Strings and (Non)-Geometry

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MAX-PLANCK-GESELLSCHAFT

Ginzburg Conference on Physics, Moscow, May 28 - June 2, 2012

Outline:

I) Introduction

II) Point particle in a magnetic field

III) Non-geometric flux compactifications and deformed geometries

D. Lüst, JEHP 1012 (2011) 063, arXiv:1010.1361; arXiv:1205.0100
R. Blumenhagen, A. Deser, D.Lüst, E. Plauschinn, F. Rennecke, J. Phys A44 (2011), 385401, arXiv:1106.0316
C. Condeescu, I. Florakis, D. Lüst, JHEP 1204 (2012) 121, arXiv:1202.6366.

Additional work: D. Andriot, M. Larfors, D.L. P. Patalong, arXiv:1106.4015
D. Andriot, O. Hohm, M. Larfors, D.L. P. Patalong, arXiv:1202.3060, arXiv:1204.1979

IV) Outlook & open problems

I) Introduction

Question What is gravity? ⇔ What is space-time?

Problems: Quantization, Dark Matter & Energy, Hierarchy,..

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Point particles in classical Einstein gravity see smooth & continuous manifolds.

However Einstein gravity is plagued by singularities !

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However Einstein gravity is plagued by singularities !

Space (time?) can be only dissolved up to distances of order L_P .

L_P is the shortest possible distance!

String theory: Theory of Quantum Gravity

How does a string see space-time?

The short distance nature of space can be possibly tested by string scattering at high energies.

Shortest possible scale in string theory: L_s

We expect that geometry is changing at distances of the order of the string length.

String theory: Theory of Quantum Gravity

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Stringy (non)- geometry: deformed geometry:

- Non-commutative geometry: $[X_i, X_j] \simeq \mathcal{O}(L_s)$
- Non-associative geometry: $[[X_i, X_j], X_k] \simeq \mathcal{O}(L_s)$

II) Point particle in a magnetic field

Configuration space: $\mathcal{M} = T^*\mathcal{Q}\,, \qquad \vec{B} = \mathrm{rot}\;\vec{A}$

Langrange function: $L = \frac{1}{2}(p_i)^2 = \frac{1}{2}(\dot{x}^i - A^i)^2$

Canonical momenta: $p_i = \frac{\partial L}{\partial \dot{x}^i} = \dot{x}^i - A^i$

$$\pi^{ij} = \{x^i, x^j\} = 0\,, \quad \pi_{ij} = \{p_i, p_j\} = 0\,, \quad \{x^i, p_j\} = \delta_i^j$$

Mechanical momenta: $\overline{p}^i = \dot{x}^i = p^i + A^i$

$$\pi^{ij} = \{x^i, x^j\} = 0\,, \quad \bar{\pi}_{ij} = \{\overline{p}_i, \overline{p}_j\} = \epsilon_{ijk} B^k\,, \quad \{x^i, p_j\} = \delta_i^j$$

Non-commutative (Poisson) algebra

Point particle in the field of a magnetic monopole:

(R. Jackiw)

$$\vec{B} \in H^2(\mathcal{Q}),\ H = dB = \star\rho_{magn}$$ (B is non-closed)

ρ_{magn} ... charge density of a magnetic monopole.

$$\pi^{ij} = \{x^i, x^j\} = 0\,, \quad \bar{\pi}_{ij} = \{\bar{p}_i, \bar{p}_j\} = H_{ijk}x^k\,, \quad \{x^i, p_j\} = \delta_i^j$$

This leads to:

$$\bar{\pi}_{ijk} = \{\{\bar{p}_i, \bar{p}_j\}, \bar{p}_k\} + \text{perm.} = H_{ijk}$$

Twisted Poisson structure.

(C. Klimcik, T. Strobl, (2002); A. Alekseev, T. Strobl, (2005); C. Saemann, R. Szabo, arXiv:1106.1890)

As we will see, we will get a twisted Poisson structure for closed strings, however for the position operators instead of the momentum operators.

