Turbulent Instabilities in Quark-Gluon PLasma

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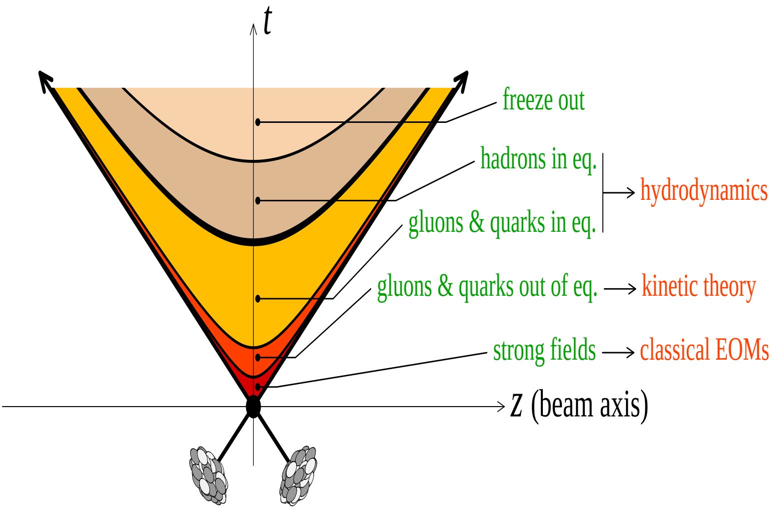
Little Bang: stages of evolution



Motivation

- Turbulent plasma is defined as a system comprising particles and, in addition to the weak mean field generated by these particles, random set of field modes
- QED plasma observed in experiments is always turbulent
- Properties of turbulent plasma are radically different from the textbook "Vlasov"one: anomalously small shear viscosity and electric conductivity, etc.
- Early stages of high energy nuclear collisions are overloaded with various instabilities both in the field (glasma) and kinetic (anisotropic plasma) regimes

Turbulent polarization: QED plasma

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- Turbulent plasma: perturbation of equilibrated system of particles by weak turbulent fields
- Collisionless Vlasov approximation

$$p^\mu \left[\partial_\mu - eq\left(F^R_{\mu\nu} + F^T_{\mu\nu}\right)\frac{\partial}{\partial p_\nu}\right] f(p,x,q) = 0$$

$$\partial^\mu \left(F^R_{\mu\nu} + F^T_{\mu\nu}\right) = j_\nu$$

$$j_\nu(x) = e\sum_{q,s}\int dP p_\nu q f(p,x,q)$$

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- Turbulent correlators:

$$\langle F^T_{\mu\nu} \rangle = 0$$

$$\langle F^{T\mu\nu}(x) F^{T\mu'\nu'}(y) \rangle = K^{\mu\nu\mu'\nu'}(x,y),$$

$$K^{\mu\nu\mu'\nu'}(x) = K_0^{\mu\nu\mu'\nu'} \exp\left[-\frac{t^2}{2\tau^2} - \frac{r^2}{2a^2} \right].$$

Turbulent polarization: QED plasma

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- Turbulent polarization:

$$\Pi^{\mu\nu}(k) = \frac{\delta\langle j^{\mu}(k|F^{R}, F^{T})\rangle_{F^{T}}}{\delta A^{R}_{\nu}}$$

$$\langle j^{\mu}(k\,|F^{R}, F^{T})\rangle_{F^{T}} = e\sum_{q,s}\int dP p_{\nu} q \langle\delta f(p, k, q|F^{R}, F^{T})\rangle_{F^{T}}$$

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- Kinetic equation:

$$f = f^{eq} + Gp^{\mu} F_{\mu\nu} \partial_p^{\mu} f \ , \qquad G \equiv \frac{eq}{\imath((pk) + \imath\epsilon)}$$

- The turbulent contribution for induced current comes from the cubic terms that are of the first order in $F^R_{\mu\nu}$ and of the second order in $F^T_{\mu\nu}$.

$$\begin{aligned} \delta f &= \sum_{m=0} \sum_{n=0} \rho^m \tau^n \delta f_{mn} \\ F^{\mu\nu} &= \sum_{m=0} \sum_{n=0} \rho^m \tau^n F^{\mu\nu}_{mn} \end{aligned}$$

- We are thus interested in computing δf_{12}

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- Relevant contribuitons to the distribution function:

$$\delta f \simeq \delta f_{\mathrm{HTL}} + \langle \delta f_{12} \rangle_{\mathrm{I}} + \langle \delta f_{12} \rangle_{\mathrm{II}}$$

where

$$\begin{aligned}
\delta f_{\mathrm{HTL}} &= G p_\mu F_{10}^{\mu\nu} \partial_{\mu,p} f^{\mathrm{eq}} \\
\langle \delta f_{12} \rangle_{\mathrm{I}} &= G p_\mu \langle F_{01}^{\mu\nu} \partial_{\nu,p} G p_{\mu'} F_{10}^{\mu'\nu'} \partial_{\nu',p} G p_\rho F_{01}^{\rho\sigma} \rangle \partial_{\sigma,p} f^{\mathrm{eq}} \\
\langle \delta f_{12} \rangle_{\mathrm{II}} &= G p_\mu \langle F_{01}^{\mu\nu} \partial_{\nu,p} G p_{\mu'} F_{01}^{\mu'\nu'} \partial_{\nu',p} G p_\rho F_{10}^{\rho\sigma} \rangle \partial_{\sigma,p} f^{\mathrm{eq}}
\end{aligned}$$

