Topological transitions in electronic spectra of vortex clusters in type-II superconductors

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Outline

- **Introduction**

Types of topological electronic transitions in the band spectra of crystals.
Fermi surface in vortex state of superconductors.

- **Examples of topological electronic transitions in vortex matter.**

  de Haas – van Alfen oscillations in vortex state.
  Magnetic breakdown criterion.
  Quasiparticle tunneling between vortices and reconnection of Fermi surface segments.

- **Opening of Fermi surface segments in vortex cores.**

  Creation of vortex-antivortex pairs,
  vortex near the surface or defect.
  Landau criterion for vortex depinning.
  Two step scenario of vortex escape from the defect.

Topological electronic transitions (Lifshits-1960)

Void formation/disappearance

Quasimomentum space.

$$E(\vec{k}) = E_F$$

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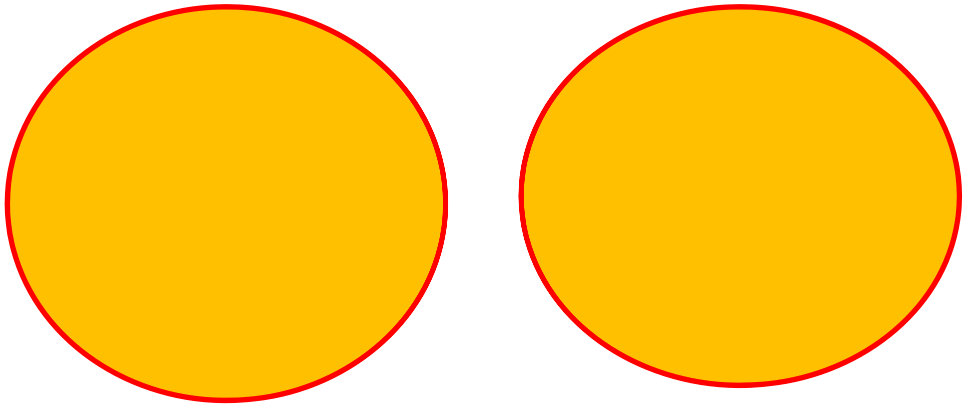
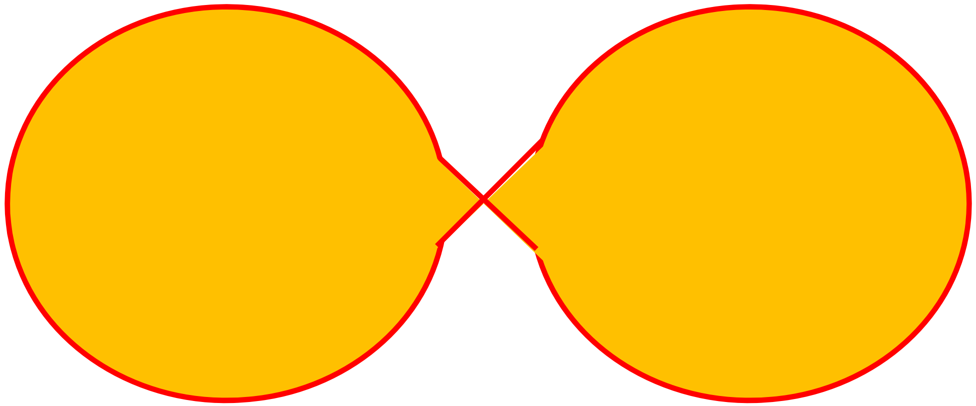


Quasimomentum space.

$$E(\vec{k}) = E_F$$

Topological electronic transitions (Lifshits-1960)

Neck formation/disruption



Quasimomentum space.

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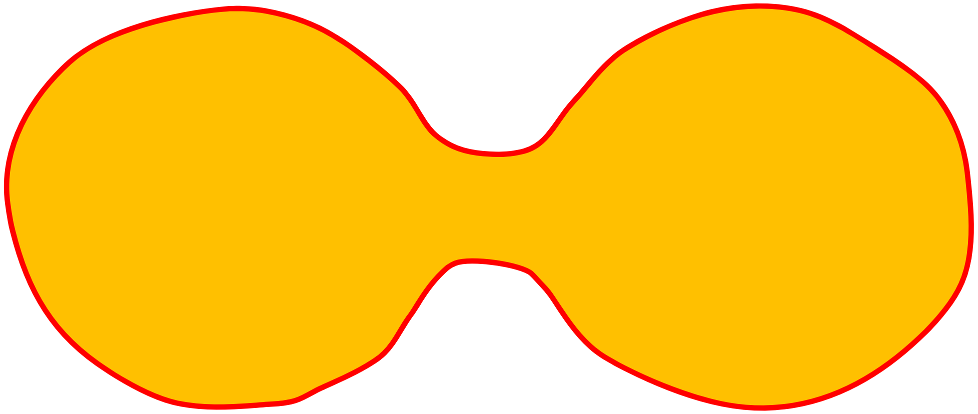
Topological electronic transitions (Lifshits-1960)

Neck formation/disruption



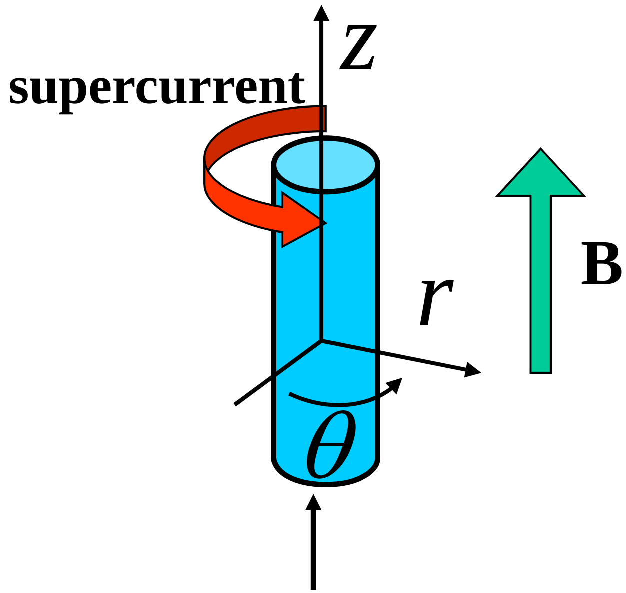
Experiment: peculiarities of DOS and transport characteristics

Quasimomentum space.

$$E(\vec{k}) = E_F$$

Fermi surface in vortex state of superconductors

Vortex line



$$\Delta = |\Delta(r)|\, e^{i\theta}$$

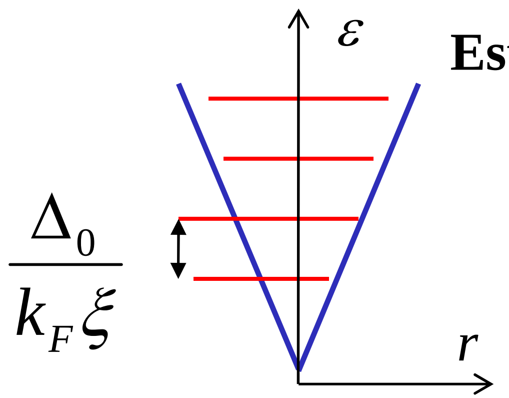
phenomenological theory of the mixed state

$$-\xi^2(T)\left(\nabla + \frac{2\pi i}{\Phi_0}\vec{A}(\vec{r})\right)^2 \Psi = \Psi - |\Psi|^2\,\Psi$$

ξ **- Coherence length (core radius)**

Bound fermionic states in vortex core

Superconducting gap profile:
potential well for electrons



Estimate of minigap in excitation spectrum

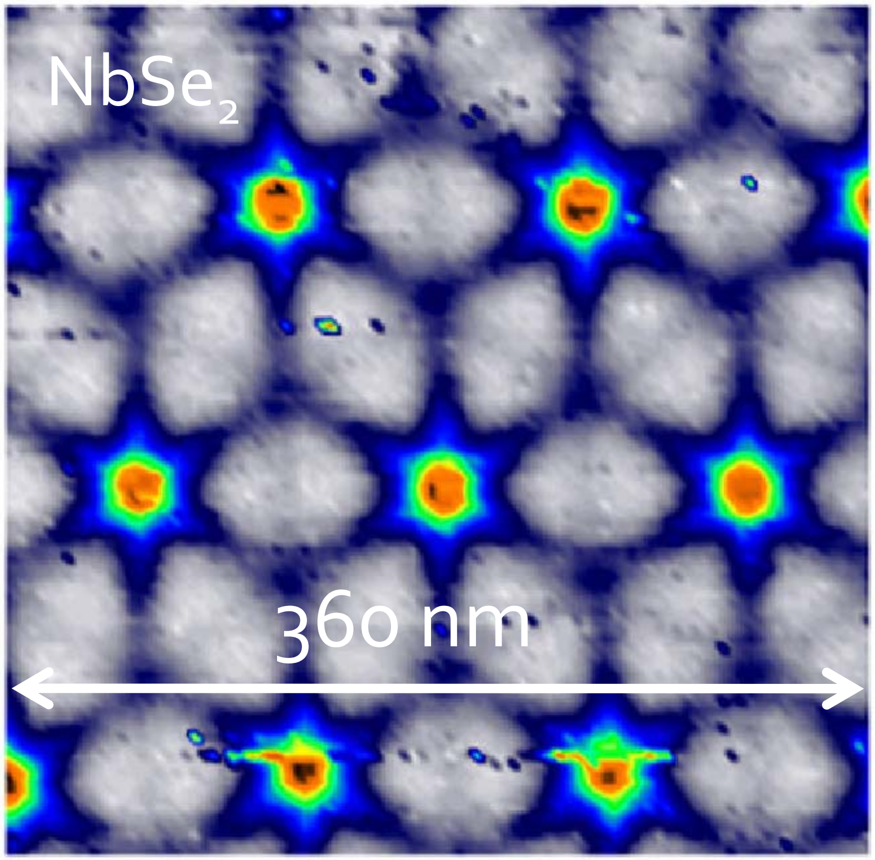
$$\varepsilon_{\min} \sim \frac{\hbar^2}{m\xi^2} \sim \frac{\hbar^2 \Delta_0}{m\hbar v_F \xi} \sim \frac{\Delta_0}{k_F \xi}$$

STM vortex images

PRL **101**, 166407 (2008) PHYSICAL REVIEW LETTERS week ending 17 OCTOBER 2008

Superconducting Density of States and Vortex Cores of 2H-NbS_2

I. Guillamón,[1] H. Suderow,[1] S. Vieira,[1] L. Cario,[2] P. Diener,[3] and P. Rodière[3]

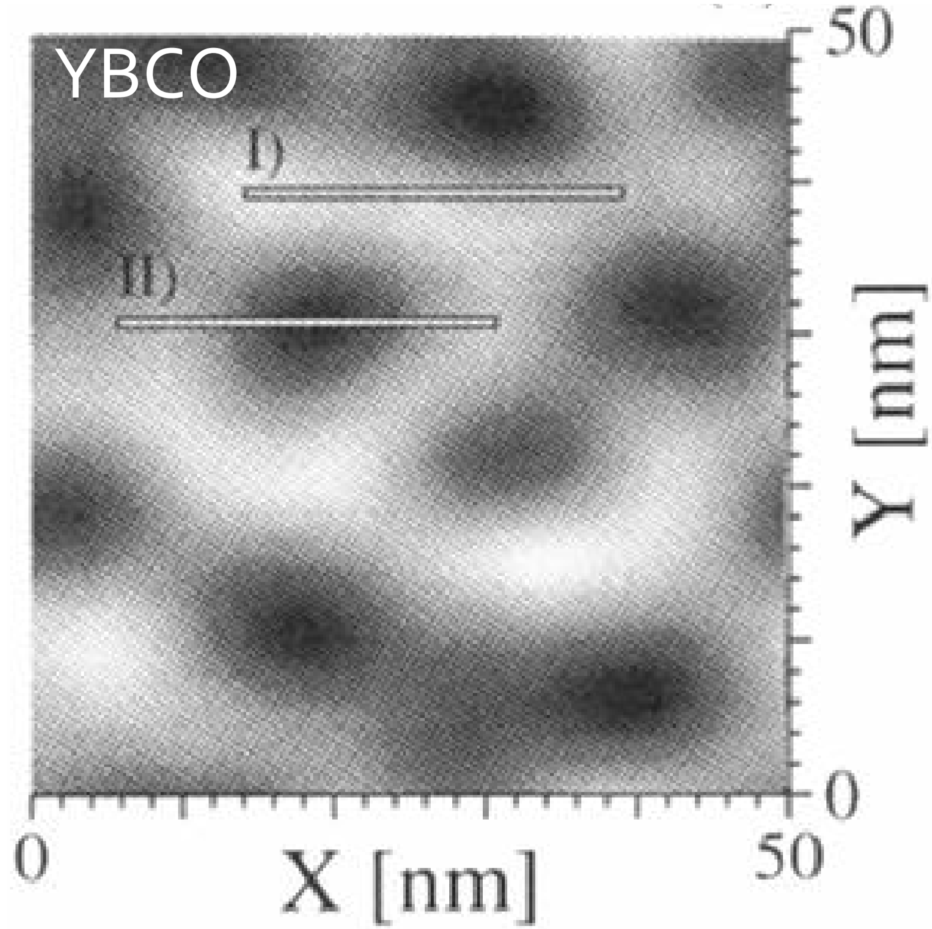


*PRL, **101**, 166407 (2008)*

VOLUME 75, NUMBER 14 PHYSICAL REVIEW LETTERS 2 OCTOBER 1995

Direct Vortex Lattice Imaging and Tunneling Spectroscopy of Flux Lines on $YBa_2Cu_3O_{7-\delta}$

I. Maggio-Aprile, Ch. Renner, A. Erb, E. Walker, and Ø. Fischer



*PRL, **75**, 2754 (1995)*

Fermi surface for electrons inside the core

Convenient parametrization of a classical trajectory:

1. Orbital momentum

$$\mu = [\vec{r}, \vec{k}_\perp]\vec{z}_0 = k_\perp r \sin(\theta_p - \theta)$$

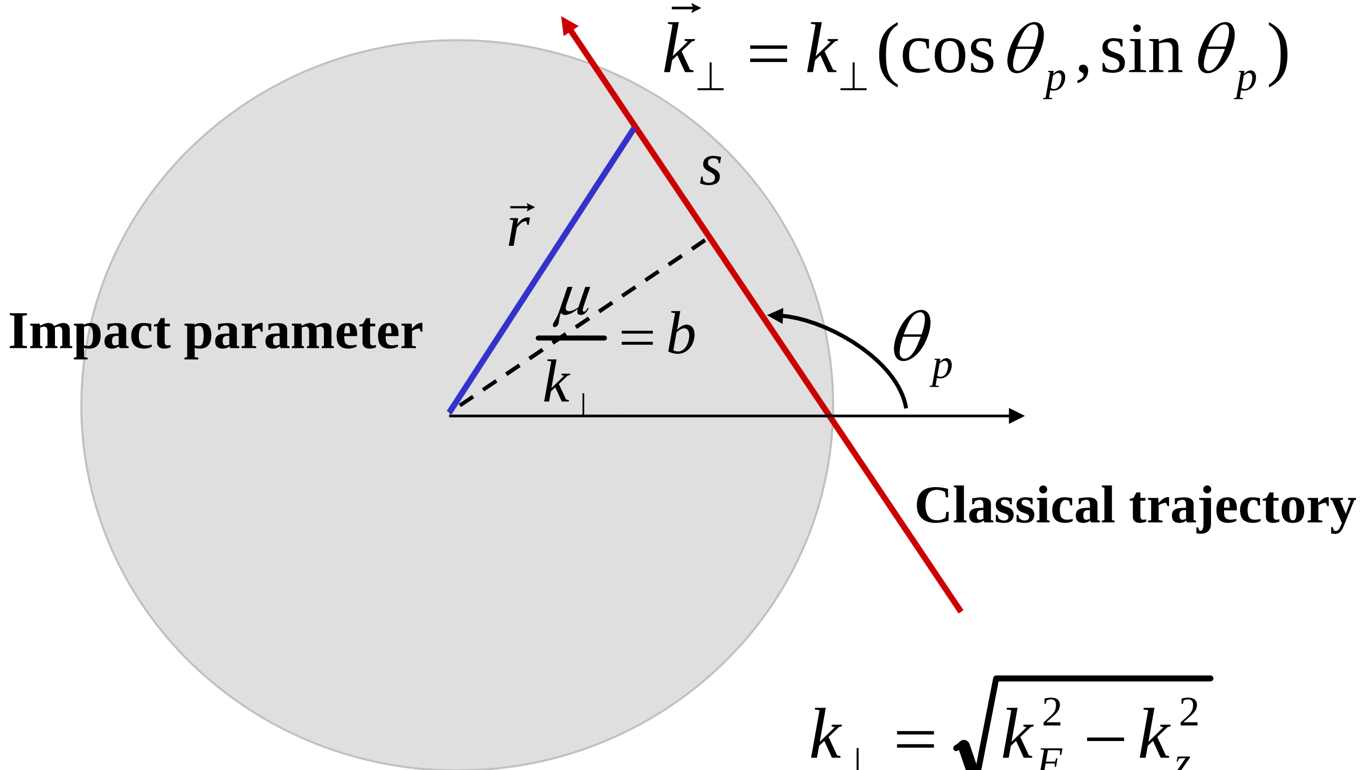
2. Trajectory orientation angle

$$\theta_p$$

Quasiclassical approximation

$$\lambda_F << \xi$$

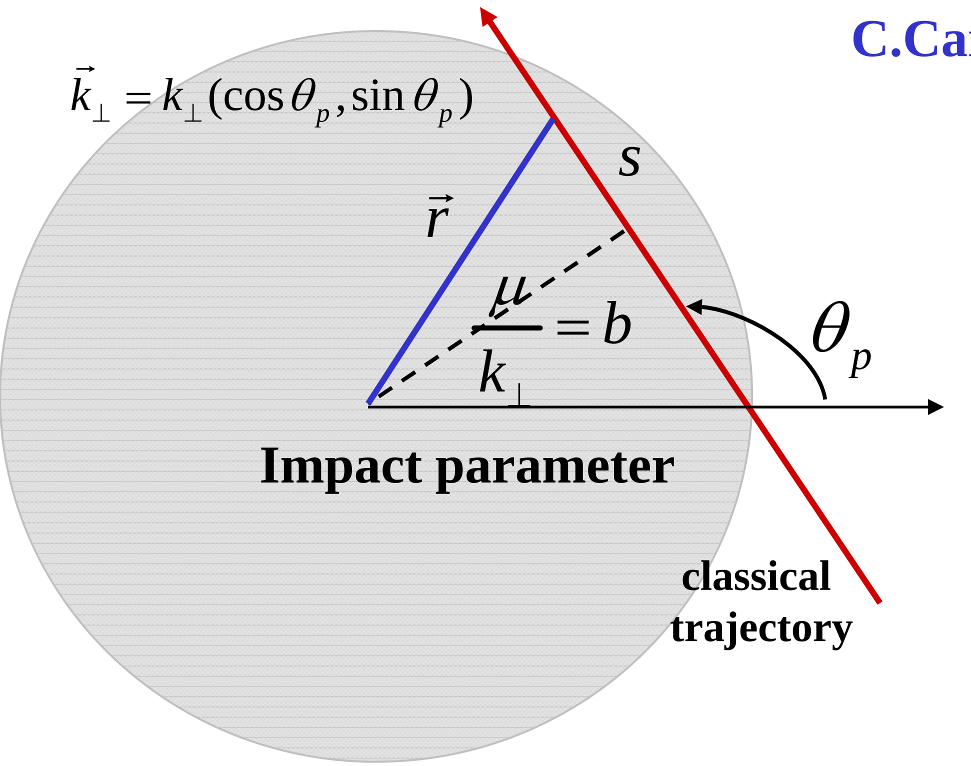
$$\vec{k}_\perp = k_\perp(\cos\theta_p, \sin\theta_p)$$



$$k_\perp = \sqrt{k_F^2 - k_z^2}$$

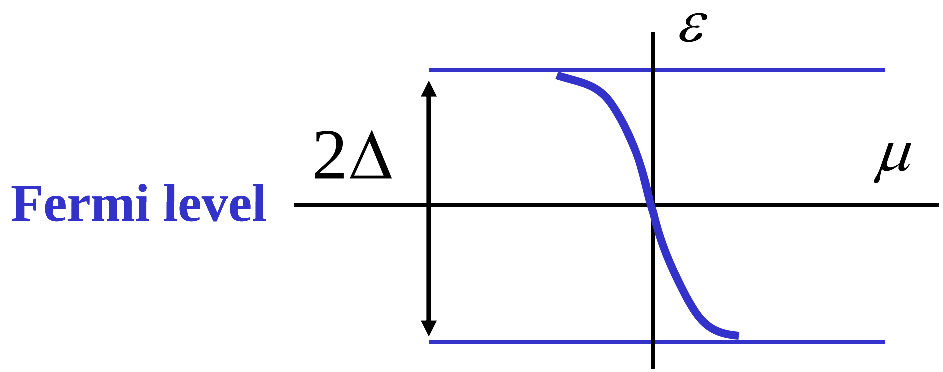
Bound quasiparticle states.

C.Caroli, P.G.de Gennes, J.Matricon (1964)



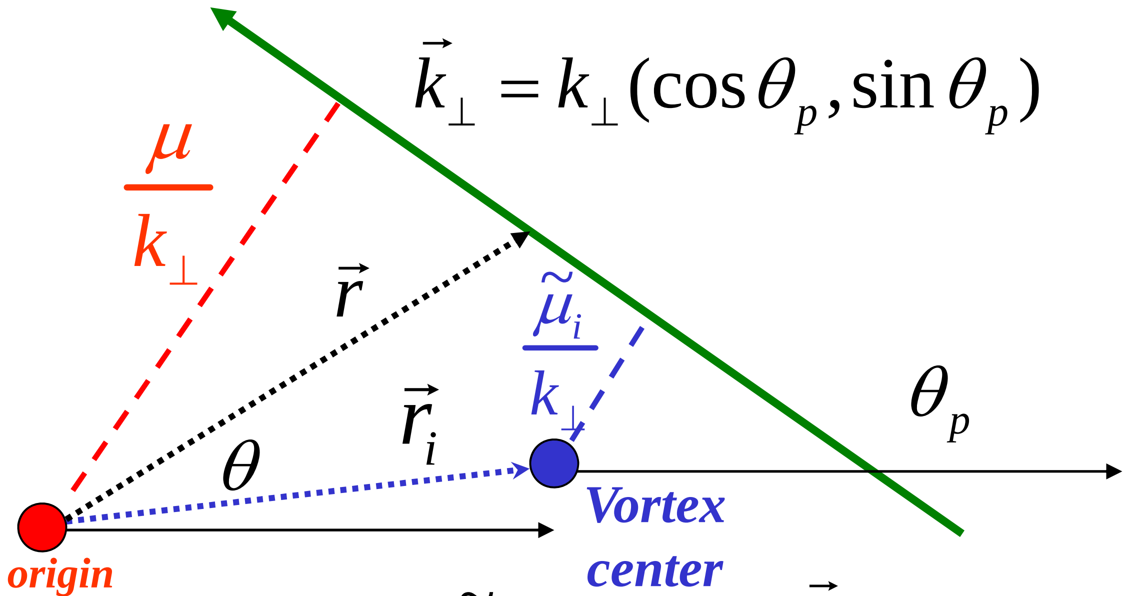
$$k_\perp = \sqrt{k_F^2 - k_z^2}$$

Anomalous spectral branch.



$$\varepsilon_\mu(k_\perp) = -\omega\mu \approx -\frac{\mu\Delta_0}{k_\perp\xi}$$

Fermi surface for electrons inside the core

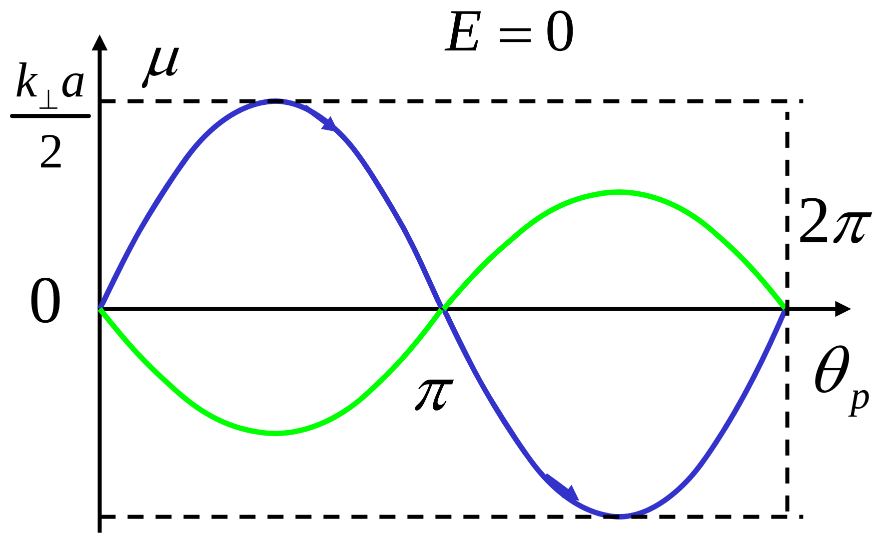


$$\vec{k}_\perp = k_\perp(\cos\theta_p, \sin\theta_p)$$

$$\tilde{\mu}_i = \mu - [\vec{r}_i, \vec{k}_\perp]\vec{z}_0$$

$$\varepsilon_i = -\omega\tilde{\mu}_i = -\omega\mu + \omega[\vec{r}_i, \vec{k}_\perp]\vec{z}_0$$

$$\mu_i(\theta_p) = -E/\omega + \omega[\vec{r}_i, \vec{k}_\perp]\vec{z}_0$$



Topological electronic transitions in vortex state?

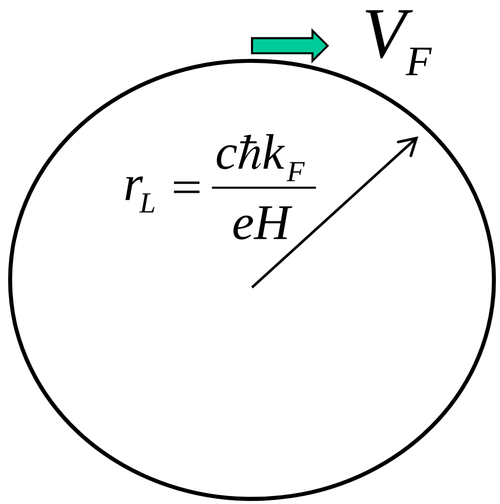
Opening of Fermi surface segments = creation of vortex-antivortex pair

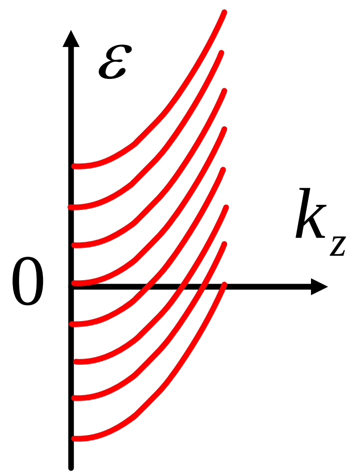
Merging and reconnection of Fermi surface segments = phase transitions in vortex matter governed by magnetic field and transport current

G.E.Volovik, JETP Lett. (1989):
Lifshits-type transitions between vortex states with different number of zero energy modes

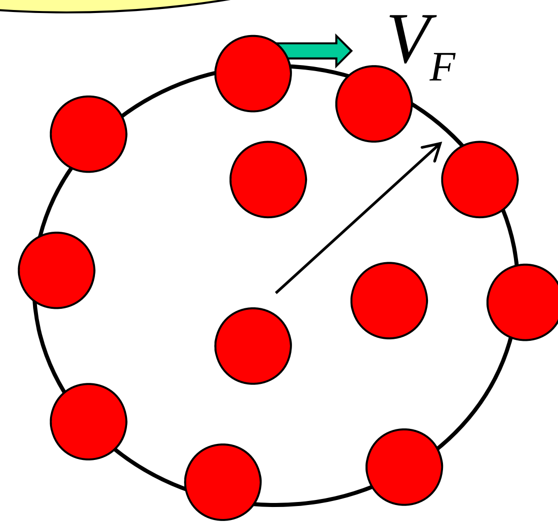
Simple example of topological transitions: de Haas-van Alfen oscillations.

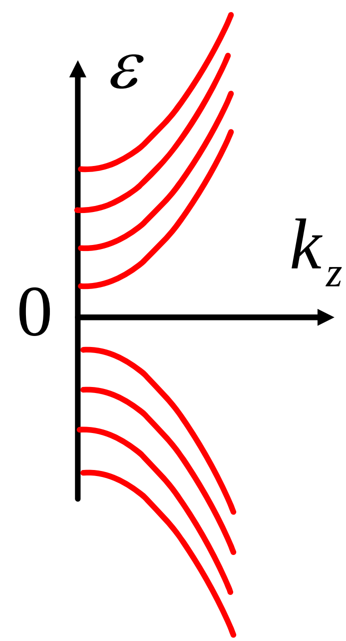
Normal metal.
Landau spectrum.
Cyclotron orbits.





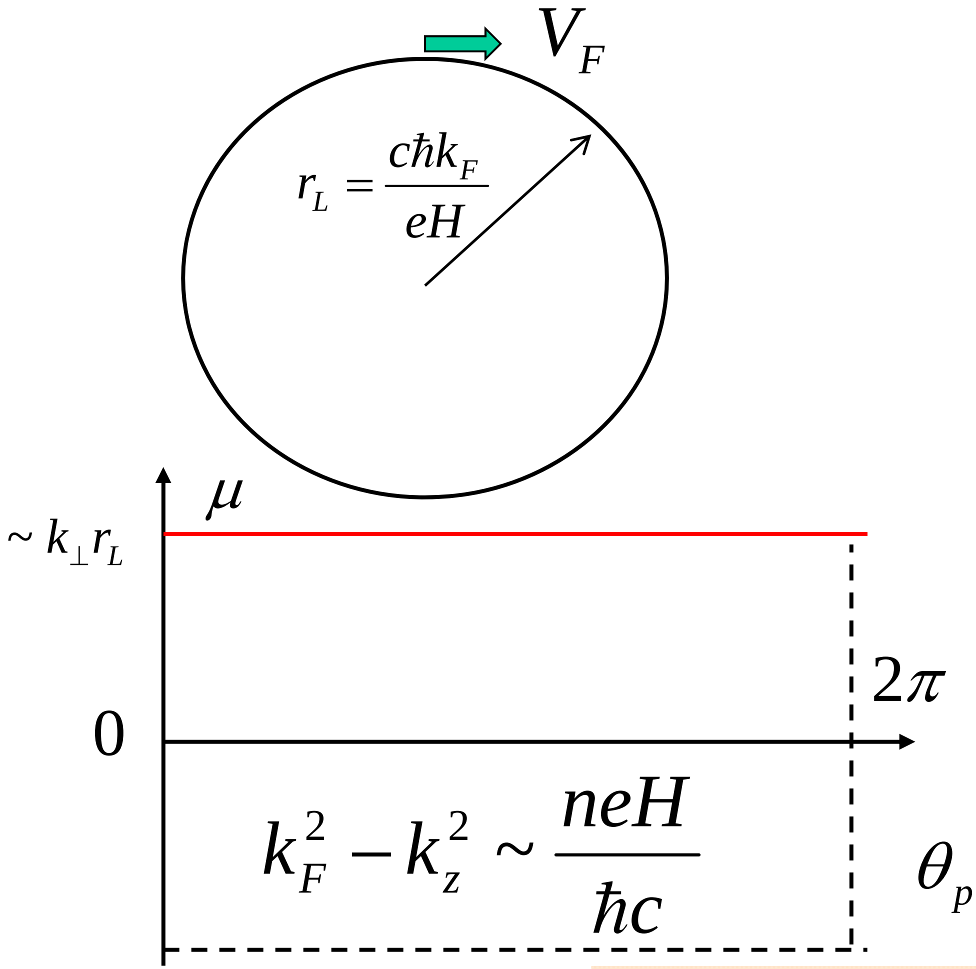
Superconductor.
Caroli – de Gennes – Matricon spectrum with minigap.
Cyclotron orbits are destroyed.



