Soft breaking of BRST symmetry in field-antifield formalism

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- Gribov-Zwanziger theory
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Gribov-Zwanziger theory
[Gribov (1978), Zwanziger (1993)]

Yang-Mills action [Yang,Mills (1953)]

$$S_0(A) = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}, \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc}A^b_\mu A^c_\nu$$

Gauge invariance

$$\delta S_0 = 0, \quad \delta A^a_\mu = D^{ab}_\mu \xi^b, \quad D^{ab}_\mu = \delta^{ab}\partial_\mu + f^{acb}A^c_\mu$$

Faddeev-Popov action [Faddeev,Popov (1967)]

$$S_{FP}(\Phi) = S_0(A) + \bar{C}^a K^{ab}(A) C^b + \chi^a(A) B^a$$

$$\Phi^A = (A^a_\mu, B^a, C^a, \bar{C}^a)$$

$$K^{ab}(A) = \frac{\delta \chi^a(A)}{\delta A^c_\mu} D^{cb}_\mu$$

Gribov-Zwanziger theory

BRST symmetry [Becchi,Rouet,Stora (1975),Tyutin (1975)]

$$\delta_B S_{FP}(\Phi) = 0$$

$$\begin{aligned}
\delta_B A^a_\mu(x) &= D^{ab}_\mu C^b(x)\mu \\
\delta_B C^a(x) &= \frac{1}{2} f^{abc} C^b(x) C^c(x)\mu \\
\delta_B \bar{C}^a(x) &= B^a(x)\mu \\
\delta_B B^a(x) &= 0
\end{aligned}$$

Nilpotency

$$\delta_B^2 \Phi^A = 0$$

Landau gauge

$$\partial^\mu A^a_\mu = 0, \quad K^{ab} = \partial^\mu D^{ab}_\mu = \partial^\mu \partial_\mu + f^{acb} A^c_\mu \partial^\mu$$

Gribov region

$$\Omega \equiv \{A^a_\mu,\ \partial^\mu A^a_\mu = 0, K^{ab} > 0\}$$

Zwanziger functional

$$M(A) = \gamma^2 \left(f^{abc} A^b_\mu (K^{-1})^{ad} f^{dec} A^{e\mu} + D(N^2 - 1) \right)$$

$$(K^{-1})^{ad} K^{db} = \delta^{ab}$$

D is dimension of space-time and γ is the so-called thermodynamic or Gribov parameter.

Gribov-Zwanziger theory

Gap equation

$$\frac{\partial \mathcal{E}_{vac}}{\partial \gamma} = 0$$

Vacuum energy $\mathcal{E}_{vac}$

$$\exp\left\{\frac{i}{\hbar}\mathcal{E}_{vac}\right\} = \int D\Phi\ \exp\left\{\frac{i}{\hbar}S_{GZ}(\Phi)\right\}$$

Gribov-Zwanziger action

$$S_{GZ}(\Phi) = S_{FP}(\Phi) + M(A)$$

Non-invariance

$$\frac{\delta M}{\delta A^a_\mu} D^{ab}_\mu \xi^b \neq 0, \qquad \delta_B S_{GZ} \neq 0$$

Open problems:

- In construction of the GZ action the Landau gauge is used only. What is beyond the Landau gauge?
- The construction is connected with Yang-Mills theories only. What is beyond the Yang-Mills theories?
- For the GZ theory the BRST symmetry is broken. How does this breakdown affect on physical quantities?

Our main assumptions:

- In Yang-Mills theories Gribov horizon exists not only in the Landau gauge.
- Gribov horizon may exist for general gauge theories.
- Gribov region can be described in the form of an additional functional to full action of a given gauge system. This functional destroys the BRST symmetry.

Field-antifield formalism
[Batalin, Vilkovisky, (1981,1983)]

General gauge theories

$$S_0 = S_0(A), \quad A^i, \quad i = 1, 2, \ldots, n, \quad \varepsilon(A^i) = \varepsilon_i,$$

$$\delta A^i = R^i_\alpha(A)\xi^\alpha, \quad S_{0,i}(A)R^i_\alpha(A) = 0,\ \alpha = 1, 2, \ldots, m \quad 0 < m < n\ ,$$

$$\Phi\ \equiv\ \{\Phi^A\} = \{A^i, \ldots\} \qquad \varepsilon(\Phi^A) = \varepsilon_A\ ,$$

$$\Phi^*\ \equiv\ \{\Phi^*_A\} = \{A^*_i, \ldots\}, \quad \varepsilon(\Phi^*_A) = \varepsilon_A + 1$$

$$\bar{S} = \bar{S}(\Phi, \Phi^*), \quad \frac{1}{2}(\bar{S}, \bar{S}) = i\hbar\,\Delta\bar{S}$$

$$\bar{S}|_{\Phi^* = \hbar = 0} = S_0(A)$$

Field-antifield formalism

Antibracket

$$(F,G) \equiv \frac{\delta F}{\delta \Phi^A}\,\frac{\delta G}{\delta \Phi^*_A} - (F \leftrightarrow G)\,(-1)^{[\varepsilon(F)+1]\cdot[\varepsilon(G)+1]}$$

