Superconductivity in low-density electron systems.

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in collaboration with

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I. Introduction

According to Prof. P.W. Anderson there are two most important questions in modern theory of non-phonon SC:

1) To convert the sign of the Coulomb interaction

2) To understand the properties of the normal state in high-*Tc* materials and other unconventional SC-systems

We agree with these statements. We will address them in Galitskii-Bloom Fermi-gas approach for low density electron systems.

We will prove an existence of SC at low-density limit, in purely repulsive Fermi-systems where we are far from AFM and structural instabilities. Moreover in this limit we can develop regular perturbation theory.

The small parameter of the problem is gas parameter: $|a| p_F << 1; \hbar = 1$

a – is the s-wave scattering length; p_F – is Fermi-momentum.

T_C-s which we obtain are not very low. Our theory often works even for rather high densities because of intrinsic nature of SC-instability.

<u>Our philosophy</u>: exactly solve low-density limit and then go to higher densities.

<u>The basic mechanism</u> which we address in the talk is enhanced Kohn-Luttinger mechanism of SC (PRL 1965). We will check the normal state of low-density electron systems with respect to marginality.

Unconventional superconductive systems.

Examples of p-wave superconductors:

1) 3He-A and 3He-B, $S_{tot} = l = 1$; $T_C \sim 1$ mK
2) ^{6}Li and ^{40}K in magnetic traps (p-wave Feshbach resonance),
   $T_C \sim (10^{-6} \div 10^{-7})$ K
3) $U_{1-x}Th_xBe_{13}$, UNi_2Al_3, $T_C \sim (0.5 \div 1)$ K, heavy mass $m^* \sim (100 \div 200)m_e$
4) Organic superconductor α-$(BEDT\text{-}TTF)_2I_3$, $T_C \sim 5$ K
5) Sr_2RuO_4, $T_C \sim 1$ K (part of the community assumes d-wave pairing)

Examples of d-wave superconductors:

1) UPt_3, $T_C \sim 0.5$ K, heavy mass $m^* \sim 200\ m_e$
2) high-T_C superconductors, $T_C \sim (40 \div 150)$ K, $La_{2-x}Sr_xCuO_4$, $YBa_2Cu_3O_{7-\delta}$ (part of the community assumes s-wave pairing)

Neutron stars (98% of bineutrons, 2% of biprotons):

For bineutrons $S_{tot} = l = 1$, but there is a strong spin-orbital coupling

$$J = \left|\vec{S}_{tot} + \vec{l}\right| = 2, \quad T_C \sim (10^8 \div 10^{10})\ \mathrm{K}$$

Unconventional **multiband s-wave** SC: MgB_2, $BaFe_2(As_{1-x}P_x)_2$

The basic model is Fermi-gas model

In FG with attraction:

$a < 0, S = l = 0$ - s-wave pairing. $T_{CO} \approx 0.28\varepsilon_F e^{-\frac{\pi}{2|a| p_F}}$

Gor'kov, Melik-Barhudarov, 1961

In FG with repulsion:

$a > 0$ – repulsion between two particles in vacuum. I will show that due to effective interaction of two particles in substance (via polarization of fermionic background) we can convert the sigh of interaction in p-wave channel and obtain

$S = l = 1$ - p-wave $T_{C1} \sim \varepsilon_F e^{-\frac{1}{(ap_F)^2}}$

Fay, Lazer ,1969
Kagan, Chubukov, 1988

The most important is to understand what is effective interaction U_{eff}?

$$U_{eff}(\vec{p},\vec{k}) = \frac{4\pi a}{m} + \left(\frac{4\pi a}{m}\right)^2 \Pi(\vec{p}+\vec{k})$$

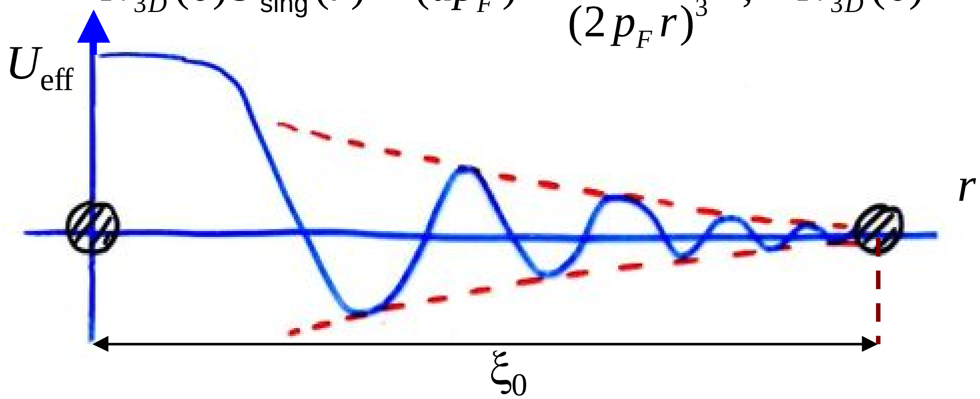
$$\Pi(q) = \int \frac{d^3\vec{p}}{(\varepsilon_{p+q} - \varepsilon_p)} \begin{bmatrix} \varepsilon_{p+q} > 0 \\ \varepsilon_p < 0 \end{bmatrix}$$ - is a polarization operator

Besides a regular part it contains a Kohn's anomaly of the form:

$$\Pi_{\text{sing}}(q) \sim (q - 2p_F)\ln(q - 2p_F)$$ in 3D

Kohn's anomaly and Friedel oscillations

In real space Kohn's anomaly leads to Friedel oscillations (RKKY interaction) $N_{3D}(0)U^{eff}_{\text{sing}}(r) \sim (ap_F)^2 \frac{\cos(2p_F r)}{(2p_F r)^3}; \quad N_{3D}(0) = \frac{mp_F}{2\pi^2}$ is 3D density of states



As a result we start from pure hard-core repulsion in vacuum and get competition between repulsion and attraction in substance.

Singular part of U_{eff} "plays" in favor of attraction for orbital momenta $l>>1$

Regular part of U_{eff} "- in favor of repulsion (Kohn, Luttinger 1965)

S-wave SC is suppressed by hard-core repulsion. However for $l \neq 0$ the attractive contribution is dominant.

<u>The exact solution yields:</u>

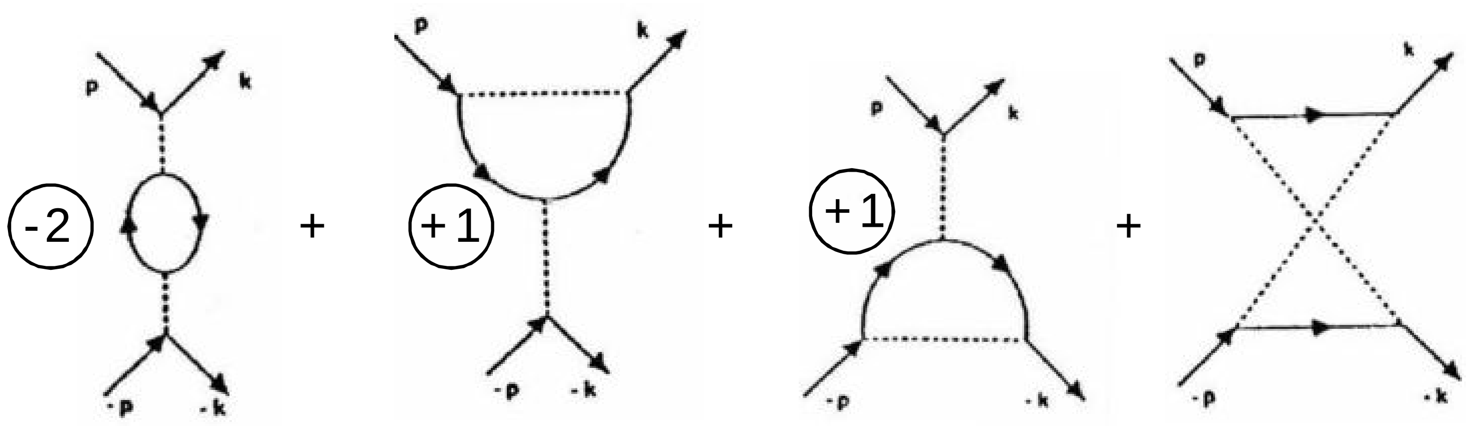
$$T_{C1} \sim \varepsilon_F e^{-\frac{5\pi^2}{4(2\ln 2-1)(ap_F)^2}}$$

What is U_{eff}?

U_{eff} – is an irreducible bare vertex – the diagrams which contribute to U_{eff} cannot be separated by two lines running in the same direction (Thus U_{eff} does not contain the Cooper loop).

$$U_{eff}(\vec{p},\vec{k}) = \frac{4\pi a}{m} + \left(\frac{4\pi a}{m}\right)^2 \Phi(\vec{p},\vec{k})$$

$\Phi(\vec{p},\vec{k}) =$



First three diagrams cancel each other exactly for contact interaction and as a result:

$\Phi(\vec{p},\vec{k}) = \Pi(\vec{p}+\vec{k})$ is an exchange diagram

p-wave pairing

$\Pi(\vec{p}+\vec{k})$ is polarization operator in "crossed channel" (for $\tilde{q} = |\vec{p} \oplus \vec{k}|$ instead $q = |\vec{p} - \vec{k}|$).

For $|\vec{p}| = |\vec{k}| = p_F$ the "crossed" momentum $\tilde{q}^2 = 2p_F^2(1 \oplus \cos\theta)$, where $\theta = \widehat{\vec{p}\vec{k}}$.

Algebraically $\Pi(\tilde{q})$ reads:

$$\Pi(\tilde{q}) = \frac{N(0)}{2}\left[1 + \frac{4p_F^2 - \tilde{q}^2}{4p_F\tilde{q}} \ln\frac{2p_F + \tilde{q}}{|2p_F - \tilde{q}|}\right]$$ - static Lindhard function (1954)

Integration of $\Pi(\tilde{q})$ with first Legandre polynomial $P_1(\cos\theta) = \cos\theta$ yields the desired result

for p-wave harmonic $\Pi_1 = \int_{-1}^{1} P_1(\cos\theta)\frac{d\cos\theta}{2}\Pi(\tilde{q}) \approx \frac{N(0)}{5}(1 - 2\ln 2) < 0$

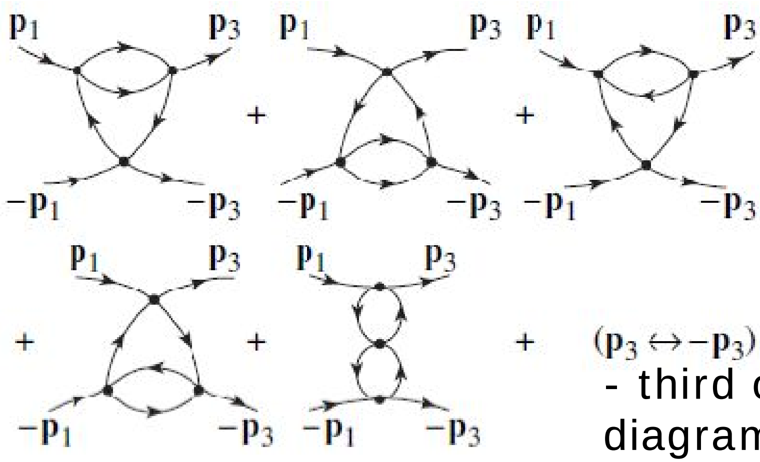
The critical temperature:

$T_{C1} \sim \frac{2e^C}{\pi}\varepsilon_F \exp\left\{-\frac{13}{\lambda^2}\right\}$, where $\lambda = \frac{2ap_F}{\pi}$ in 3D is a gas-parameter of Galitskii

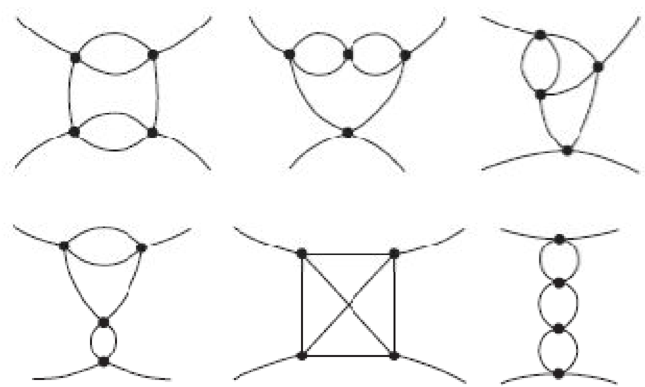
Evaluation of the preexponential factor

To evaluate the preexponential factor we need to calculate the diagrams of third and fourth order in ap_F:

Efremov, Mar'enko, Baranov, Kagan, 2001



- third order diagrams



- fourth order diagrams

as well as to take into account the retardation effects (*Alexandrov, Golubov, 1992*). The accurate calculations yields:

$$N(0)U_{eff}^{l=1} = -\left(\frac{\lambda^2}{13} + \frac{\lambda^3}{3} + \frac{\lambda^4}{4}\right)$$

and thus

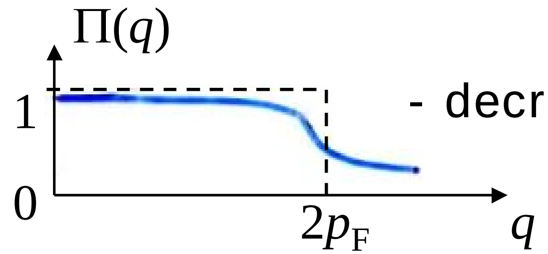
$$T_{C1} = \frac{2}{\pi} e^{C} \varepsilon_F \exp\left\{-\frac{13.0}{\lambda^2} + \frac{42.0}{\lambda} + 190\right\} \text{ for } \lambda \le 1/4$$

where we neglect the contribution of vacuum p-wave scattering amplitude. Hence third and fourth order diagrams correspond to attraction but it is difficult to evaluate T_c exactly for $\lambda > 1/4$ *(remark of Dirk Rainer)*

Model-independent considerations of Prof. P. Nozieres

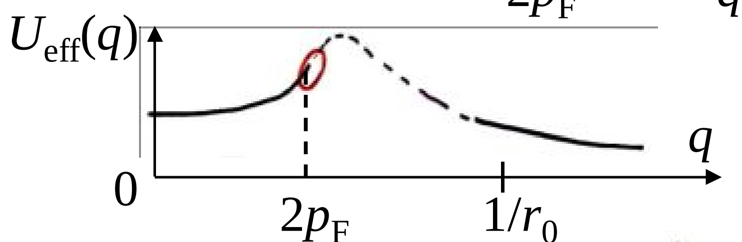
$$U_{eff}(q) = \frac{U_0(q)}{\varepsilon(q)}; \quad \varepsilon(q) = 1 + U_0(q)\Pi(q)$$ - dielectric function

