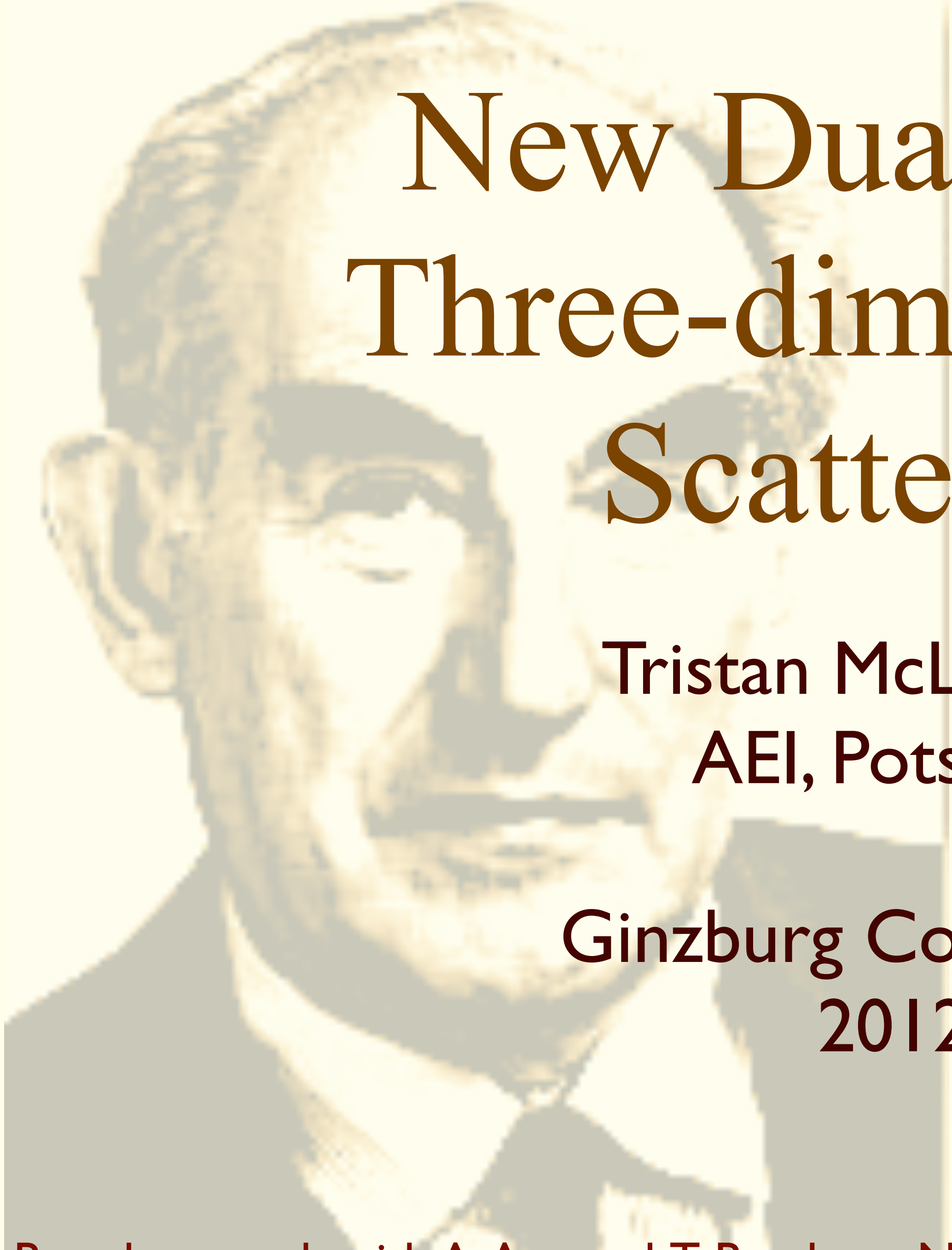

New Dualities in Three-dimensional Scattering

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Based on work with A. Agarwal, T. Bargheer, N. Beisert, S. He, Y-t Huang, F. Loebbert.