Delta-operator

$$\Delta \equiv (-1)^{\varepsilon_A}\frac{\delta_l}{\delta \Phi^A}\,\frac{\delta}{\delta \Phi^*_A}, \quad \Delta^2 = 0, \quad \varepsilon(\Delta) = 1$$

Extended action

$$S_{ext}(\Phi,\Phi^*) = \bar{S}\Big(\Phi,\ \Phi^* + \frac{\delta\psi}{\delta\Phi}\Big)$$

Gauge fixing functional

$$\psi = \psi(\Phi), \quad \varepsilon(\psi) = 1$$

Quantum master equation

$$\frac{1}{2}(S_{ext}, S_{ext}) = i\hbar\,\Delta S_{ext}$$

Field-antifield formalism

Generating functional

$$Z(J,\Phi^*) = \int D\Phi \;\exp\Big\{\frac{i}{\hbar}\big(S_{ext}(\Phi,\Phi^*) + J_A\Phi^A\big)\Big\}$$

Ward identity for Z

$$J_A\frac{\delta Z(J,\Phi^*)}{\delta\Phi^*_A} = 0$$

BRST symmetry

$$I(\Phi,\Phi^*) = D\Phi \;\exp\Big\{\frac{i}{\hbar}S_{ext}(\Phi,\Phi^*)\Big\}, \quad \delta_B I(\Phi,\Phi^*) = 0$$

$$\delta_B\Phi^A = \mu\frac{\delta S_{ext}}{\delta\Phi^*_A}, \quad \delta_B\Phi^*_A = 0$$

Field-antifield formalism

Gauge invariance

$$Z_{\psi}(0) \equiv Z(0,0), \qquad Z_{\psi+\delta\psi}(0) = Z_{\psi}(0)$$

Ward identity for Γ

$$(\Gamma, \Gamma) = 0$$

Gauge dependence

$$\delta\Gamma = (\Gamma, \langle\delta\psi\rangle), \quad \langle\delta\psi\rangle = \delta\psi(\widehat{\Phi})$$

$$\widehat{\Phi}^A = \Phi^A + i\hbar\,(\Gamma''^{-1})^{AB}\frac{\delta_l}{\delta\Phi^B}, \quad (\Gamma'')_{AB} = \frac{\delta_l}{\delta\Phi^A}\Big(\frac{\delta\Gamma}{\delta\Phi^B}\Big)$$

Gauge independence on-shell

$$\delta\Gamma\,\Big|_{\frac{\delta\Gamma}{\delta\Phi}=0} = 0$$

Modified action

$$S = S_{ext} + M, \quad M = M(\Phi, \Phi^*)$$

$$\frac{1}{2}(M, M) = -i\hbar\,\Delta M$$

$$M = M_0 + O(\hbar), \quad (M_0, M_0) = 0$$

$$m_0 = M_0 \,|_{\Phi^*=0}, \quad m_{0,i}R^i_\alpha \neq 0$$

For the Gribov-Zwanziger theory m_0 coincides with the Zwanziger functional

Modified master equation

$$\frac{1}{2}(S, S) - i\hbar\,\Delta S = (S, M)$$

Generating functionals and Ward identities

Generating functional

$$Z(J,\Phi^*) = \int D\Phi \ \exp\Big\{\frac{i}{\hbar}\big(S(\Phi,\Phi^*) + J_A\Phi^A\big)\Big\}$$

Ward identity for Z

$$\frac{\hbar}{i}\Big(J_A + M_A\Big)\frac{\delta Z(J,\Phi^*)}{\delta\Phi^*_A} - J_A M^{A*} Z(J,\Phi^*) = 0$$

$$M_A = M_A\Big(\frac{\hbar}{i}\frac{\delta}{\delta J},\Phi^*\Big) \equiv \frac{\delta M(\Phi,\Phi^*)}{\delta\Phi^A}\Big|_{\Phi\to\frac{\hbar}{i}\frac{\delta}{\delta J}}$$

$$M^*_A = M^{A*}\Big(\frac{\hbar}{i}\frac{\delta}{\delta J},\Phi^*\Big) \equiv \frac{\delta M(\Phi,\Phi^*)}{\delta\Phi^*_A}\Big|_{\Phi\to\frac{\hbar}{i}\frac{\delta}{\delta J}}$$

Generating functional of vertex functions

$$\Gamma(\Phi,\,\Phi^*) = W(J,\Phi^*) - J_A\Phi^A, \quad \Phi^A = \frac{\delta W}{\delta J_A}, \quad Z = \exp\{i/\hbar W\}$$