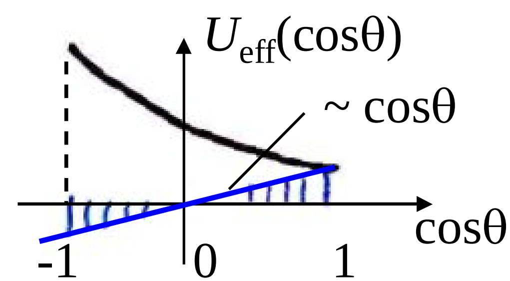
Usually:



- decreases for $q > 2p_F$

That is why $U_{eff}(q)$ increases for $0 \le q \le 2p_F$





$$P_1(\cos\theta) = \cos\theta = 1 - \frac{q^2}{2p_F^2} \qquad \cos\theta = \begin{cases} 1; & q = 0 \\ -1; & q = 2p_F \end{cases}$$

$$U_{eff}^{\,l=1} = \int_{-1}^{1} U(x)x\,dx < 0$$ - if $U_{eff}(q)$ for $x \in [-1, 0]$ is lager then $U_{eff}(q)$ for $x \in [0, 1]$

Hence for all effective potentials increasing for q from 0 to $2p_F$ we have p-wave SC in isotropic case and the Kohn's anomaly does not play a decisive role here (in contrast with the case *l>>1*) Note that in 3D FG-model $U_{eff}(\cos\theta)$ decreases on the interval [-1,1] due to crossing $\vec{q} \to \tilde{\vec{q}}$

Two dimensional case

$N_{2D}(0)U_{eff}(\tilde{q}) = f_0 + f_0^2\Pi(\tilde{q})\frac{4\pi}{m}; \quad f_0 = \frac{1}{2\ln\left(1/p_F r_0\right)}$ - a gas parameter in 2D,

$N_{2D}(0) = \frac{m}{2\pi}$ is density of states in 2D

$N_{2D}(0)U_{eff}(r) \sim f_0^2 \frac{\cos(2p_F r)}{(2p_F r)^2}$ - strong 2D Friedel oscillations. However

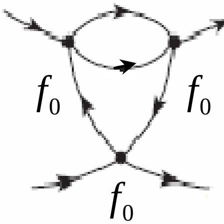
$N_{2D}(0)U_{eff}(\tilde{q}) \sim f_0^2 \,\mathrm{Re}\sqrt{\tilde{q} - 2p_F} = 0 \;\; for \; \tilde{q} \le 2p_F$

Strong 2D Kohn's anomaly has a one-sided character and is ineffective for SC problems where $\tilde{q} \le 2p_F$.

SC appears only in THIRD ORDER of perturbation theory

$$T_{C1} \sim \varepsilon_F \exp\left\{-\frac{1}{4.1 f_0^3}\right\}$$

Chubukov, 1993



$N_{2D}(0)U_{eff}(\tilde{q}) \sim f_0^3 \,\mathrm{Re}\sqrt{2p_F - \tilde{q}}$ - in the 3rd order in f_0 - the Kohn's anomaly becomes effective for SC

Later on *Kagan, Mar'enko et al*. (*Physics B 2000*), made more rigorous calculations and obtained $N_{2D}(0)U_{eff}^{l=1} = -6.1 f_0^3$ (instead of - $4.1 f_0^3$) evaluating all skeleton diagrams of the third order in f_0 in 2D.

Repulsive-U Hubbard model

$$\hat{H} = -t\sum_{<i\,j>\sigma} c^{+}_{i\sigma} c_{j\sigma} + U\sum_{i} n_{i\uparrow} n_{i\downarrow};$$

$n_{i\sigma} = c^{+}_{i\sigma} c_{i\sigma}$ - is the density of electrons on site *i* with spin-projection σ

3D Hubbard model:

For $nd^{3} < 1$: triplet p-wave pairing due to Kohn-Luttinger effect

$$T_{C1} \sim \varepsilon_F e^{-\frac{13}{\lambda^2}}$$ (Baranov, Kagan, JETP 1991)

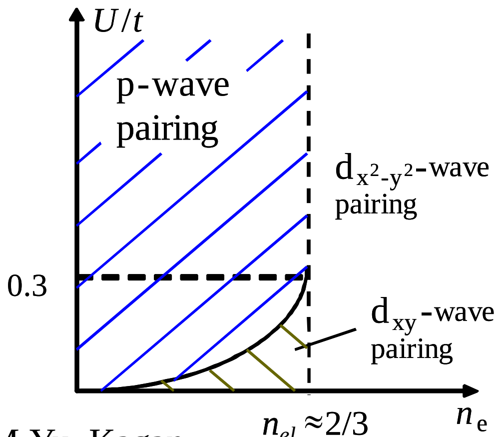
$\lambda = 2ap_F/\pi$; $a \approx d$ – intersite distance for the lattice at $U >> W$ - strong Hubbard repulsion (Kanamori T-matrix approximation 1963). Note that in the absence of a lattice $a \sim r_0$ (hard-core radius of the potential).

2D Hubbard model

For $nd^2 < 1$ $T_{C1} = \varepsilon_F e^{-\frac{1}{6.1 f_0^3}}$, where $f_0 = \frac{1}{\ln\frac{4W}{\varepsilon_F}}$; $n_e = \frac{2\varepsilon_F}{W}$ – dimensionless density of electrons

(Fukujama et al., Jour. of Phys. Soc. Jap. 1991)

Qualitative phase diagram at low density in 2D



M.Yu. Kagan,
Habilitation Thesis, 1994

For $U >> t$ we have p-wave pairing for electron desities $n_e = \frac{2\varepsilon_F}{W} \leq 2/3$ (Chubukov) and $d_{x^2-y^2}$-wave pairing for $n_e \geq 2/3$

For $U \leq 0,3t$ and $n_e \leq 2/3$ we have also a region of d_{xy}-pairing (Baranov, Kagan, Zeit. fur Physik B, 1992)

For d_{xy}-pairing the quartic corrections to parabolic spectrum are important:

$$\varepsilon(p) = -2t(\cos p_x d + \cos p_y d) - \mu \approx \frac{p^2 - p_F^2}{2m} - \frac{p_x^4 d^2 + p_y^4 d^2}{24m}; \; m = \frac{1}{2td^2}$$

$\Delta_{d_{xy}} = \Delta_0^d \sin p_x d \sin p_y d \sim \Delta_0 \sin 2\varphi$ (in contrast to $\Delta_{d_{x^2-y^2}} \sim \Delta_0 \cos 2\varphi$), φ is the angle between momentum $\vec{p}$ and x - axis on the square lattice. The critical temperature for d_{xy} - pairing reads:

$$T_C^{d_{xy}} \sim \varepsilon_F \exp\left\{-\frac{20}{f_0^2 n_e^2}\right\} \quad \text{(Baranov,Kagan 1992)}$$

Our result for d_{xy} pairing was confirmed lateron in RG-approach by Schulz et al PRB 1996.

d-wave pairing in the t-t` Hubbard model.

In the t-t' Hubbard model with the quasiparticle spectrum

$\varepsilon(p) = -2t(\cos p_x d + \cos p_y d) + 4t'\cos p_x d \cos p_y d - \mu$

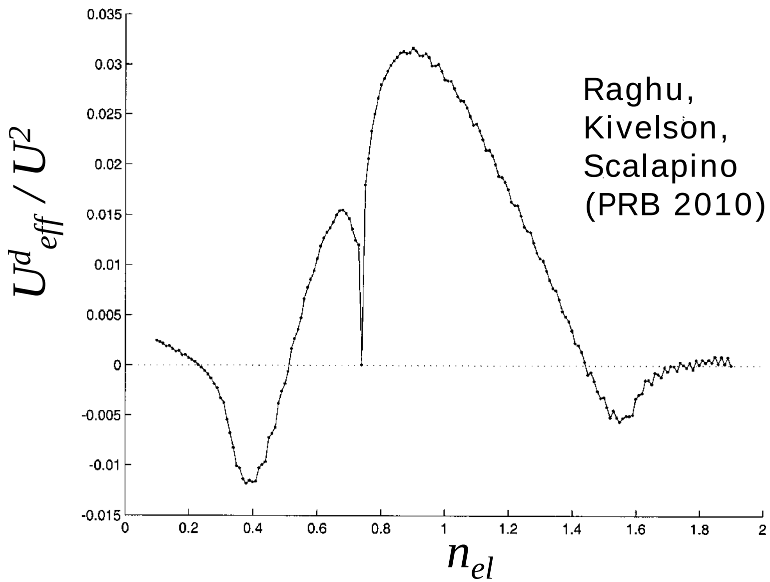
in the weak-coupling Born case $U < W = 8t$ $(|t'/t| \sim 0,3)$ according to Raghu, Kivelson and Scalapino effective interaction in the d-wave channel U_{eff}^{d} at intermediate and large densities $n_{el} \geq 0,6$ behaves in a very non-trivial way in a second order of perturbation theory for $U_{eff} = U + U^2\Pi(\vec{p} + \vec{k})$. Namely it has a large central maximum at $n_{el} \sim 0,9 \div 1,$ smaller maximum at $n_{el} \sim 0,6$ and rather deep minimum inbetween for $n_{el}^{V.H.} \sim 0,7$ (where Van Hove singularity is positioned for $|t'/t| \sim 0,3$).

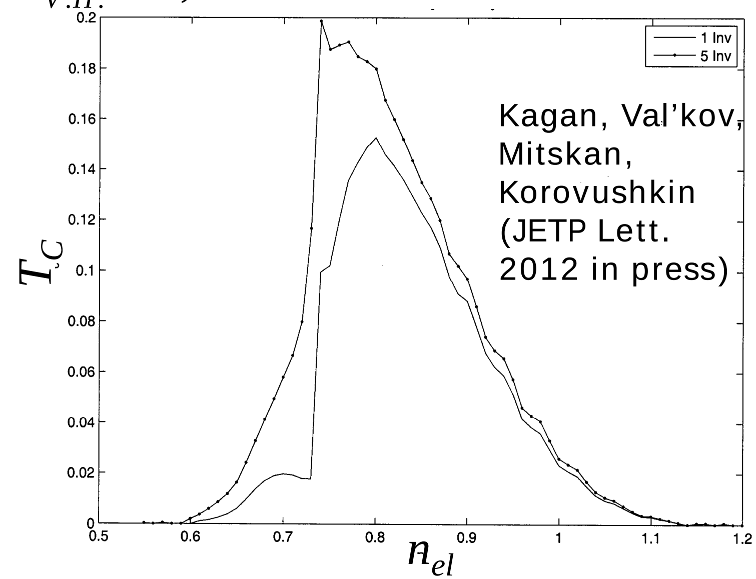
Density dependence of the critical temperature

In the same time the main exponent in the expression of the d-wave critical temperature

$|\lambda_d| = N_{2D}(0)\,|U_{eff}^d|$ $\left(T_C^d \sim \varepsilon_F \exp\left\{-\frac{1}{|\lambda_d|}\right\};\ \varepsilon_F \sim \frac{Wn_{el}}{2}\right)$ has a monotonous behavior

with a maximum at $n_{el} \sim 0,82$. This result is due to presence of an additional Van Hove anomaly in the density of states $N_{2D}(0)$ for $n_{V.H.} \sim 0,7$.

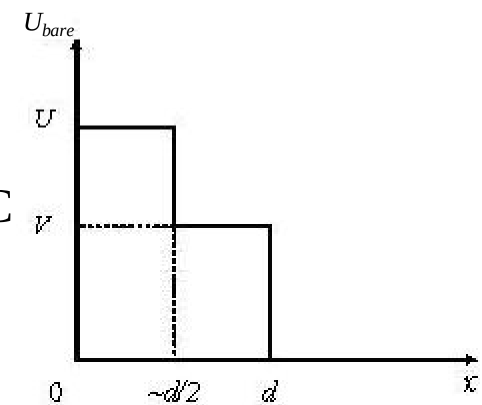




Evaluation of the main exponent for T_C^d in maximum at $n_{el} = 0,82$ with $t \sim 0,3eV$ and $|t'/t| \sim 0,3$ for $U \sim 6t$ yields $T_C^d \sim (80 \div 100)K$. The preexponential factor in T_C requires the evaluation of third and fourth order in U diagrams for U_{eff} as well as the retardation effects of the Eliashberg type in the kernel of the integral Bethe-Salpeter equation (Baranov, Efremov, Mar'enko, Kagan, JETP 2000)

SC in Shubin-Vonsovskii model at low density

It is a useful toy-model to study the effects of intersite Coulomb repulsion on SC and Mott-Hubbard transition as well as the physics of nanoscale phase separation (Kagan et al. JETP Lett. 2011)



$\hat{H} = -t\sum_{<ij>\sigma} c^{+}_{i\sigma} c_{j\sigma} + U\sum_{i} n_{i\uparrow} n_{i\downarrow} + \frac{V}{2}\sum_{<ij>} n_i n_j$ - Shubin-Vonsovskii model is the most repulsive and the most unbeneficial model for SC. However at low electron densities it is again unstable towards SC p-wave pairing even in strong-coupling case $U>V>W$ and low electron density $nd^D<<1$.

Moreover in the main order in the gas-parameter $\lambda = \frac{2dp_F}{\pi}$ in 3D and $f_0 = \frac{1}{\ln(4W/\varepsilon_F)}$ in 2D the critical temperature will be given by the same expressions:

$T_{C1} \sim \varepsilon_F \exp\left\{-\frac{13}{\lambda^2}\right\}$ in 3D and $T_{C1} \sim \varepsilon_F \exp\left\{-\frac{1}{6{,}1 f_0^3}\right\}$ as in the absence of Coulomb repulsion (for $V=0$)

The presence of Coulomb repulsion V changes only the preexponential factor.

