STOCHASTIC DYNAMO IN RANDOM ACOUSTIC FIELDS

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The problem on excitation of the magnetic field (stochastic dynamo) by random acoustic field of velocities is considered on the basis of the functional method of the method of successive approximations. Under conditions of absence of acoustic wave attenuation in the first (diffusion) approximation, the statistical Lyapunov characteristic parameter of magnetic field energy $\alpha = -\lim_{t\to\infty} \frac{\partial}{\partial t} \langle \ln E(\boldsymbol{r}, t) \rangle$ vanishes. This means that no structure formation (clustering) is present in the magnetic field realizations in the scope of this approximation. The possibility of clustering magnetic field energy is governed by the sign of the statistical Lyapunov characteristic parameter calculated in the second approximation of the method. It is shown that clustering driven by the acoustic field velocity is realized with probability one, i.e., almost in every individual realization. The characteristic setup time of clustering is evaluated.

Introduction

An important issue in the theory of magnetohydrodynamic turbulence is the treatment of diffusion at the initial phases of development. The basic equation is the induction equation for a solenoidal magnetic field $\boldsymbol{H}(\boldsymbol{r},t)$ in the kinematic approximation

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial \boldsymbol{r}}\boldsymbol{u}(\boldsymbol{r},t)\right)\boldsymbol{H}(\boldsymbol{r},t)=\left(\boldsymbol{H}(\boldsymbol{r},t)\cdot\frac{\partial}{\partial \boldsymbol{r}}\right)\boldsymbol{u}(\boldsymbol{r},t),\qquad \boldsymbol{H}(\boldsymbol{r},0)=\boldsymbol{H}_0. \tag{1}$$

Here $\boldsymbol{u}(\boldsymbol{r},t)$ is the field of turbulent velocities, which are considered homogeneous and isotropic in space and stationary in time gaussian field with the given statistical properties.

Dynamic system (1) is conservative, and the magnetic flux $\int d\boldsymbol{r}\,\boldsymbol{H}(\boldsymbol{r},t)$ remain constant during the evolution. For homogeneous initial condition $\boldsymbol{H}(\boldsymbol{r},0)=\boldsymbol{H}_0$, that we consider here, the following equality is a corollary of the conservatism of dynamic systems (1) $\langle\boldsymbol{H}(\boldsymbol{r},t)\rangle=\boldsymbol{H}_0$, where $\langle\cdots\rangle$ denotes averaging over an ensemble of realizations of random velocity field $\{\boldsymbol{u}(\boldsymbol{r},t)\}$.

A specific feature of Eq. (1) is the parametric excitation with time *in each realization* of the magnetic field energy $E(\boldsymbol{r},t) = \boldsymbol{H}^2(\boldsymbol{r},t)$ (for a turbulent fluid flow), which is called the *stochastic dynamo*.

Such a parametric excitation is accompanied by the increase of all statistical characteristics of the problem solution (such as moment and correlation functions of any order) with time.

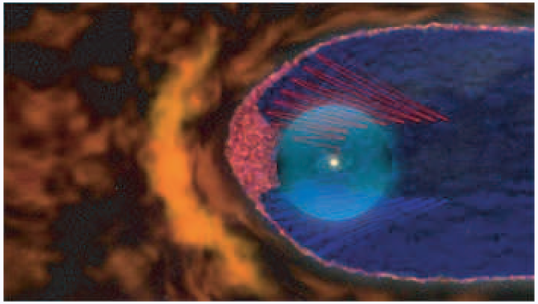
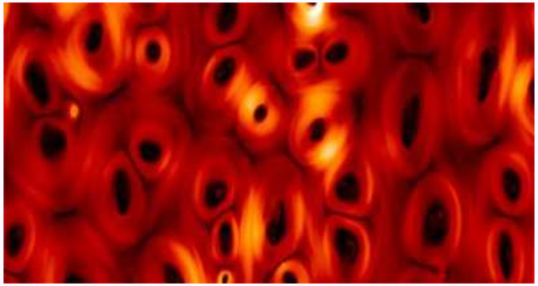
On the other hand, *separate field realizations* of magnetic field energy can show the stochastic nonstationary phenomenon of *clustering* in phase and physical spaces.