Ward identity for Γ

$$\frac{1}{2}(\Gamma,\Gamma) = \frac{\delta\Gamma}{\delta\Phi^A}\widehat{M}^{A*} + \widehat{M}_A\frac{\delta\Gamma}{\delta\Phi^*_A}$$

$$\widehat{M}_A \;\equiv\; \frac{\delta M(\Phi,\Phi^*)}{\delta\Phi^A}\Big|_{\Phi\to\widehat{\Phi}}, \quad \widehat{M}^{A*} \;\equiv\; \frac{\delta M(\Phi,\Phi^*)}{\delta\Phi^*_A}\Big|_{\Phi\to\widehat{\Phi}}$$

$$\widehat{\Phi}^A = \Phi^A + i\hbar\,(\Gamma^{''-1})^{AB}\frac{\delta_l}{\delta\Phi^B}, \quad (\Gamma^{''})_{AB} = \frac{\delta_l}{\delta\Phi^A}\Big(\frac{\delta\Gamma}{\delta\Phi^B}\Big)$$

Gauge dependence

Variation of action

$$\delta S_{ext} = \frac{\delta \delta\psi}{\delta \Phi^A} \frac{\delta S_{ext}}{\delta \Phi^*_A}, \quad \psi(\Phi) \to \psi(\Phi) + \delta\psi(\Phi)$$

$$\delta S = \delta S_{ext} + \delta M$$

Gauge variation of Z

$$\delta Z(J, \Phi^*) = \frac{i}{\hbar} \int D\Phi \, \Big(\frac{\delta \delta\psi}{\delta \Phi^A} \frac{\delta S_{ext}}{\delta \Phi^*_A} + \delta M \Big) \, \exp \Big\{ \frac{i}{\hbar} \big(S(\Phi, \Phi^*) + J_A \Phi^A \big) \Big\}$$

Gauge variation of Γ

$$\delta\Gamma = \frac{\delta\Gamma}{\delta\Phi^A}\widehat{F}^A\langle\delta\psi\rangle - \widehat{M}_A\widehat{F}^A\langle\delta\psi\rangle + \langle\delta M\rangle$$

$$\widehat{F}^A = -\frac{\delta}{\delta\Phi^*_A} - (-1)^{\varepsilon_B(\varepsilon_A+1)}(\Gamma''^{-1})^{BC}\left(\frac{\delta_l}{\delta\Phi^C}\frac{\delta\Gamma}{\delta\Phi^*_A}\right)\frac{\delta_l}{\delta\Phi^B}$$

Gauge dependence on-shell

$$\frac{\delta\Gamma}{\delta\Phi^A} = 0 \qquad \longrightarrow \qquad \delta\Gamma \neq 0$$

A weak hope

$$\langle\delta M\rangle = \widehat{M}_A\widehat{F}^A\langle\delta\psi\rangle$$

In tree approximation

$$\delta M = \frac{\delta M}{\delta\Phi^A}\widehat{F}^A_0\delta\psi, \quad \widehat{F}^A_0 = -(-1)^{\varepsilon_B(\varepsilon_A+1)}(S''^{-1})^{BC}\left(\frac{\delta_l}{\delta\Phi^C}\frac{\delta S}{\delta\Phi^*_A}\right)\frac{\delta_l}{\delta\Phi^B}$$

Application to the Gribov-Zwanziger theory

$$\chi^a(A,B,\xi) = \chi^a(A) + \frac{\xi}{2}B^a = \partial^\mu A^a_\mu + \frac{\xi}{2}B^a$$

$$\psi = \bar{C}^a \chi^a(A,B,\xi), \quad \delta\psi = \frac{1}{2}\bar{C}^a B^a \delta\xi$$

$$S_{FP}(\Phi,\xi) = S_0(A) \; + \; \bar{C}^a K^{ab}(A) C^b + \chi^a(A,B,\xi) B^a$$

$$S_{GZ}(\Phi,\xi) = S_{FP}(\Phi,\xi) \; + \; M(A,\xi)$$

$$\delta M(A,\xi) \neq \frac{1}{2}\frac{\delta M}{\delta \Phi^A}\widehat{F}_0^A \bar{C}^a B^a \delta\xi$$

Conclusions

- A definition of soft breaking of BRST symmetry within the field-antifield formalism was given. It includes the Gribov-Zwanziger theory as a very special (but important) case of Yang-Mills theories in the Landau gauge.
- It was shown the gauge dependence of effective action even on shell. It means that S-matrix depends on gauge. In particular, vacuum expectation values of gauge invariant operators such as $F^{a2}_{\mu\nu}$ do depend on gauge.
- It was proven that a consistent formulation of gauge theories with soft breaking of BRST symmetry does not exist.

Thank you!