Superconformal Chern Simons matter theories

- Such three-dimensional theories (for example $\mathcal{N} = 6$ ABJM & $\mathcal{N} = 8$ BLG) describe the low energy dynamics of multiple M2-branes.
  - $\mathcal{N} = 6$ ABJM has many properties in common with 4D $\mathcal{N} = 4$ SYM. Spectrum of anomalous dimensions is integrable in the planar limit & it possesses a holographic dual string theory.
    - Scattering amplitudes also share many features with $\mathcal{N} = 4$ SYM however the $\mathcal{N} = 6$ ABJM theories are less constrained by supersymmetry and provide interesting generalisations.
      - Some evidence that onshell they are related to three-dimensional supergravity in flat space e.g.

$$\text{``}\mathcal{N} = 8 \text{ BLG''}^2 \equiv \quad \mathcal{N} = 16 \ E_{8(8)} \text{ sugra}$$

$\mathcal{N}$ = 6 ABJM theory

Onshell field content: four complex bosons and four fermions

$$\phi^{\hat{I}}(p)^{A}{}_{\bar{A}} \quad \& \quad \psi_{\hat{I}}(p)^{A}{}_{\bar{A}} \qquad \hat{I} = 1, \ldots, 4$$

transforming in bifundamental rep. of $U(N_c) \times U(N_c)$ gauge group.

Useful to introduce real* spinors for onshell momenta

$$p^{\alpha\beta} = \lambda^{\alpha}\lambda^{\beta} \qquad \alpha = 1, 2$$

and Graßmann variables η^I , $I = 1, 2, 3$ for onshell superfield

$$\Phi = \phi^4 + \eta^I \psi_I + \tfrac{1}{2}\epsilon_{IJK}\eta^I\eta^J\phi^K + \tfrac{1}{3!}\epsilon_{IJK}\eta^I\eta^J\eta^K\psi_4$$

& conjugate fermionic superfield

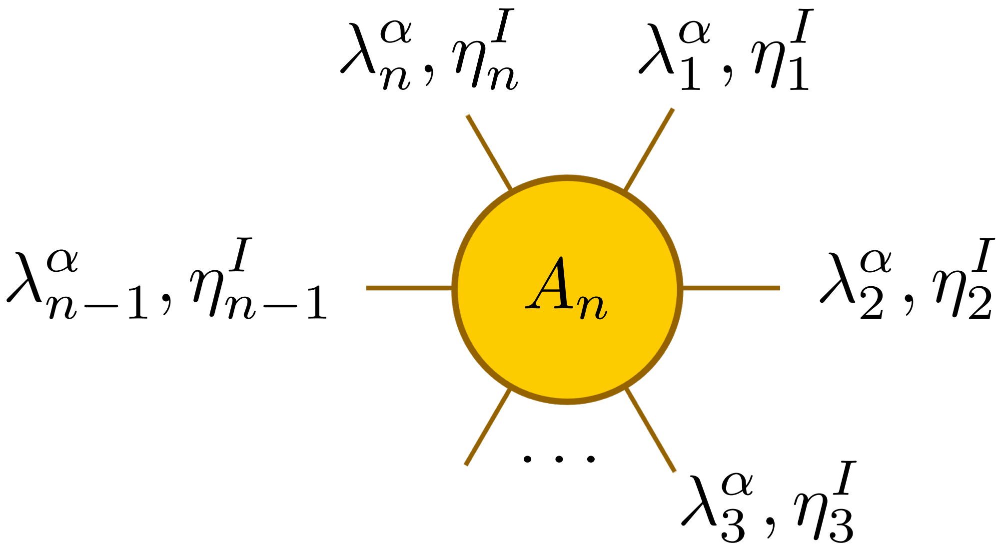
$$\bar{\Phi} = \bar{\psi}^4 + \ldots$$

* caveat emptor

We can define colour ordered, planar amplitudes

$$\mathcal{A}(\bar{\Phi}(p_1)^{\bar{A}_1}{}_{A_1}, \Phi(p_2)^{B_2}{}_{\bar{B}_2}, \dots, \Phi(p_n)^{B_n}{}_{\bar{B}_n}) \quad = \quad A(\bar{1}, 2, \dots, n)\delta^{B_2}{}_{A_1}\delta^{\bar{A}_3}{}_{\bar{B}_2}\dots\delta^{\bar{A}_1}{}_{\bar{B}_n} + \dots$$

sum over all permutations of even and odd sites modulo cyclic permutations by two sites.