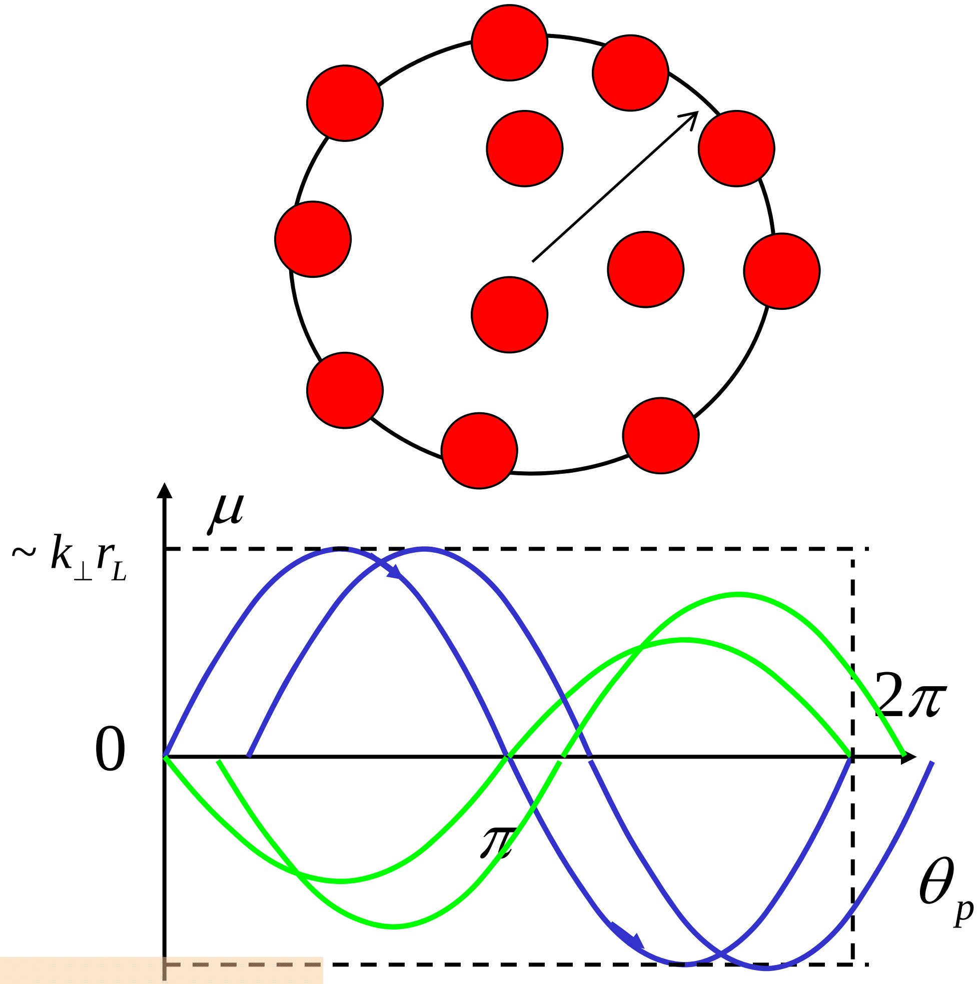


Simple example of topological transitions: de Haas-van Alfen oscillations.

Normal metal



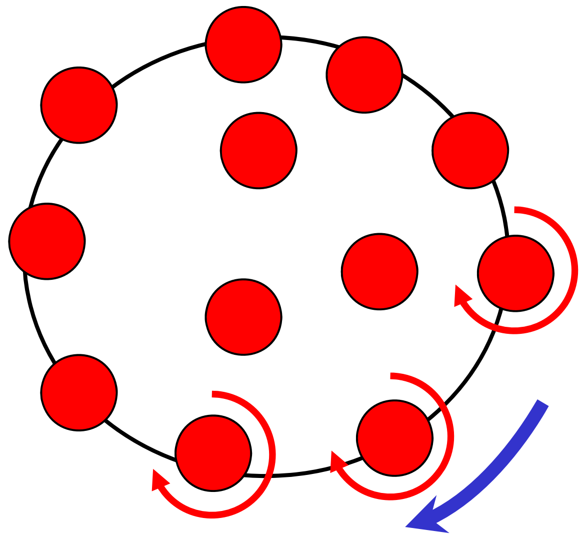
Superconductor.



$$\varepsilon \sim (n + 1/2)\omega$$

$$\int_0^{2\pi n_\theta} \mu(\theta_p) d\theta_p = 2\pi(n + \beta)$$

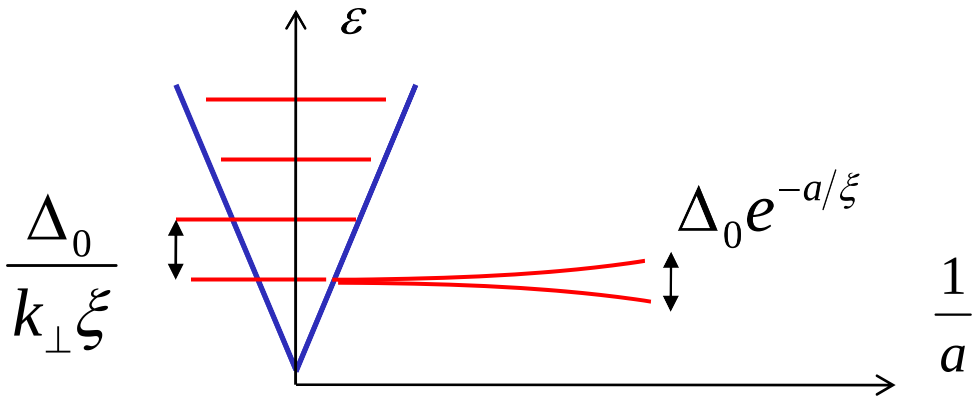
Precession of classical trajectory inside vortex core competes with rotation along the cyclotron trajectory



Question: how to restore the cyclotron motion?

Answer: to take account of quasiparticle tunneling between the vortex cores.

Qualitative arguments.
Critical intervortex distance:
minigap = energy level splitting due to tunneling



$$a_c \approx \frac{\xi}{2} \ln(k_F \xi)$$

Typical intervortex distance $a \sim \sqrt{\frac{\phi_0}{H}}$

$$k_F \xi \sim 10^2 \div 10^3$$

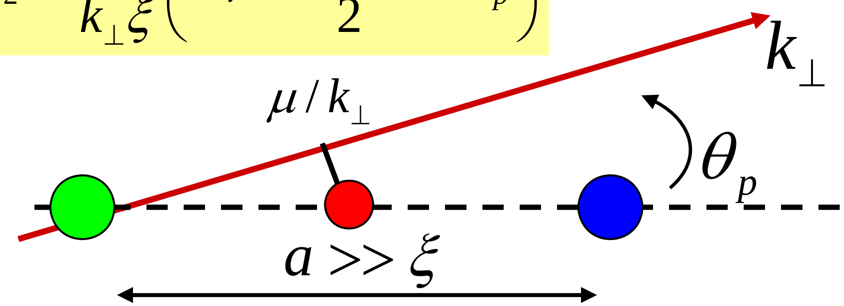
$$\frac{a_c}{\xi} \approx 2 \div 3$$

$$H^* \sim \frac{\phi_0}{a_c^2} \sim \frac{H_{c2}}{[\ln(k_F \xi)]^2}$$

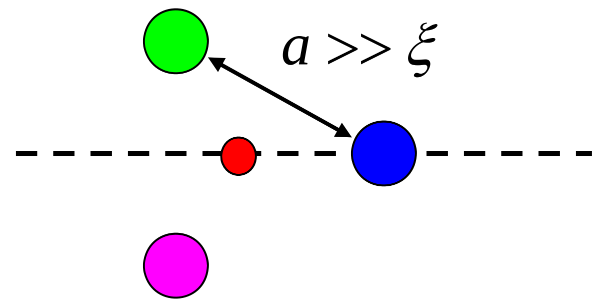
Examples of reconnection of Fermi lines:

2 vortices.

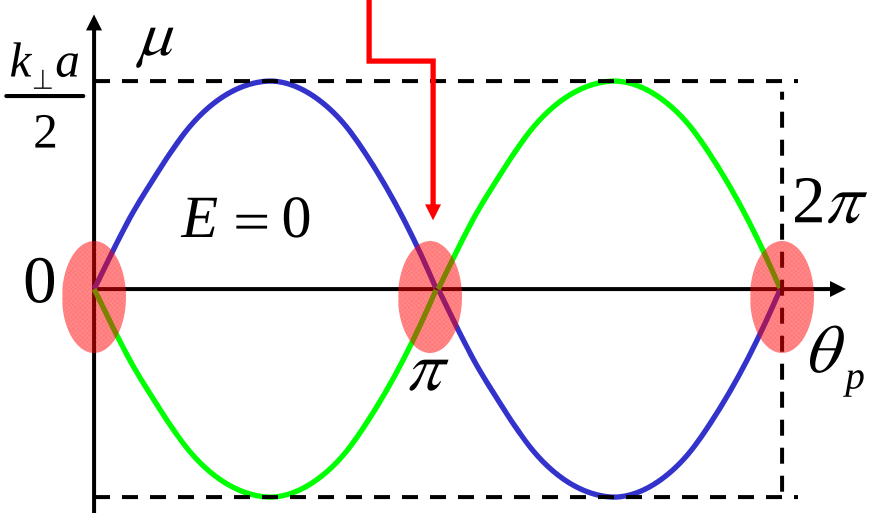
$$\varepsilon_{1,2}=\frac{\Delta}{k_\perp \xi}\left(-\mu \pm \frac{k_\perp a}{2}\sin\theta_p\right)$$

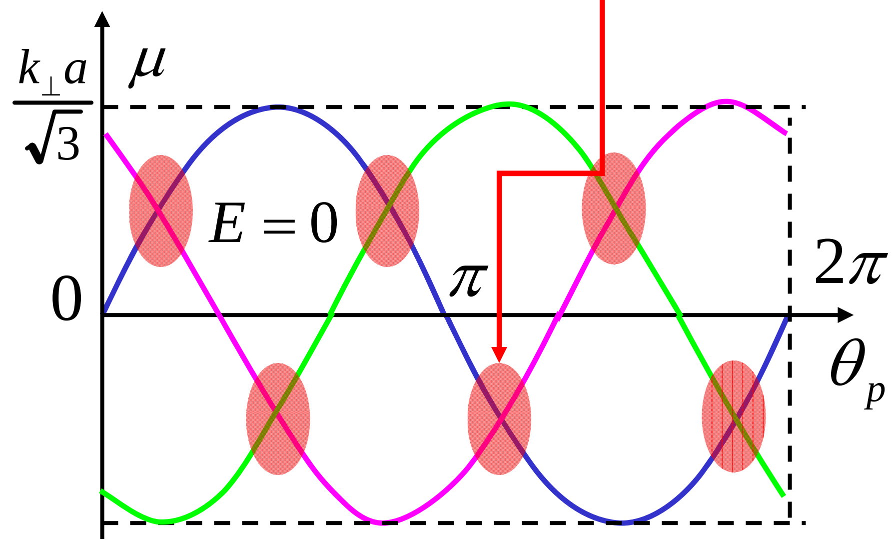


3 vortices.



Trajectories pass through neighboring vortex cores





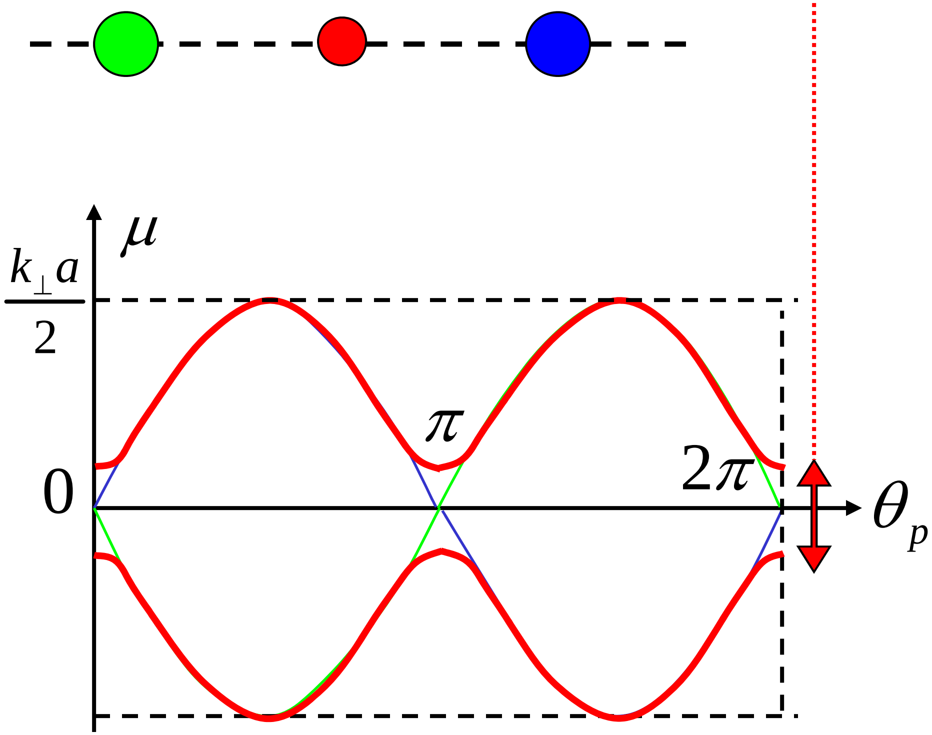
Tunneling between vortices. Splitting of CdGM energy branches.

$$\delta E \sim \Delta_0 \exp\left(-\frac{k_F a_{ij}}{k_\perp \xi} \right)$$

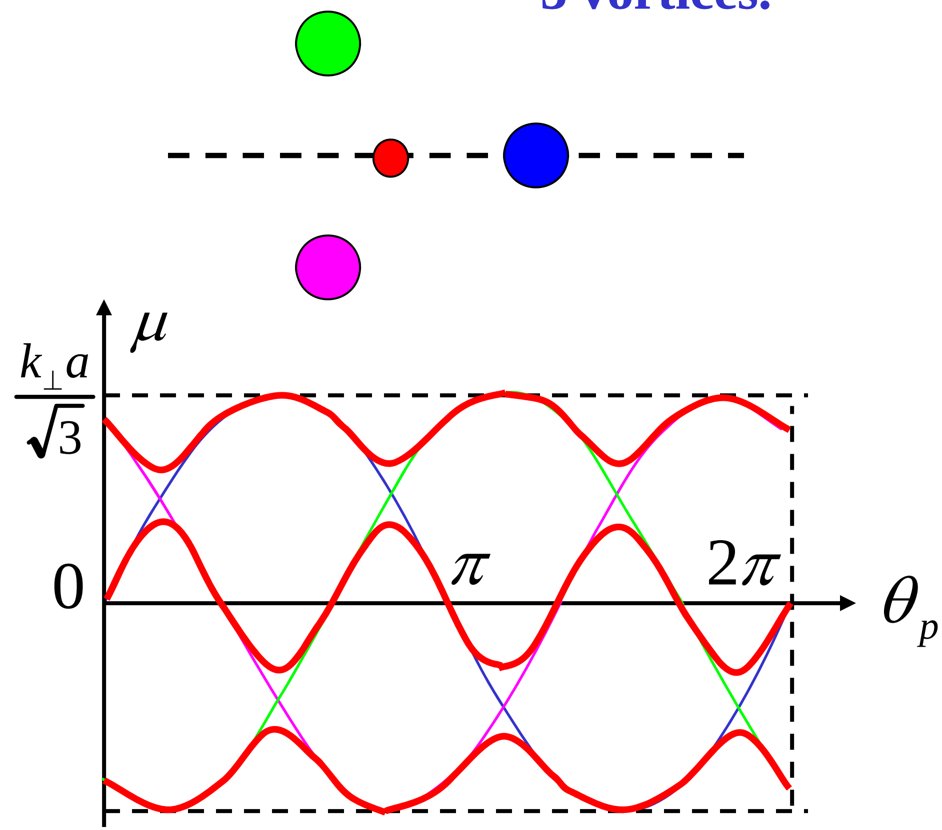
$$a_{ij} = |\vec{r}_i - \vec{r}_j|$$

$$\delta\mu(a_{ij}) \sim \frac{\delta E}{\omega} \sim k_\perp \xi \exp\left(-\frac{k_F a_{ij}}{k_\perp \xi} \right)$$