III) Non-geometric flux compactifications

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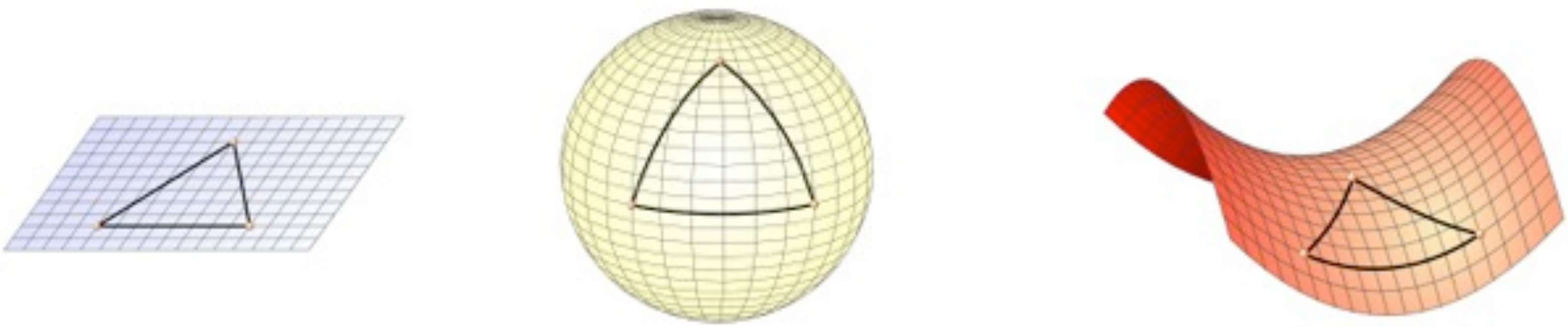
- Flat space: Triangle: $\alpha + \beta + \gamma = \pi$
- Curved space: Triangle: $\alpha + \beta + \gamma > \pi (< \pi)$



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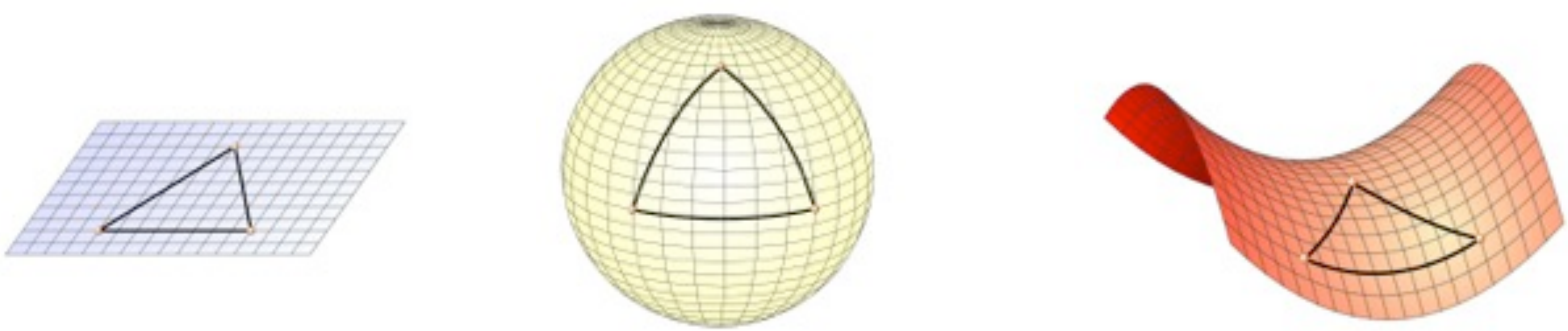

Manifold: need different coordinate charts, which are patched together by coordinates transformations, i.e. group of diffeomorphisms: $\mathrm{Diff}(M): \quad f: U \to U'$

Properties of Riemannian manifolds:

- distances between two points can be arbitrarily short.

- coordinates commute with each other:

$$[X_i, X_j] = 0$$

This is the situation, if one is using point particles to probe distance and the geometry of space.

Now we want to understand, how extended closed strings may possibly see the (non)-geometry of space.