At larger densities $n_e \geq 0{,}5$ there are, however, extended regions of phase-separation in Shubnikov-Vonsovskii-Verwey model in the strong-coupling limit $U>>V>>W$

Born approximation

In Born approximation $W > U > V$ both types of Mott-Hubbard and Verwey phase-separation are absent close to $n_{el} \sim 1/2$ and $n_{el} \sim 1$ and we can again construct SC phase-diagram in the second order of perturbation theory for U_{eff}. We can work at arbitrary densities $0 < n_{el} < 1$ with an account of the distant hoppings (t' and t'') in the quasiparticle spectrum

$\varepsilon(p) = -2t(\cos p_x d + \cos p_y d) + 4t' \cos(p_x d)\cos(p_y d) + 2t''(\cos 2p_x d + \cos 2p_y d) - \mu,$

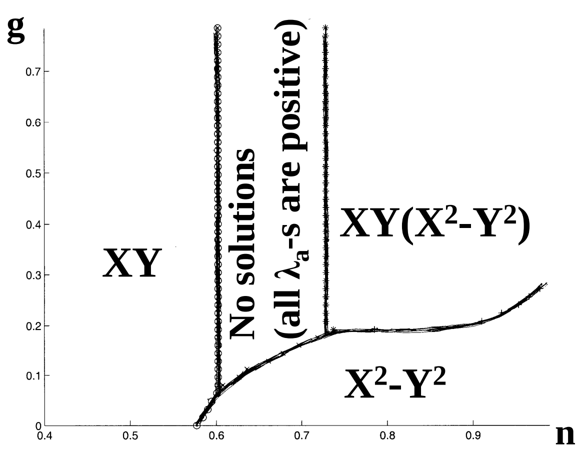
$|t| > |t'| > |t''|$, $W = 8t$ and the additional Coulomb repulsion V on the neighboring sites in the bare interaction $U_{bare}(q) = U + 2V(\cos q_x d + \cos q_y d)$; $\vec{q} = \vec{p} - \vec{p}\,'$

The work along these lines is in progress now (Kagan, Val'kov, Korovushkin, Mitskan).

Preliminary considerations show that while $V(q)$ will suppress $d_{x^2-y^2}$ – pairing close to half-filling at least in the main (first order) of perturbation theory, the distant hopping t'' which shifts Van Hove singularity to smaller densities can produce an opposite effect of the T_C - increase.

First order results of Kivelson et al.

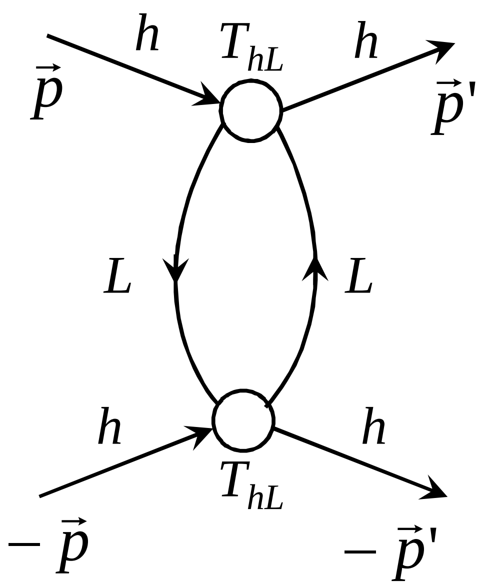
For $V \neq 0$ it is convenient besides the dimensionless coupling constants $\lambda_a = U_{eff}^a N_{2D}(0)$ which define critical temperatures T_{Ca} in different channels (xy, $x^2 - y^2$ and so on) to introduce according to Kivelson et al one more important dimensionless constant $g = \frac{VW}{U^2}$ where the bandwidth $W = 8t$ in 2D. Then with an account of Coulomb interaction V only in the first order corrections (which are essentially repulsive) and for $t' = t'' = 0$ the qualitative phase-diagram looks like as follows:



Raghu, Berg, Chubukov, Kivelson PRB 2011

Second order corrections in V according to our preliminary estimates a little bit modify the region of no solutions (where all the coupling constants λ_a-s are positive) in qualitative agreement with the results of Kivelson et al.

Superconductivity in the two-band model with one narrow band



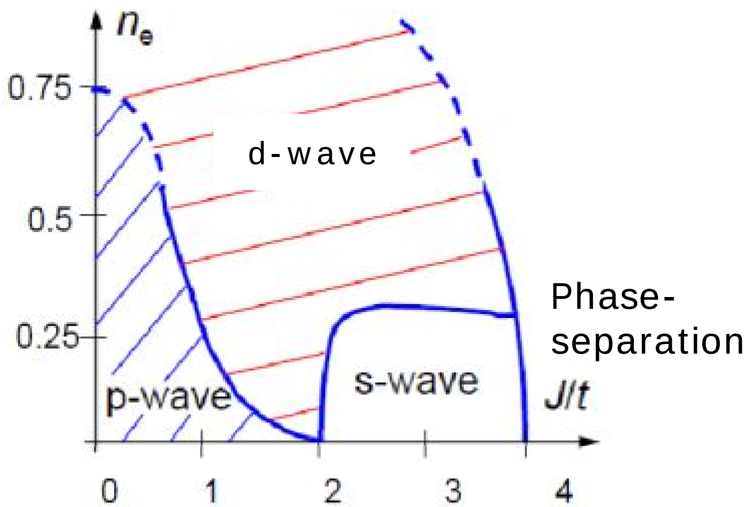
In the two band model with one narrow band SC critical temperature is mostly governed by the pairing of heavy electrons via polarization of light electrons thus the most important is interband interaction U_{hL}.

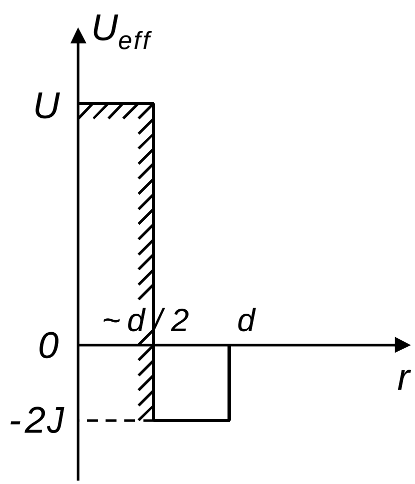
For ε_{Fh}~*(30-50)K* – typical for HF-compounds or semimetals (superlattices, heterostructures in 2D) T_{C1} in maximum (for $n_h/n_L \sim 4$) can reach *(1-5)K* (Kagan, Val'kov JETP 2010) which is quite nice.

The two SC gaps for heavy and light electrons are opened simultaneously below this temperature.

SC Phase-diagram of the t-J model

Kagan, Rice, 1994





$$\hat{H} = -t\sum_{<i\,j>\sigma} c^+_{i\sigma} c_{j\sigma} + U\sum_i n_{i\uparrow} n_{i\downarrow} + J\sum_{<ij>}\left(\vec{S}_i \vec{S}_j - \frac{1}{4} n_i n_j\right) -$$

t – J model with released constraint

$n_{i\sigma} = c^+_{i\sigma} c_{i\sigma}$ - is onsite density

$\vec{S}_i = \frac{1}{2} c^+_{i\mu} \vec{\sigma}_{\mu\nu} c_{i\nu}$ is electron spin on site i

$\vec{\sigma} = \{\sigma_1, \sigma_2, \sigma_3\}$ - are Pauli matrices.

We assume that $U >> \{J; t\}$. In fact it is a model with strong onsite repulsion U and small AFM attraction $\sim J$ on the neighboring sites (effectively we haveVan der Waals interaction potential)

We consider generalized t - J model with arbitrary J/t - ratio.

d-wave pairing in the t-J model

The equation for T_C reads for $J < t : 1 = Jd^2 \iint \frac{dp_x}{2\pi}\frac{dp_y}{2\pi}\varphi_d^2 \frac{th\frac{(\varepsilon_p - \mu)}{2T_C}}{2(\varepsilon_p - \mu)}$,

where $\varphi_d = (\cos p_x d - \cos p_y d)$ - an eigenfunction for $d_{x^2-y^2}$ - pairing on the 2D square lattice.

$\varepsilon_p - \mu = -2t(\cos p_x d + \cos p_y d) - \mu$ - is the uncorrelated quasiparticle spectrum in 2D.

As a result we get for d-wave pairing $T_{Cd} \sim \varepsilon_F \exp\left\{-\frac{\pi t}{2Jn_e^2}\right\}$ - Kagan, Rice 1994

For $n_e \geq (0{,}6 \div 0{,}7)$ and not very small $J/t \geq (1/3 \div 1/2)$ it becomes dominant over p-wave SC.

As an extrapolation for $J/t \sim (1/2 \div 1/3)$ typical for high-T_C materials

and $n_e \sim 0{,}85$ - at optimal doping we get a rough estimate:

$T_{Cd} \sim \varepsilon_F e^{-5} \sim 10^2 K$ for $\varepsilon_F \sim 10^4 K$ which is quite reasonable.

d-wave pairing at small hole densities *$x=(1-n_{el})<<1$*

In the opposite case of small hole densities $x=(1-n_{el})<<1$ the similar equation for T_C with the spin-polaronic spectrum $\varepsilon(p)$ was derived by N. M. Plakida (JETP Lett. 2001, JETP 2003) using diagrammatic technique for the Hubbard operators. In the weak-coupling BCS-case for $T_C < E_F(x)$ the critical temperature in the paramagnetic region reads: $T_C^d \sim \sqrt{WE_F(x)}\exp\left\{-\frac{1}{\lambda}\right\}$, where $\lambda \sim JN_0(x) \sim 0{,}3$ is a coupling constant $E_F(x)$ is Fermi-energy, $N_0(x)$ is an averaged density of states. T_C is maximal at $x \sim x_{opt}$ where $E_F(x_{opt}) \sim \frac{W}{2}$ and where we have a crossover from a small hole-like Fermi-surface to a large electronic one.

In maximum again $T_C^d \sim 10^2\,K$. Note that N.M. Plakida considered generalized one-band t-J model derived by Zhang-Rice construction from two-band Emery model for $U_{dp}=0$ and $J<t$. In this case the local constraint is also not very important and we can neglect the kinematical interaction of Zaitsev et al. (JETP 1976, FTT 1987)

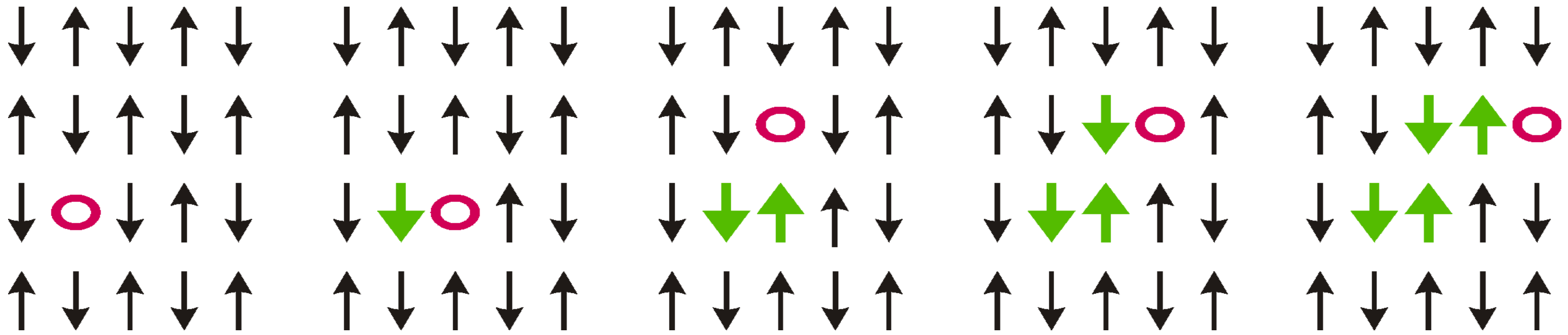
Possible bosonic region of the phase-diagram of the t-J model.

In the extreme underdoped case very close to half-filling $x<<x_{opt}$ we have the physics of the pseudogap at $T_C \leq T \leq T_*$ and a bosonic-type Uemura plot for $T_C(x)$. If we assume the bosonic character of the pseudogap (connected with the preformed pairs and not with AFM-fluctuations), then we could expect the formation of local pairs consisting of two spin-polarons at higher temperatures $T_* \sim |E_b|$ and their BEC at lower temperatures $T_C \leq E_F(x) \leq T_*$ (In this region of phase-diagramm $E_F(x) \sim Jx$ according to Patrick Lee et al. PRL 1989.) In this limiting case my philosophy is more close to the ideas of R.B. Laughlin (1998) on spin-charge confinement than to the philosophy of P.W. Anderson and P.A. Lee on spin-charge separation in the 2D t-J model.

Laughlin assumed the spin-charged confinement in the strongly interacting Fermi-Bose mixture of spinons and holons at small hole density in analogy with the confinement in quark-gluon plasma in QCD. The spin-charge confinement leads to the creation of composite holes (or spin-polarons or strings). The basic result here is connected with the ideas of Bulaevskii, Nagaev, Khomskii (1969); Brikman, Rice (1970) on AFM-strings.

String-like solution for a composite hole

(Brinkman, Rice 1970; Bulaevskii et al., 1969)



$$-\frac{\hbar^2}{2m}\Delta\Psi + \frac{z\,JS^2}{2} r\Psi = E\,\Psi; \quad \Psi \sim Ai(r)$$ - Airy function

$$V(r) = \frac{zJS^2}{2} r$$ - a spinon-holon interaction

$$r_0 \sim \left(\frac{t}{z\,JS^2}\right)^{1/3}; \quad E_0 \sim \left(z\,JS^2\right)^{2/3} t^{1/3}$$

An account of the quantum fluctuations leads to the dispersion of composite hole with a spectrum: $E_h = E_0 + J(\cos k_x d + \cos k_y d)^2$

(P. Fulde "Strong correlations in molecules and solids")

Interaction

Residual interaction of the two composite holes for a small hole concentration $x << 1$ has a dipole-dipole character.

$$V(r) \sim \frac{\lambda}{r^2}$$

Shraiman, Siggia, 1990

It can lead to a shallow bound state of the two composite holes (two spin polarons) in the $d_{x^2-y^2}$-wave channel.

Belinicher et al., 1995; Chernychev, 2001

It is quite appealing to consider T_C versus x dependence for high-T_C superconductors as the BCS-BEC crossover for the pairing of two composite holes (two spin polarons) in the d-wave channel.

Conclusions

1. On a large class of models (3D and 2D Fermi-gas model, 3D and 2D Hubbard model, Shubin-Vonsovskii t-U-V model) we demonstrated the possibility of p-wave and d-wave SC-transitions in purely repulsive Fermi-systems at low electron densities.

2. We also demonstrated a possibility to increase *Tc* of the p-wave pairing considerably already at low density in a spin-polarized case and in a two-band situation. The results are important for superfluid ^3He-A, Sr_2RuO_4, organic superconductors as well as for layered semimetals and superlattices.

