Semiclassical Transitions With Black Hole in the Intermediate State

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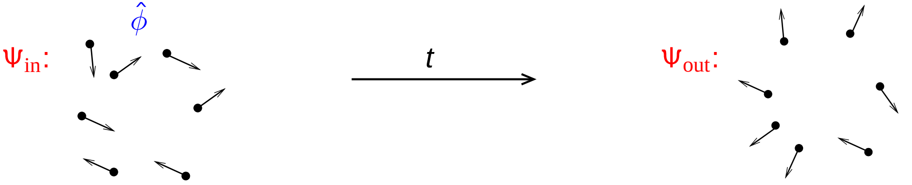
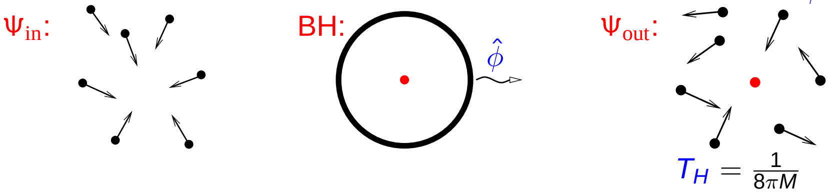
Hawking effect and asymptotic states

Scattering: *Hawking '75*



Collapse & Evaporation:



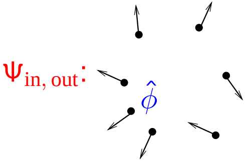
Gravitational $\hat{\mathcal{S}}$–matrix: $\Psi_{\text{out}} = \hat{\mathcal{S}}\Psi_{\text{in}}$?

flat $\leftrightarrow$ flat

't Hooft '85

Why S–matrix?

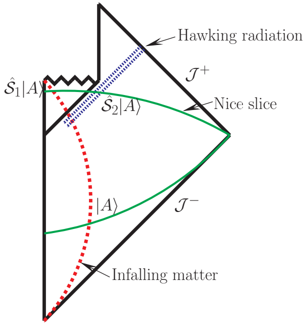
- $\Psi_{\text{in, out}}$ are <u>well understood</u>.



- Information paradox

  $\hat{\mathcal{S}}^\dagger \hat{\mathcal{S}} \neq 0$?

  It is worth calculating $\hat{\mathcal{S}}$!

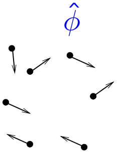


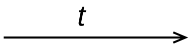
Hand-waving argument!

Calculating $\hat{\mathcal{S}}$–matrix elements

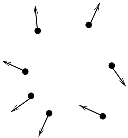
Scattering:

$\Psi_{\rm in}$:





$\Psi_{\rm out}$:

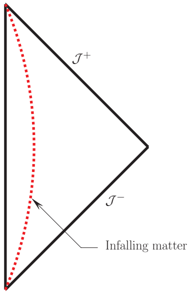


$$\langle \Psi_{\rm out} | \hat{\mathcal{S}} | \Psi_{\rm in} \rangle = \int dg_{\mu\nu} d\phi \, \Psi^*_{\rm out} \, \mathrm{e}^{i(S_g[g] + S_\phi[\phi])} \, \Psi_{\rm in}$$

Semiclassics: $S_g + S_\phi \gg 1 \Rightarrow \boxed{g_{\mu\nu}(x), \phi(x)}$ saddle point

$$\frac{\delta}{\delta g_{\mu\nu}}, \; \frac{\delta}{\delta \phi} \left[S_g + S_\phi - i \ln \Psi_{\rm in,\, out} \right] = 0$$

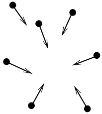
$$\Rightarrow \boxed{\langle \Psi_{\rm out} | \hat{\mathcal{S}} | \Psi_{\rm in} \rangle = \mathcal{A} \cdot \mathrm{e}^{i(S_g[g] + S_\phi[\phi]) + \ln \Psi_{\rm out} + \ln \Psi_{\rm in}}}$$



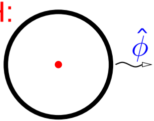
$g_{\mu\nu}(x), \phi(x)$

Collapse & Evaporation:

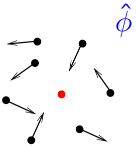
Ψ_{in}:



BH:



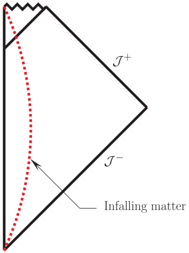
Ψ_{out}:



The same integral:

$$\langle \Psi_{\mathrm{out}} | \hat{\mathcal{S}} | \Psi_{\mathrm{in}} \rangle = \int dg_{\mu\nu} d\phi \, \Psi^*_{\mathrm{out}} \, \mathrm{e}^{i(S_g[g] + S_\phi[\phi])} \, \Psi_{\mathrm{in}}$$

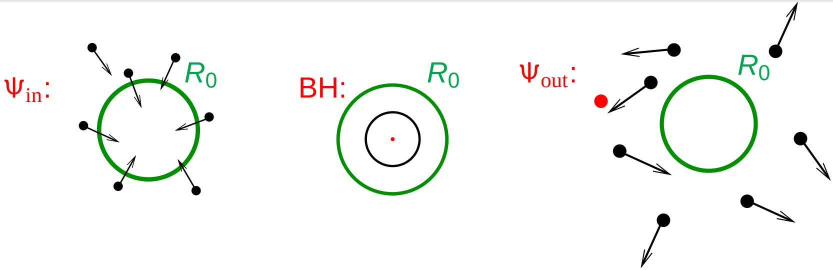
BUT: No saddle point $g_{\mu\nu}(x)$, $\phi(x)$ describing complete evolution !



$$\langle \Psi_{\mathrm{out}} | \hat{S} | \Psi_{\mathrm{in}} \rangle = ?$$

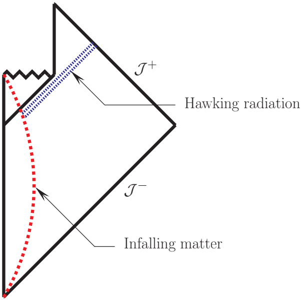
$g_{\mu\nu}(x)$, $\phi(x)$

Adding constraint



$$\Sigma[g,\phi] = \int dt\, M_{R_0}(t)$$

Add constraint $\Sigma[g,\phi] = \Sigma_0$
to GR equations
$\Downarrow$
Correct asymptotics of solution!



$g_{\mu\nu}(x)$, $\phi(x)$

Faddeev-Popov method

$$1 = \int_0^\infty d\Sigma_0\, \delta(\Sigma_0 - \Sigma[g,\phi]) = \int_0^\infty d\Sigma_0 \int_{-i\infty}^{i\infty} d\epsilon\; \mathrm{e}^{\epsilon(\Sigma_0 - \Sigma[g,\phi])}$$

$$\langle \Psi_{\rm in} | \hat{\mathcal{S}} | \Psi_{\rm out} \rangle = \int dg\, d\phi \cdot \mathrm{e}^{i(S_g + S_\phi)} \cdot \Psi_{\rm in} \Psi^*_{\rm out}$$

Strategy:

Bezrukov, DL '03

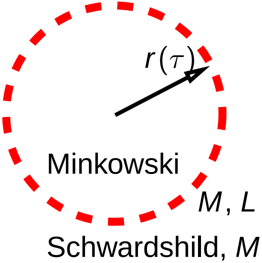
DL, Panin, Sibiryakov '07

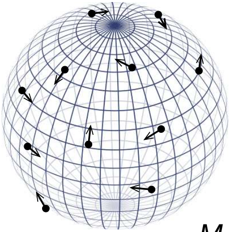
1. Evaluate path integral at fixed Σ_0.
   $\Psi_{\rm in}$ and $\Psi_{\rm out}$ live in flat space!

$$\boxed{S_\epsilon = S + i\epsilon\Sigma}$$

2. Evaluate integral over ϵ : $\Sigma_0 = \Sigma[g,\phi]$
3. Integrate over Σ_0: $\epsilon \to +0$.
4. $\mathcal{P} = \sum_{\rm out} |\langle \Psi_{\rm in} | \hat{\mathcal{S}} | \Psi_{out} \rangle|^2 \approx \lim_{\epsilon \to 0} \exp\left[-2\mathrm{Im}\left(S_\epsilon - i \ln \Psi_i\right)\right]$