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- Non-geometric Q-fluxes: spaces that are locally still Riemannian manifolds but not anymore globally.

Transition functions between two coordinate patches are not only diffeomorphisms but also T-duality transformations:

$$\mathrm{Diff}(M) \quad \rightarrow \quad \mathrm{Diff}(M) \times SO(d,d)$$

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Physics is nevertheless smooth and well-defined!

T-duality:

Consider compactification on a circle with radius R:

$$X(\tau,\sigma) = X_L(\tau+\sigma) + X_R(\tau-\sigma)$$

$$\begin{aligned} X_L(\tau+\sigma) &= \frac{x}{2} + p_L(\tau+\sigma) + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\alpha_n e^{-in(\tau+\sigma)}\,, \\ X_R(\tau-\sigma) &= \frac{x}{2} + p_R(\tau-\sigma) + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\tilde{\alpha}_n e^{-in(\tau-\sigma)} \end{aligned}$$

(KK momenta)

$$\begin{aligned} p_L &= \frac{1}{2}\left(\frac{M}{R} + (\alpha')^{-1}NR\right), \\ p_R &= \frac{1}{2}\left(\frac{M}{R} - (\alpha')^{-1}NR\right) \end{aligned}$$

$$\begin{aligned} p &= p_L + p_R = \frac{M}{R} \\ \tilde{p} &= p_L - p_R = (\alpha')^{-1}NR \end{aligned}$$

(dual momenta - winding modes)

T-duality: $T: \quad R \longleftrightarrow \frac{\alpha'}{R}, \quad M \longleftrightarrow N$

$$T: \quad p \longleftrightarrow \tilde{p}\,, \quad p_L \longleftrightarrow p_L\,, \quad p_R \longleftrightarrow -p_R\,.$$

- Dual space coordinates: $\tilde{X}(\tau,\sigma) = X_L - X_R$

$(X,\tilde{X})$: Doubled geometry: (O. Hohm, C. Hull, B. Zwiebach (2009/10))

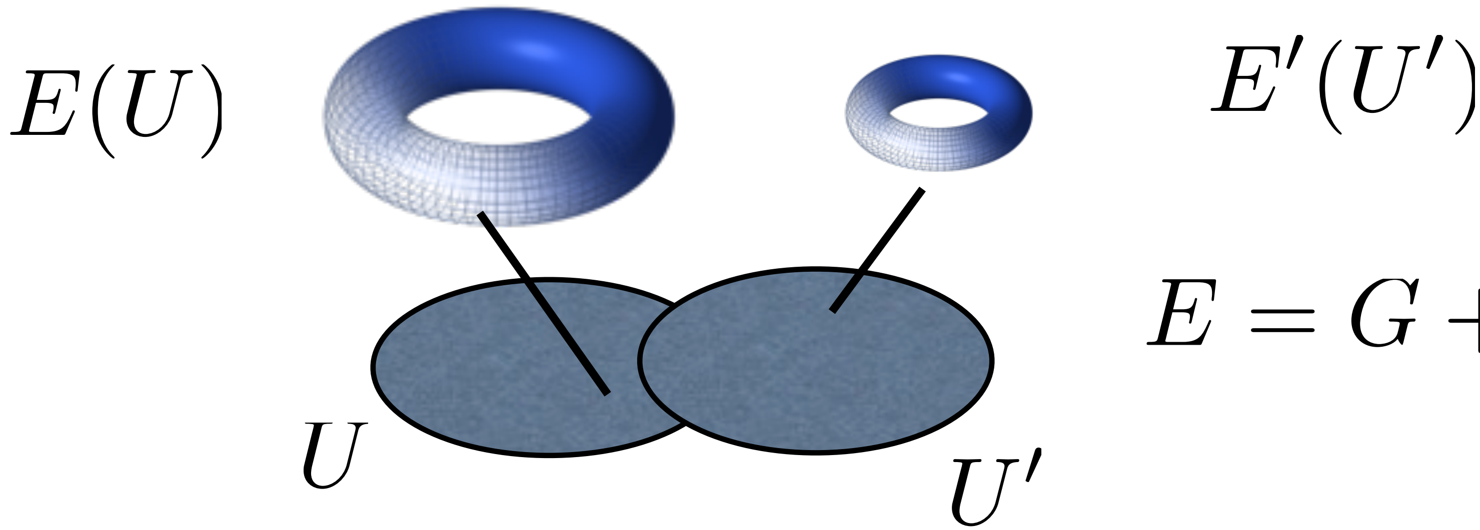
T-duality is part of stringy diffeomorphism group.