3. In the two-band Hubbard model with one narrow band we considered anomalous resistivity and electron-polaron effect in the normal state of the model as well as enhanced Kohn-Luttinger mechanism of p-wave pairing below Tc. The results are important for uranium-based heavy-fermion compounds and optimally doped CMR-systems.

4. Searching for high-Tc SC we constructed qualitative phase-diagram of the t-J model. In the t-J model for the range of parameters typical for high-*Tc* materials we get d-wave pairing with reasonable critical temperature in the range of 100K. We also proposed a new scenario for SC in strongly undrdoped case connected with the concept of BCS-BEC crossover for pairing of two composite holes (two strings) in the d-wave channel.

The two-particle problem for composite holes

If we solve the two-particle problem for composite holes (two strings) interacting via dipole-dipole potential we will find in accordance with *Belinicher* the binding energy for $d_{x^2-y^2}$ pairing:

$$|E_b| \sim 0.02\, t \sim T^*$$

In the same time the BEC critical temperature for small hole concentration reads:

$$T_C^{BEC} \sim J\, x < T^*,$$

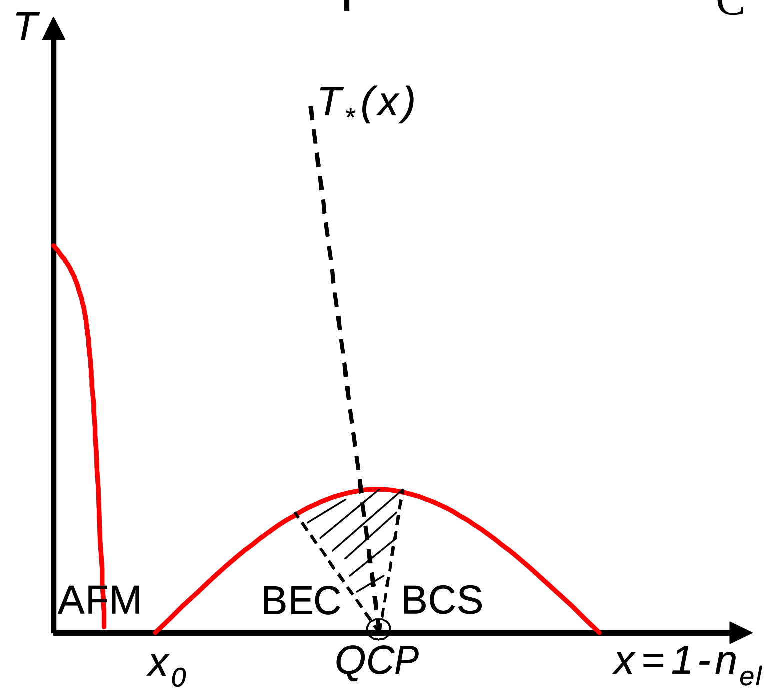
where $J \sim (0.3 \div 0.5)\, t$.

In the opposite limit of larger hole concentration as we already mentioned we have for $d_{x^2-y^2}$ pairing the BCS-results of Kagan, Rice; Plakida.

BCS-BEC crossover in d-wave channel

These two results should qualitatively interpolate at $x \sim x_{opt}$. Here however the gas of composite holes (strings) is not diluted – the string polarons start to overlap. Thus there is no linear dependence of T_C^{BEC} from x.

Another point is that possibly not all the composite holes are delocalized and participate in the formation of Bose-condensate, so from the experimental perspective T_C^{BEC} seems to behave as $J(x-x_0)$ for



QCP is possible at T=0 (Sachdev, Di Castro)

$$\frac{|x - x_0|}{x_0} << 1$$, where x_0 is a concentration of holes for which SC first appears. Thus possibly x_0 is a concentration of localized holes (which is close to the percolation ideas of *Abrikosov*).

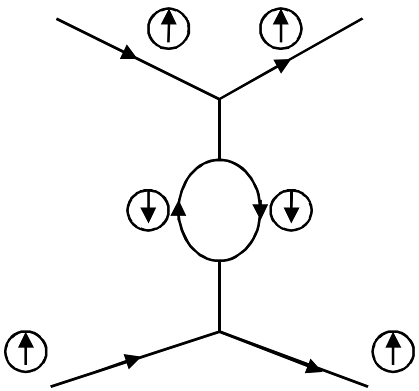
The possibility to increase T_C already at low density

1) to apply external magnetic field (or strong spin-polarization) (Kagan,Chubukov 1989)

2) to consider the two-band situation (Kagan, 1992)

In both cases the most important idea – is to separate the channels.

In magnetic field the Cooper pair is formed by two spins "up" and effective interaction is prepared by two spins "down". As a result the Kohn's anomaly is strongly increased. For H = 0: $(q-2p_F)\ln|q-2p_F| = (\pi-\theta)^2\ln(\pi-\theta)$ and only second derivative in (π - θ) is singular (diverges). Note that: $\vec{q}=\vec{p}-\vec{p}';\theta=\angle(\vec{p}\,\vec{p}')$, while for $H \neq 0$ we get: $(q_\uparrow - 2p_{F\downarrow})\ln|q_\uparrow - 2p_{F\downarrow}| = (\theta-\theta_C)\ln|\theta-\theta_C|$ and already first derivative in $(\theta - \theta_C)$ diverges.



Note that θ_C differs from π proportionally to $\left(\frac{p_{F\uparrow}}{p_{F\downarrow}}-1\right)$

Unfortunately there is a competing process, namely

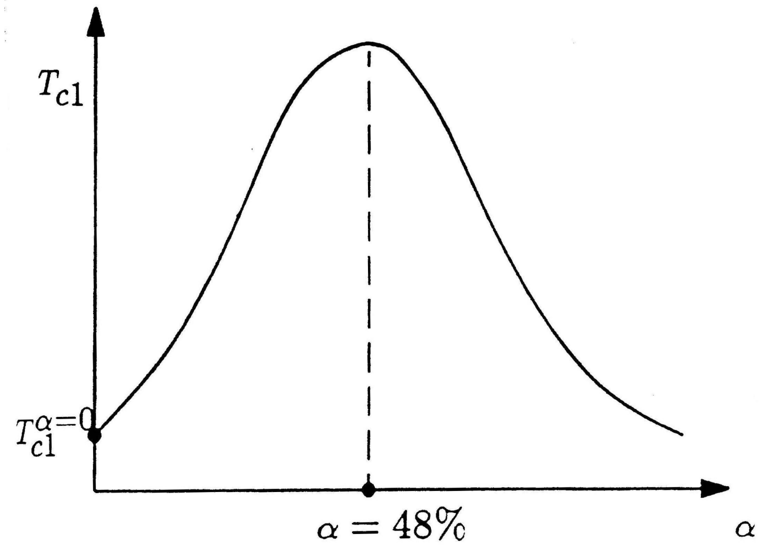
the decrease of the density of states of the "down" spins:

$$N_\downarrow(0) = \frac{mp_{F\downarrow}}{2\pi^2}$$

As a result of this competition $T_C^{\uparrow\uparrow}$ has reentrant behavior with large maximum.

Experimental confirmation of our ideas

This theory is confirmed by experiments of *Frossati et al*., in Kamerlingh-Onnes Laboratory in Leiden:



^{3}He: $T_C^{\uparrow\uparrow}$ (α = 6%) = 3.2 mK

T_C (α = 0) = 2.7 mK

20% increase of T_C

for polarization degree $\alpha = \dfrac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} = 0{,}06$

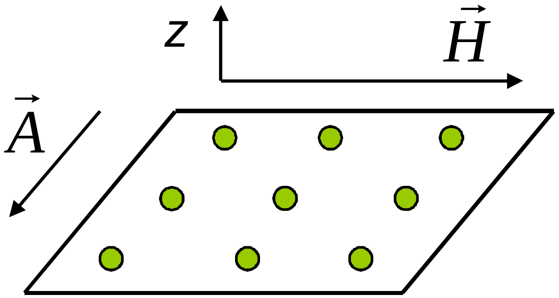
In maximum we predict:

$T_C^{\uparrow\uparrow}$ = 6.4 T_C for ^{3}He and

$T_C^{\uparrow\uparrow}$ = 10^5 T_C for ^{3}He-^{4}He mixtures

M.Yu. Kagan, A.V. Chubukov, *JETP Lett*. 1989
M.A. Baranov, A.V. Chubukov, M.Yu. Kagan, *Int. Jour. Mod. Phys. B.* 1992

2D electron gas in parallel magnetic field

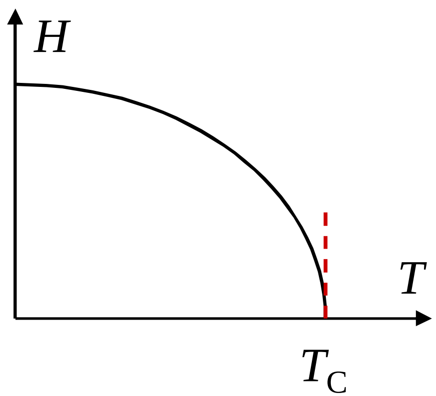


In 2D electron gas in $\parallel$ magnetic field: $\vec{H} = H\vec{e}_x$

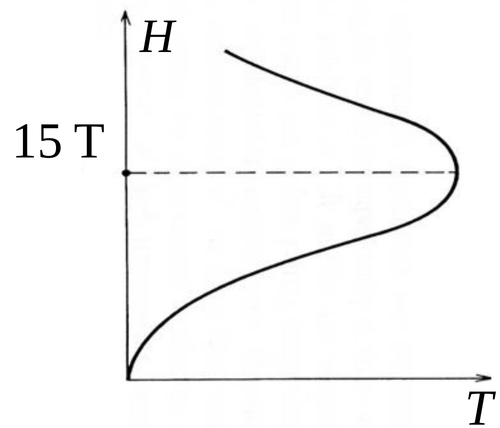
The vector-potential $\vec{A} = Hz\vec{e}_y$ - does not change the motion of electrons in (x,y) plane.

The diamagnetic Meissner effect is totally suppressed. Hence 2D SC in || magnetic field becomes equivalent to neutral ^{3}He-monolayer

3D usual SC



2D SC in $\parallel$ magnetic field – reentrant behavior



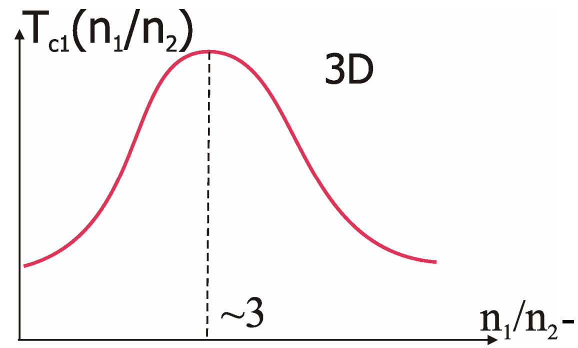
Kagan, Baranov, 1993

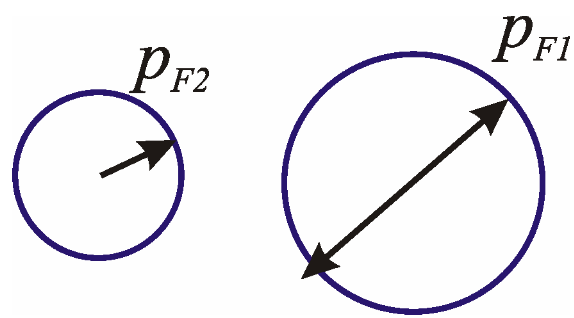
For low-density systems with $\varepsilon_F \sim 30$ K, $H \sim 15$ T we can get $T_{C1} \sim 0.5$ K

Possible SC in very pure heterostructures

Two sorts of fermions

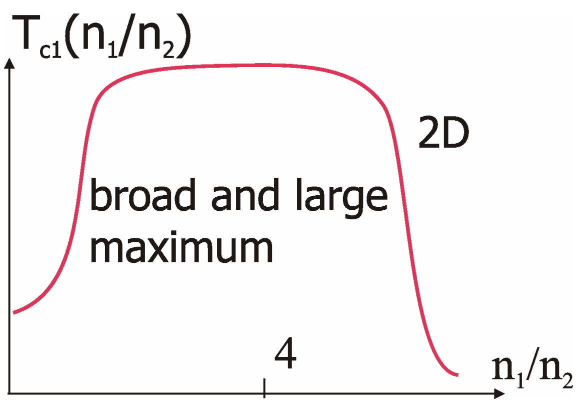
The main idea is again that: two particles of sort 1 form Cooper pairs, while the particles of sort 2 prepare an effective interaction





- relative density

In 3D: $T_{C1}^{\max} = T_{C1}(n_1 / n_2 \approx 3) \approx \varepsilon_{F1} \exp\left\{-\frac{7}{\lambda^2}\right\}$ - main exponent increases almost in 2 times.



In 2D: $T_{C}^{\max} = T_{C1}(n_1 / n_2 = 4) = \varepsilon_{F1} \exp\left\{-\frac{1}{2 f_0^2}\right\}$

The Kohn's anomaly $\mathrm{Re}\sqrt{q_{\uparrow} - 2 p_{F\downarrow}}$ becomes effective for SC already in the second order in f_0.

M.Yu. Kagan, Phys.Lett.A. 1992 D.V. Efremov et al JETP 2001

Two-band Hubbard model with one narrow band. I

This model is very rich. It describes adequately mixed valence systems such as uranium-based HF and possibly also some other novel superconductors and transition-metal systems with orbital degeneracy such as complex magnetic oxides in optimally doped case. Moreover it contains such highly nontrivial effect as ELECTRON POLARON EFFECT (*Nozieres, Kondo, Yu. Kagan, Prokof'ev*) in the homogeneous state.