Clustering of a field is identified as the emergence of compact areas with large values of this field against the residual background of areas where these values are fairly low.

I illustrate structure formation in magnetic field by the extract from an internet-page:

"*What does puzzle astrophysicists so strongly?*

Contrary to hypotheses formed for fifty years, at the boundary of planetary system observers encountered not a linear and gradually decreasing magnetic field (or magnetic *laminar*), but a boiling foam of locally magnetized areas each of hundreds of millions kilometers in extent, which form a non-stationary cellular structure in which magnetic field lines are permanently breaking and recombining to form new areas—*magnetic "bubbles"*.

Statistical characteristics of the velocity field

In the general case, random field $\boldsymbol{u}(\boldsymbol{r},t)$ is assumed to be the divergent $(\operatorname{div}\boldsymbol{u}(\boldsymbol{r},t) \neq 0)$ Gaussian field with zero-valued mean and correlation and spectral tensors $(\tau = t - t')$

$$\langle u_i(\boldsymbol{r},t)u_j(\boldsymbol{r}_1,t')\rangle = \sigma_{\boldsymbol{u}}^2 B_{ij}(\boldsymbol{r}-\boldsymbol{r}',\tau) = \sigma_{\boldsymbol{u}}^2 \int d\boldsymbol{k}\, E_{ij}(\boldsymbol{k},\tau)e^{i\boldsymbol{k}(\boldsymbol{r}-\boldsymbol{r}')}, \qquad (2)$$

where $\sigma_{\boldsymbol{u}}^2 = \left\langle \boldsymbol{u}^2(\boldsymbol{r},t)\right\rangle$ is the variance of the velocity field. For a spatially homogeneous and isotropic random velocity field, the spatial spectral tensor has the form

$$E_{ij}(\boldsymbol{k},\tau) = E_{ij}^{\mathrm{s}}(\boldsymbol{k},\tau) + E_{ij}^{\mathrm{p}}(\boldsymbol{k},\tau),$$

where the spectral components have the following structure

$$E_{ij}^{\mathrm{s}}(\boldsymbol{k},\tau) = E^{\mathrm{s}}(k,\tau)\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right), \quad E_{ij}^{\mathrm{p}}(\boldsymbol{k},\tau) = E^{\mathrm{p}}(k,\tau)\frac{k_i k_j}{k^2}.$$

Here, $E^{\mathrm{s}}(k,\tau)$ and $E^{\mathrm{p}}(k,\tau)$ are the solenoidal and potential components of the spectral density of the velocity field, respectively.

Introduce the parameters D^{s} and D^{s} that describe statistical properties of problem solution in the diffusion approximation,

$$D^{\mathrm{s}} = \frac{1}{d-1}\int\limits_0^\infty d\tau \;\langle \boldsymbol{\omega}(\boldsymbol{r}, t+\tau)\boldsymbol{\omega}(\boldsymbol{r}, t)\rangle, \quad D^{\mathrm{p}} = \int\limits_0^\infty d\tau \;\left\langle \frac{\partial \boldsymbol{u}(\boldsymbol{r}, t+\tau)}{\partial \boldsymbol{r}}\frac{\partial \boldsymbol{u}(\boldsymbol{r}, t)}{\partial \boldsymbol{r}}\right\rangle, \tag{3}$$

where d is the dimension of space and $\boldsymbol{\omega}(\boldsymbol{r}, t) = \boldsymbol{\nabla}\times\boldsymbol{u}(\boldsymbol{r}, t)$ is the curl of the velocity field.