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Ex. Four-points tree-level

$$A_4(\bar{\Phi}_1, \Phi_2, \bar{\Phi}_3, \bar{\Phi}_4) = \frac{\delta^{(3)}(P)\delta^{(6)}(Q)}{\langle 12\rangle\langle 23\rangle}$$

[Bargheer, Loebbert, Meneghelli]

Lorentz invariants: $\langle ij\rangle = \lambda_i^\alpha \epsilon_{\alpha\beta}\lambda_j^\beta$

(Super)momenta:

$$P^{\alpha\beta} = \sum_{a=1}^{n} \lambda_a^\alpha \lambda_a^\beta \,, \quad Q^{I\alpha} = \sum_{a=1}^{n} \eta_a^I \lambda_a^\alpha$$

Symmetries

Amplitudes have $\mathcal{N} = 6$ superconformal symmetry i.e. OSp(6|4)

$$J^A A_n = 0 \ , \qquad J^A = \sum_{a \in \text{legs}} J_a^A$$

e.g. $S_\alpha^I = \eta^I \partial_\alpha$. N.B. "anomalies" on multi-collinear configurations.

Yangian symmetries: $$J^{(1)A} = f^A_{BC} \sum_{a<b} J_a^B J_b^C$$

[Bargheer, Loebbert, Meneghelli]

Can interpret part of the Yangian as dual conformal symmetry with dual space:

[Huang & Lipstein]

$$x_{a,a+1} = x_a - x_{a+1} = \lambda_a \lambda_a \ ,$$

$$\theta_{a,a+1} = \lambda_a \eta_a \ , \quad y^{IJ}_{a,a+1} = \eta_a^I \eta_a^J$$

dual R-sym coords

- BCFW recursion relations generates all tree-level amplitudes (in principle) and proves Yangian symmetry to all tree-level amplitudes. [Gang, Huang, Koh, Lee, & Lipstein]

- Orthogonal Graßmannian formulation. [Lee]

- Four point tree and one-loop amplitudes in the mass-deformed version which preserves all susy and the symmetry algebra have

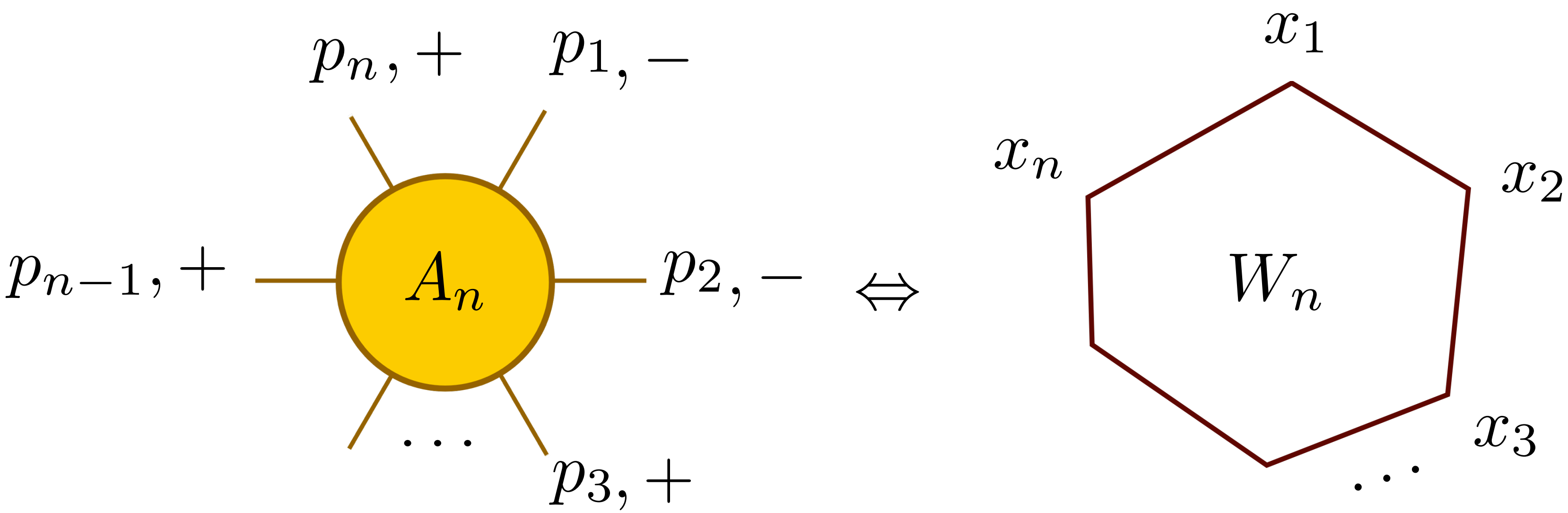
$$SL(2,\mathbb{R}) \ltimes PSU(2|2)^2 \ltimes \mathbb{R}^3$$