Let us verify this model with respect to marginality and anomalous resistivity characteristics:

Two-band Hubbard model

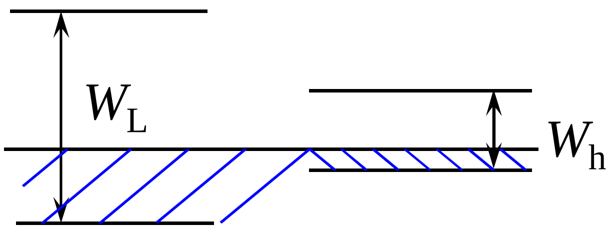
In real space the Hamiltonian reads:

$$\hat{H}' = -t_h \sum_{<ij>} a^+_{i\sigma} a_{j\sigma} - t_L \sum_{<ij>} b^+_{i\sigma} b_{j\sigma} - \varepsilon_0 \sum_{i\sigma} n^{\sigma}_{ih} - \mu \sum_{i\sigma} (n^L_{i\sigma} + n^h_{i\sigma}) +$$

$$+ U_{hh} \sum_i n^{\uparrow}_{ih} n^{\downarrow}_{ih} + U_{LL} \sum_i n^{\uparrow}_{iL} n^{\downarrow}_{iL} + \frac{U_{hL}}{2} \sum_i n_{iL} n_{ih}$$

Electron Polaron Effect in the two-band model

For low temperatures $T << W_h^* << W_L$, inverse scattering times $1/\tau \sim T^2$ and resistivity $R \sim T^2$ - FL-picture, but:



$$\frac{m_h^*}{m_h} \sim \left(\frac{m_h}{m_L}\right)^{\frac{b}{1-b}}$$ – electron polaron effect

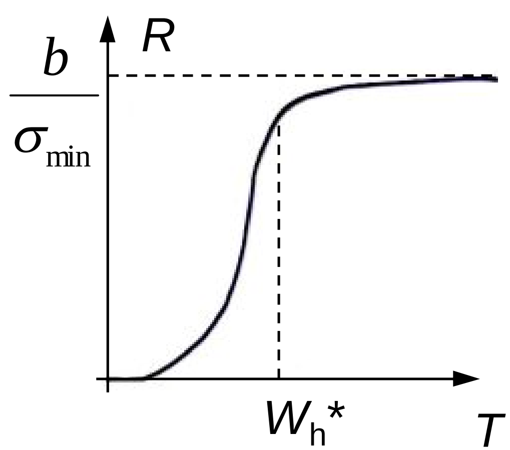
(Yu. Kagan, Prokof'ev)

Polaron exponent b is connected with interband Hubbard interaction U_{hL} and can reach the value ½ in the unitary limit yielding $m^*_h \sim 100 m_e$ typical for HF-compounds. For higher temperatures $T > W_h^*$:

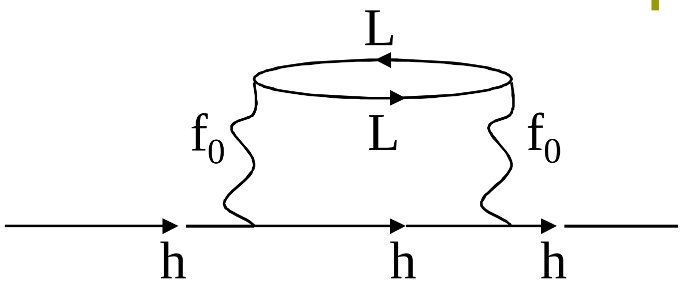
$1/\tau_h \sim 1/\tau_{hL} \sim bT$ - heavy component is marginal (heavy electrons move diffusively in light electrons surrounding). However light electrons scatter on the heavy ones as if on a static impurity $1/\tau_L \sim 1/\tau_{Lh} \sim$ const – non marginal.

$$R \sim \frac{b}{\sigma_{min}} \frac{1}{\left(1 + \left(\frac{W_h^*}{T}\right)^2\right)}$$ - goes on saturation in 3D case (UNi_2Al_3)

(Electron-electron scattering and no real impurities)



Electron-polaron effect



Σ_{hL} – heavy light self-energy

It corresponds to the non-adiabatical part of many-particle wave function which describes the heavy particle dressed in a cloud of virtual electron-hole pairs of light particles. Nonadiabaticity of the cloud in some energy interval manifests itself when the heavy particle moves from one elementary cell to a neighboring one. Formally EPE is connected with interband Hubbard interaction U_{hL}. In the second order of perturbation theory $m_h^* / m_h = Z_{\mathrm{h}}^{-1} = 1 + b \ln \dfrac{m_h}{m_L}$, where $b = 2 f_0^2$,

$Z_{\mathrm{h}} = 1 - \dfrac{\partial \Sigma_{\mathrm{hL}}(\omega, \varepsilon_q)}{\partial \omega}$ is Z-factor of heavy particle.

In Born approximation $f_0 = U_{hL} \nu_L(\varepsilon_F)$. In more general case of low density and strong Hubbard interaction $\mathrm{U}_{\mathrm{hL}} > \mathrm{W}_{\mathrm{L}}$: $f_0 = \dfrac{2dp_F}{\pi}$ - Galitskii gas parameter, d is interstate distance.

In 2D $f_0 = \dfrac{1}{2 \ln 1/dp_F}$ - gas-parameter of Bloom.

Resistivity in 3D

Exact solution of coupled kinetic equations with an account

of umklapp processes yields for $p_{Fh} \sim p_{FL} \sim p_F \sim 1/d$:

$$\sigma_{hL} \sim \sigma_{Lh} \sim \frac{\sigma_{\min}}{b}\left(\frac{W_h^*}{T}\right)^2 \quad \text{for } T < W_h^*$$

We can also show that $\sigma_L \square \sigma_{Lh}$ and $\sigma_h \square \sigma_{hL}$

Thus R~$\dfrac{1}{(\sigma_h + \sigma_L)} \sim \dfrac{b}{\sigma_{\min}}\left(\dfrac{T}{W_h^*}\right)^2$, where $\sigma_{\min} = \dfrac{e^2 p_F}{\hbar}$ is minimal Mott-Regel conductivity n 3D

At high temperatures $T > W_h^*$: $\quad {}^1\!/_{\tau_{Lh}} \sim bW_L; \quad {}^1\!/_{\tau_{hL}} \sim bT$

$$\sigma_{Lh} \sim \frac{n_L e^2}{m_L}\tau_{Lh} \sim \frac{n_L e^2}{m_L b W_L} \sim \frac{\sigma_{\min}}{b},$$

$\sigma_{hL} \sim \dfrac{\sigma_{\min}}{b}\left(\dfrac{W_h^*}{T}\right)^2$ - with an account for Einstein relation $\dfrac{\partial n_h}{\partial \varepsilon} \sim \dfrac{W_h^*}{T}$ for $T > W_h^*$

Hence R~$\dfrac{1}{(\sigma_{Lh} + \sigma_{hL})} \sim \dfrac{b}{\sigma_{\min}\left[1 + \left(\dfrac{W_h^*}{T}\right)^2\right]}$

Other mechanisms of mass-enhancement, tendency towards phase separation

EPE is connected with Z-factor of heavy particle (with $\left.\frac{\partial \Sigma_{hL}(\omega, \varepsilon_{\vec{q}})}{\partial \omega}\right|_{\omega \to 0}$)

However in 3D-case momentum dependence of heavy-light self energy $\left.\frac{\partial \Sigma_{hL}(\omega, \varepsilon_{\vec{q}})}{\partial \varepsilon_{\vec{q}}}\right|_{q \to p_F}$ also becomes very substantial. Hence the full expression for m^*_h/m_h in the second order of perturbation theory reads

$$\frac{m_h^*}{m_h} = 1 + b \ln \frac{m_h}{m_L} + \frac{b}{18} \frac{m_h n_h}{m_L n_L}$$

and possess the additional term which is linear in the bare mass-ratio m_h/m_L. If in LD approximation $m_h \sim 10 m_L$, then this term becomes dominant over EPE ~ln m_h/m_L for large density mismatch $n_h \geq 5 n_L$.

It is very interesting that the same parameter $b \frac{m_h n_h}{m_L n_L} \geq 1$ governs the tendency towards phase-separation in the two-band model yielding negative partial compressibility $\chi_{hh}^{-1} \sim c_h^2 \sim (n_h / m_h)(\partial \mu_h / \partial n_h)$ where μ_h is chemical potential of the heavy particle. This result is in qualitative agreement with predictions of Kugel et al (PRB 2007).

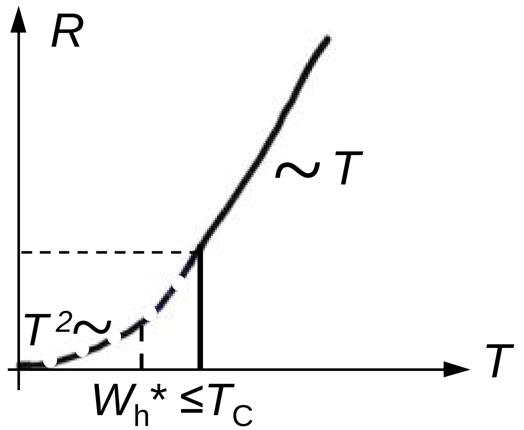
Weak-localization Effects and Anomalous Resistivity in 2D

In 2D case light component also has a tendency towards marginality due to weak localization corrections of Altshuler-Aronov type: (due to multiple scattering of light electrons on the heavy ones as if on impurities in dirty metals)

$$\sigma_{loc} \sim \frac{\sigma_{\min}}{b}\left[1 - b\ln\frac{W_L}{bT}\right]$$

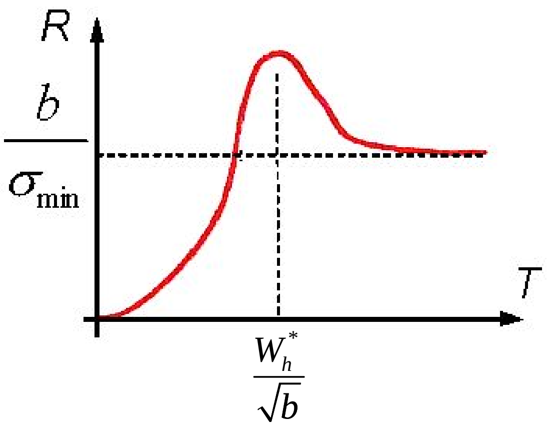
That is why a marginal behavior in the two-band model has chances to arise only as a high-temperature asymptotics in 2D where:
the additional narrowing of the heavy-band and additional localization of the light-band are governed by the same parameter: $b\ln(m_h/m_L) \geq 1$ for $bT \sim W_h^*$

May be we can expect then the following $R(T)$-curve?



This behavior can be interesting for optimally doped cuprates

The real life is however more interesting. We obtain the maximum and localization tail at higher temperatures - resembles the curve for R(T) in optimally doped layered CMR-systems



(M.Yu Kagan, V.V, Val'kov, JETP 2011)

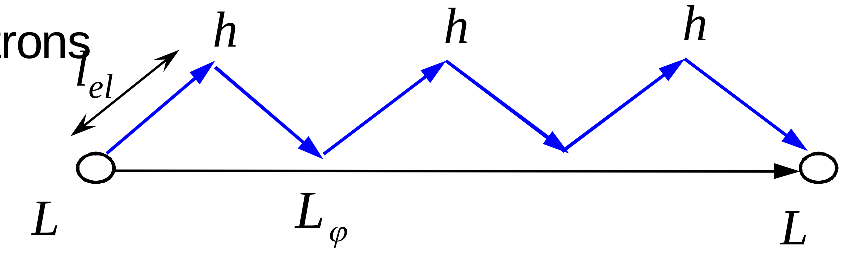
Altshuler-Aronov Effect in 2D

Weak-localization corrections due to quantum-mechanical backward scattering in 2D read:

$\Delta\sigma_L/\sigma_{0L} \sim b\ln\frac{\tau_\varphi}{\tau}$, where $\sigma_{0L} = \frac{\sigma_{\min}}{b}$ is classical conductivity of the light band,

$\sigma_{\min} = \frac{e^2}{\hbar}$ - Mott-Regel minimal conductivity in 2D

$\tau_\varphi = \tau_{ee} = \tilde{\tau}_{LL}$ - is decoherence time for light electrons



$\tau = \tau_{ei} = \tau_{Lh}$ - is elastic time

$l_{el} = v_{FL}\tau_{Lh}$ - elastic length

$L_\phi = \sqrt{D\tau_\varphi} = v_{FL}\sqrt{\tau_{Lh}\tilde{\tau}_{LL}}$ - diffusive length

$1/\tau_{Lh} \sim bW_L$ - elastic time

$1/\tilde{\tau}_{LL} = b^2T$ - Altshuler-Aronov effect in "dirty" metal in 2D

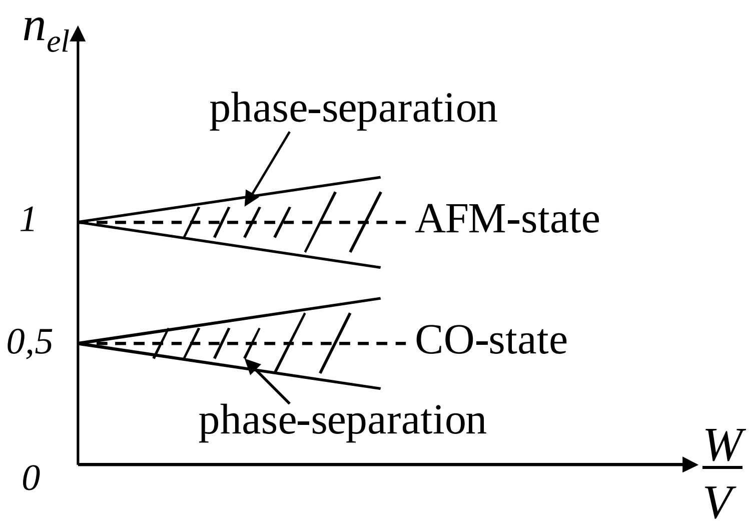
(electron-electron scattering time becomes marginal in dirty limit when between two scattering events for light electrons, a light electron scatters a lot of time on heavy electrons as if on almost elastic impurities)

Hence: $\sigma_L = \frac{\sigma_{\min}}{b}(1 - b\ln\frac{W_L}{bT})$

Localization and phase-separation in the Shubin-Vonsovskii model at larger densities.

For larger densities we have 2 types of localization in the t-U-V model: for n_{el}->1/2 and $U>>V>>t$ there is Verwey localization with checkerboard CO-state and for n_{el}->1 – Mott-Hubbard localization.

Close to $n_{el}=1/2$ and close to $n_{el}=1$ we have extended regions of nanoscale phase separation (Kagan, Kugel, Khomskii, JETP 2001).



$$n_{el} = 1/2 - \delta$$

metallic droplets in CO-matrix

(Kagan, Kugel, Sov. Phys. Uspekhi 2001)

Phase-separation and SC in Shubin-Vonsovskii model

For $n=1/2-\delta$ – phase-separation on almost empty clusters with one electron inside CO-matrix:

XOXOXO(OOXOO)XOXOXO(OOXOO)XOXOXO

Kagan, Kugel,
Sov. Phys. Uspekhi 2001

For $n=1/2+\delta$ – phase-separation on almost full clusters with one hole inside CO-matrix:

XOXOXO(XXOXX)XOXOXO(XXOXX)XOXOXO

The formation of stripes is also possible here.
The granulas with one hole (or one electron) start to overlap and CO-state is totally destroyed (totally melted) at critical concentration:

$$\frac{1}{2}-\delta_C \sim \frac{1}{\pi}\left(\frac{t}{V}\right)^{1/2} \text{ in 2D}$$

For moderate $V\geq 2t$ the region of phase-separation is rather extended ($\delta_C \sim 0{,}25$) and stretches to rather high densities from $(1/2-\delta_C)$ to $(1/2+\delta_C)$.
Of couse, close to $n_{el}=1$ we have phase-separation on FM-metallic granulas inside insulating AFM-matrix.