$$T: \quad X \longleftrightarrow \tilde{X}\,, \quad X_L \longleftrightarrow X_L\,, \quad X_R \longleftrightarrow -X_R$$

- Shortest possible radius: $R \geq R_c = \sqrt{\alpha'}$

T-fold: Patching uses T-duality.

e.g. torus fibrations



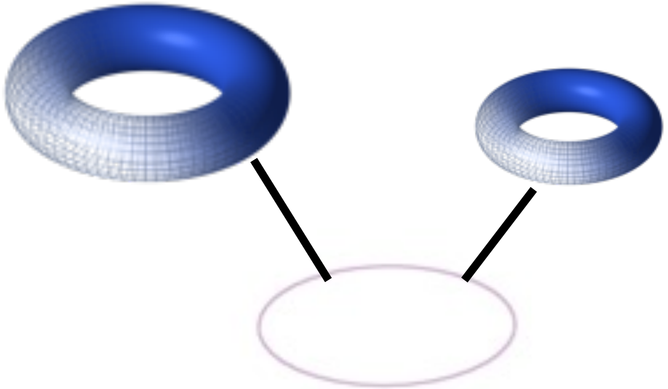
$$E = G + B$$

Geometric background: $E' = aEa^t \;\text{ in }\; U \cap U' \,,\;\; a \in GL(d, Z)$

Non-geometric background:

$$E' = \frac{aE + b}{cE + d} \;\;\text{in}\;\; U \cap U'$$

Example: torus bundle over S^1 :



Metric: $$ds^2 = dx_3^2 + \frac{1}{1+x_3^2}(dx_1^2 + dx_2^2)$$

B-field: $$B_{x_1,x_2} = \frac{x_3}{1+x_3^2}$$

Monodromy: x_3 is periodic:

$$E(x_3 + 2\pi) = \frac{aE(x_3)+b}{cE(x_3)+d} \in SO(2,2;Z)$$

Mathematical framework:

- Doubled field theory: uses completely SO(d,d) invariant formalism.

- Generalized complex geometry: uses doubled tangent space $T \oplus T^*$.

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Standard effective action is in general not well-defined for non-geometric backgrounds:

$$\mathcal{S}_{NS} \sim \int dx^{10} \left(R - \frac{1}{12} H^2 + \cdots \right)$$

Well-defined (10D) effective action for non-geometric backgrounds can be constructed.

D. Andriot, M. Larfors, D.L. P. Patalong, arXiv:1106.4015

D. Andriot, O. Hohm, M. Larfors, D.L. P. Patalong, arXiv:1202.3060, 1204.1979

- Relation to gauged supergravity in 4D
- Moduli stabilization, de Sitter solutions, ...

We will consider a class of four different 3-dimensional flux backgrounds, which are related by T-duality:

(Shelton, Raylor, Wecht, 2005; Dabholkar, Hull, 2005)

Chain of 3 T-dualities:

$$F^{(3)}: \qquad H \underset{T_{x_1}}{\longleftrightarrow} \omega \underset{T_{x_2}}{\longleftrightarrow} Q \underset{T_{x_3}}{\longleftrightarrow} R$$