Berezin, Okhrimenko '00





$$S_m = -\int d\tau \sqrt{m^2 + \frac{L^2}{r^2}}$$

$$S_g = \frac{1}{16\pi} \int d^4x\, \mathcal{R} \sqrt{-g}$$
$$+ \text{ boundary terms}$$

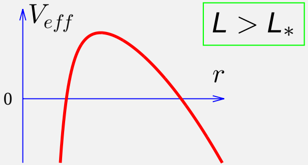
$M, L, m = 0$

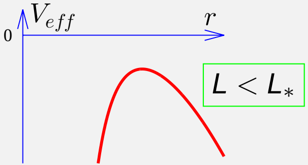
$$\left(\frac{dr}{d\tau}\right)^2 \overbrace{- \left(\frac{rM}{L} + \frac{L}{2r^2}\right)^2 + 1}^{V_{eff}} = 0$$

Shell evolution

$\epsilon = 0$

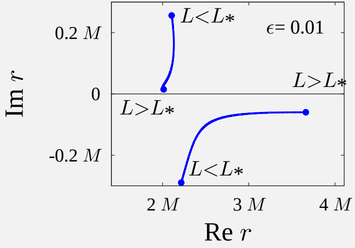
$$V_{eff} = -\left(\frac{rM}{L} + \frac{L}{2r^2}\right)^2 + 1 = 0$$



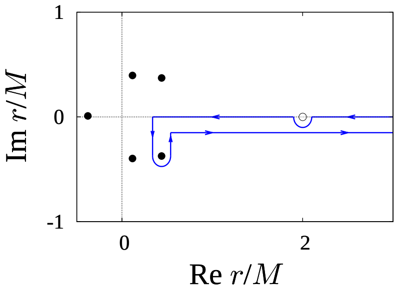


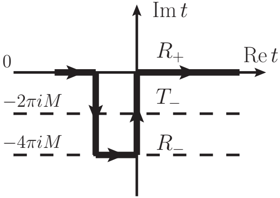
$\epsilon \neq 0$

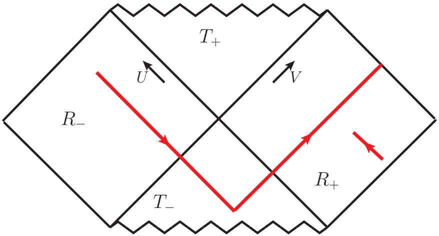
$$S_\epsilon = S + i\epsilon\Sigma$$
$$\Sigma[g, r] = \int dt\, \theta(R_0 - r(\tau))$$



Always reflection!

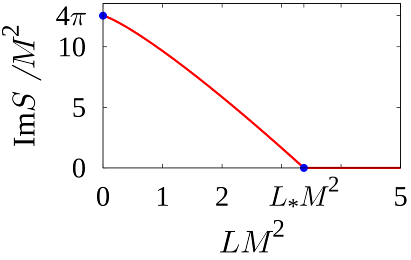






U, V – Kruskal coordinates

Result: $\epsilon \to 0$



Probability at $L = 0$: $\mathcal{P} = \lim_{\epsilon \to +0} \mathrm{e}^{-2\mathrm{Im}\, S_\epsilon + \text{b.t.}} = \mathrm{e}^{-4\pi M^2}$

$$\mathcal{P} = \exp(-A/4) = \exp(-S_B)$$

Parikh, Wilczek '00

Conclusions

New semiclassical method:

- Derived from first principles.
- Gives $\langle \Psi_{\rm out} | \hat{S} | \Psi_{\rm in} \rangle$.
- Includes backreaction.
- Easy to implement: solve classical Eqs. with complex terms.
- Can be applied to complex models: $\phi(r,t)$, $g_{\mu\nu}(r,t)$.

Cf. Berezin, Boyarsky, Neronov '99

Dust shell:

- Our result is consistent with Hawking radiation & Parikh, Wilzcek & Bekeinstein entropy.
- Black hole is a true energy eigenstate: $\mathcal{P} \neq 1$ *Cf. Vilkovisky '06*

Other models?