Note that determination of different statistical averages concerned with solving stochastic equations assumes necessity of splitting correlations of random field $\boldsymbol{u}(\boldsymbol{r}, t)$ with different functionals of this field like $R[t; \boldsymbol{u}(\boldsymbol{r}, \tau)]$, where $0 < \tau \leq t$. In the case of the Gaussian field homogeneous in space and stationary in time with the correlation function (2) this splitting is performed by the Furutsu–Novikov formula

$$\langle u_i(\boldsymbol{r}, t) R[t; \boldsymbol{u}(\boldsymbol{r}, \tau)]\rangle = \sigma_{\boldsymbol{u}}^2 \int\limits_0^t dt' \int d\boldsymbol{r}' B_{ij}(\boldsymbol{r}-\boldsymbol{r}', t-t')\left\langle \frac{\delta R[t; \boldsymbol{u}(\boldsymbol{r}, \tau)]}{\delta u_j(\boldsymbol{r}', t')}\right\rangle.$$

In particular, in the diffusion approximation, the Lyapunov characteristic parameter $\alpha = -\lim_{t\to\infty} \frac{\partial}{\partial t} \langle \ln E(\boldsymbol{r}, t) \rangle$ for magnetic field energy is given by the formula

$$\alpha = 2\sigma_{\boldsymbol{u}}^2 \frac{d-1}{d+2} \left(D^p - D^s\right), \tag{4}$$

and, in this case, clustering of magnetic field energy is realized ***in separate realizations*** with probability one only if $\alpha > 0$, i.e. under the condition $D^{\mathrm{p}} > D^{\mathrm{s}}$. The characteristic time of cluster structure formation is governed by the Lyapunov characteristic exponent $t \sim 1/\alpha$.

In the opposite case, for $D^{\mathrm{p}} \leq D^{\mathrm{s}}$, particular realizations show only general increase of magnetic field energy with time in the whole of space. In particular, ***clustering is absent*** for a noncompressible flow ($D^{\mathrm{p}} = 0$), but ***clustering is present*** for a potential field.

For the random Gaussian acoustic velocity field $\boldsymbol{u}(\boldsymbol{r}, t)$ statistically homogeneous and isotropic in space and stationary in time, the correlation and spectral tensors ($\tau = t - t'$) have the form

$$\langle u_i(\boldsymbol{r}, t) u_j(\boldsymbol{r}', t') \rangle = \sigma_{\boldsymbol{u}}^2 B_{ij}(\boldsymbol{r} - \boldsymbol{r}', \tau) = \sigma_{\boldsymbol{u}}^2 \int d\boldsymbol{k}\, E_{ij}(\boldsymbol{k}) f(\boldsymbol{k}, \boldsymbol{r}, \tau),$$

where $\sigma_{\boldsymbol{u}}^2 = \left\langle \boldsymbol{u}^2(\boldsymbol{r}, t) \right\rangle$ is the variance of the velocity field and functions $f(\boldsymbol{k}, \boldsymbol{r}, \tau)$ and $E_{ij}(\boldsymbol{k})$ are as follows

$$f(\boldsymbol{k}, \boldsymbol{r}, \tau) = e^{-\lambda_{\mathrm{p}} k^2 \tau} \cos\{\boldsymbol{k}\boldsymbol{r} - \omega(k)\tau\} \quad E_{ij}(\boldsymbol{k}) = E(k) \frac{k_i k_j}{k^2}.$$

Integral of this function at $\boldsymbol{r} = 0$ with respect to time equals

$$I(k) = \int_0^\infty dt\;\, f(k, \mathbf{0}, t) = \frac{\lambda_p}{\lambda_{\mathrm{p}}^2 k^2 + c^2},$$

and, consequently, diffusion coefficient D^{p} vanishes for $\lambda_{\mathrm{p}} \to 0$.

Note that, in the general case, the integral $\int_0^\infty dt\;\, \cos\{\omega(k)t\} = \pi\delta\left(\omega(k)\right)$ describes resonance features of an acoustic field in the higher approximations of the perturbation theory.

Basic equations

We demonstrate the general idea of the method of successive approximations by the example of the equation of magnetic field induction (1).

