Issues Concerning Loop Corrections to the Primordial Power Spectra arXiv: 1204.1784

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Single Scalar Inflation

$$\mathcal{L} = \frac{1}{16\pi G}\mathrm{R}\sqrt{-\mathrm{g}} - \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi \mathrm{g}^{\mu\nu}\sqrt{-\mathrm{g}} - V(\varphi)\sqrt{-\mathrm{g}}$$

- Slow roll parameter: $\epsilon(t) \equiv -\frac{\dot{H}}{H^2}$,$0<\epsilon<1$
- 1st horizon crossing: $k/a(t_k)=H(t_k)$
- Convention of Maldacena & Weinberg

$$g_{ij}(t,\vec{x}) \equiv a^2(t)e^{2\zeta(t,\vec{x})}\tilde{g}_{ij}(t,\vec{x})$$

$$\tilde{g}_{ij}(t,\vec{x}) \equiv \left(e^{h(t,\vec{x})}\right)_{ij} = \delta_{ij} + h_{ij} + \frac{1}{2}h_{ik}h_{kj} + \ldots$$

Primordial Power Spectra

$$\Delta^2_{\mathcal{R}}(k) \equiv \frac{k^3}{2\pi^2} \lim_{t \gg t_k} \int d^3x \, e^{-i\vec{k}\cdot\vec{x}} \Big\langle \Omega \Big| \zeta(t,\vec{x}) \zeta(t,\vec{0}) \Big| \Omega \Big\rangle$$

$$\Delta^2_{h}(k) \equiv \frac{k^3}{2\pi^2} \lim_{t \gg t_k} \int d^3x \, e^{-i\vec{k}\cdot\vec{x}} \Big\langle \Omega \Big| h_{ij}(t,\vec{x}) h_{ij}(t,\vec{0}) \Big| \Omega \Big\rangle$$

- Gauges: $G_0(t,\vec{x}) \equiv \varphi(t,\vec{x}) - \varphi_0(t) = 0$

  $G_i(t,\vec{x}) \equiv \partial_j h_{ij}(t,\vec{x}) = 0$

- Quadratic (fixed & constrained) action:

$$\mathcal{L}_{\zeta^2} = \frac{(D-2)\,\epsilon\, a^{D-1}}{16\pi G} \Big\{ \dot{\zeta}^2 - \frac{1}{a^2} \partial_k \zeta \partial_k \zeta \Big\}$$

$$\mathcal{L}_{h^2} = \frac{a^{D-1}}{64\pi G} \Big\{ \dot{h}_{ij} \dot{h}_{ij} - \frac{1}{a^2} \partial_k h_{ij} \partial_k h_{ij} \Big\}$$

Primordial Power Spectra-- Tree order

- D=4,

$$\Delta^2_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2} \lim_{t \gg t_k} \Big\{ 4\pi G \times |u_\zeta(t,k)|^2 + O(G^2) \Big\}$$

$$\Delta^2_h(k) = \frac{k^3}{2\pi^2} \lim_{t \gg t_k} \Big\{ 32\pi G \times 2 \times |u(t,k)|^2 + O(G^2) \Big\}$$

- If ϵ=const.,

$$u_\zeta(t,k) = \frac{u(t,k)}{\sqrt{\epsilon}} \quad , \quad u(t,k) = -iC(\epsilon) \times \frac{H(t_k)}{\sqrt{2k^3}} \quad \rightarrow$$

$$\Delta^2_h = C(\epsilon) \times \frac{16 G H^2(t_k)}{\pi} + O(G^2)$$

$$\Delta^2_{\mathcal{R}} = C(\epsilon) \times \frac{G H^2(t_k)}{\pi \epsilon(t_k)} + O(G^2)$$

- The tensor-to-scalar ratio: $r \equiv \frac{\Delta^2_h}{\Delta^2_{\mathcal{R}}} = 16\epsilon$

Primordial Power Spectra—Compare it with the data

- South Pole Telescope(95% confidence): $r=16\epsilon < 0.17 \rightarrow \epsilon < 0.011 \rightarrow (1/\epsilon) > 94$
- $\Delta^2_{\mathcal{R}}=2.441\times 10^{-9}$ & $r= \Delta^2_h/\Delta^2_{\mathcal{R}} <0.17 \rightarrow$
  - $\Delta^2_h < 4.15\times 10^{-10}$
  - $GH^2 \approx \pi/16 \times r \times \Delta^2_{\mathcal{R}} < 10^{-10}$
  - $GH^2 < 10^{-10} \rightarrow H < 10^{14}$ GeV $\rightarrow$ QG perturbative, but not negligible.