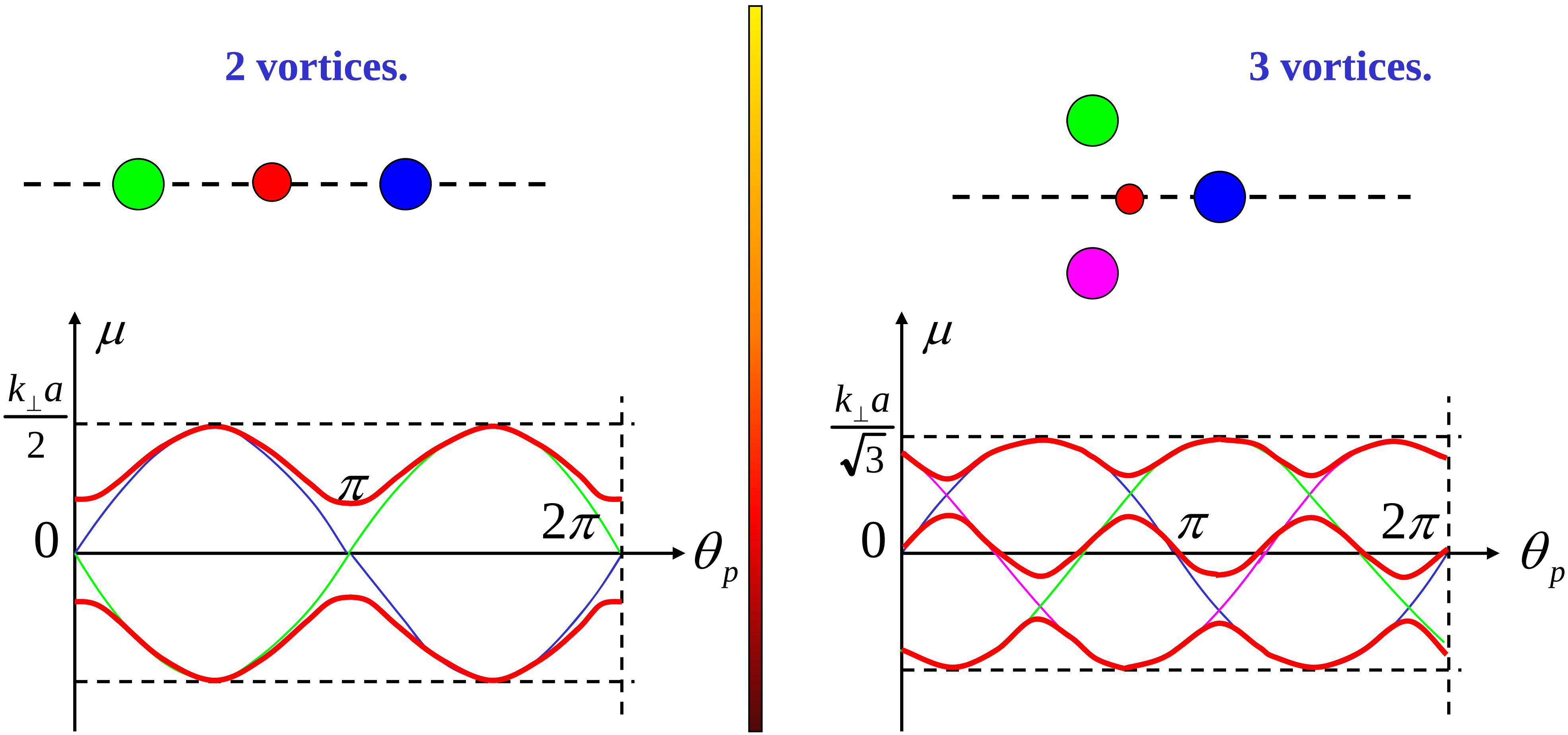
2 vortices.



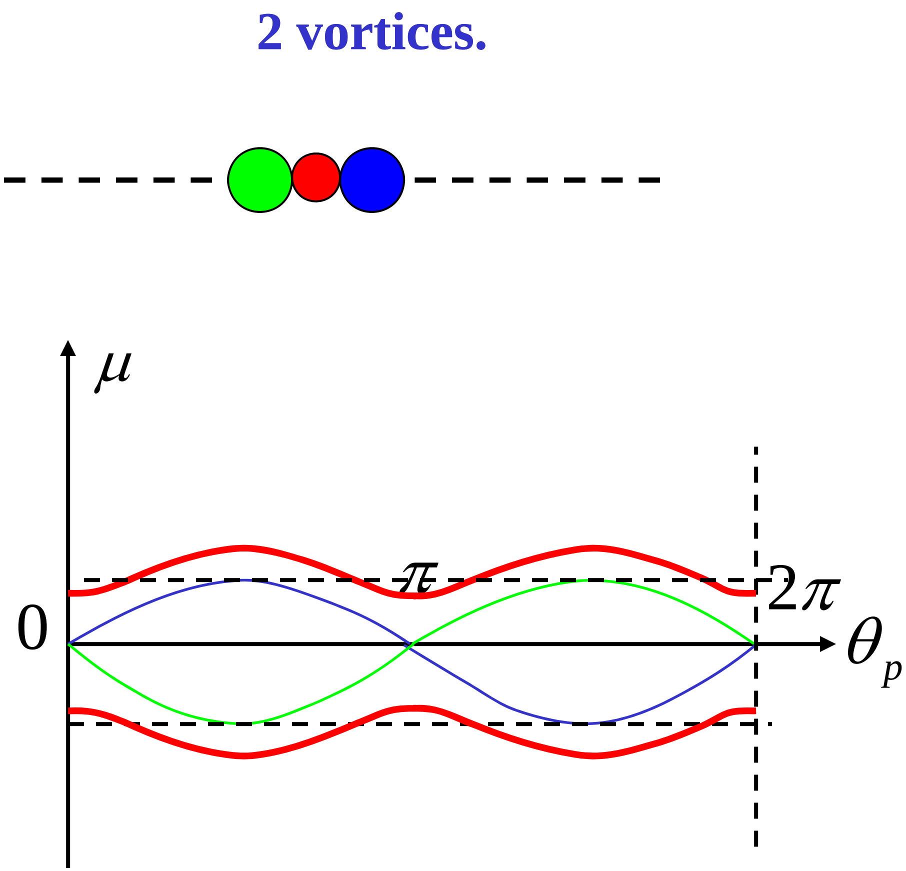
3 vortices.

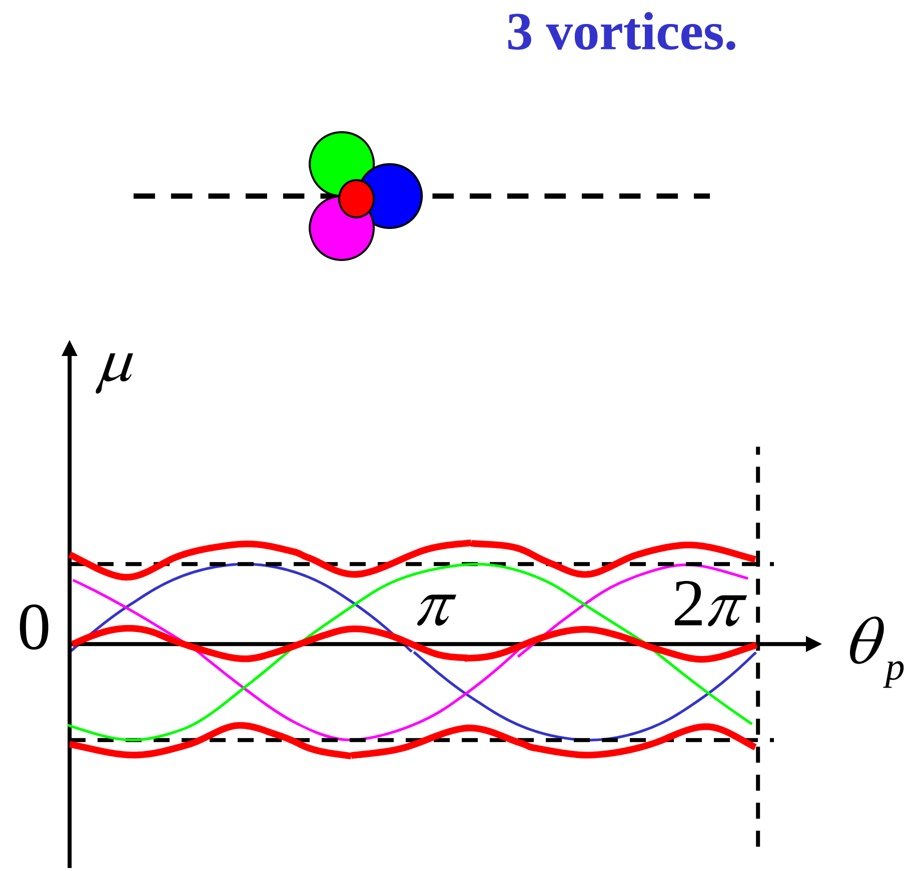


Crossover to a multiquantum vortex



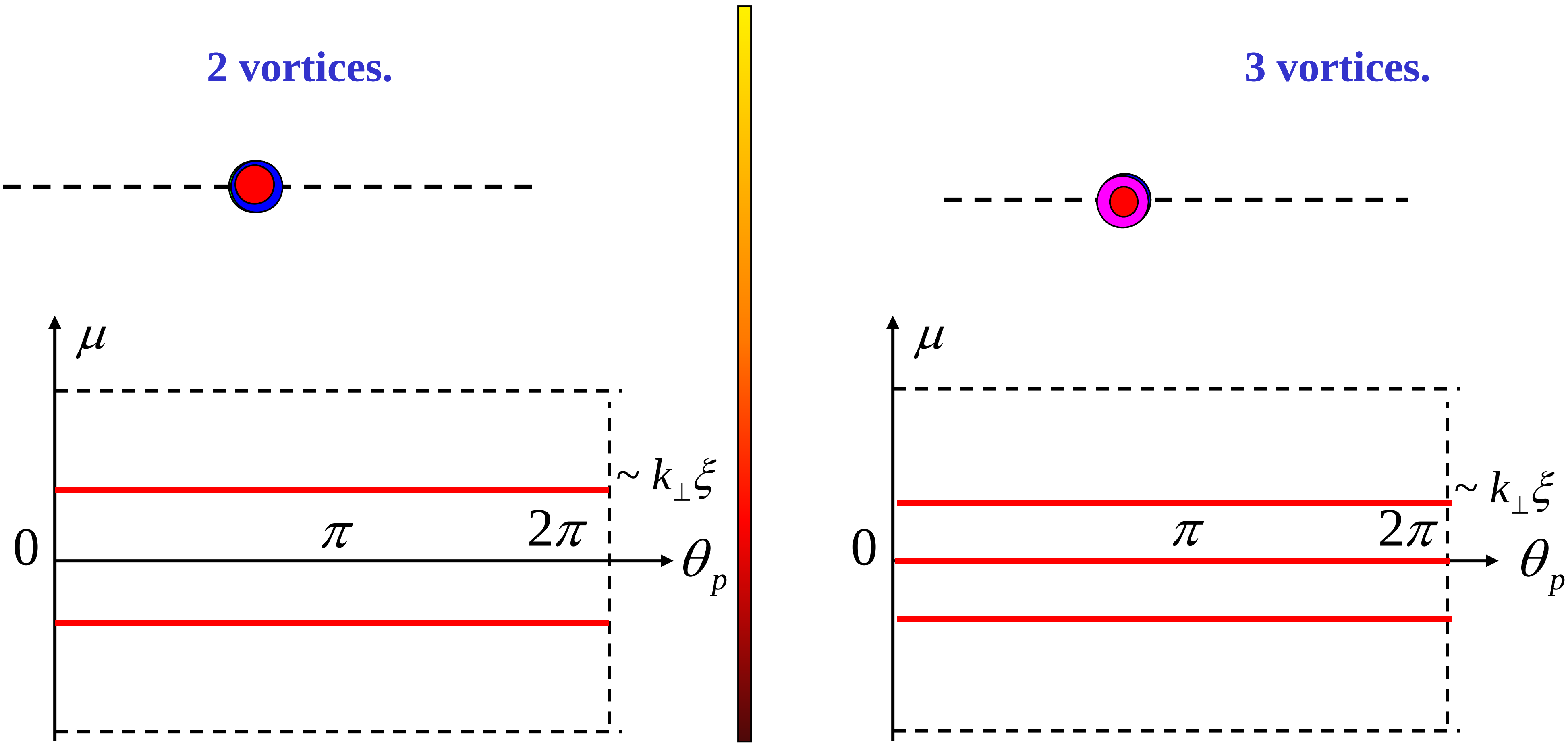
Crossover to a multiquantum vortex





Crossover to a multiquantum vortex

$$a = 0$$



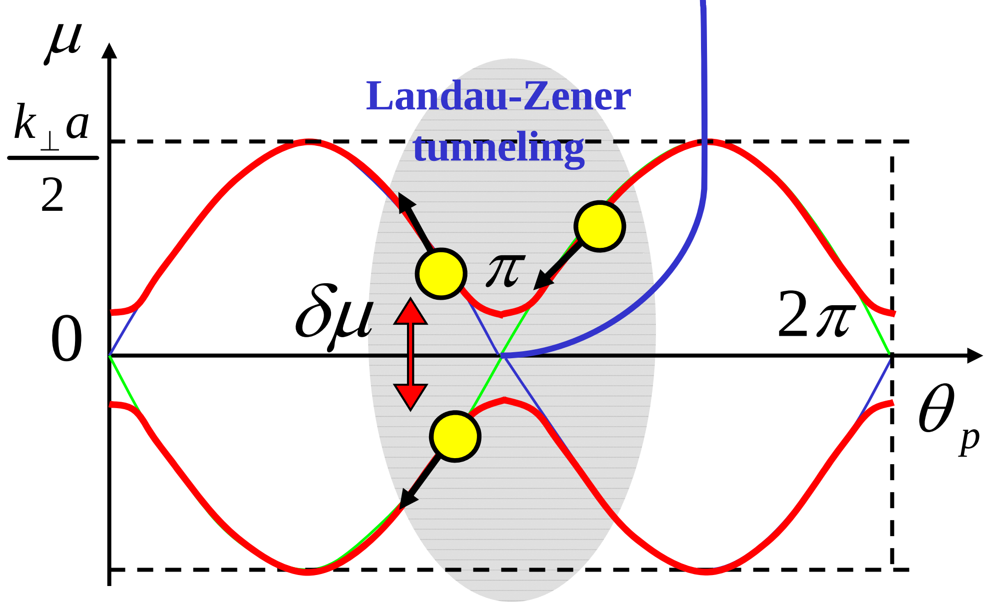
Quantum mechanics of precessing trajectories

$$[\theta_p, \hat{\mu}] = i$$

Uncertainty principle: $\Delta\mu\Delta\theta_p \sim 1$

$$\Delta\mu \sim k_\perp a_{ij} \Delta\theta_p$$

Quantum mechanical uncertainty of the angular momentum near the branch crossing points = $\Delta\mu \sim \sqrt{k_\perp a_{ij}}$