The local behavior of magnetic field realizations $\boldsymbol{H}(\boldsymbol{r},t)$ in the random velocity field $\{\boldsymbol{u}(\boldsymbol{r},t)\}$ is described in terms of magnetic field probability density. To determine this density, we introduce the indicator function of magnetic field $\varphi(\boldsymbol{r},t;\boldsymbol{H}) = \delta(\boldsymbol{H}(\boldsymbol{r},t)-\boldsymbol{H})$ concentrated on surface $\boldsymbol{H}(\boldsymbol{r},t) = \boldsymbol{H} = \text{const}$. This function satisfies the linear Liouville equation

$$\frac{\partial}{\partial t}\varphi(\boldsymbol{r},t;\boldsymbol{H}) = \widehat{N}(\boldsymbol{r},t;\boldsymbol{H})\varphi(\boldsymbol{r},t;\boldsymbol{H}), \qquad (5)$$

with the initial condition $\varphi(\boldsymbol{r},0;\boldsymbol{H}) = \varphi_0(\boldsymbol{H}) = \delta(\boldsymbol{H}(\boldsymbol{r},0)-\boldsymbol{H}_0)$, where operator $\widehat{N}(\boldsymbol{r},t;\boldsymbol{H})$

$$\widehat{N}(\boldsymbol{r},t;\boldsymbol{H}) = -\frac{\partial}{\partial r_k}u_k(\boldsymbol{r},t) - \frac{\partial u_k(\boldsymbol{r},t)}{\partial r_l}\frac{\partial}{\partial H_k}H_l + \frac{\partial u_k(\boldsymbol{r},t)}{\partial r_k}\left(1+\frac{\partial}{\partial H_l}H_l\right).$$

Rewrite the Liouville equation in the form of an integral equation

$$\varphi(\boldsymbol{r},t;\boldsymbol{H}) = \varphi_0(\boldsymbol{H}) + \int\limits_0^t d\tau \widehat{N}(\boldsymbol{r},\tau;\boldsymbol{H})\varphi(\boldsymbol{r},\tau;\boldsymbol{H}).$$

The first variational derivative of the indicator function

$$S_i(\boldsymbol{r},t;\boldsymbol{r}',t';\boldsymbol{H}) = \frac{\delta\varphi(\boldsymbol{r},t;\boldsymbol{H})}{\delta u_i(\boldsymbol{r}',t')}$$

satisfies, for $0 \le t' \le t$, the following stochastic integral equation

$$S_i(\boldsymbol{r},t;\boldsymbol{r}',t';\boldsymbol{H}) = \widehat{N_i}(\boldsymbol{r},\boldsymbol{r}';\boldsymbol{H})\varphi(\boldsymbol{r},t';\boldsymbol{H})\theta(t-t') + \int\limits_{t'}^t d\tau \widehat{N}(\boldsymbol{r},\tau;\boldsymbol{H})S_i(\boldsymbol{r},\tau;\boldsymbol{r}',t';\boldsymbol{H}),$$

where new operator $\widehat{N_i}(\boldsymbol{r},\boldsymbol{r}';\boldsymbol{H}) = \dfrac{\delta\widehat{N}(\boldsymbol{r},t;\boldsymbol{H})}{\delta u_i(\boldsymbol{r}',t')}$ can be represented in the form

$$\widehat{N}_i(\boldsymbol{r},\boldsymbol{r}';\boldsymbol{H}) = -\delta(\boldsymbol{r}-\boldsymbol{r}')\frac{\partial}{\partial r_i} - \frac{\partial\delta(\boldsymbol{r}-\boldsymbol{r}')}{\partial r_l}\frac{\partial}{\partial H_i}H_l + \frac{\partial\delta(\boldsymbol{r}-\boldsymbol{r}')}{\partial r_i}\frac{\partial}{\partial H_l}H_l.$$