  been calculated. [Agarwal, Beisert, TMcL]

- Dual to $AdS_4 \times CP^3$ type-IIA string theory
  - evidence of classical integrability
  - integrable spectrum (ABA & TBA/Y-system) [Gromov & Vieira] [Gromov, Kazakov, Vieira][Bombardelli, Fioravanti and Tateo]

- In $\mathcal{N}=4$ SYM dual superconformal symmetry is related to the self-duality of $AdS_5 x S^5$ under a combination of bosonic and fermionic T-dualities. [Berkovits&Maldacena] [Beisert, Ricci, Tseytlin &Wolf]

- Maps planar amplitudes to polygonal, light-like (super)-Wilson loops. [Alday&Maldacena]
  - Simplest case: MHV amplitudes to bosonic Wilson loops.



  - For N^kMHV amplitudes we need to consider super-Wilson loops. [Mason & Skinner] [Caron-Huot]

- ABJM has no notion of chirality, no MHV subsector.
  - In ABJM what is five sided Wilson loop dual to?
  - T-dualities (bosonic and fermionic) are singular in $AdS_4 \times CP^3$.

- Nonetheless ∃ evidence for some form of the duality in ABJM
  - At four points: one-loop amplitude vanishes as does bosonic Wilson loop, two-loops matches.
  - n-pt bosonic Wilson loop vanishes at one-loop and matches functional form of $\mathcal{N} = 4$ SYM answer at two-loops.

[Agarwal, Beisert & TMcL], [Henn, Plefka, Wiegandt],[Chen&Huang],[Bianchi et al], [Wiegandt]

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[Agarwal, Beisert & TMcL], [Henn, Plefka, Wiegandt],[Chen&Huang],[Bianchi et al], [Wiegandt]

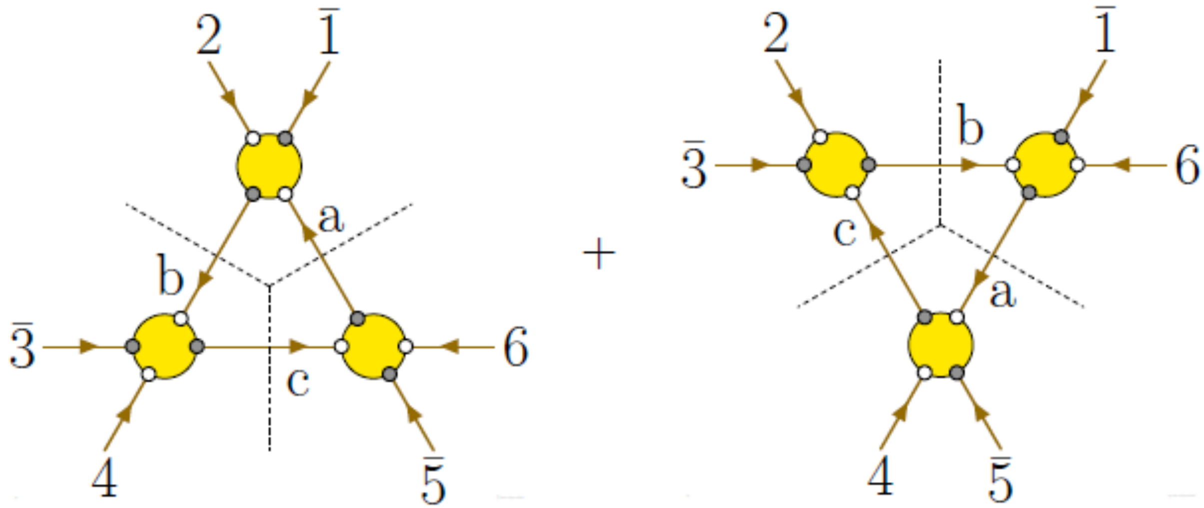
Need to consider higher point amplitudes.