Criterion of intervortex tunneling efficiency:

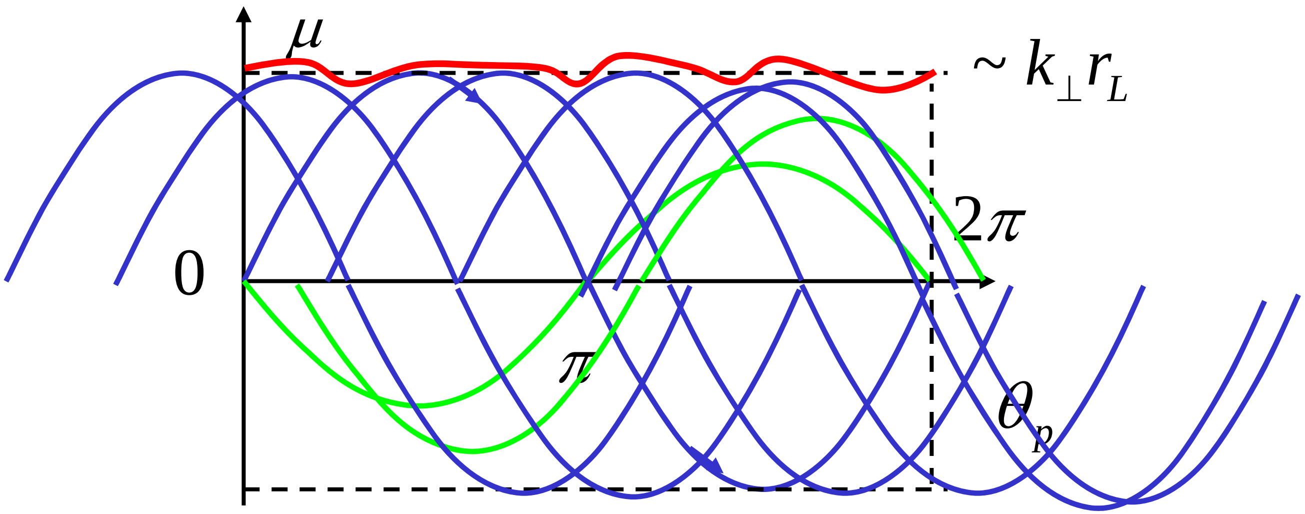
$$\delta\mu >> \Delta\mu$$

$$\frac{k_\perp \xi^2}{a_{ij}} \exp\left(-\frac{2k_F a_{ij}}{k_\perp \xi}\right) >> 1$$

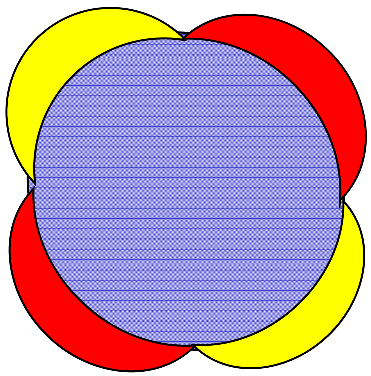
Reconnection of Fermi surface segments due to the intervortex tunneling

Magnetic breakdown criterion.

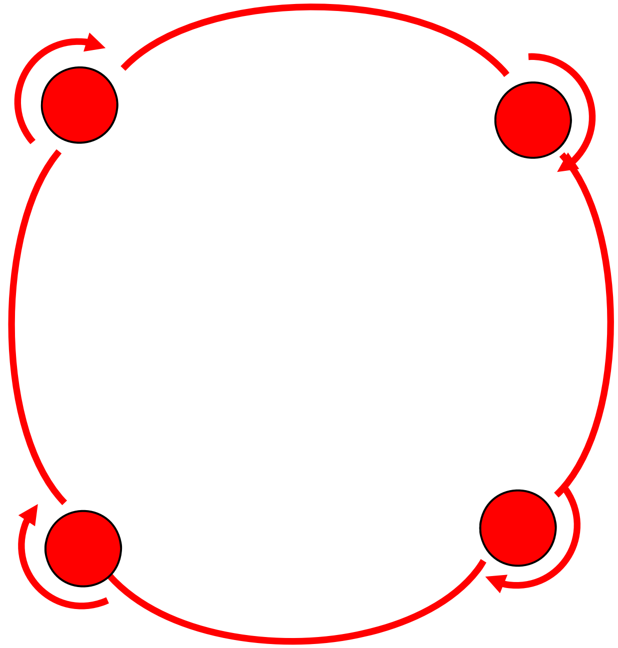
$$H > H^* \sim \frac{H_{c2}}{[\ln(k_F \xi)]^2}$$



Useful generalization: d-wave pairing.
Cyclotron arcs are connected by
precession of trajectories inside the cores



$$\Delta = \frac{\Delta_0 k_x k_y}{k_F^2}$$



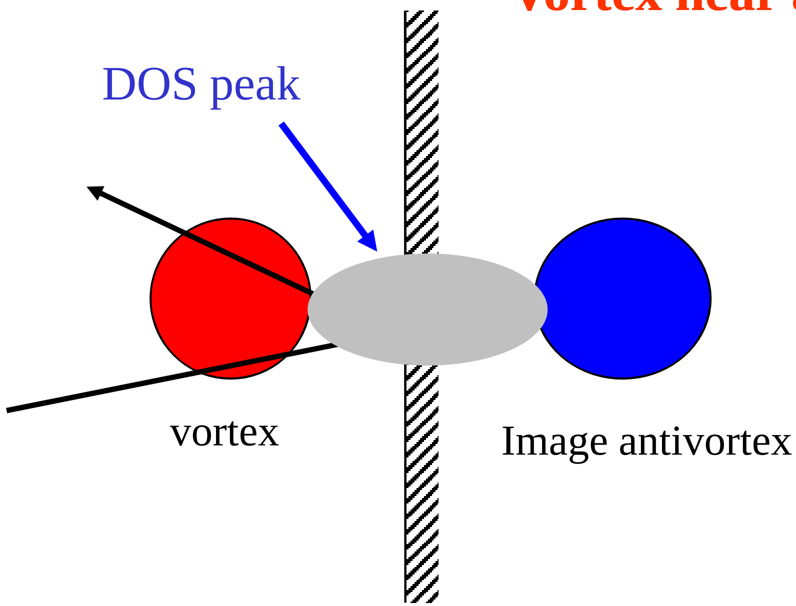
$$\delta\theta \sim \frac{a_c}{a}$$

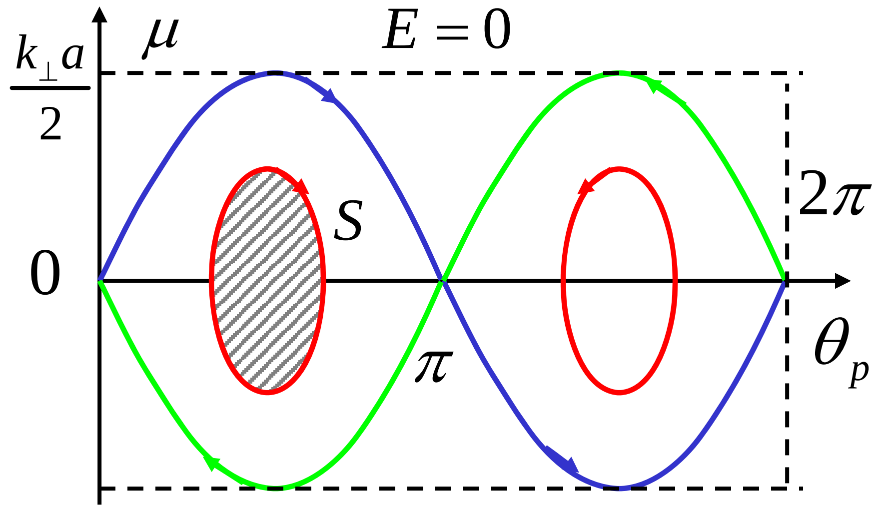
$$\int_0^{2\pi n_\theta} \mu(\theta_p) d\theta_p \sim k_F r_L \frac{a_c}{a} \sim 2\pi(n+\beta)$$

$$k_F^2 \sqrt{\frac{H}{H^*}} \sim \frac{neH}{\hbar c}$$

Examples of topological transitions.
Opening of Fermi surface segments in vortex cores.

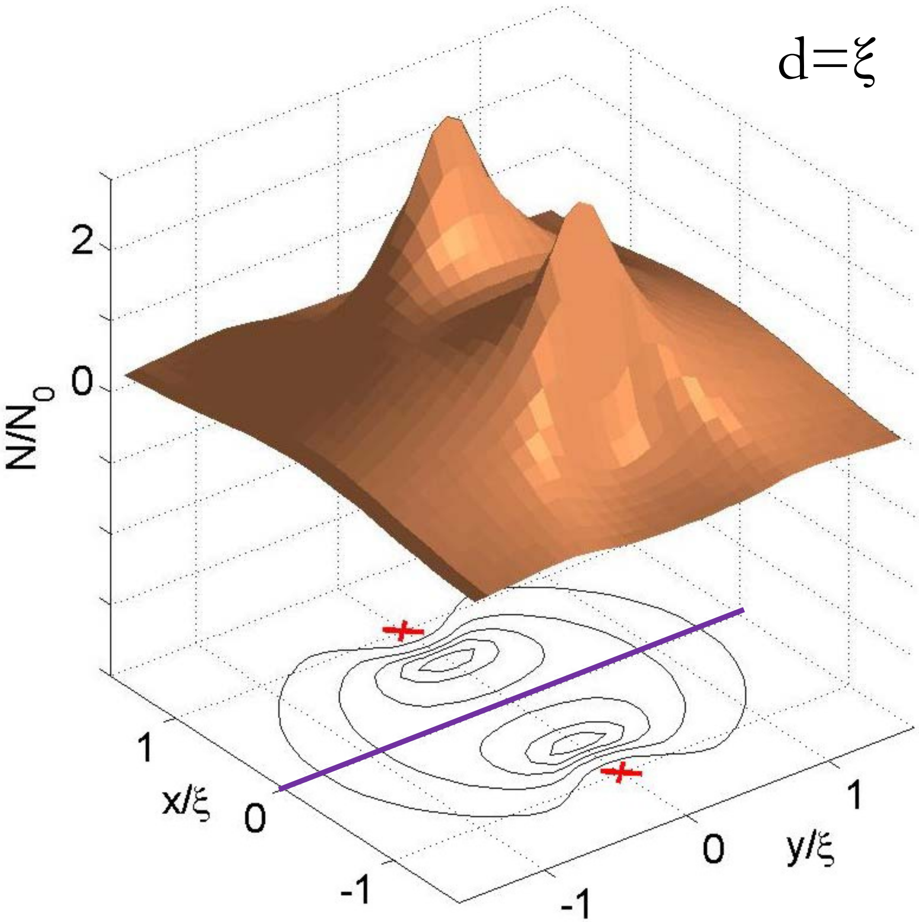
Vortex near a flat surface

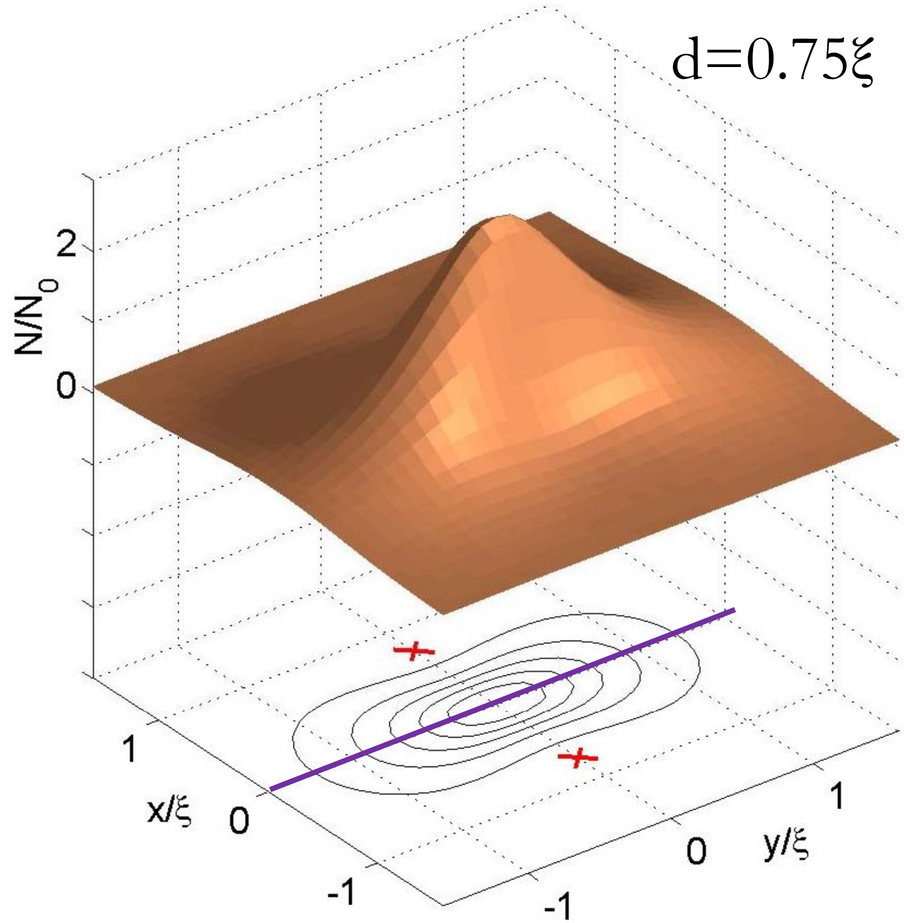




Anomalous branch disappears. DOS decreases

Local DOS for a vortex positioned near the surface





LDOS peak is shifted towards the boundary

Several boundaries: splitting of LDOS peak becomes possible

Vortex pinning by columnar defects

Artificial columnar pinning centers:

Proton or heavy-ion irradiation : 2R ~ 50A

Inclusion of normal particles or nanorods: 2R ~ 50-100A

Submicrometer holes (nanorods): 2R ~ 0.1-1 μm

Attraction of vortex to a cavity (hole)

G.S.Mkrtchyan and V.V.Shmidt (1972)
A.Buzdin and D.Feinberg (1996)
A.Buzdin and M.Daumens (1998, 2000)
H.Nordborg and V.Vinokur (2000)

Microscopic theory of vortex pinning by scattering centers

E.V.Thuneberg et al. (1982,1984)

$$\sigma << \xi_0^2$$

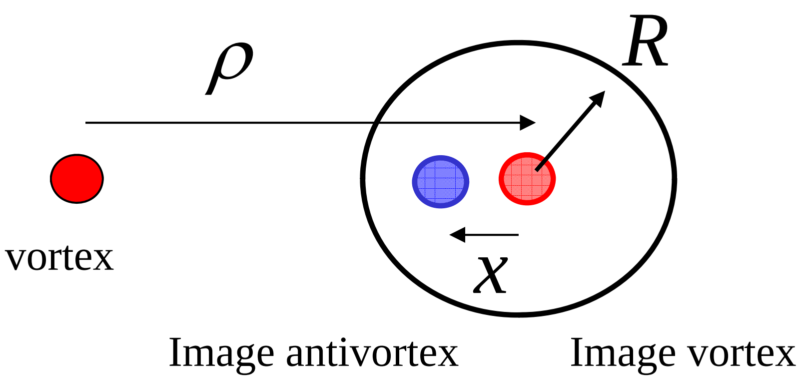
Multiquanta vortices trapped by the cavities