In a similar way, the second variational derivative

$$S_{ij}(\boldsymbol{r},t;\boldsymbol{r}',t';\boldsymbol{r}'',t'';\boldsymbol{H}) = \frac{\delta^2\varphi(\boldsymbol{r},t;\boldsymbol{H})}{\delta u_i(\boldsymbol{r}',t')\delta u_j(\boldsymbol{r}'',t'')}$$

satisfies the stochastic integral equation

$$\begin{aligned} &S_{ij}(\boldsymbol{r},t;\boldsymbol{r}',t';\boldsymbol{r}'',t'';\boldsymbol{H}) \\ &\qquad = \widehat{N}_i(\boldsymbol{r},\boldsymbol{r}';\boldsymbol{H})S_j(\boldsymbol{r},t';\boldsymbol{r}'',t'';\boldsymbol{H})\theta(t-t')\theta(t'-t'') \\ &\qquad + \widehat{N}_j(\boldsymbol{r},\boldsymbol{r}'';\boldsymbol{H})S_i(\boldsymbol{r},t'';\boldsymbol{r}',t';\boldsymbol{H})\theta(t-t'')\theta(t''-t') \\ &\qquad + \int\limits_{\max\{t',t''\}}^{t} d\tau\, \widehat{N}(\boldsymbol{r},\tau;\boldsymbol{H})S_{ij}(\boldsymbol{r},\tau;\boldsymbol{r}',t';\boldsymbol{r}'',t'';\boldsymbol{H}). \end{aligned}$$

The one-point probability density of the solution to the dynamic equation (1) coincides with the indicator function averaged over an ensemble of random velocity field realizations

$$P(\boldsymbol{r},t;\boldsymbol{H}) = \langle\varphi(\boldsymbol{r},t;\boldsymbol{H})\rangle\,.$$

Statistical averaging

Average equation (5) over an ensemble of realizations of random field $\{\boldsymbol{u}(\boldsymbol{r},t)\}$. Using the Furutsu–Novikov formula, we obtain an nonclosed equation in probability density independent of spatial point $\{\boldsymbol{r}\}$,

$$\frac{\partial}{\partial t}P(t;\boldsymbol{H}) = -\sigma_{\boldsymbol{u}}^2 \int\limits_0^t dt' \int d\boldsymbol{r}' \frac{\partial B_{ki}(\boldsymbol{r}-\boldsymbol{r}',t-t')}{\partial r_l} \frac{\partial}{\partial H_k} H_l \left\langle S_i(\boldsymbol{r},t;\boldsymbol{r}',t';\boldsymbol{H})\right\rangle$$
$$+ \sigma_{\boldsymbol{u}}^2 \int\limits_0^t dt' \int d\boldsymbol{r}' \frac{\partial B_{ki}(\boldsymbol{r}-\boldsymbol{r}',t-t')}{\partial r_k} \left(1 + \frac{\partial}{\partial H_l} H_l\right) \left\langle S_i(\boldsymbol{r},t;\boldsymbol{r}',t';\boldsymbol{H})\right\rangle . \quad (6)$$

Averaging then the stochastic equation in the first variational derivative over an ensemble of realizations of random field $\{\boldsymbol{u}(\boldsymbol{r},t)\}$, we obtain an equation in average value of the first variational derivative $\left\langle S_l\left(\boldsymbol{r},t;\boldsymbol{r}',t';\boldsymbol{H}\right)\right\rangle$

$$
\begin{aligned}
&\left\langle S_i\left(\boldsymbol{r},t;\boldsymbol{r}',t';\boldsymbol{H}\right)\right\rangle - \widehat{N_i}(\boldsymbol{r},\boldsymbol{r}';\boldsymbol{H})P(t';\boldsymbol{H})\theta(t-t') \\
&\quad= -\sigma_{\boldsymbol{u}}^2\int\limits_{t'}^{t} d\tau \int\limits_{0}^{\tau} dt'' \int d\boldsymbol{r}'' \frac{\partial}{\partial r_k} B_{kj}(\boldsymbol{r}-\boldsymbol{r}'',\tau-t'')\left\langle S_{ij}(\boldsymbol{r},\tau;\boldsymbol{r}',t';\boldsymbol{r}'',t'';\boldsymbol{H})\right\rangle \\
&- \sigma_{\boldsymbol{u}}^2\int\limits_{t'}^{t} d\tau \int\limits_{0}^{\tau} dt'' \int d\boldsymbol{r}'' \frac{\partial B_{kj}(\boldsymbol{r}-\boldsymbol{r}'',\tau-t'')}{\partial r_l}\frac{\partial}{\partial H_k} H_l\left\langle S_{ij}(\boldsymbol{r},\tau;\boldsymbol{r}',t';\boldsymbol{r}'',t'';\boldsymbol{H})\right\rangle \\
&\qquad + \sigma_{\boldsymbol{u}}^2\int\limits_{t'}^{t} d\tau \int\limits_{0}^{\tau} dt'' \int d\boldsymbol{r}'' \frac{\partial B_{kj}(\boldsymbol{r}-\boldsymbol{r}'',\tau-t'')}{\partial r_k}\left(1+\frac{\partial}{\partial H_l}H_l\right) \\
&\qquad\qquad \times\left\langle S_{ij}(\boldsymbol{r},\tau;\boldsymbol{r}',t';\boldsymbol{r}'',t'';\boldsymbol{H})\right\rangle .
\end{aligned}
$$