The dense electron plasma

In 3D dense electron plasma Chubukov, Kagan (Jour. of Phys: Cond. Mat 1989) predicted a cascade of SC-transitions with orbital moment of the Cooper pair l being dependent upon electron density (r_S)

$$l > l_C = \frac{|\ln r_S|}{\sqrt{r_S}}\left(1 + \frac{7}{2}\frac{\ln|\ln r_S|}{|\ln r_S|} + \ldots\right),$$

where $r_S = \frac{1{,}92}{p_F a_B} \ll 1$ is correlative radius,

T_C is low in this range of electron densities.

Qualitatively the same results here were obtained recently by Alexandrov, Kabanov (PRL 2011)

The dilute electron plasma

The possibility of p-wave SC with $T_{C1} \sim (10^{-3} \div 10^{-2}) K$ for 3D electron plasma of intermediate and small electron densities (simple and noble metals, semimetals) was also predicted in the papers of J. Appel et al (PRB 1969), S.Kuchenhoff and P. Woelfle (PRB 1988) and A.V. Chubukov, M.Yu. Kagan (1989).

The most realistic region of densities for p-wave pairing in 3D dilute plasma is $20 \leq r_S \leq 35$. Some groups (A.S. Alexandrov, V.Kabanov, 2011; K. Efetov, I. Aleiner 2010) belive in Khodel-Shaginyan type of Fermi-surface reconstruction at these densities.

The question is very difficult in particular in 2D where rigorously speaking we should sum up an infinite parquette class of diagrams for the irreducible bare vertex U_{eff} in the Cooper channel.

Applications of the theory in low-temperature physics

In superfluid ^{3}He 20% enhancement of $T_{C1}^{\uparrow\uparrow}$ (A1-phase) was experimentally observed by *Frossati et al*., in Leiden

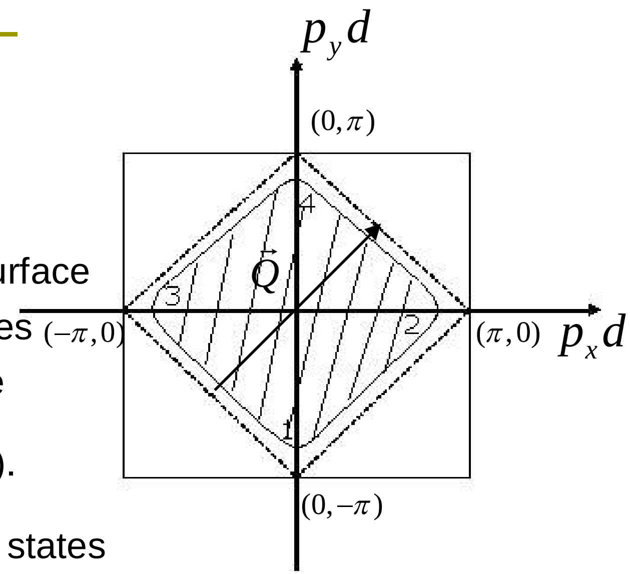
In ^{3}He-^{4}He mixtures the scattering length a changes sign at ^{3}He concentration $x = 1 - 2\%$. For $x < 1\%$ $a < 0$ – attraction (s-wave pairing predicted by *Bashkin, Meyerovich*). $T_{C0} = 10^{-4}$-10^{-5} K for $H = 0$ and $x \sim 1\%$. For $x > (2 \div 4)\%$ $a > 0$ – repulsion (p-wave pairing *Kagan, Chubukov*). $T_{C1} = 10^{-5}$ K for $H = 15$ T and maximal solubility $x_{max} \sim 9.5\%$.

For fermionic ^{6}Li $a = -2.3\times10^3$ Å – attraction. $T_{C0} = 10^{-6}$ K is predicted by *Stoof* on the BCS-side ($a < 0$) of the Feshbach resonance.

s-wave pairing is suppressed when the disbalance between hyperfine components is large enough $(n_1 - n_2)/(n_1 + n_2) > T_{C0}/\varepsilon_F$, where n_1 and n_2 are the densities of two hyperfine components of nuclear spin I (*M.Yu.Kagan, Yu.Kagan, M.Baranov, 1996*). In this situation arises p-wave pairing with :

$$T_{C1} \sim \varepsilon_{F1} e^{-7/\lambda^2} \sim 10^{-7} \text{ K for } \lambda \sim 1$$

SC in 2D Hubbard model at larger electron densities $n_{el} \leq 1$



At larger densities $n_{el} \leq 1$ close to half-filling the spectrum of electrons

becomes almost hyperbolic $\varepsilon(p) \approx \pm\left(\frac{p_x^2 - p_y^2}{2m}\right)$

close to the corner points (0,π) (0,-π) and (π,0) (-π,0) where the Fermi-surface almost touches the Brillouin zone. As a result the Kohn's anomaly becomes logarithmically strong as in 1D case (almost flat parts of the Fermi-surface

are present and we have almost perfect nesting $\varepsilon_P = -\varepsilon_{P+Q}$ for $\vec{Q} = \left(\frac{\pi}{a}, \frac{\pi}{d}\right)$).

Additional increase for T_C is due to Van-Hove singularity in the density of states which is also logarithmically strong.

Thus in the weak-coupling case for $f_0 = \frac{U}{8\pi t} << 1$ and in the second order of the perturbation theory

both polarization operator and a Cooper loop will contain $\ln^2$ contribution and thus effective interaction

$U_{eff} \sim f_0 + f_0^2 \ln^2 \frac{t}{\mu}$ ($f_0 \ln^2 \frac{t}{\mu}$ is an effective parameter of perturbation theory for U_{eff}) we have $d_{x^2-y^2}$

SC with the critical temperature: $f_0^2 \ln^3 \frac{t}{\mu} \ln \frac{\mu}{T_C^d} \sim 1$ or equivalently: $T_C^d \sim \mu \exp\left\{-\frac{1}{f_0^2 \ln^3 \frac{t}{\mu}}\right\}$ (Kozlov 1989),

where the chemical potential $\mu << t$ is a natural cutoff of the large logarithm (for $\mu > T_C^d$).

Parquette solution at weak-coupling and very close to half-filling.

Very close to half-filling when $\mu \sim T_C$ we have so-called doubly-logarithmical parquette solution with the competition between SC and SDW-instability in particle-particle and particle-hole channels.

Here for $\mu \sim T_C$ and $f_0^{\,2} \ln^4\left(\frac{t}{\mu}\right) \sim f_0^{\,2} \ln^4\left(\frac{t}{T_C^d}\right) \sim 1$ we have $T_C^d \sim t \exp\left\{-\frac{const}{\sqrt{f_0}}\right\}$ - an elegant result of Dzyaloshinskii, Yakovenko (JETP, 1988).

One more interesting observation for the square lattices with the spectrum $\varepsilon(p) = -2t(\cos p_x d + \cos p_y d)$ close to half-filling (where $\mu \to 0$) belongs to Kopaev and Belyavskii (PRB 2007, 2009).

The spectrum satisfies also mirror nesting condition $\varepsilon(-p + \frac{Q}{2}) = -\varepsilon(p + \frac{Q}{2})$ (which directly follows from standard nesting condition $\varepsilon(p+Q) = -\varepsilon(p)$) and favours SC with large total momentum of a Cooper pair $\vec{K} = \vec{Q}$ in a clean case (no impurities).

Note that according to Kivelson (Cambridge, SCES-meeting 2011) the border between AFM and SC phases of the high-Tc materials (described by 2D Hubbard model) in weak-coupling case $U \ll t$ and very close to half-filling (for doping concentrations $x = 1 - n_{el} \ll 1$) is given by: $x_C \sim \exp\{-2\pi\sqrt{t/U}\}$ at zero temperatures $T \to 0$ with the same $1/\sqrt{f_0} \sim \sqrt{t/U}$ in the exponent for x_C as T_C^d in Dzyaloshinskii-Yakovenko story.

Some important papers on normal and SC-state of the Hubbard-model

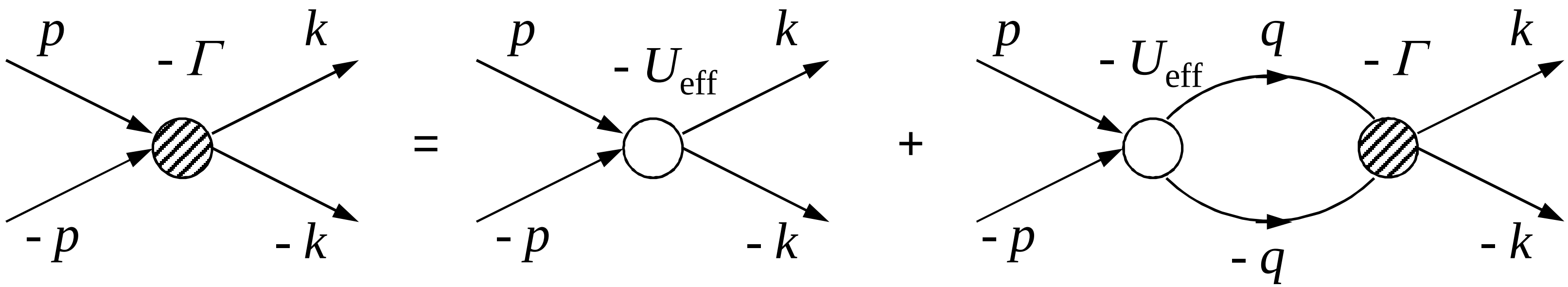
In the West

- Hubbard Proc. Roy. Soc. 1963,1964,1965
- Kanamori Prog. Theor. Phys. 1963
- Nagaoka Phys. Rev. 1966
- Brinkman, Rice PRB 1970
- Lee et al. PRL 1973, 1992
- Vollhardt Rev. Mod. Phys. 1984
- Varma et. al. PRB 1986
- Scalapino et al. PRB 1986
- Woelfle et al. PRB 1987
- Hirsch et al PRB 1987
- Schrieffer, Kampf PRB 1990
- Anderson PRL 1990, 1991
- Pines et al. PRL 1991, 1998
- Kivelson et al. PRL 2011

In Russia

- Zubarev Sov. Phys. Uspekhi 1960
- Bulaevskii, Nagaev, Khomskii JETP 1968;
- Vaks et al. JETP 1968
- Zaitsev et al JETP 1976, FTT 1987
- Wiegmann et al. JETP 1977, 1989
- Ovchinnikov, Val'kov JETP 1985
- Ioffe, Larkin PRB 1988, 1989
- Belinicher et al. PRB 1993
- Plakida et al. PRB 1994, 1995
- Barabanov, Maksimov et al JETP 1996, PRB 1998
- Izumov et al. Sov. Phys. Uspekhi 1997, 1999
- Sadovskii et al. JETP 1999, 2000

Briefly about a mathematical technique. I



$$-\Gamma(\vec{p},\vec{k}) = -U_{eff}(\vec{p},\vec{k}) + T\sum_n \int U_{eff}(\vec{p},\vec{q}) G_M(\xi_S,\vec{q}) G_M(-\xi_S,-\vec{q}) \Gamma(\vec{q},\vec{k}) \frac{d^3\vec{q}}{(2\pi)^3}$$

Bethe-Solpeter equation for total vertex Γ in the Cooper channel

$$\Omega_{tot} = \xi_{n1} + \xi_{n2} = 0; \quad \vec{K}_{tot} = \vec{p}_1 + \vec{p}_2 = 0$$

$G_M(\xi_n,\vec{q}) = [i\xi_n - \varepsilon(q)]^{-1}$ - is the Green function in temperature technique,

$\xi_n = (2n+1)\pi T$ – is Matsuba frequency for fermions, T – is the temperature;

$n = 0; \pm 1; \pm 2$ – integer;

$\varepsilon(q) = \frac{1}{2m} q^2 - \mu$ - is the quasiparticle spectrum; $\mu = \varepsilon_F$ - is chemical potential;

$|\vec{p}| = |\vec{k}| = p_F$ - the incoming and outgoing momenta are lying on the Fermi-surface

Briefly about a mathematical technique. II

$$\Gamma^l = U^l_{eff} - U^l_{eff} K \Gamma^l;$$

l – is a value of the angular momentum

$$\Gamma^l = \frac{U^l_{eff}}{1 + U^l_{eff} K}$$

$1 + U^l_{eff} K = 0$ - the pole of Γ^l yields equation for T_C(Landau-Thouless criteria)

$$K = \int G^M_p G^M_{-p} dp = N(0) \ln \frac{2e^C \varepsilon_F}{\pi T_C}$$ - the Cooper loop, C is Euler constant

The most important is the sign of U_{eff}

The search for marginal Fermi-liquid behavior in basic models at low density

1. 2D Fermi-gas at $T > T_C$ there is an anomaly in Landau f-function:

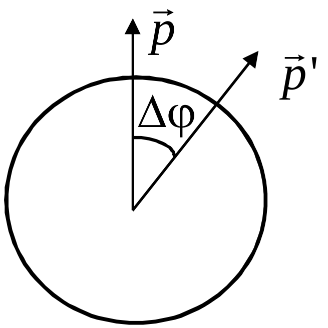
$$f_{\uparrow\downarrow}(\vec{p},\vec{p}\,') \sim \frac{1}{\sqrt{Q}}; \quad Q = \left|\vec{p} - \vec{p}\,'\right| \to 0 \text{ -}$$

(Prokof'ev, Stamp, 1992)
(Baranov, Kagan, Mar'enko, 1993)

- $\vec{p}$ and $\vec{p}\,'$ are almost parallel

It is integrable and much weaker than Anderson prediction $f_{\uparrow\downarrow} \sim \frac{1}{Q}$. (PRL 1990)

$\varepsilon = \frac{p - p_F}{p_F}$ - the distance from the Fermi-circle



$$f_{pp'} \sim \frac{1}{\sqrt{\varepsilon}} \quad \text{for} \begin{cases} p \neq p_F \\ p' = p_F \end{cases}$$

the anomaly is weak

$\Delta\varphi \sim \varepsilon^{3/2}$ – the anomaly exists in a narrow angles region around $\varphi = 0$ (where $\vec{p} \,||\, \vec{p}\,'$)

It leads to nontrivial temperature corrections to m^*, χ, and zero-sound velocity C_0:

$$\frac{m^*(T) - m^*(0)}{m^*(0)} \sim \frac{\chi(T) - \chi(0)}{\chi(0)} \sim \frac{C_0(T) - C_0(0)}{C_0(0)} \sim g^2 \frac{T}{\varepsilon_F},$$

but does not destroy FL-picture in 2D Fermi-gas.