Six-point one-loop amplitude

All one-loop amplitudes can be written as linear combination of scalar triangle integrals:

$$A_n^{(1)} = \sum_i d_i \, \mathcal{I}_{3,i}$$

We can find coefficient by calculating maximal cuts:



e.g.

$$d_1 = \frac{1}{2} \sum_{\text{sol}} \int \prod d^3\eta_i \; A_4^{(0)}(\bar{1}, 2, -\bar{\text{b}}, \text{a}) \, A_4^{(0)}(\bar{3}, 4, -\bar{\text{c}}, \text{b}) \, A_4^{(0)}(\bar{5}, 6, -\bar{\text{a}}, \text{c}).$$

Six-point one-loop amplitude

Final result [Bargheer, Beisert, Loebbert, TMcL]:

$$A_6^{(1)}(\bar{1},2,\bar{3},4,\bar{5},6) = \frac{\pi}{4} c_6(\bar{1},\bar{2},\bar{3},\bar{4},\bar{5},\bar{6}) A_6^{(0)}(\bar{6},1,\bar{2},3,\bar{4},5)$$

where

$$c_6(\bar{1},\bar{2},\bar{3},\bar{4},\bar{5},\bar{6}) = \mathrm{sgn}\langle 12\rangle \mathrm{sgn}\langle 34\rangle \mathrm{sgn}\langle 56\rangle + \mathrm{sgn}\langle 61\rangle \mathrm{sgn}\langle 23\rangle \mathrm{sgn}\langle 45\rangle .$$

- Answer is proportional to Yangian invariants, however there are additional discontinuities when two particles become collinear.
- Consistent with predictions of “anomalous” symmetries.
- Also found in a Feynman graph calculation. [Bianchi, Leoni, Mauri, Penati, Santambrogio]
- Doesn’t match bosonic Wilson loop, more akin to N=4 SYM NMHV amplitude. Optimistic conclusion:

Need a new super-Wilson loop.

SCS as a three-algebra theory

Can express the ABJM theory by rewriting the $U(N_c) \times U(N_c)$ color structure as a three-algebra [Bagger & Lambert]:

Complex vector space with basis T^a, $a = 1, \ldots, N_c{}^2$ and trilinear bracket

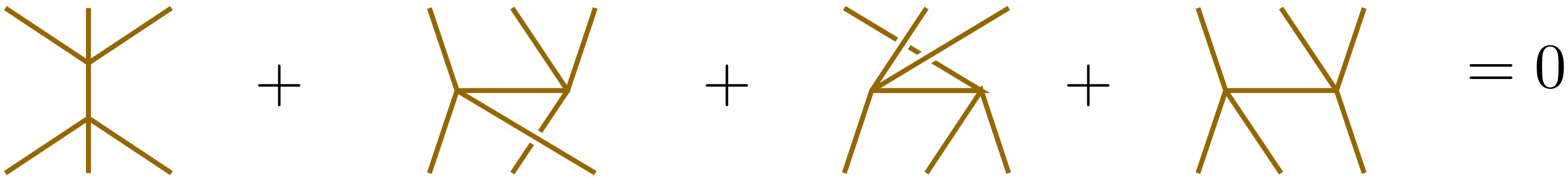
$$[T^a, T^b; \bar{T}^c] = f^{ab\bar{c}}{}_d T^d \quad \text{s.t.} \quad f^{ab\bar{c}}{}_d = -f^{ba\bar{c}}{}_d$$