Single scalar inflation: Interactions

- Maldacena: $\mathcal{L}_{\zeta^3} \sim \epsilon^2 \zeta^3$
- Seery, Lidsey & Sloth: $\mathcal{L}_{\zeta^4} \sim \epsilon^2 \zeta^4$
- Jarhus & Sloth: $\mathcal{L}_{\zeta^5} \sim \epsilon^3 \zeta^5$, $\mathcal{L}_{\zeta^6} \sim \epsilon^3 \zeta^6$
- Xue, Gao & Brandenberger:
  $\mathcal{L}_{\zeta^2 h} \sim \epsilon \zeta^2 h$, $\mathcal{L}_{\zeta h^2} \sim \epsilon \zeta h^2$, $\mathcal{L}_{\zeta^2 h^2} \sim \epsilon \zeta^2 h^2$
- ζ^{2N} or ζ^{2N-1} suppressed by ε^N
  - No 1/ε's in loops & non-Gaussianity

IR divergence versus IR Logs

- IR ∞ from $h_{ij}(t,x)$ ~ constant
- IR Logs from continual horizon crossing
- IR ∞ pure gauge but not IR Logs
- ζ(t,x) not invariant ➔ <ζζ> gauge dep.
- Leading ln[a(t)] possibly gauge indep.
  - Spin 2 parts of prop. agree in all gauges

Gauge invariance not a Panacea

- Gauge dependent ≠ unphysical
  - S-matrix from gauge dep. Green’s funcs
- Gauge invariance ≠ physical
  - 1 is invariant, but uninformative
- Gauge invariant ≠ gauge independent
  - δ[gauge] (non-inv) = δ[gauge] (invariant)
  - “Invariantizers” use gauge fixing for time!

Danger of gauge invariance

- No local gauge invariants in GR
- Nonlocal field redef's change physics!
  - E.g. $\mathcal{D}\varphi = I[\varphi]$
  - Yang-Feldman: $\varphi = \varphi_0 + \mathcal{D}^{-1} I[\varphi]$
  - Free field: $\varphi_0[\varphi] = \varphi - \mathcal{D}^{-1} I[\varphi]$
  - Relates any 2 theories w. same DOF's
    - E.g., GR = EM!

Invariantizing ζ-ζ Correlator

- Old: use $G_0 = 0$ to fix t & $G_i = 0$ for x^i
- New: use $G_0 = 0$ to fix t
  - But fix x^i invariantly (under spatial diffeo)

$$\Delta^2_{\mathcal{R}}(k) \longrightarrow \frac{k^3}{2\pi^2} \lim_{t \gg t_k} \int d^3V \, e^{-i\vec{k}\cdot\vec{V}} \Big\langle \Omega \Big| \zeta\Big(t, \vec{X}[\widehat{g}](1, \vec{V})\Big) \zeta(t, \vec{0}) \Big| \Omega \Big\rangle$$

  - E.g., $X^i[g](\tau,V)$ geodesic from $x^i = 0$ in direction V^i

Problems with Geodesics

- Renormalization
  - More UV divergences (1/(D-4) → $1/(D-4)^2$)

$$\zeta(t,\vec{0}) \times \int_0^1 d\tau\, \zeta(t,\tau\vec{V}) \times \int_0^1 d\tau'\, \zeta(t,\tau'\vec{V}) \times \zeta(t,\vec{V})$$

$$\int_0^1 d\tau\, i\Delta_\zeta\big(t,\vec{0};t,\tau\vec{V}\big) \int_0^1 d\tau'\, i\Delta_\zeta\big(t,\vec{V};t,\tau'\vec{V}\big)$$

  - No theory for renormalizing nonlocal composites!
  - No guarantee against UV-IR mixing
- No ε suppression
  - E.g., 1-loop <$\zeta\zeta$> from ζ^4 vertex
    - old: $(GH^2/\epsilon)^3 \times (\epsilon^2/GH^2) = (GH^2/\epsilon) \times (GH^2)$
    - new: $(GH^2/\epsilon)^2 = (GH^2/\epsilon) \times (GH^2/\epsilon)$
  - Tree order <$\zeta\zeta\zeta$> from ζ^3 vertex
    - old: $(GH^3/\epsilon)^3 \times (\epsilon^2/GH^2) = (GH^2/\epsilon) \times (GH^2)$
    - new: $(GH^2/\epsilon)^2 = (GH^2/\epsilon) \times (GH^2/\epsilon)$

A Better Fix

- Still fix time with $G_0 = 0$
- Still fix space point with $X^i[h](V)$

$$\Delta^2_{\mathcal{R}}(k) \longrightarrow \frac{k^3}{2\pi^2} \lim_{t \gg t_k} \int d^3V \, e^{-i\vec{k}\cdot\vec{V}} \left\langle \Omega \middle| \zeta\big(t, \vec{X}[\widetilde{g}(t)](\vec{V})\big) \zeta(t, \vec{0}) \middle| \Omega \right\rangle$$

  - But define $X^i[h](V)$ using ∂_i of a scalar GF
    - $\Delta[h]\, G[h](x;y) = \delta^3(x-y)$
    - Fix $X^i[h](V)$ э $\partial_i G[h](X;0) \sim V^i$
  - No extra UV ∞'s because 1D → 3D
  - No extra 1/ε because only h_{ij}

Acknowledge approximations

- Even tree results not exact
  - Don't know mode functions for general $\varepsilon(t)$
- QG tough → extra approximations
  - $\epsilon < 0.011$ small, but not zero
  - Derivatives small, but not zero
  - IR divergence constant, but IR log not

Summary

- IR div. differs from IR growth.
- The leading Log might be gauge-independent.
- Not all gauge dependent quantities are unphysical.
- Not all gauge invariant quantities are physical.
- Nonlocal observables can null real effects.
- Avoid altering the pattern of ϵ suppression.
- Non-local composite op. introduces extra 1/(D-4).
- The Challenge in cosmology
  - IR finiteness
  - Renormalizability
  - A reasonable observables corresponding to what could be measured.