Turbulent polarization: QED plasma

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- Turbulent polarization

$$\Pi_{ij}(\omega, \mathbf{k}|\ l) = \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) \Pi_T(\omega, |\mathbf{k}|\,|\ l) + \frac{k_i k_j}{k^2} \Pi_L(\omega, |\mathbf{k}|\,|\ l)$$

$$l \equiv \sqrt{2} \frac{\tau a}{\sqrt{\tau^2 + a^2}}$$

Turbulent polarization: QED plasma

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- Turbulent polarization

$$\Pi_{L(T)}(\omega, \mathbf{k}|\ l) = \Pi^{\rm HTL}_{L(T)}(\omega, \mathbf{k}) + \Pi^{\rm turb}_{L(T)}(\omega, \mathbf{k}|\ l)$$

$$\Pi^{\rm turb}_{L(T)}(\omega, |\mathbf{k}|\,|\ l) = \sum_{n=1}^{\infty} \frac{(|\mathbf{k}|\ l)^n}{\mathbf{k}^2} \left[\phi^{(n)}_{L(T)}\left(\frac{\omega}{|\mathbf{k}|}\right) \langle E^2_{\rm turb} \rangle + \chi^{(n)}_{L(T)}\left(\frac{\omega}{|\mathbf{k}|}\right) \langle B^2_{\rm turb} \rangle \right]$$

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- Hard Thermal Loops

$$\Pi_L^{\mathrm{H}TL}(\omega, |\mathbf{k}|) = -m_D^2 x^2 \left[1 - \frac{x}{2}\; L(x)\right]$$

$$\Pi_T^{\mathrm{H}TL}(\omega, |\mathbf{k}|) = m_D^2 \frac{x^2}{2} \left[1 + \frac{1}{2x}\;(1 - x^2)\; L(x)\right]$$

$$L(x) \equiv \ln\left|\frac{1+x}{1-x}\right| - \imath\pi\theta(1-x); \quad m_D^2 = e^2 T^2/3$$

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- Turbulent instability

$$
\begin{aligned}
\mathrm{Im}\Pi_T(\omega,\mathbf{k}|\ l) &\simeq -\pi m_D^2 \frac{x}{4}(1-x^2)\theta(1-x) \\
&+ \frac{(|\mathbf{k}|\ l)}{\mathbf{k}^2}\left(\left\langle E^2\right\rangle \mathrm{Im}\phi_{\mathrm{I\ T}}(x) + \left\langle B^2\right\rangle \mathrm{Im}\chi_{\mathrm{I}}(x)\right) \\
\mathrm{Im}\Pi_L(\omega,\mathbf{k}|\ l) &\simeq -\pi m_D^2 \frac{x^3}{2}\theta(1-x) \\
&+ \frac{(|\mathbf{k}|\ l)}{\mathbf{k}^2}\left(\left\langle E^2\right\rangle \mathrm{Im}\phi_{\mathrm{I\ L}}(x) + \left\langle B^2\right\rangle \mathrm{Im}\chi_{\mathrm{I\ L}}(x)\right),
\end{aligned}
$$

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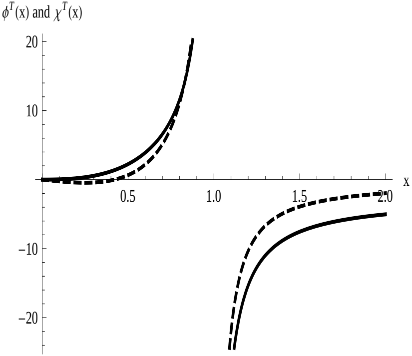
- Instability in the spacelike domain $x < 1$:



The functions $\frac{6\pi\sqrt{\pi}}{e^4}\mathrm{Im}\left[\phi_T^{(1)}(x)\right]$ (solid line) and $\frac{6\pi\sqrt{\pi}}{e^4}\mathrm{Im}\left[\chi_T^{(1)}(x)\right]$ (dashed line).

