory of electron pairing and resulting superconductivity in bulk "poor conductors" in which Fermi energy E_F is located in the region of localized states not so far from the Anderson mobility edge E_c. We assume attractive interaction between electrons near the Fermi surface. We review the existing theories and experimental data and argue that a large class of disordered films is described by this model.

Our theoretical analysis is based on analytical treatment of pairing correlations, described in the basis of the exact single-particle eigenstates of the 3D Anderson model, which we combine with numerical data on eigenfunction correlations. Fractal nature of critical wavefunction's correlations is shown to be crucial for the physics of these systems.

We identify three distinct phases: 'critical' superconductive state formed at $E_F = E_c$, superconducting state with a strong pseudo-gap, realized due to pairing of weakly localized electrons and insulating state realized at E_F still deeper inside a localized band. The 'critical' superconducting phase is characterized by the enhancement of the transition temperature with respect to BCS result, by the inhomogeneous spatial distribution of superconductive order parameter and local density of states. The major new feature of the pseudo-gapped state is the presence of two independent energy scales: superconducting gap Δ, that is due to many-body correlations and a new "pseudo-gap" energy scale Δ_P which characterizes typical binding energy of localized electron pairs and leads to the insulating behavior of the resistivity as a function of temperature above superconductive T_c. Two gap nature of the pseudo-gapped superconductor is shown to lead to specific features seen in scanning tunneling spectroscopy and point-contact Andreev spectroscopy. We predict that pseudo-gapped superconducting state demonstrates anomalous behavior of the optical spectral weight. The insulating state is realized due to the presence of local pairing gap but without superconducting correlations; it is characterized by a hard insulating gap in the density of single electrons and by purely activated low-temperature resistivity $\ln R(T) \sim 1/T$.

Based on these results we propose a new "pseudo-spin" scenario of superconductor–insulator transition and argue that it is realized in a particular class of disordered superconducting films. We conclude by the discussion of the experimental predictions of the theory and the theoretical issues that remain unsolved.

[2] M. Ma, P.A. Lee, Phys. Rev. B 32 (1985) 5658.
[3] A. Kapitulnik, G. Kotliar, Phys. Rev. Lett. 54 (1985) 473; G. Kotliar, A. Kapitulnik, Phys. Rev. B 33 (1986) 3146.
[4] L.N. Bulaevskii, M.V. Sadovskii, Pisma ZhETF 39 (1984) 524; L.N. Bulaevskii, M.V. Sadovskii, J. Low Temp. Phys. 59 (1985) 89; M.V. Sadovskii, Phys. Rep. 282 (1997) 225.

Competition between superconductivity and Anderson localization was studied originally in mid-80s [2–4]. Their major conclusion was that Anderson theorem is valid and superconductivity survives provided that the condition $T_c \gg \delta_L$ is satisfied.

We will show below that the analysis presented in [2–4] is not complete in two important respects.

Disorder always leads to spatial fluctuations of parameters which enter the Ginzburg–Landau functional; the major effect is due to fluctuations of $\alpha(T,\mathbf{r})$. Universal mesoscopic fluctuations (which provide a lower bound for the strength of this effect) were studied in Ref. [4] for usual disordered superconductors and more recently in [31] for 2D films with the strong Finkelstein effect. Here we follow

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Superconductor-insulator transition and energy localization