A.I.Buzdin (1993)
A.Bezryadin, A.I. Buzdin, B. Pannetier (1994)
A.Bezryadin, B. Pannetier (1995)
V.Bruyndoncx et al (1999)

Pinning mechanisms

$$R >> \xi$$

Electrodynamic mechanism



$$x\rho = R^2$$

$$R << \xi$$

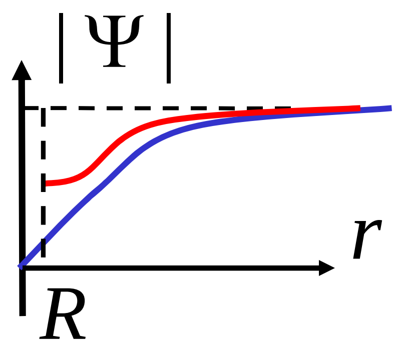
Gain in the vortex core energy

Local contribution

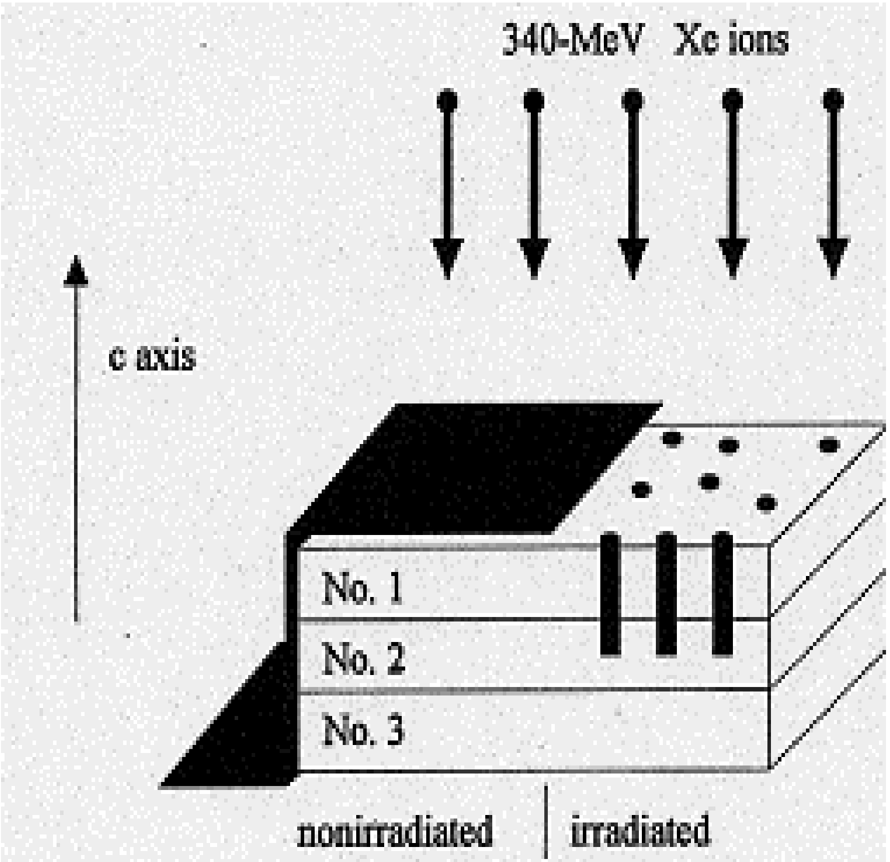
$$\delta E \sim \frac{H_{cm}^2}{8\pi} V_{defect}$$

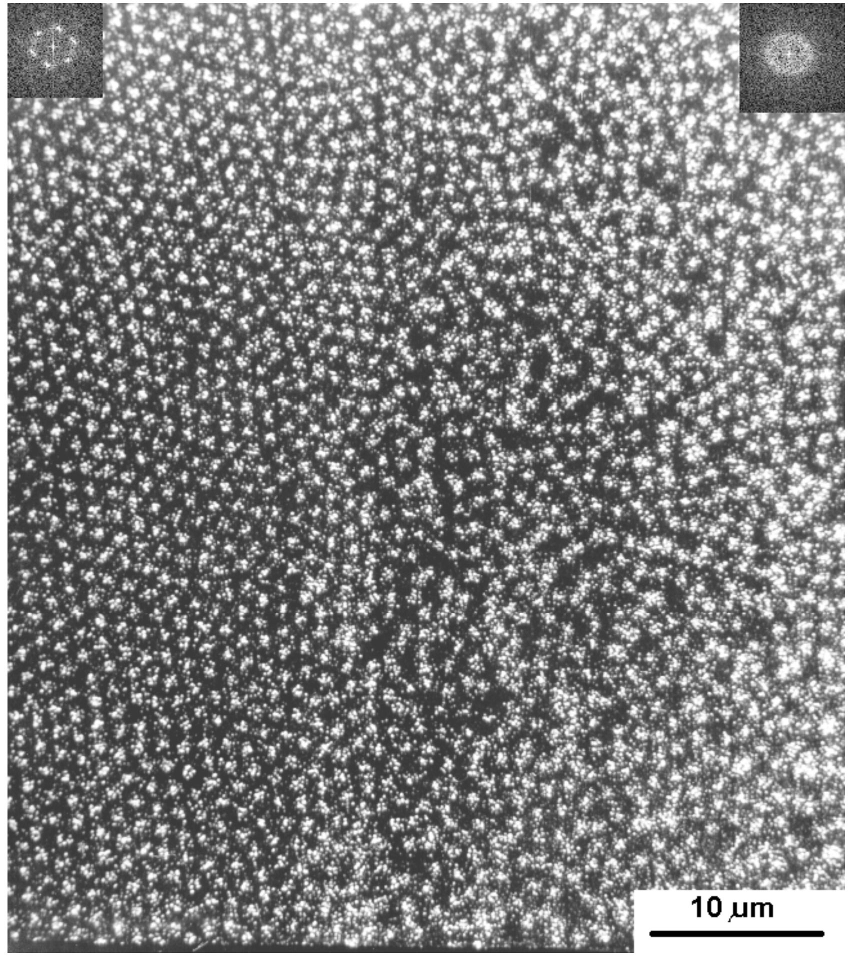
Nonlocal contribution

$$\delta E > \frac{H_{cm}^2}{8\pi} V_{defect}$$

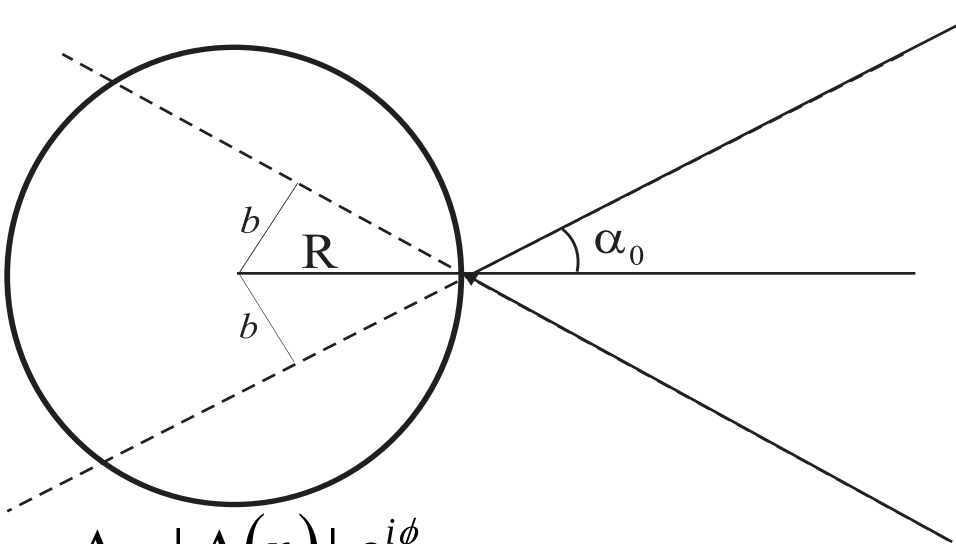


Flux Line Lattice destruction by columnar defects in BSCCO



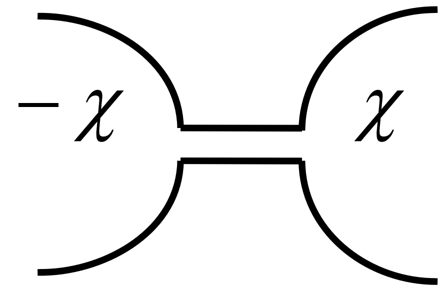


Vortex trapped by a defect with radius $R >> \lambda_F$
Analogy with quasiparticle spectrum of Josephson junction

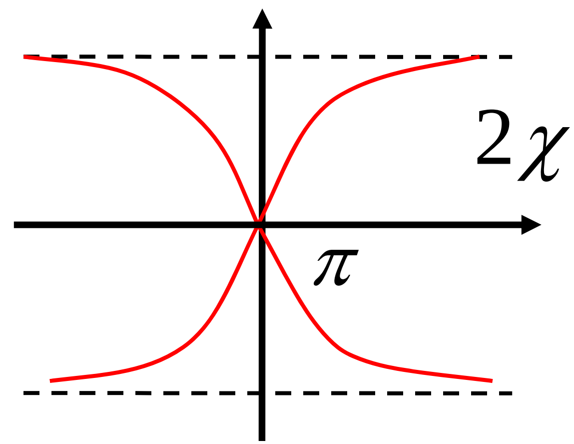


$$\Delta = |\Delta(r)| e^{i\phi}$$

Boundary condition: $\hat{\Psi}(R) = 0$

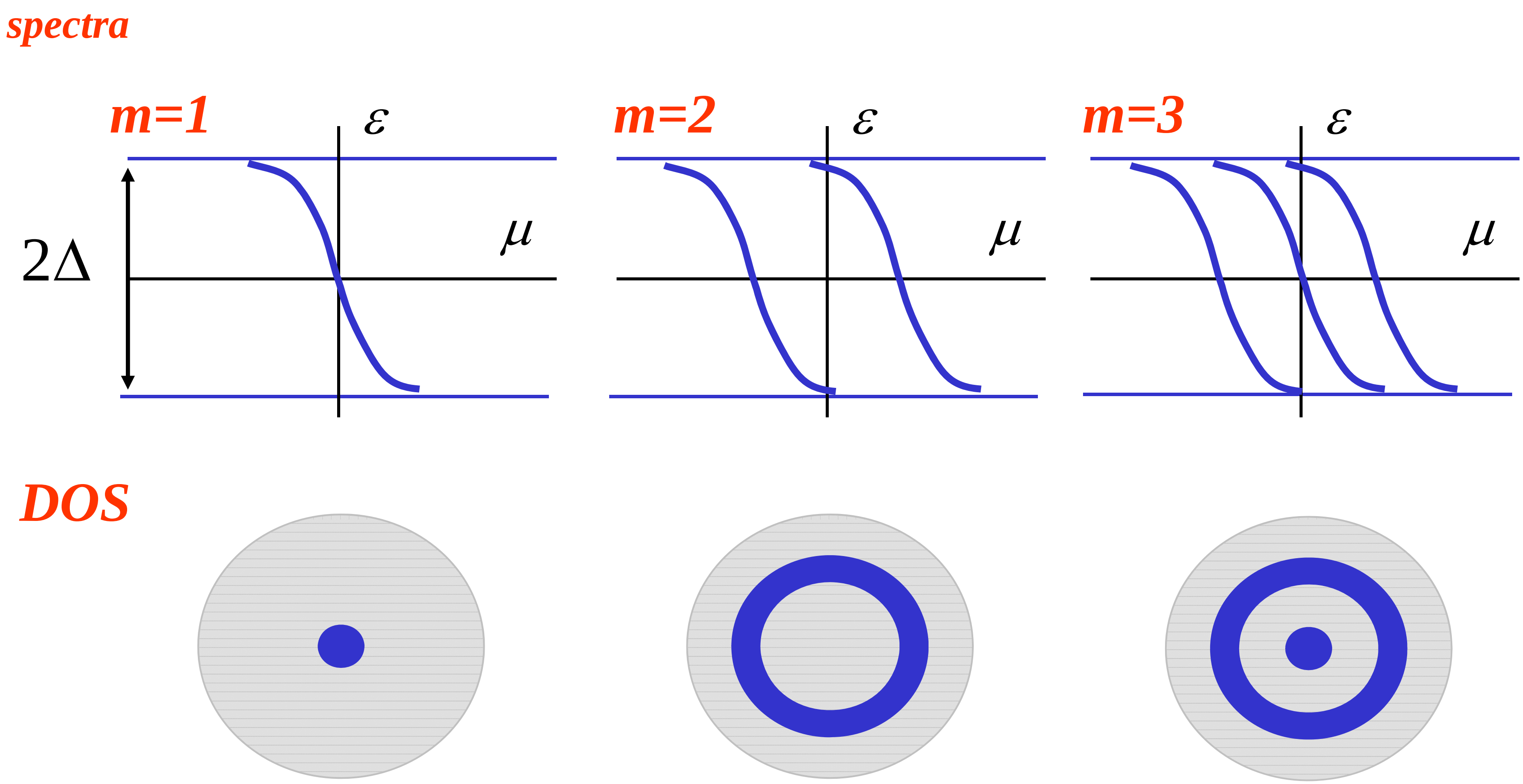


$$E/\Delta = \pm\cos\chi$$



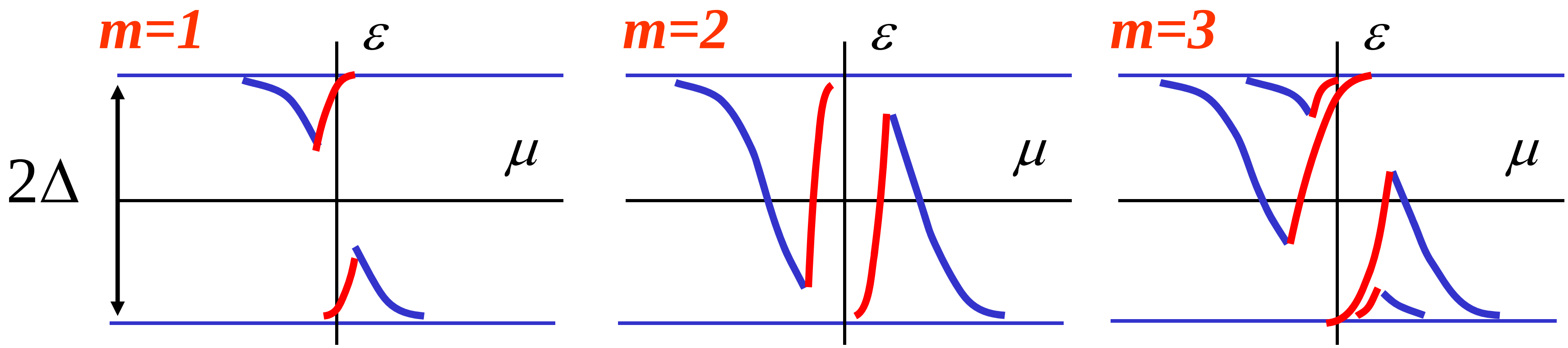
$$E/\Delta_0 = \cos\chi = \cos\alpha_0 \qquad E/\Delta_0 = sign(b)\sqrt{1-\frac{b^2}{R^2}}$$

Anomalous specral branches for different vorticities.