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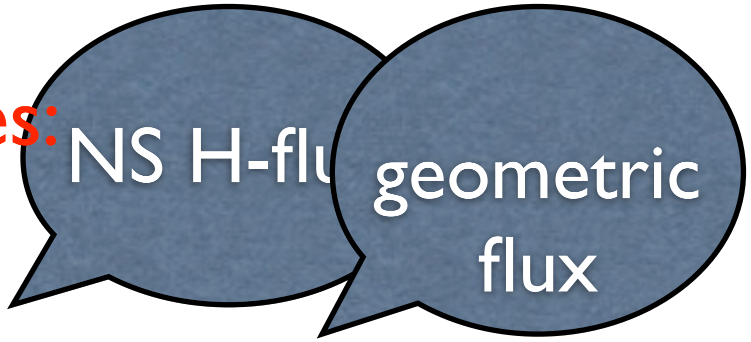

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Non-associative „Space“ with R-flux

$$[[X_i, X_j], X_k] \neq 0$$

Three-dimensional flux backgrounds:

Fibrations: 2-dim. torus that varies over a circle:

$$T^2_{x^1,x^2} \hookrightarrow M^3 \hookrightarrow S^1_{x^3}$$

The fibration is specified by its monodromy properties.

Two T-dual cases:

(i) Geometric spaces (manifolds): geometric ω - flux

complex structure is non-constant:

$$x^3 \to x^3 + 2\pi \;\Rightarrow\; \tau(x^3 + 2\pi) = \frac{a\tau(x^3) + b}{c\tau(x^3) + d}$$

$$\tau(x^3+2\pi) = -1/\tau(x^3)$$

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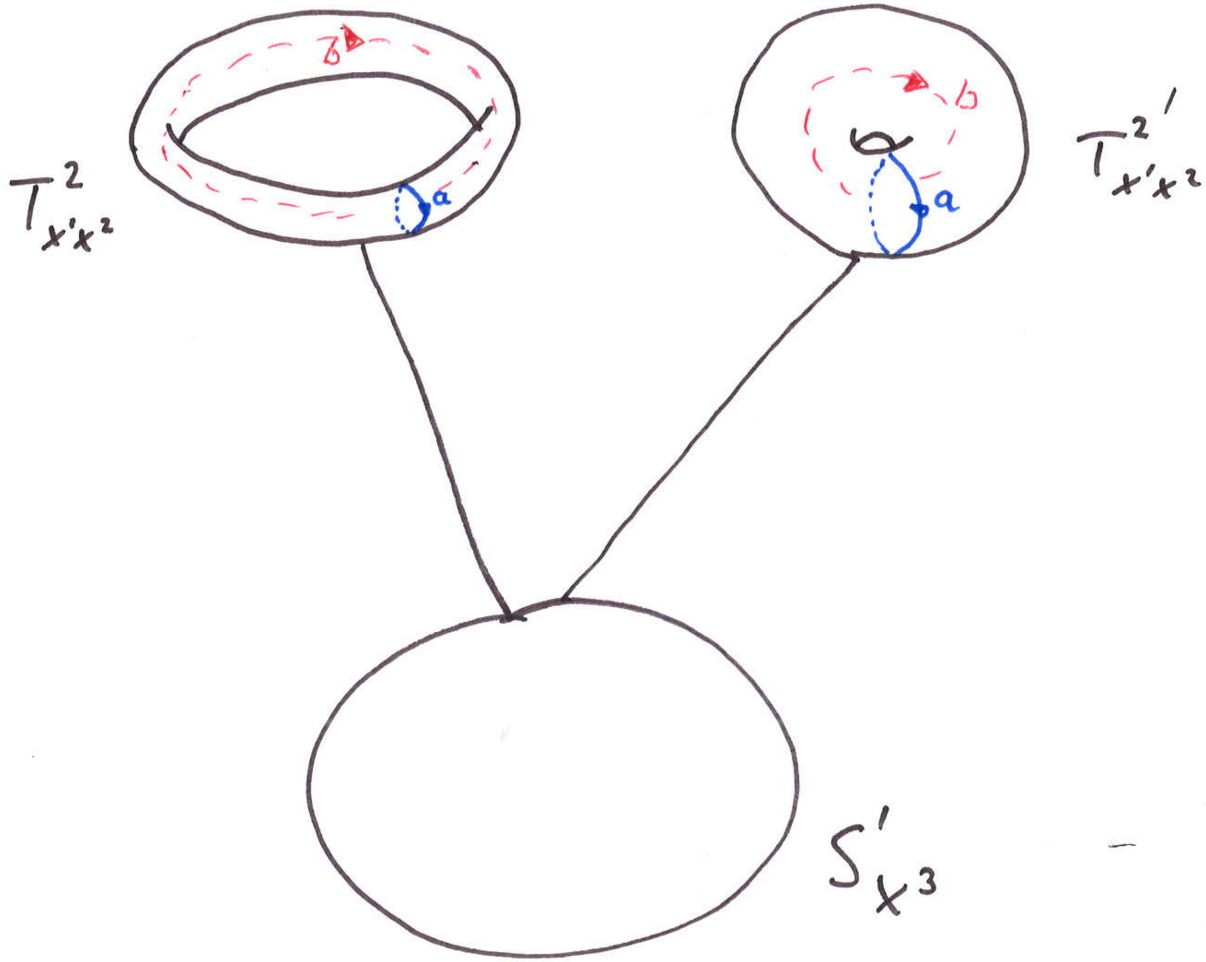