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We develop an analytical theory for generic disorder-driven quantum phase transitions. We apply this formalism to the superconductor-insulator transition and we briefly discuss the applications to the order-disorder transition in quantum magnets. The effective spin-$\frac{1}{2}$ models for these transitions are solved in the cavity approximation which becomes exact on a Bethe lattice with large branching number $K \gg 1$ and weak dimensionless coupling $g \ll 1$. The characteristic feature of the low-temperature phase is a large self-formed inhomogeneity of the order-parameter distribution near the critical point $K \geq K_c(g)$, where the critical temperature T_c of the ordering transition vanishes. We find that the local probability distribution $P(B)$ of the order parameter B has a long power-law tail in the region where B is much larger than its typical value B_0. Near the quantum-critical point, at $K \to K_c(g)$, the typical value of the order parameter vanishes exponentially, $B_0 \propto e^{-C/[K-K_c(g)]}$ while the spatial scale N_{inh} of the order parameter inhomogeneities diverges as $[K-K_c(g)]^{-2}$. In the disordered regime, realized at $K < K_c(g)$ we find actually two distinct phases characterized by different behavior of relaxation rates. The first phase exists in an intermediate range of $K^*(g) < K < K_c(g)$. It has two regimes of energies: at low excitation energies, $\omega < \omega_d(K,g)$, the many-body spectrum of the model is *discrete*, with zero-level widths, while at $\omega > \omega_d$ the level acquire a nonzero width which is self-generated by the many-body interactions. In this phase the spin model provides by itself an intrinsic thermal bath. Another phase is obtained at smaller $K < K^*(g)$, where all the eigenstates are discrete, corresponding to full many-body localization. These results provide an explanation for the activated behavior of the resistivity in amorphous materials on the insulating side near the superconductor-insulator transition and a semiquantitative description of the scanning tunneling data on its superconductive side.

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Localization of preformed Cooper pairs in disordered superconductors

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The most profound effect of disorder on electronic systems is the localization of the electrons transforming an otherwise metallic system into an insulator. If the metal is also a superconductor then, at low temperatures, disorder can induce a pronounced transition from a superconducting into an insulating state. An outstanding question is whether the route to insulating behaviour proceeds through the direct localization of Cooper pairs or, alternatively, by a two-step process in which the Cooper pairing is first destroyed followed by the standard localization of single electrons. Here we address this question by studying the local superconducting gap of a highly disordered amorphous superconductor by means of scanning tunnelling spectroscopy. Our measurements reveal that, in the vicinity of the superconductor–insulator transition, the coherence peaks in the one-particle density of states disappear whereas the superconducting gap remains intact, indicating the presence of localized Cooper pairs. Our results provide the first direct evidence that the superconductor–insulator transition in some homogeneously disordered materials is driven by Cooper-pair localization.

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ЛОКАЛИЗАЦИЯ И СВЕРХПРОВОДИМОСТЬ

Л.Н.Булаевский, М.В.Садовский

Показано, что система, находящаяся в состоянии андерсоновской локализации в нормальном состоянии, может стать сверхпроводящей ниже критической температуры T_c. Получены коэффициенты уравнения Гинзбурга – Ландау для сверхпроводящего перехода в области андерсоновской локализации и исследовано поведение верхнего критического магнитного поля H_{c2} в металлической и диэлектрической области в зависимости от степени беспорядка.

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РОСТ ПРОСТРАНСТВЕННЫХ ФЛУКТУАЦИЙ В СВЕРХПРОВОДНИКАХ ВБЛИЗИ ПЕРЕХОДА АНДЕРСОНА

Л.Н.Булаевский, М.В.Садовский

Найден интервал температур около T_c, где сильны пространственные флуктуации сверхпроводящего параметра порядка, вызванные примесями. Далеко от порога андерсоновской локализации этот интервал очень узок по сравнению с интервалом сильных термодинамических флуктуаций, и сверхпроводящий параметр порядка есть самоусредняющаяся величина. Вблизи порога локализации флуктуации из-за беспорядка велики во всей области проявления сверхпроводимости.

SUPERCONDUCTIVITY AND LOCALIZATION

Michael V. Sadovskii, ©*World Scientific, Singapore, 2000*

These estimates are in complete accordance with the results of our discussion of Ginzburg—Landau approximation [Bulaevskii L.N., Sadovskii M.V. (1984); Bulaevskii L.N., Sadovskii M.V. (1985)] and in fact we now have the complete qualitative picture of superconductivity in Anderson insulator both for $T \sim T_c$ and $T \to 0$, i.e. in the ground state.