The right-hand side of this equation includes average values of the second variational derivatives.

Average value of second variational derivative is given by the expression

$$
\begin{aligned}
&\langle S_{ij}(\boldsymbol{r},\tau;\boldsymbol{r}',t';\boldsymbol{r}'',t'';\boldsymbol{H})\rangle \\
&\qquad = \widehat{N}_i(\boldsymbol{r},\boldsymbol{r}';\boldsymbol{H})\,\langle S_j(\boldsymbol{r},t';\boldsymbol{r}'',t'';\boldsymbol{H})\rangle\,\theta(\tau-t')\theta(t'-t'') \\
&\qquad + \widehat{N}_j(\boldsymbol{r},\boldsymbol{r}'';\boldsymbol{H})\,\langle S_i(\boldsymbol{r},t'';\boldsymbol{r}',t';\boldsymbol{H})\rangle\,\theta(\tau-t'')\theta(t''-t') \\
&\qquad\qquad + \int\limits_{\max\{t',t''\}}^{\tau} d\tau_1\,\left\langle \widehat{N}(\boldsymbol{r},\tau_1;\boldsymbol{H})S_{ij}(\boldsymbol{r},\tau_1;\boldsymbol{r}',t';\boldsymbol{r}'',t'';\rho)\right\rangle,
\end{aligned}
$$

in which the last term (proportional to $\sigma_{\boldsymbol{u}}^2$) contains the third variaional derivatives. We will neglect the last term because we will use the method of successive approxiations considering the effects of the order of $\sigma_{\boldsymbol{u}}^4$.

Thus, we arrived at the closed system of equations in the probability density and average value of the first variational derivative with a very great number of terms.

Solving this system by the method of successive approximations to small terms of the order of $\sigma_{\boldsymbol{u}}^2$, we arrive at the operator equation in the probability density, which is valid in the second order of the method of successive approximations

$$\frac{\partial}{\partial t}P(t;\boldsymbol{H}) = \widehat{M}(t;\boldsymbol{H})P(t;\boldsymbol{H}),$$

where operator $\widehat{M}(t;\boldsymbol{H}) = \widehat{M}_1(t;\boldsymbol{H}) + \widehat{M}_2(t;\boldsymbol{H})$. The first term $\widehat{M}_1$ correspond to the ***diffusion approximation*** and is the term of the first order in parameter $\sigma_{\boldsymbol{u}}^2$. The second term $\widehat{M}_2$ corresponds to the approximation of the second order.

First (diffusion) approximation

In the scope of the first approximation, equation in the probability density of random acoustic field has the form

$$\frac{\partial}{\partial t}P(t;\boldsymbol{H}) = \sigma_{\boldsymbol{u}}^2 \frac{D_{\boldsymbol{H}}^{(1)}}{d(d+2)} \left\{ \frac{\partial^2}{\partial H_i \partial H_i} H_{\mathbf{p}} H_{\mathbf{p}} + \left(d^2 - 2\right) \frac{\partial^2}{\partial H_k \partial H_{\mathbf{p}}} H_k H_{\mathbf{p}} \right\} P(t;\boldsymbol{H}),$$

where the diffusion coefficient in $\boldsymbol{H}$–space has the form

$$D_{\boldsymbol{H}}^{(1)} = \sigma_{\boldsymbol{u}}^2 \frac{\lambda_{\mathbf{p}}}{c^2} \int d\boldsymbol{k} \;\; k^2 E(k).$$