Anomalous specral branches for different vorticities.
Influence of defect

spectra

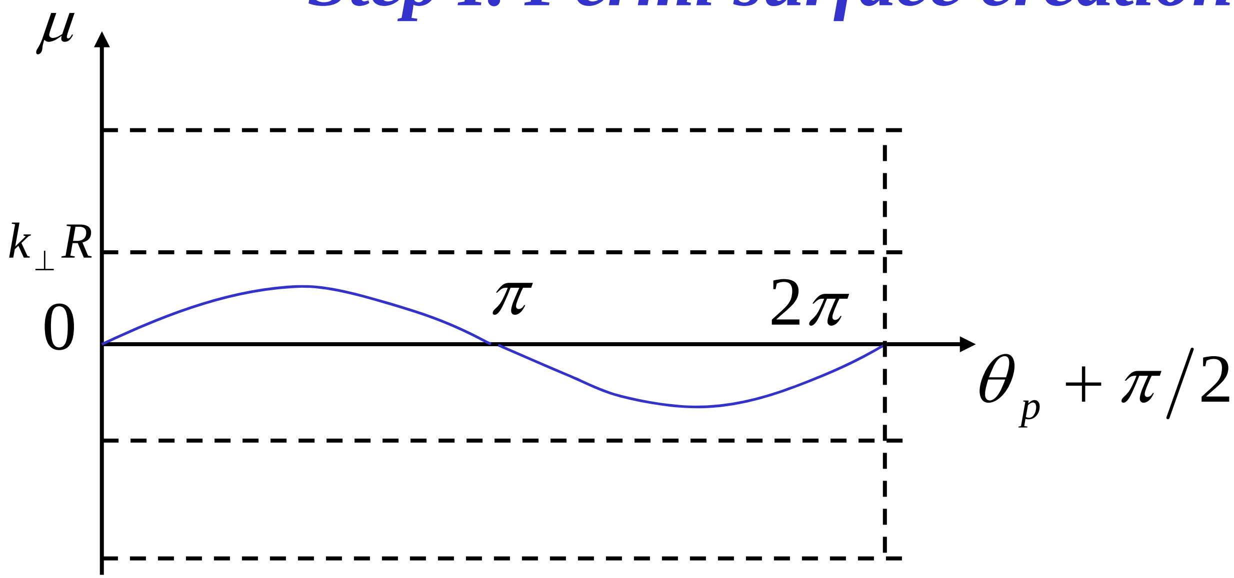


DOS

Increasing minigap

Vortex escape as a Fermi surface transformation

Step I. Fermi surface creation

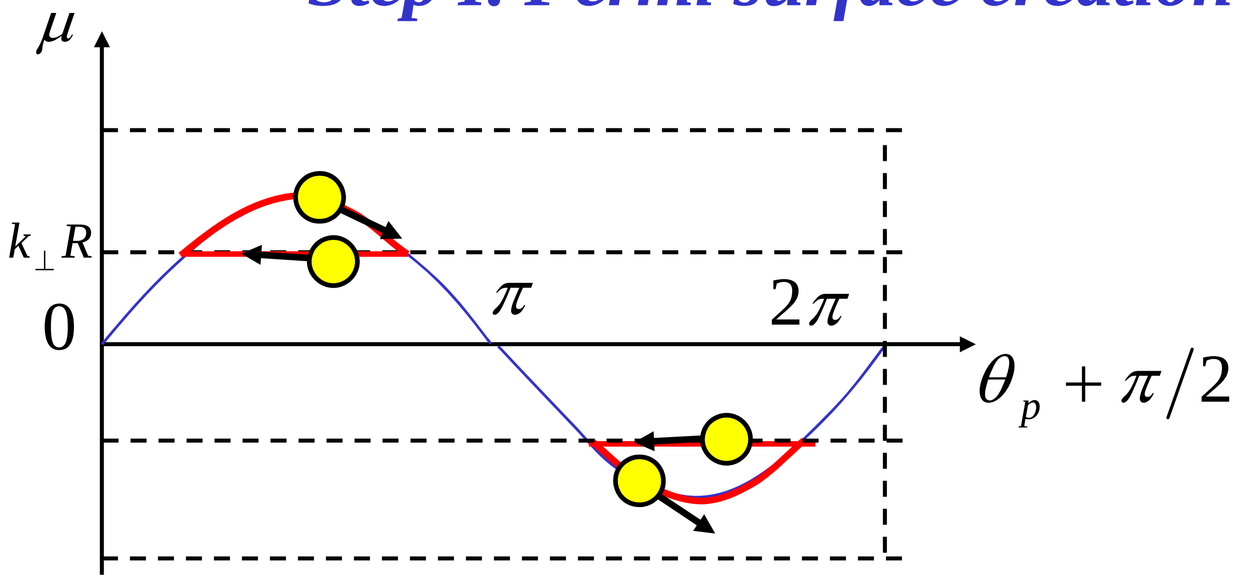


$$E = -\omega\mu + \hbar k_\perp V_s \cos\theta_p$$

$$\mu = \frac{\hbar k_\perp V_s}{\omega}\cos\theta_p \qquad V_s < \frac{\omega R}{\hbar}$$

Vortex escape as a Fermi surface transformation

Step I. Fermi surface creation

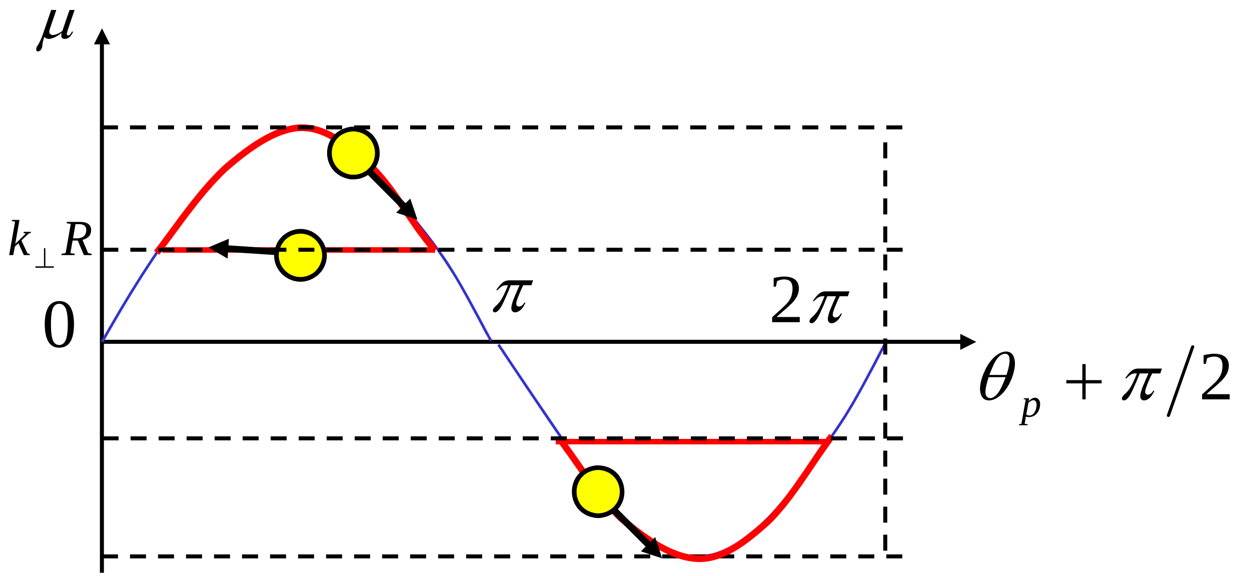


$$E = -\omega\mu + \hbar k_\perp V_s \cos\theta_p$$

$$\mu = \frac{\hbar k_\perp V_s}{\omega}\cos\theta_p \qquad V_s > \frac{\omega R}{\hbar}$$

Vortex escape as a Fermi surface transformation

Step I. Fermi surface creation. Vortex is bound to the cavity



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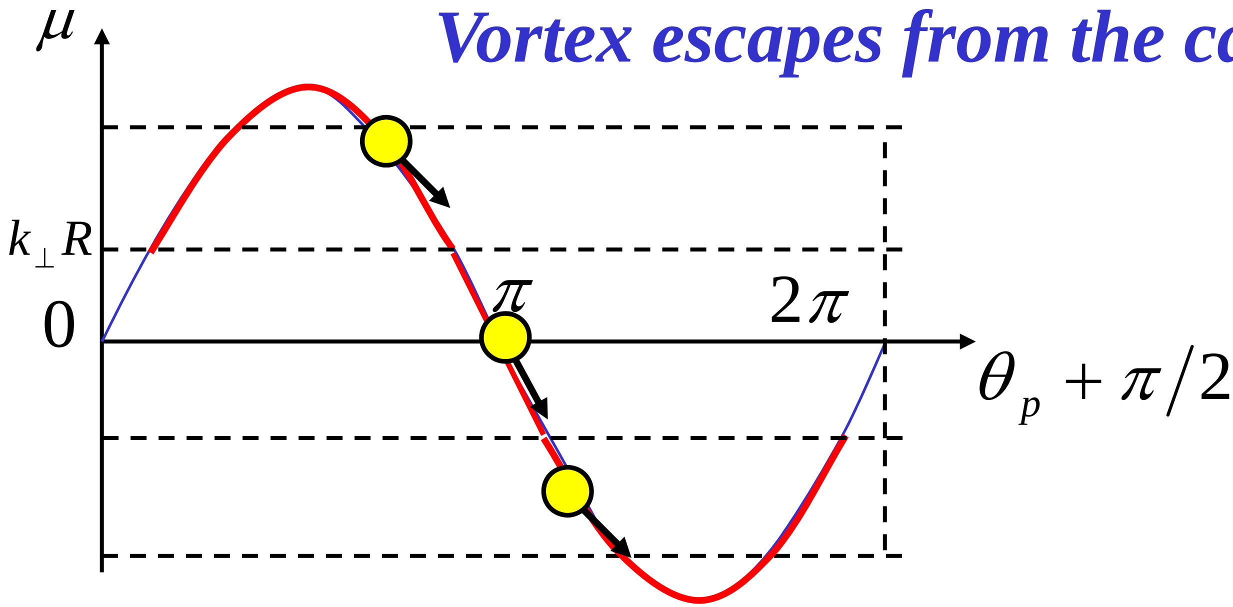
Vortex escape as a Fermi surface transformation

Step II. Merging of two parts of Fermi surface. Vortex escapes from the cavity.

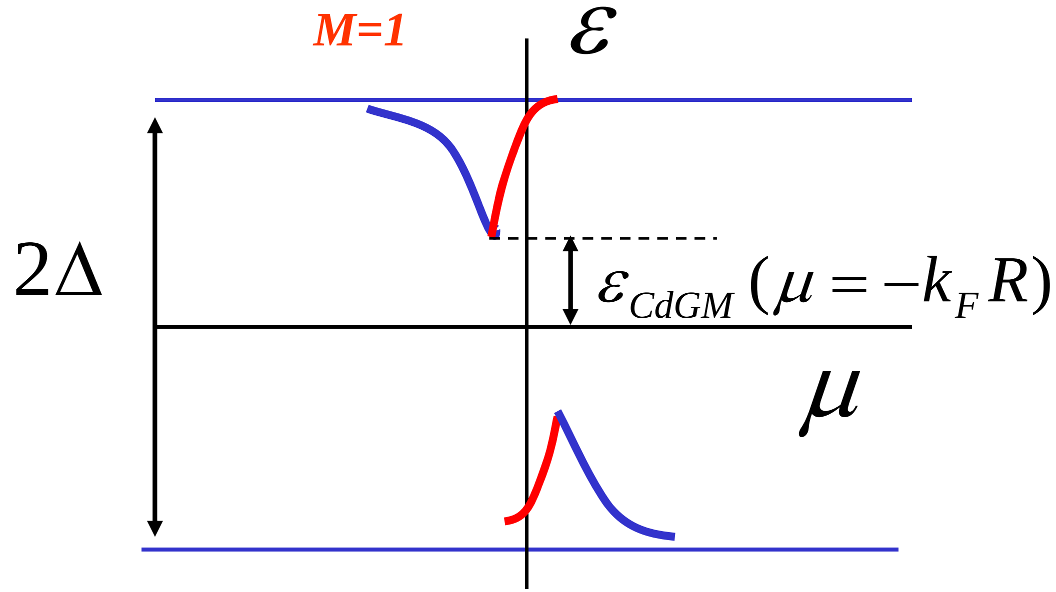


$$E = -\omega\mu + \hbar k_\perp V_s \cos\theta_p$$

$$\mu = \frac{\hbar k_\perp V_s}{\omega}\cos\theta_p \qquad V_s > \frac{\omega R}{\hbar}$$

$$\Delta\mu > k_\perp R$$

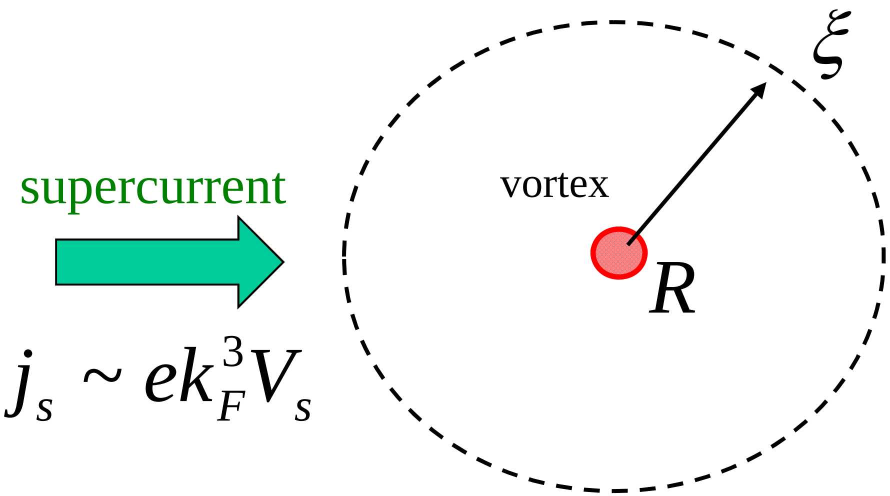
Depinning current I.