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- Instability in the spacelike domain $x < 1$ for strong enough turbulent fields
- Effect due to stochastic inhomogeneity and thus similar to stochastic transition radiation (vanishes in the limit $l \rightarrow 0$)

Turbulent polarization: QED plasma

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- The plasmons are characterized by dispersion relations $\omega_{\mathrm{T(L)}}(|\mathbf{k}|)$ that are read from the zeroes of the corresponding components of dielectric permittivity:

$$\mathbf{k}^2 \left(1 - \frac{\Pi_{\mathrm{L}}(k^0, |\mathbf{k}|)}{\omega^2}\right)\Bigg|_{k^0=\omega_{\mathrm{L}}(|\mathbf{k}|)} = 0$$

$$\mathbf{k}^2 - (k^0)^2 + \Pi_{\mathrm{T}}((k^0, |\mathbf{k}|)\ |_{k^0=\omega_{\mathrm{T}}(|\mathbf{k}|)} = 0$$

- HTL limit:

$$\omega_{\mathrm{L}}^2(|\mathbf{k}|)_{\mathrm{HTL}} = \omega_{\mathrm{pl}}^2 \left(1 + \frac{3}{5}\left(\frac{|\mathbf{k}|}{\omega_{\mathrm{pl}}}\right)^2 + O\left(\left(\frac{|\mathbf{k}|}{\omega_{\mathrm{pl}}}\right)^4\right)\right)$$

$$\omega_{\mathrm{T}}^2(|\mathbf{k}|)_{\mathrm{HTL}} = \omega_{\mathrm{pl}}^2 \left(1 + \frac{6}{5}\left(\frac{|\mathbf{k}|}{\omega_{\mathrm{pl}}}\right)^2 + O\left(\left(\frac{|\mathbf{k}|}{\omega_{\mathrm{pl}}}\right)^4\right)\right)$$

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- Turbulent corrections:

$$
\begin{aligned}
\omega_{\rm L}^2(|\mathbf{k}|)_{\rm turb} &= (\omega_{\rm pl\ L}^{\rm turb})^2 \left(1+\frac{3}{5}y_{\rm L}^2\right) \\
&- \frac{e^4 l^2}{6\pi^2}\left(\frac{24}{5}\langle E^2\rangle + \frac{64}{15}\langle B^2\rangle\right) y_{\rm L}^2 + O\left(y_{\rm L}^4\right) \\
\omega_{\rm T}^2(|\mathbf{k}|)_{\rm turb} &= (\omega_{\rm pl\ T}^{\rm turb})^2 \left(1+\frac{3}{5}y_{\rm T}^2\right) \\
&- \frac{e^4 l^2}{6\pi^2}\left(\frac{24}{7}\langle E^2\rangle + \frac{32}{15}\langle B^2\rangle\right) y_{\rm T}^2 + O\left(y_{\rm T}^4\right)
\end{aligned}
$$

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- $$y_{\rm L} = \frac{|\mathbf{k}|}{\omega_{\rm pl\,L}^{\rm turb}}; \quad y_{\rm T} = \frac{|\mathbf{k}|}{\omega_{\rm pl\,T}^{\rm turb}} \ ,$$

  and

$$\begin{aligned}
(\omega_{\rm pl\,L}^{\rm turb})^2 &= \omega_{\rm pl\,L}^2 - \frac{e^4 l^2}{6\pi^2}\left(\frac{16}{3}\langle E^2\rangle + \frac{8}{3}\langle B^2\rangle\right) \\
(\omega_{\rm pl\,T}^{\rm turb})^2 &= \omega_{\rm pl\,T}^2 - \frac{e^4 l^2}{6\pi^2}\left(\frac{128}{15}\langle E^2\rangle + \frac{8}{3}\langle B^2\rangle\right)
\end{aligned}$$

Turbulent polarization: QCD plasma

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- New contribution to turbulent thermalization:

$$\Pi^{\rm turb}_{L(T)}(\omega, k) = \frac{m_g^2}{k^2}\langle A^2\rangle \sum_{n=1}^{\infty}(kl)^n \eta^{(n)}_{L(T)}\left(\frac{\omega}{k}\right)$$

$$\eta^{(1)}_T(x) = \frac{\imath\sqrt{\pi}g^2(N^2-1)}{16N}\left(x + \frac{1}{2}(1-x^2)L(x)\right)$$

$$\eta^{(1)}_L(x)\frac{\imath\sqrt{\pi}g^2(N^2-1)}{16N}2x\left(-1 + \frac{x}{2}L(x)\right)$$

Turbulent polarization: QCD plasma

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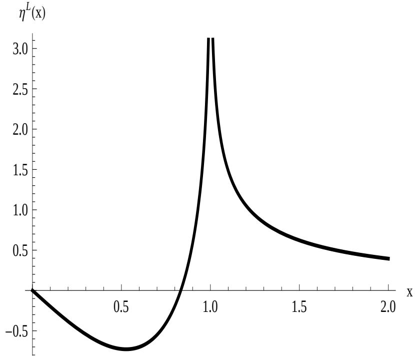
- Instability in the timelike domain $x > 1$:



Function $\frac{16N}{\sqrt{\pi}g^2(N^2-1)}\mathrm{Im}\left[\eta_L^{(1)}(x)\right]$

Turbulent polarization: QCD plasma

Conclusions

- New instabilities of stochastically nonstationary inhomogeneous ultrarelativistic QCD plasma were described