ON KINETIC THEORY OF ENERGY LOSSES IN RANDOMLY INHOMOGENEOUS MEDIUM

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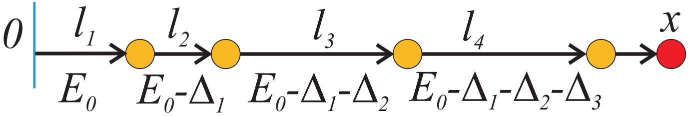
Energy loss: motivation

- Dense non-abelian medium created in ultrarelativistic heavy ion collisions is extremely inhomogeneous (turbulent) on the event-by-event basis
- Energy loss of hard particles, of both radiative and collisional origin, in heavy ion collisions is a key variable studied in experiments
- Many other interesting analogies, e.g. propagation of cosmic rays in turbulent fields

Energy loss: model



- High energy particle with energy E_0 is incident on the medium. The border of the medium is at $x = 0$
- Energy loss Δ occurs through a sequence of losses $\{\Delta_i\}$, $i = 1, \cdots, n$ through scattering on randomly placed scattering centers located along the particle trajectory at distances $l_1, l_2, \cdots, l_n, l_{n+1}$

$$\Delta = \Delta_1 + \Delta_2 + \cdots + \Delta_n. \tag{1}$$

- The goal is to calculate the distribution $f(\Delta, x)$ of energy loss Δ at some depth x

Energy loss in a homogeneous medium: Landau equation

- Landau kinetic equation on $f(\Delta, x)$ in a homogeneous medium:

$$f(\Delta,x) = \delta(\Delta) + \frac{1}{a}\int_0^x dx' \int_0^\infty d\varepsilon w(\varepsilon)\left[f(\Delta-\varepsilon,x') - f(\Delta,x')\right]$$

- $w(\varepsilon)$ is a probability distribution of loosing an energy ε in a single scattering event
- $1/a$ is a constant linear density of scattering centers

Energy loss in a random medium

- CTRW-like evolution equation for energy loss in a random medium:

$$f\left(\Delta,x\right) = \delta\left(\Delta\right)\Psi\left(x\right) + \int_0^x dx'\psi\left(x-x'\right)\int_0^\Delta d\varepsilon w\left(\varepsilon\right)f\left(\Delta-\varepsilon,x'\right) \quad (2)$$

- $\Psi\left(x\right) = \int_x^\infty dy\psi(y)$ is the probability of having no scattering events in the interval $[0,x]$
- Evolution equation (2) is conveniently solved by the double Laplace transform

$$\tilde{f}\left(p,q\right) \equiv \int_0^\infty d\Delta e^{-p\Delta}\int_0^\infty dx e^{-qx} f\left(\Delta,x\right)$$

Energy loss in a random medium

- Let us introduce a new function $\tilde{g}(q)$:

$$\tilde{g}(q) \equiv \tilde{\psi}(q) / [1 - \tilde{\psi}(q)] \tag{3}$$

- Equation for $\tilde{f}(p, q)$:

$$\tilde{f}(p, q) = 1/q + \tilde{g}(q) [\tilde{w}(p) - 1] \tilde{f}(p, q) \tag{4}$$

- The equation (4) is equivalent to the following version of the evolution equation:

$$\begin{aligned} f(\Delta, x) &= \delta(\Delta) + \int_0^x dx' g(x - x') \int_0^\infty d\varepsilon w(\varepsilon) \\ &\cdot \left[f(\Delta - \varepsilon, x') - f(\Delta, x') \right] \end{aligned} \tag{5}$$

Energy loss in a random medium: comparison

- Landau equation

$$f(\Delta,x) = \delta(\Delta) + \frac{1}{a}\int_0^x dx' \int_0^\infty d\varepsilon\, w(\varepsilon)\left[f(\Delta-\varepsilon,x') - f(\Delta,x')\right]$$

- CTRW-like equation

$$\begin{aligned} f(\Delta,x) &= \delta(\Delta) + \int_0^x dx' g(x-x') \int_0^\infty d\varepsilon\, w(\varepsilon) \\ &\cdot \left[f(\Delta-\varepsilon,x') - f(\Delta,x')\right] \end{aligned} \qquad (6)$$

Energy loss in a random medium: analytical solution

- Let us take

$$\tilde{g}(q) = (aq)^{-\alpha}, \quad \alpha = D - 2$$
$$\tilde{w}(p) \simeq 1 - p\bar{\varepsilon}, \quad \bar{\varepsilon} = \int \varepsilon w(\varepsilon) d\varepsilon$$

- Then

$$f(\Delta, x) = \frac{1}{\bar{\varepsilon}} \left(\frac{a}{x}\right)^{\alpha} W_{\alpha}\left[\frac{\Delta}{\bar{\varepsilon}} \left(\frac{a}{x}\right)^{\alpha}\right],$$
$$W_{\alpha}(z) = \sum_{l=0}^{\infty} \frac{(-z)^{l}}{l!\Gamma(1-\alpha-\alpha l)} \qquad (7)$$

- and (!)

$$\langle \Delta(x) \rangle = [\bar{\varepsilon}/\Gamma(1+\alpha)] (x/a)^{\alpha}, \qquad 0 < \alpha < 1 \qquad (8)$$

Energy loss in a random medium: conclusions

- A CTRW-like kinetic equations generalizing the Landau kinetic equation for energy straggling was suggested.
- Sublinear dependence of the mean energy loss on distance found for a particular case of self-similar randomly inhomogeneous medium