Additionally one has a trace form and a reality condition

$$h^{\bar{a}b} = \mathrm{Tr}(\bar{T}^a, T^b) \qquad \& \qquad f^{ab\bar{c}\bar{d}} = f^{*\bar{c}\bar{d}ab}$$

Key property is fundamental identity (c.f. Jacobi identity)

$$f^{ef\bar{g}}{}_b f^{*\bar{a}\bar{d}cb} + f^{fe\bar{a}}{}_b f^{*\bar{g}\bar{d}cb} + f^{ec\bar{d}}{}_b f^{*\bar{g}\bar{a}fb} + f^{cf\bar{d}}{}_b f^{*\bar{g}\bar{a}eb} = 0$$



SCS as a three-algebra theory

Can express the ABJM theory by rewriting the $U(N_c) \times U(N_c)$ color structure as a three-algebra [Bagger & Lambert]:

Enhanced $\mathcal{N} = 8$ susy (BLG-theory) when vector space is real and structure constants are totally antisymmetric

$$[T^a, T^b, T^c] = f^{abc}{}_d T^d \qquad \text{s.t.} \qquad f^{abcd} = f^{[abcd]}$$

Only one finite dimensional example.

SCS as a three-algebra theory

Superfields transform as fundamental representations of three-alg.
Four point amplitudes:

$$\mathcal{N}=6: \quad \mathcal{A}(\bar{1},2,\bar{3},4)=\frac{4\pi i}{k}\frac{\delta^{(3)}(P)\delta^{(6)}(Q)}{\langle 12\rangle\langle 23\rangle}f^{a_2a_4\bar{a}_1\bar{a}_3}$$

$$\mathcal{N}=8: \quad \mathcal{A}(1,2,3,4)=\frac{4\pi i}{k}\frac{\delta^{(3)}(P)\delta^{(8)}(Q)}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}f^{a_1a_2a_3a_4}$$

[Huang Lipstein]

In general is written as a sum of quartic graphs:

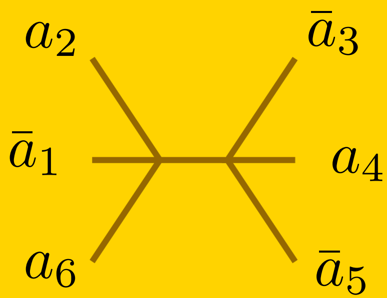
$$\mathcal{A}_n \propto \sum_{i\in \text{graphs}} \frac{n_i c_i}{\prod_{\alpha_i}\ell^2_{\alpha_i}}$$

$c_i \,:\, f^{a_n a_2 \bar{a}_1}{}_{b_1} f^{*\bar{a}_3\bar{a}_5 b_1}{}_{\bar{b}_2}\cdots$

ℓ_i^2 : inverse propagators

n_i : kinematic numerators

e.g.



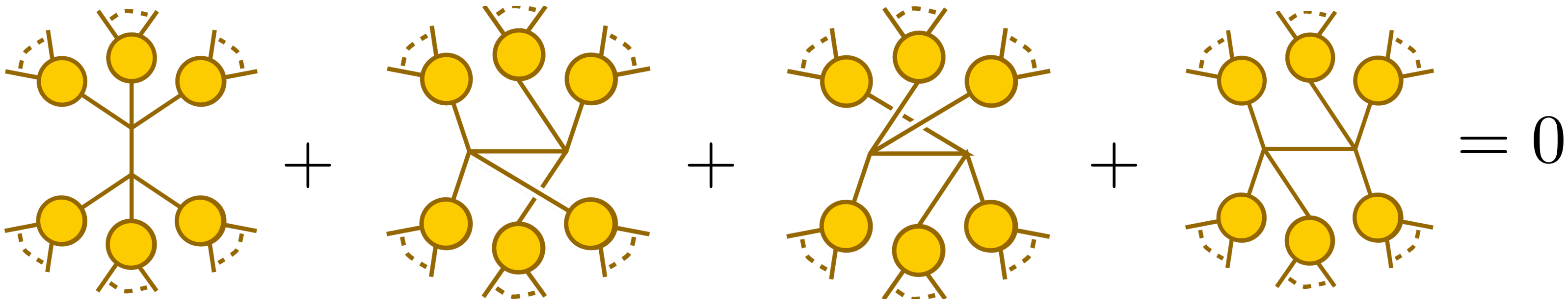
$c_1 \,:\, f^{a_6 a_2 \bar{a}_1}{}_{b_1} f^{*\bar{a}_3\bar{a}_5 a_4 b_1}$