Correspondingly, the equation in probability density of magnetoc field energy $E(\boldsymbol{r},t) = \boldsymbol{H}^2(\boldsymbol{r},t)$ is lognormal, and the Lyapunov characteristic exponent is

$$\alpha = 2\frac{d-1}{d+2} D_{\boldsymbol{H}}^{(1)}.$$

Consequently, clustering of magnetic field energy is realized with probability one, i.e., it occurs in almost all realizations. The setup time of cluster structure depends on dissipative factor. In the absence of wave field attenuation, parameter $\alpha = 0$ and, as we mentioned earlier, the second approximation is required for answering whether clustering is present in particular realizations, or not.

Second approximation for magnetic field

The above equation with operator $\widehat{M}(t; \boldsymbol{H})$ is very cumbersome in the general case. Bearing in mind that our main interest consists in answering whether magnetic field energy will show clustering or general increase with time in particular realizations, we confine ourselves to calculating paramer α in the second approximation. From the structure of operator $\widehat{M}(t; \boldsymbol{H})$ follows that only few terms will contribute to the Lyapunov characteristic parameter α. Namely, only seven terms can be obtained for operator $\widehat{M_2}(t; \boldsymbol{H})$.

Multiplying this equation by $\ln E$, integrating it by parts with respect to $\boldsymbol{H}$, and calculating asymptotic form of time-dependent integrals for $t \to \infty$, we obtain that average logarithm of energy is negative in the three-dimensional case ($d = 3$)

$$\frac{\partial}{\partial t} \langle \ln E(\boldsymbol{r}, t) \rangle = -\sigma_{\boldsymbol{u}}^4 \frac{76\pi^2}{c^3} \int k^4 d\boldsymbol{k}\ E^2(k).$$

Consequently, the Lyapunov exponent in the second order of the method of successive approximations has the form

$$E^*(t) = E_0 e^{\langle \ln E(\boldsymbol{r},t) \rangle} = E_0 e^{-\alpha_2 t},$$

where the Lyapunov characteristic parameter is

$$\alpha_2 = \frac{\sigma_{\boldsymbol{u}}^2}{c^2} \int d\boldsymbol{k}\ \ k^2 E(k) \left[\frac{4}{5} \lambda_{\mathbf{p}} + 76\pi^2 \frac{\sigma_{\boldsymbol{u}}^2}{c} k^2 E(k) \right].$$

Thus, we have calculated the Lyapunov exponent for magnetic field energy in random acoustic velocity field in the second order of the method of successive approximations In this approximation, the Lyapunov exponent decreases with time, which is evidence of clustering of magnetic field energy with probability one, i.e., almost in every realization of magnetic field energy. The characteristic setup time of cluster structure of magnetic field energy is governed by the Lyapunov characteristic parameter α_2, namely $t \sim 1/\alpha_2$.

THANK YOU VERY MUCH!

Remarks.

General idea of the method of perturbation theory is suggested in paper V.I. Klyatskin, V.I. Tatarskii, Radiophysics and Quantum Electronics 14 1100 (1971).

General theory of clustering of magnetic field energy is described in review V.I. Klyatskin, Physics–Uspekhi, 54 (5) (2011) and monographs V.I. Klyatskin, *Lectures on Dynamics of Stochastic Systems*, Elsevier, Amsterdam (2011), В.И. Кляцкин *Очерки по динамике стохастических систем*, M: URSS (2012).

Application of the method of perturbation theory to diffusion of a passive scalar tracer is given in monographs V.I. Klyatskin *Stochastic Equations through the Eye of the Physicist: Basic Concepts, Exact Results and Asymptotic Approximations*, Elsevier, Amsterdam (2005) and in two Russian books (2002, 2008).

An extensive description will be published in journal Theoretical and Mathematical Physics, 2012.