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(ii) Non-geometric spaces (T-folds): non-geometric Q-flux

size + B-field is non-constant:

$$x^3 \to x^3 + 2\pi \;\Rightarrow\; \rho(x^3 + 2\pi) = \frac{a\rho(x^3) + b}{c\rho(x^3) + d}$$

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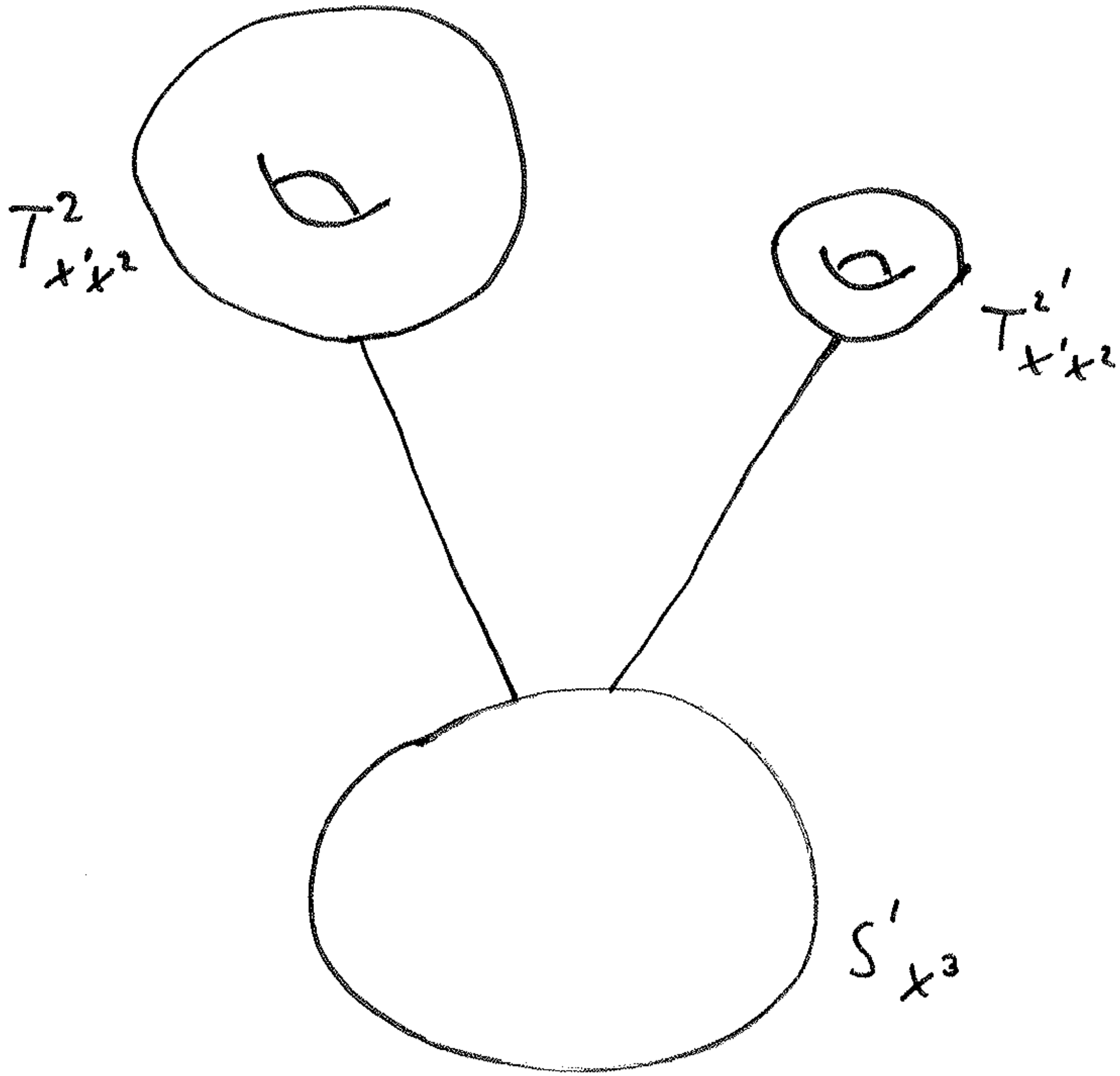