Antibound state of Hubbard-Anderson in the 2D Hubbard model at low electron density (M.Yu. Kagan, V.V. Val'kov, P. Woelfle 2011)

For $U >> W$ - strong-coupling and $n_{el} = \frac{2\varepsilon_F}{W} << 1$ - at low electron density

$\omega_{\text{ab}} \approx U(1 - n_{el})$ - antibound state (Anderson 1990, Hubbard 1963)

The T-matrix for $\omega \sim \omega_{\text{ab}}$ acquires a bosonic like form with a pole:

$$T(\omega, \vec{q}) \approx \frac{U\omega}{\left(\omega + \frac{q^2}{4m^*} + \mu_B + io\right)}$$

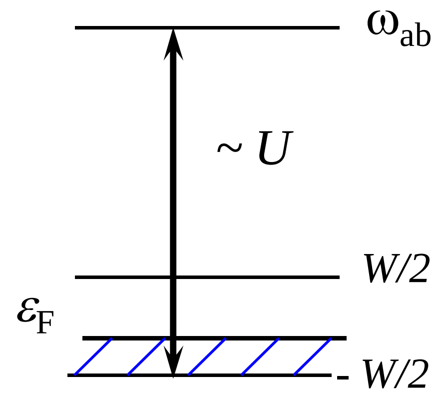
$$m^*/m \sim \frac{(1 - n_{el})}{2n_{el}}$$

($2m^*$ is an effective mas of a "bosonic pair" with hole-like dispersion)

$\mu_{\text{B}} = 2\mu - |E_B|$ - is bosonic chemical potential

$\mu \approx -\frac{W}{2} + \varepsilon_F$ for $n_{el} << 1$

$|E_b| = U(1 - n_{el}) + n_{el}W$ - an energy of the antibound state (Fukujama 1991; Kagan, Woelfle 1992)



The dressed Green-function

Self-energy in the T-matrix approximation acquires a form:

$$\mathrm{Re}\,\Sigma(\omega,\vec{q}) = \frac{\omega U n_{el}/2}{(\omega + \mu_B + io)};\ \mathrm{Im}\,\Sigma(\omega,\vec{q}) = -\pi N_{2D}(0) U \mu_B \left[\theta(\omega+\mu_B) - \theta(\omega+\mu_B-\varepsilon_F)\right]$$, where

$N_{2D}(0) = \frac{m}{2\pi}$ is 2D density of states; $m = \frac{1}{2td^2}$

The dressed Green-function $G^{-1}(\omega,\vec{q}) = G_0^{-1}(\omega,\vec{q}) - \Sigma(\omega,\vec{q})$ has the two-pole structure similar to Hubbard-I approximation already at low electon densities.

$$G(\omega,\vec{q}) = \frac{Z_{LHB}}{\omega - (\varepsilon_q - \mu) Z_{LHB} + io} + \frac{Z_{UHB}}{\omega - U\left(1 - \frac{n_{el}}{2}\right) - (\varepsilon_q - \mu) Z_{UHB} + io},$$ where

$$Z_{LHB} = \left(1 - \frac{n_{el}}{2}\right);\ Z_{UHB} = \frac{n_{el}}{2};\ Z_{LHB} + Z_{UHB} = 1$$

The Hartree-Fock correction to the thermodynamic potential

$\Delta\Omega \sim \int \Sigma(\omega,\vec{p})\, G_0(\omega,\vec{p}) \frac{d\omega}{2\pi} \frac{d^2\vec{p}}{(2\pi)^2} \sim Z_{UHB}\varepsilon_F n_{el} \sim n_{el}^3 > 0$ - positive shift of the thermodynamic potential at $T = 0$

The antibound state leads to non-trivial corrections to Landau Fermi-liquid theory in 2D but does not destroy it. In the low energy sector for $\omega \le \varepsilon_F << U$ Galitskii-Bloom theory for SC Fermi-gas works pretty well

Extended s-wave pairing for J > t and low densities.

$\Delta_s = \Delta_0^s(\cos p_x d + \cos p_y d)$ - extended s-wave pairing arises in the t-J model at low electron densities for $J > J_{C0} = 2t$ (Emery, Kivelson, Lin PRL 1990)

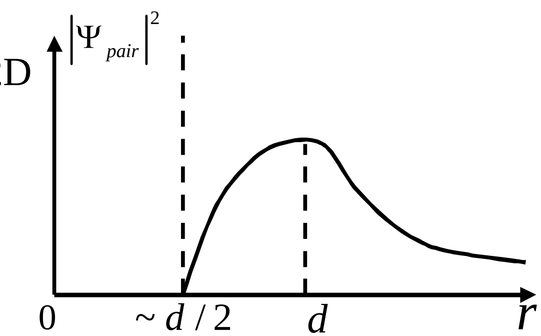
The pair Ψ-function is zero for $r \le \frac{d}{2}$ - in the region of strong Hubbard interaction $U >> \{J, t\}$ and thus $U\Psi = 0$ in the effective Shroedinger equation. It has a maximum for $r \sim d$ - it is centerd on the neighboring sites. Thus pair Ψ-function has region of zero values but does not change sign. For $J > J_{C0} = 2t$ there is a bound state of two electrons with an energy $E_b = -8W \exp\left\{-\frac{J\pi}{J - J_{C0}}\right\}$ for $J \ge J_{C0}$ (Kagan, Rice)

and $E_b = -J - \frac{20t^2}{J}$ for $J >> t$ (Emery, Kivelson, Lin)

In the BCS-case $|E_b| < \varepsilon_F$: $T_{CS} = \sqrt{2\varepsilon_F |E_b|}$ - famous Miyake formula (PRB 1986)

In the BEC-case $|E_b| > \varepsilon_F$ we have two temperatures: $T_* \sim \frac{|E_b|}{\ln \frac{|E_b|}{\varepsilon_F}}$ - Saha crossover temperature which describes creation of local pairs(dimers) and superconductive critical temperature

$T_{CS} \sim \frac{\varepsilon_F}{4 \ln \ln \frac{|E_b|}{\varepsilon_F}}$ given by Fisher-Hohenberg formula for slightly non-ideal 2D Bose-gas of local pairs (dimers) (PRL 1988)



Phase-separation at large J/t and low densities

The energy of BEC-phase becomes negative at large J/t - we have liquid of local pairs (or dimers): $E_{BEC} - E_N = -N\frac{|E_b|}{2} < 0.$

If we further increase J/t at low densities than the formation of quartets, or larger complexes will become energetically beneficial on the square lattice.

But as it was shown by Emery, Kivelson, Lin this liquid phase for $J/t > 3{,}8$ (earlier than the threshold for quartet formation) becomes unstable towards total phase-separation on PM-cluster with $n_{el} \to 0$ and AFM-cluster with $n_{el} \to 1.$

$J_{P.S.} = 3{,}8t$ can be defined from the inequality:

$$\frac{E_{AFM} - E_N}{N} = -1{,}18J \leq \frac{E_{BEC} - E_N}{N} = -\frac{|E_b|}{2}$$

P-wave pairing for J < t and low densities

$\Delta_p = \Delta_{0p}(\sin p_x d + i \sin p_y d)$ on 2D square lattice.

The critical temperature of p-wave pairing reads:

$$T_C^P \sim \varepsilon_F \exp\left\{ -\frac{1}{6{,}1 f_0^3} \right\}$$

It is governed by Kohn-Luttinger mechanism as in 2D Hubbard model

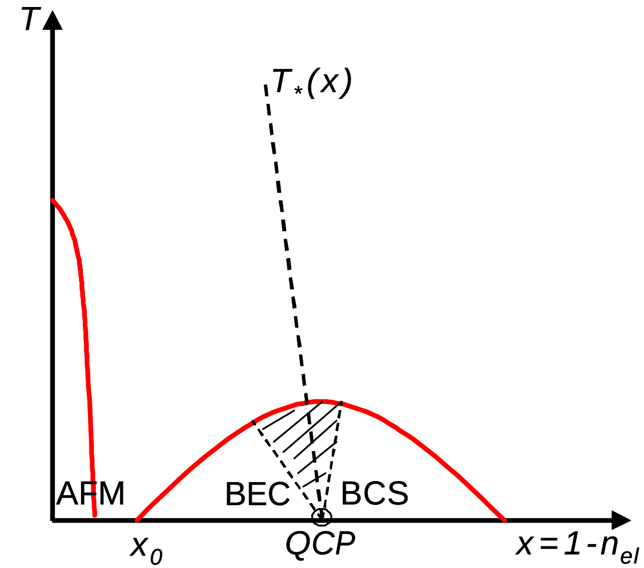
The coupling constant $f_0 = \frac{1}{\left(\ln \frac{4W}{\varepsilon_F} + \frac{\pi J}{J_{CO} - J} \right)}$ for $J < J_{CO} = 2t$ (Kagan, Rice)

For $J/t \to 0$: $f_0 = \frac{1}{\ln \frac{4W}{\varepsilon_F}}$ as in the 2D Hubbard model

p-wave pairing is dominant for $J < t$ and low electron densities $n_{el} = \frac{2\varepsilon_F}{W} << 1.$

Appealing features of the BCS-BEC crossover scenario.

BCS-BEC crossover in the d-wave channel contains and in principle could explain the following features:



S. Sachdev et al. 1996
C. di Castro et al. PRL 1996

1) QCP - quantum critical point $\mu(T=0)=0$ which separates gapped BEC-pase from gappless BCS-phase with respect to quasiparticle energy in SC $d_{x^2-y^2}$ -channel

$$E_p = \sqrt{(p^2/2m - \mu)^2 + \Delta_0^2 \cos^2 2\varphi};$$

$\mu > 0$ - BCS-case, $\mu < 0$ - BEC-case

2) semiconductive behavior of the resistivity (Kagan, Beck et al. 2001)

$\rho(T) \sim T^{-\beta}$ in underdoped phase (normal bosonic metal

in our scenario for the temperatures $T_C < T < T_*$) In Kagan, Beck story $\rho(T) \sim \frac{1}{\sqrt{T}}$.

3) strange metal behavior of the resistivity $\rho(T) \sim T^{\alpha}$; $\alpha \sim (1 \div 2)$ in optimally doped and slightly overdoped phase (for $T \geq T_C$ in the Hertz-Millis critical region around QCP with respect to concentration x in our scenario)

GL-functional and global phase diagram for p-wave superfluid Fermi-gas with repulsion

For p-wave superfluids the tensor structure of a SC-gap reads (see *Vollhardt, Woelfle* book):

$$\Delta_{\alpha\beta} = \Delta(T) i(\sigma_2\sigma_i)_{\alpha\beta} A_{ik} n_k ,$$

where $\{\sigma_2, \sigma_i\}$ are the Pauli matrices, $\vec{n} = \vec{p} / p_F$ is a unit vector in momentum space, $\Delta(T)$ is a magnitude of a gap.

Finally A_{ik} is (3×3) matrix – an order parameter in triplet p-wave SC ($S_{tot} = l = 1$). Accordingly the GL-functional yields:

$$\Delta F_{GL} = \alpha\, Sp(AA^+) + \beta_1 \left| Sp A\tilde{A} \right|^2 + \beta_2 (Sp AA^+)^2 +$$
$$+ \beta_3 Sp((A^+A)(A^+A)^*) + \beta_4 Sp((AA^+)^2) + \beta_5 Sp((A^+A)(AA^+)^*),$$

where $Sp(A\tilde{A}) = \sum_{ij} A_{ij}^2$

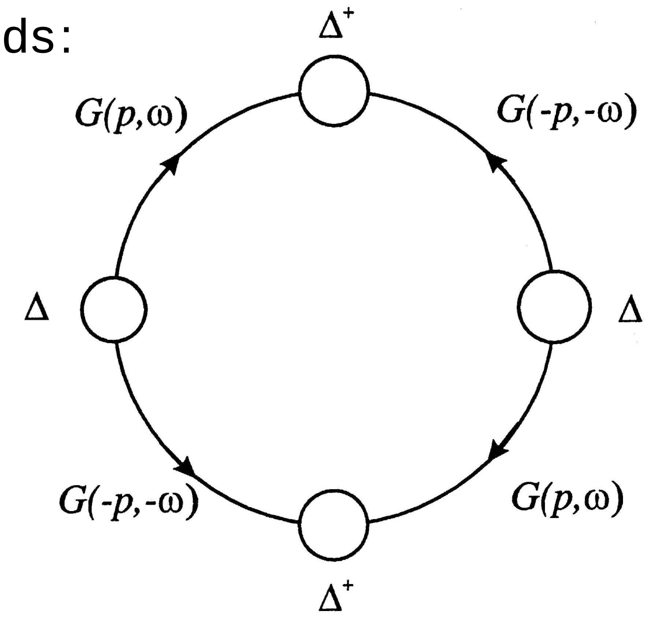
Hence GL-functional contains 5 different quartic invariants ~ A^4 and 5 different β–s.