Color-kinematics duality

Claim: there exists a duality between color and kinematics analogous to that in YM [Bern, Johansson & Carrasco]:

$$\mathcal{A}_n \propto \sum_{i \in \text{graphs}} \frac{n_i c_i}{\prod_{\alpha_i} \ell_{\alpha_i}^2}$$

Different color structures are related by Fundamental identity:



$$c_s + c_t + c_u + c_v = 0$$

There exists numerators satisfying the same relations:

$$n_s + n_t + n_u + n_v = 0$$

[Bargheer, He, TMcL]

Evidence: four points (trivial) and six points (non-trivial)

- Implies non-trivial relations between color ordered amplitudes

Doubling to Supergravity

Given numerators satisfying the fundamental identities
we can replace the color structures with another copy of the numerators:

$$\mathcal{M}_n = \sum_{i \in \text{graphs}} \frac{n_i n_i}{\prod_{\alpha_i} \ell_i^2}$$

This defines the scattering amplitudes for a theory with a spectrum
given by the square of gauge theory (c.f. KLT, BCJ)

- $\mathcal{N} = 8$ BLG case gives a theory with 128 bosons + 128 fermions and $\mathcal{N} = 16$ supersymmetry.
- Only has amplitudes with even numbers of external legs.
- This theory will have a hidden three-algebra structure in its kinematics!

$\mathcal{N} = 16$ Supergravity

- Maximally supersymmetric 3D supergravity constructed by Marcus and Schwarz has 128 bosons and 128 fermions transforming as SO(16) spinors $\Rightarrow$ correct spectrum and no odd-point amplitudes.

- Can be found by dimensional reduction and duality transformation of 4D $\mathcal{N} = 8$ supergravity:

$$D = 4 \qquad g_{\tilde{\mu}\tilde{\nu}}\,(2) \qquad A^I_{\tilde{\mu}}\,(56) \qquad \varphi^{\tilde{A}}\,(70)$$

$$g_{\tilde{\mu}\tilde{\nu}} \to g_{\mu\nu},\ B_\mu,\ \phi \qquad A^I_{\tilde{\mu}} \to A^I_\mu,\ \varphi^I \qquad \varphi^{\tilde{A}} \to \varphi^{\tilde{A}}$$

$$D = 3 \qquad g_{\mu\nu} \quad \underbrace{B_\mu \quad \phi \quad A^I_\mu \quad \varphi^I \quad \varphi^{\tilde{A}}}_{\varphi^A\ (128)\quad E_{8(8)}/S0(16)\ \text{coset}}$$

using $\qquad \triangle^2 F_{\mu\nu} = \epsilon_{\mu\nu\lambda}\partial^\lambda \varphi$

$\mathcal{N} = 16$ Supergravity

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- Can be found by dimensional reduction and duality transformation of 4D $\mathcal{N} = 8$ supergravity.

- Four-point amplitude is the square of BLG four-point:

$$\mathcal{M}_4 = \frac{i\kappa^2}{4} \frac{\delta^{(3)}(P)\delta^{(16)}(Q)}{(\langle 12\rangle\langle 23\rangle\langle 31\rangle)^2}$$

- Six-point is:

$$\mathcal{M}_6 = \sum_{i \in \text{graphs}} \frac{n_i n_i}{\prod_{\alpha_i} \ell_i^2}$$

where the numerators are those of the SCS theory and the sum is over the same quartic graphs.

Conclusions & Outlook

- Provided evidence for tree-level color-kinematics in ABJM (and BLG) theories when written as three-algebra theories.

- Provided evidence that one can, á la BCJ, "double" BLG theory into $\mathcal{N} = 16$ E_8 supergravity and hence for a hidden three-algebra structure in 3D supergravity.

✻ Does this color-kinematics duality persist to higher points? Loop integrands?

✻ Is $\mathcal{N} = 16$ 3D supergravity finite?