x^3

Specific example: Z_4-monodromy

D. L., JEHP 1012 (2011) 063, arXiv:1010.1361,
C. Condeescu, I. Florakis, D. L., arXiv:1202.6366

$$X^3(\tau, \sigma + 2\pi) = X^3(\tau, \sigma) + 2\pi N_3$$

← winding number

$$X_L(\tau, \sigma + 2\pi) \quad = \quad e^{i\theta} X_L(\tau, \sigma)\,, \quad \theta = -2\pi N_3 H$$

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Corresponding closed string mode expansion ⇒

$$X_L(\tau + \sigma) = i\sqrt{\frac{\alpha'}{2}} \sum_{n \in \mathbb{Z}} \frac{1}{n - \nu} \alpha_{n-\nu} e^{-i(n-\nu)(\tau+\sigma)}\,, \qquad \nu = \frac{\theta}{2\pi} = -N_3 H$$

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Corresponding closed string mode expansion $\Rightarrow$

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(shifted oscillators!)

Then one obtains:

$$[X_L(\tau,\sigma), \bar{X}_L(\tau,\sigma)] = \Theta$$

$$\Theta = \alpha' \sum_{n\in\mathbb{Z}} \frac{1}{n-\nu} = -\alpha'\pi\cot(\pi N_3 H)$$

Right moving torus coordinates:

$$X_R(\tau, \sigma + 2\pi) = e^{-i\theta} X_R(\tau, \sigma)$$

This is very similar to asymmetric orbifolds. A specific string solution on a freely action asymmetric orbifold was recently constructed: C. Condeescu, I. Florakis, D. L., arXiv:1202.6366

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dual momentum (winding) in third direction

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Corresponding uncertainty relation:

$$(\Delta X^1)^2 (\Delta X^2)^2 \geq L_s^6 \, (F^{(3)})^2 \, \langle \tilde{p}^3 \rangle^2$$

The spatial uncertainty in the X_1, X_2 - directions grows with the dual momentum in the third direction: non-local strings with winding in third direction.

- For the case of non-geometric R-fluxes one finally gets: $$[X^1, X^2] \simeq i L_s^3 \, F^{(3)} \, p^3$$

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Use $\quad [p^3, X^3] = -i$

$$\Longrightarrow \quad [[X^1, X^2], X^3] + \text{perm.} \simeq F^{(3)} \, L_s^3$$

Non-associative algebra! (twisted Poisson structure)

This nicely agrees with the non-associative closed string structure found by Blumenhagen, Plauschinn in the SU(2) WZW model: arXiv:1010.1263

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- Can geometry and strings be described as an emergent concept (from particle species)?

(G.Dvali, C. Gomez, D.L., work in progress)

- What is the generalization of quantum mechanics for this non-associative geometry? How to represent this algebra (octonians?)?