Weak-coupling case

In weak-coupling case $\beta_i^{W.c.}$ are connected with each other by the following relation:

$$-\beta_5^{W.c.} = \beta_4^{W.c.} = \beta_3^{W.c.} = \beta_2^{W.c.} = -2\beta_1^{W.c.}$$

The skeleton diagram for the case of $\beta_i^{W.c.}$ reads:



Analytically:

$$\beta_1^{W.c.} = -\frac{2}{15}\int \frac{d\omega}{2\pi}\frac{d^3\vec{p}}{(2\pi)^3} G_M^2(\vec{p},\omega) G_M^2(-\vec{p},-\omega)\cdot \Delta^4(T)$$

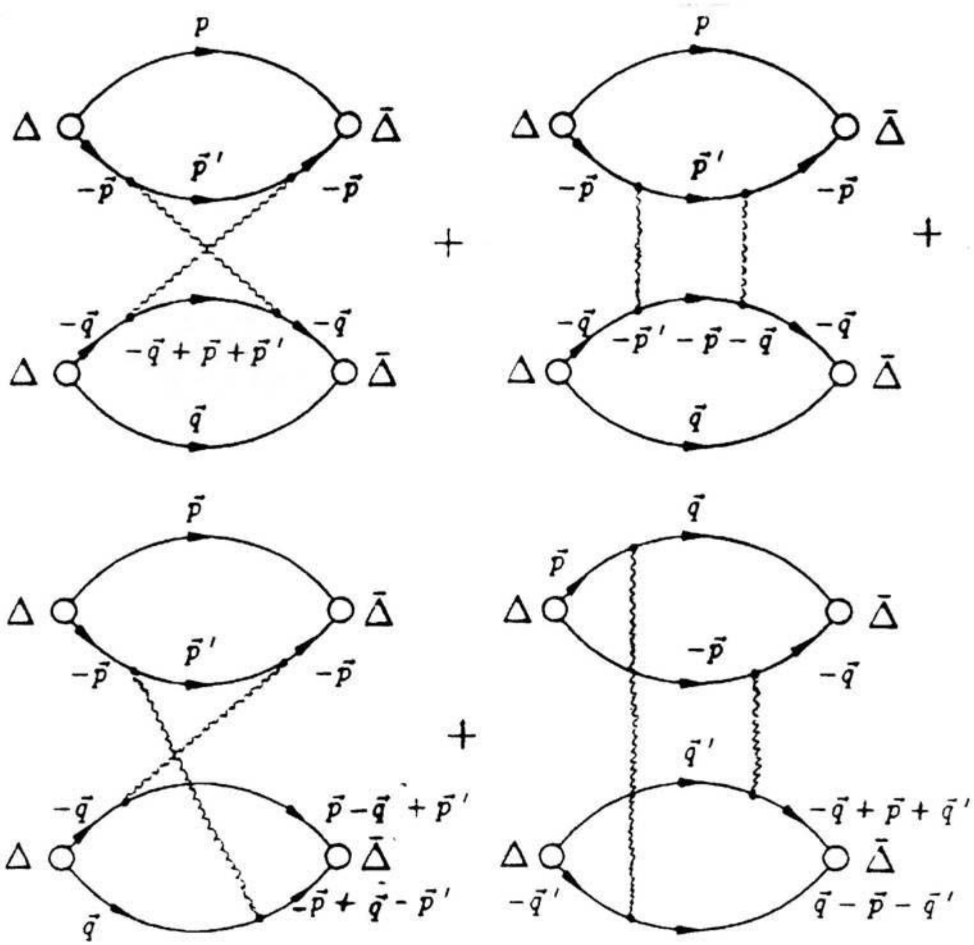
$$= \frac{N(0)}{2T_{C1}^2}\frac{7\xi(3)}{120\pi^2}\cdot \Delta^4(T),$$

where $G_M(\vec{p},\omega)$ - is fermionic Matsubara Green- function of a normal state.

The global minimum of the phase-diagram in weak coupling case corresponds to isotropic triplet B-phase with equal probabilities to have the Cooper pairs with $S_z^{tot} = -1, 0, 1$ of a total spin of a pair.

Strong coupling corrections

As it was shown by *Reiner and Serene* for superfluid [3]He strong-coupling corrections to β-s are proportional to T_{C1}/ε_F and are given by the following diagrams:



Exact values of $\beta_1 \dots \beta_5$ in superfluid Fermi-gas

For superfluid Fermi-gas *Reiner and Serene* strong-coupling corrections could be calculated explicitly *(Kagan, Baranov, Capel et al.)*

The wavy lines in strong-coupling diagrams are connected with a total two-particle vertex Γ for the Cooper channel.

In first two orders in gas parameter λ: $\Gamma \sim \lambda + \lambda^2 K + \lambda^2 \Pi$, where K is a Cooper loop and Π is polarization operator (or exchange diagram).

The ratio: $T_{C1}/\varepsilon_F = Ae^{42/\lambda}e^{-13/\lambda^2}$ is connected with exact value of T_{C1}

As a result strong-coupling corrections to β-s yield (*Kagan, Baranov,et al.*)

$$\beta_1 = \left|\beta_1^{W.c.}\right| \left\{ -1 + \frac{T_{C1}}{2\varepsilon_F}(-75.4\lambda^2 + 0.2\lambda^3) \right\}$$

$$\beta_2 = \left|\beta_1^{W.c.}\right| \left\{ 2 + \frac{T_{C1}}{2\varepsilon_F}(-70\lambda^2 + 29.4\lambda^3) \right\}$$

$$\beta_3 = \left|\beta_1^{W.c.}\right| \left\{ 2 + \frac{T_{C1}}{2\varepsilon_F}(-6.4\lambda^2 + 13.3\lambda^3) \right\}$$

$$\beta_4 = \left|\beta_1^{W.c.}\right| \left\{ 2 + \frac{T_{C1}}{2\varepsilon_F}(-48.3\lambda^2 - 108.8\lambda^3) \right\}$$

$$\beta_5 = \left|\beta_1^{W.c.}\right| \left\{ -2 + \frac{T_{C1}}{2\varepsilon_F}(-108.9\lambda^2 - 183.2\lambda^3) \right\}$$

where $\lambda = 2ap_F/\pi$ - is the gas parameter in 3D

The global minimum of GL-functional with an account of strong coupling corrections to β-s

For small values of $\lambda < 1$ the global minimum of GL-functional with an account of strong coupling corrections corresponds to isotropic B-phase. At higher densities when $\lambda \geq 1$ an anisotropic A-phase with $S_z^{tot} = \pm 1$ becomes more beneficial.

In strong magnetic fields $H > H_p = T_{C1}/\mu_B$, when we have paramagnetic suppression of $S_z^{tot} = 0$, the phase-diagram already at low densities (small values of λ) correspond to A1-phase (with $S_z^{tot} = +1$) and A2-phase (with $S_z^{tot} = \pm 1$). The hope to get a third exotic phase (different from A and B-phases) can be possibly connected with weak magnetic fields $H < H_p$ and intermediate values of $\lambda \sim 1$ (*Capel, Brussaard, unpublished*).

Note that for ^3He $\lambda = 2ap_F/\pi \sim 1.4$, where $a \sim r_0$ – hard-core radius of the ^3He-^3He interaction potential.

Note also that for $T \to 0$

$$E_S - E_N \sim -\frac{\Delta^2}{12} N(0)$$ as in a standard BCS-picture.

2D superfluid Fermi-gas with repulsion

Similar analysis in 2D (*Kagan, Mar'enko et al., 2001*) shows that an account of strong-coupling corrections stabilize an axial phase as a global minima of GL-functional in a wide range of parameters, while a 2D planar phase corresponds to a local minima.

We did not find any local minima of GL-functional corresponding to exotic phase in 2D p-wave superfuid Fermi-gas.

Two-band Emery model

The basic model for high-T_C SC is the two-band Emery model.

It describes d-electrons of Cu and p-electrons of O in the CuO-plane.

In fact it is the two-band Hubbard model with one-particle (t_{pd}) and two-particle (U_{pd}) hybridizations.

$$\hat{H} = \varepsilon_d \sum_{i\sigma} d^+_{i\sigma} d_{i\sigma} + \varepsilon_F \sum_{i\sigma} p^+_{i\sigma} p_{i\sigma} - t_{pd} \sum_{i\sigma} (p^+_{i\sigma} d_{i+1\sigma} + h.c.) - t_{pp} \sum_{i\sigma} p^+_{i\sigma} p^+_{i+2\sigma} +$$

$$+ U_{dd} \sum_i n^{\uparrow}_{id} n^{\downarrow}_{id} + U_{pp} \sum_i n^{\uparrow}_{ip} n^{\downarrow}_{ip} + \frac{U_{pd}}{2} \sum_i n_{ip} n_{i+1d}$$

The largest parameter of the model $U_{dd} \sim 8eV$;

$\Delta = \varepsilon_d - \varepsilon_p = 3eV$ - charge-transfer gap, $t_{pd} \sim 1,5eV$.

Thus $U_{dd} >> \{\Delta, t_{pd}\}$ - high-T_C materials are charge-transfer insulators (Sawatzky, Zaanen)

Kondo-lattice model

For $t_{pp} \neq 0$ but $U_{dp} = U_{pp} = 0$ we can get Kondo-lattice model from the two-band Emery model by canonical transformation of Rice, Tsunetsugu, Troyer

$\hat{H} = -t_{pp} \sum_{i\sigma} p^{+}_{i\sigma} p_{i+1\sigma} + J_{pd} \sum_{i} \vec{S}_{id} \vec{\sigma}_{i+1p} + J_{dd} \sum_{i} \vec{S}_{id} \vec{S}_{i+2d} + U_{dd} \sum_{i} n^{\uparrow}_{id} n^{\downarrow}_{id}$, where $J_{pd} = J_K \sim \frac{t^2_{pd}}{\Delta} > 0$ - is Kondo-exchange between local spin on Cu-site $\vec{S}$ and a spin of conductivity p-electron on O-site $\vec{\sigma}$, $J_{dd} \sim \frac{t^4_{pd}}{\Delta^3}$ - is AFM superexchange interaction between 2 local spins on neighboring Cu-sites. Kondo-singlets (or Zhang-Rice singlets) are local for $J_K \sim J_{pd} >> t_{pp}$. They are created at higher temperatures $T_* \sim J_K$ and play the role of holes in KLM while unpaired spins on Cu-sites play the role of electrons.

For $\frac{t^2_{pd}}{\Delta} \sim J_K >> t_{pp}$ the critical temperature in analogy with Kagan-Rice result for t-J model reads:

$$T_{cd} \sim t^{eff}_{dd} n_d \exp\left\{-\frac{t^{eff}_{dd}}{J_{dd} n^2_d}\right\} \sim \frac{t^2_{pd}}{\Delta} n_d \exp\left\{-\frac{\Delta^2}{t^2_{pd} n^2_d}\right\}$$

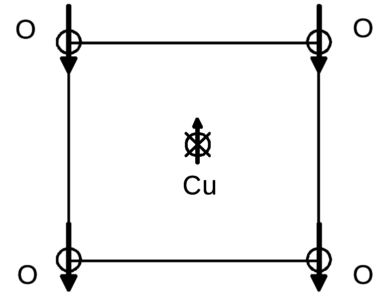
(Kagan 1999 Lecture course in MEPI)

The same result was obtained in Hubbard diagrammatic technique by Barabanov, Val'kov et al. JETP Lett. 2008

Zhang-Rice construction

For $t_{pp} = 0:\ U_{dp} = U_{pp} = 0$ and $U_{dd} >> \{\Delta; t_{pd}\}$ two-band Emery model can be reduced by Zhang-Rice construction to the generalized t-J model with $J < t$ (see also Unger, Fulde, Zaanen-Oles, Hybertsen-Schlüter, Fukujama et al.).

In this model $t_{eff} \sim \frac{t_{pd}^2}{\Delta}$ and $J_{eff} \sim \frac{t_{pd}^4}{\Delta^3}$ - superexchange. The role of holes in the generalized t-J model play Zhang-Rice singlets organized by Cu-spin in the surrounding of linear combination of 4 O-spins. The singlets are centered on Cu-sites. Thus the effective Cu-site in the generalized t-J model is CuO_4 elementary plaquette. The role of electrons in the model play unpaired Cu-spin.



elementary CuO_4-plaquette

Kondo-Hubbard model

For $t_{pp} \neq 0$ ($t_{pp} \sim 0,3eV$ in experiment); $U_{pd} \neq 0$ ($U_{pd} \sim 2eV$);
$U_{pp} \neq 0$ ($U_{pp} \sim (3-4)eV$) the two-band Emery model can be reduced to more complicated Kondo-Hubbard model (Zaanen; Maekawa 1998)

$$\hat{H} = -t_{pp}\sum_{i\sigma} p^{+}_{i\sigma} p_{i+2\sigma} + J_{pd}\sum_{i}\vec{S}_{id}\vec{\sigma}_{i+1p} + J_{dd}\sum_{i}\vec{S}_{id}\vec{S}_{i+2d} +$$

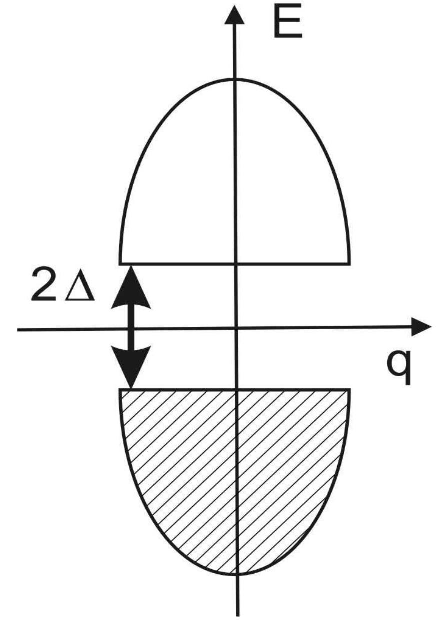
$$+U_{dd}\sum_{i} n^{d}_{i\uparrow} n^{d}_{i\downarrow} + U_{pp}\sum_{i} n^{\uparrow}_{ip} n^{\downarrow}_{ip} + \frac{U_{pd}}{2}\sum_{i} n_{id} n_{i+1p}$$

Instability of CO state for *n≠0,5*

at <u>*n = 0.5*</u>

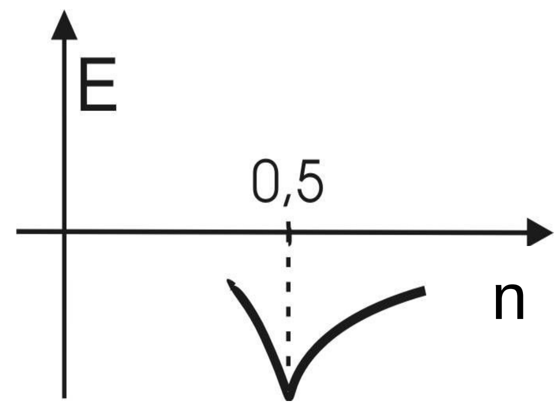
$$E_{CO} - E_N = -\frac{W^2}{6Vz}$$

$2\Delta = Vz$ - a gap in CO state



for <u>$n \neq 0.5$</u> an energy of CO state has a **cusp:**

$$\kappa^{-1} = \frac{d^2E}{dn^2} = \frac{d\mu}{dn} < 0$$



CO state is **unstable** for $n \neq 0.5$ towards phase - separation

Anomalous superconductivity and resistivity in the two-band model.

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