Small defect radius

$$R \ll \xi$$

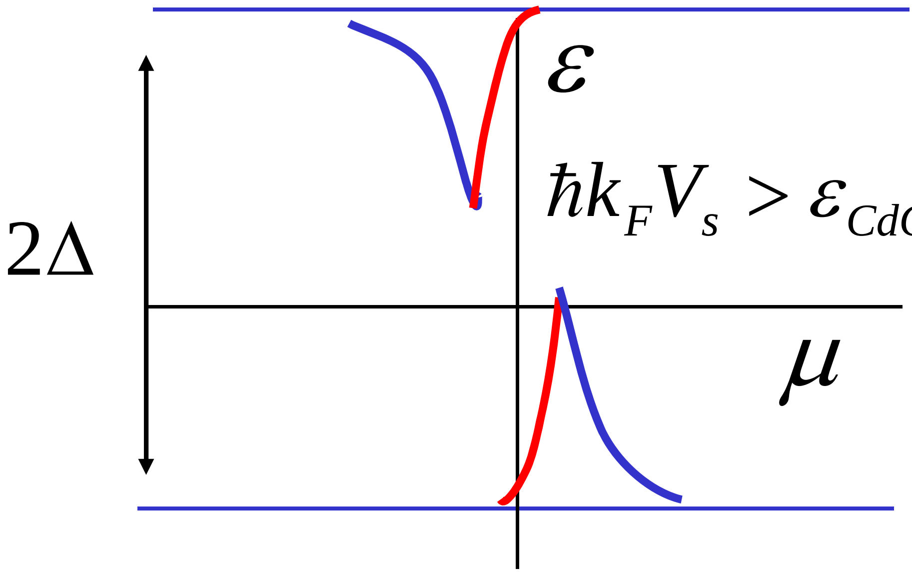
$$\varepsilon_{CdGM}(\mu=-k_F R) \approx \Delta \frac{R}{\xi}$$



Doppler shift of quasiparticle energy:

$$\varepsilon = \varepsilon_{CdGM}(\mu=-k_F R) + \hbar \vec{k}_F \vec{V}_s$$

Depinning current I.



$$\hbar k_F V_s > \varepsilon_{CdGM}(\mu = -k_F R)$$

Critical superfluid velocity: $$V_{c1} = \varepsilon_{CdGM}(\mu = -k_F R)/\hbar k_F$$

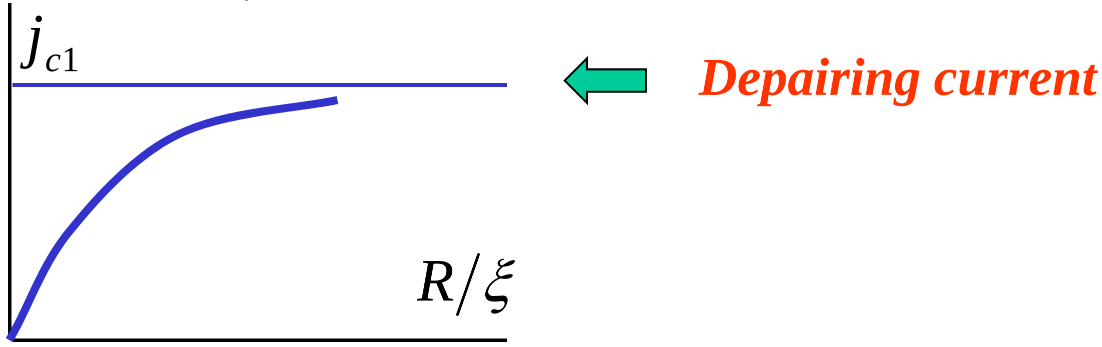
Critical supercurrent: $$j_{c1} = \frac{e}{\hbar} k_F^2 \varepsilon_{CdGM}(\mu = -k_F R)$$

Small defect radius $$j_{c1} \approx \frac{e}{\hbar} k_F^2 \Delta \frac{R}{\xi}$$

Large defect radius $$j_{c1} \approx \frac{e}{\hbar} k_F^2 \Delta$$ *- Depairing current*

Depinning current I.

$$j_{c1} = \frac{e}{\hbar} k_F^2 \varepsilon_{CdGM} (\mu = -k_F R)$$



Comparison with textbook depinning current estimates for $R < \xi$

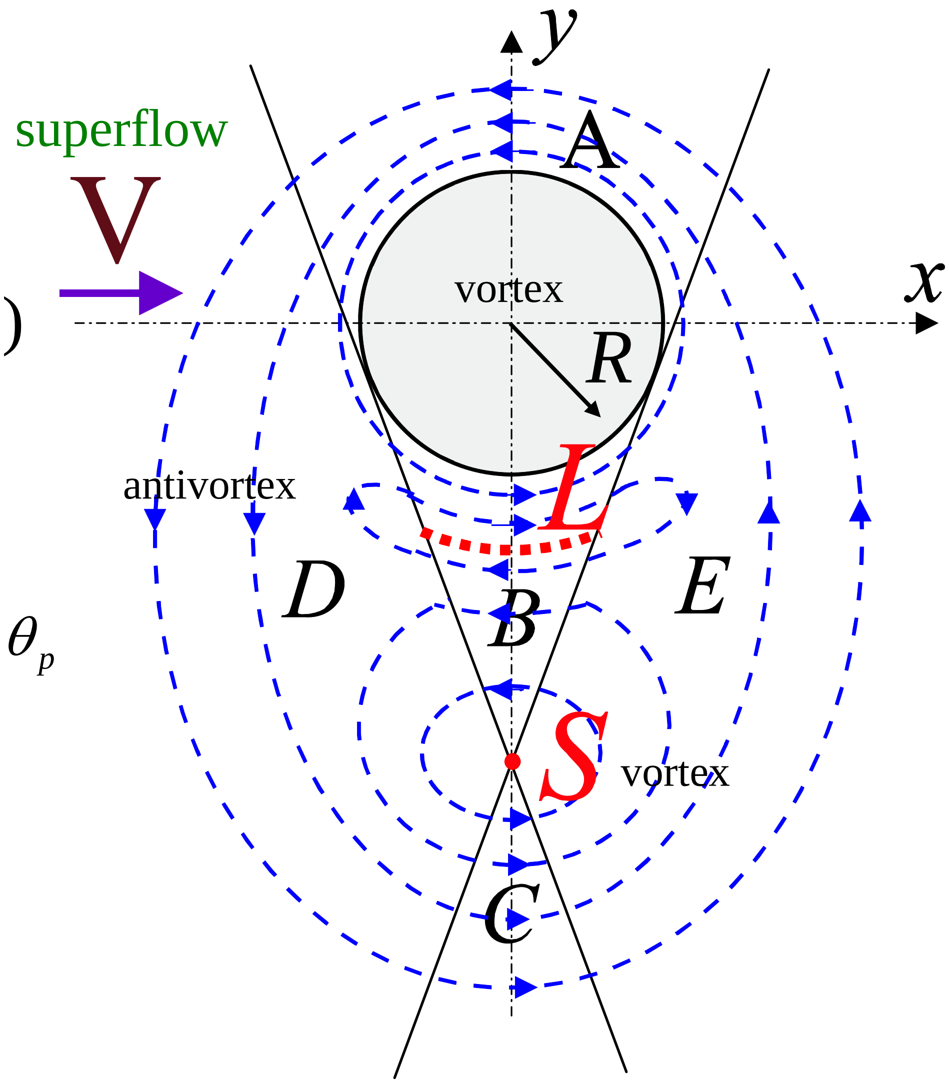
Energy gain for a vortex pinned at the cylindrical defect: $\delta U \sim H_{cm}^2 R^2$

Depinning current: $j_p = \dfrac{\delta U}{\xi} \sim H_{cm}^2 R \dfrac{R}{\xi} \sim j_{c1} \dfrac{R}{\xi} << j_{c1}$

Superconducting gap profile. Scenario of the vortex escape from a small cavity.

$$\frac{\Delta_{core}(r,\theta)}{\Delta_{\infty}} = \int_0^{2\pi} \frac{d\theta_p}{2\pi} e^{i\theta_p} \tanh\left(\frac{E}{2T}\right) h(E)$$

$$E(r,\theta,\theta_p) = \varepsilon(b = r\sin(\theta_p - \theta)) + \hbar k_F V_s \cos\theta_p$$



Depinning current II.

$$[\theta_p, \hat{\mu}] = i$$

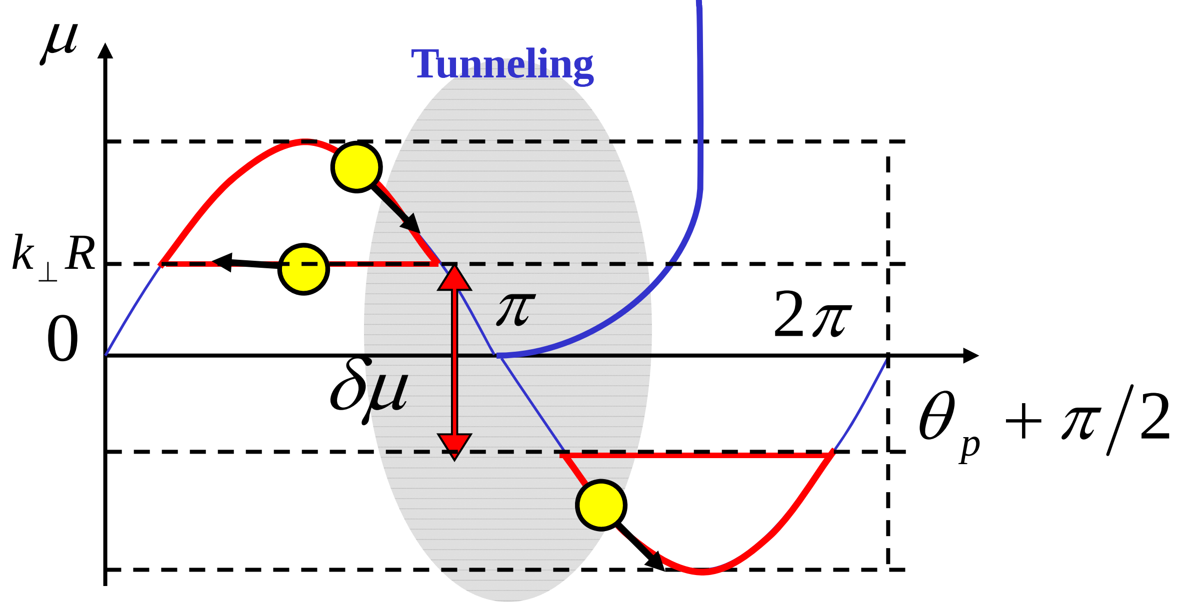
Quantum mechanics of precessing trajectories

Uncertainty principle: $\Delta\mu\Delta\theta_p \sim 1$

$$\Delta\mu \sim \hbar k_\perp V_s \Delta\theta_p / \omega$$

Quantum mechanical uncertainty of angular momentum near the branch crossing point:

$$\Delta\mu \sim \sqrt{\frac{\hbar k_\perp V_s}{\omega}}$$



Criterion of tunneling efficiency:

$$\Delta\mu > k_\perp R$$

$$V_s > \frac{k_\perp R^2 \omega}{\hbar}$$

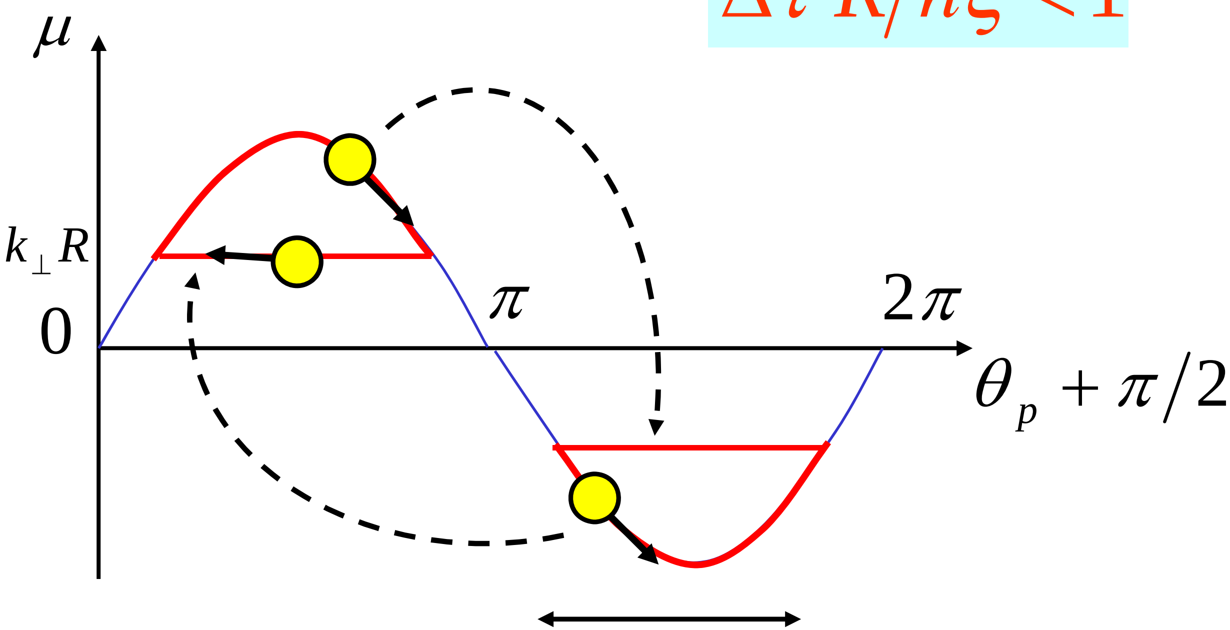
Depinning current II.

$$V_{s2} > \frac{k_F R^2 \omega}{\hbar}$$

$$j_{c2} \approx \frac{e}{\hbar} k_F^3 \Delta \frac{R^2}{\xi} \sim j_{depair} \frac{k_F R^2}{\xi} \sim j_{c1} k_F R$$

Depinning current II. Effect of impurities

$$\Delta\tau R/\hbar\xi < 1$$



$$\delta\theta_p$$

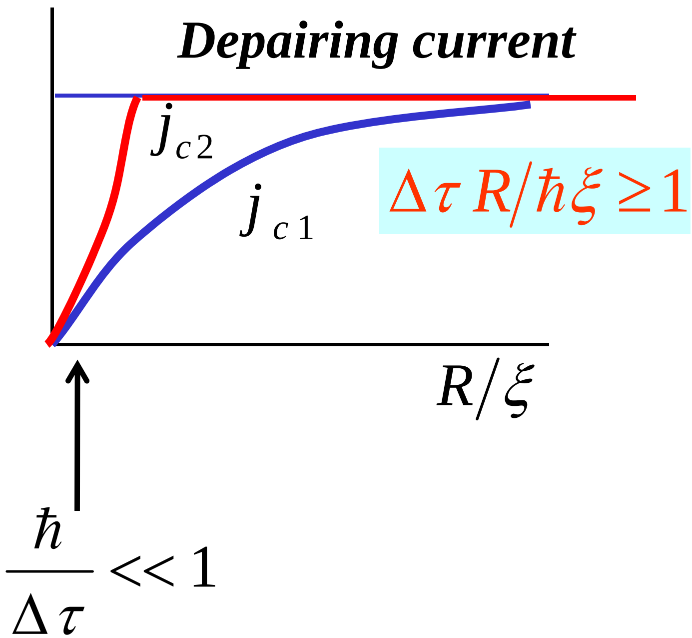
$$\frac{1}{\tau_s} \sim \frac{\delta\theta_p}{\tau} < \frac{\Delta}{\hbar}\frac{R}{\xi}$$

$$\cos\left(\delta\theta_p/2\right) = \frac{j_{c1}}{j}$$

$$j_{c2} \approx \frac{j_{c1}}{\cos\left(\gamma\Delta\tau R/\hbar\xi\right)}$$

Depinning current at low temperatures

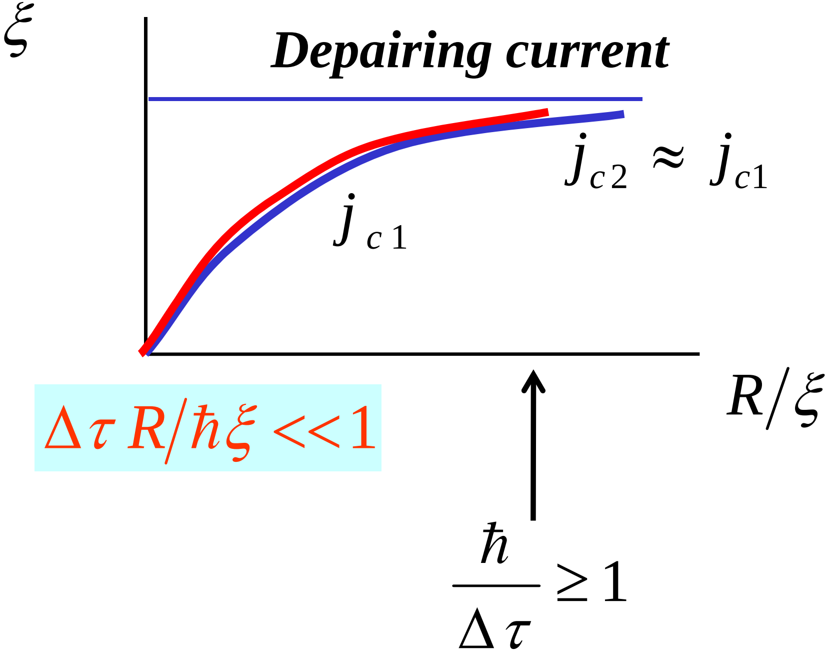
Clean limit



$$j_{c1} = j_{depair} \frac{R}{\xi}$$

Dirty limit

(in agreement with Thuneberg et al)



- **Topological electronic transitions in vortex matter**
- **de Haas – van Alfen oscillations in vortex matter**
- **Fermi surface transformations and depinning**
- **bound vortex configurations near the columnar defects**