Ultracold atoms as a new tool in condensed matter physics

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Magnetic and electric forces

Strong effects of extermal fields.

Magnetic moment in a magnetic field:

$$U = -\mu B$$

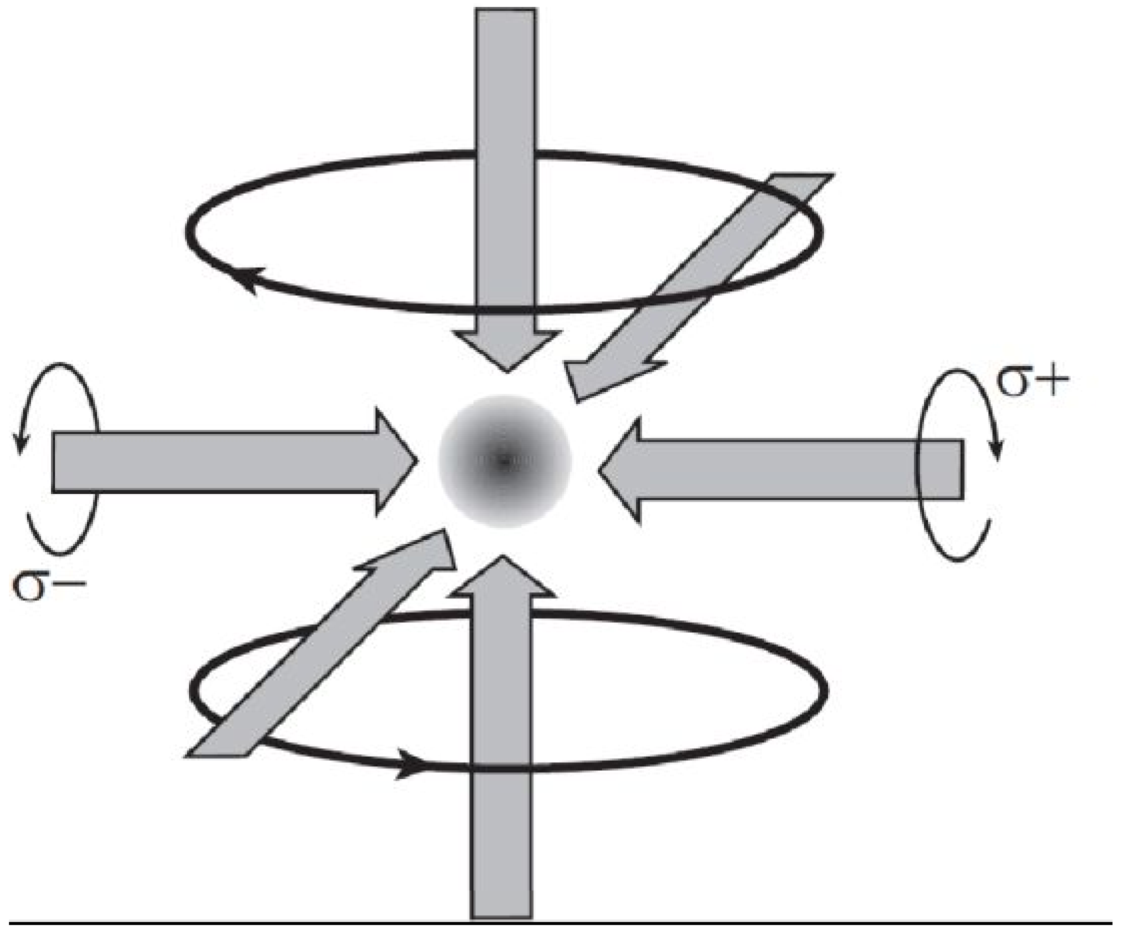
Force in a.c. electric field:

$$U(\mathbf{r}) = -\frac{1}{2}\alpha(\omega)E^2(\mathbf{r},t)$$

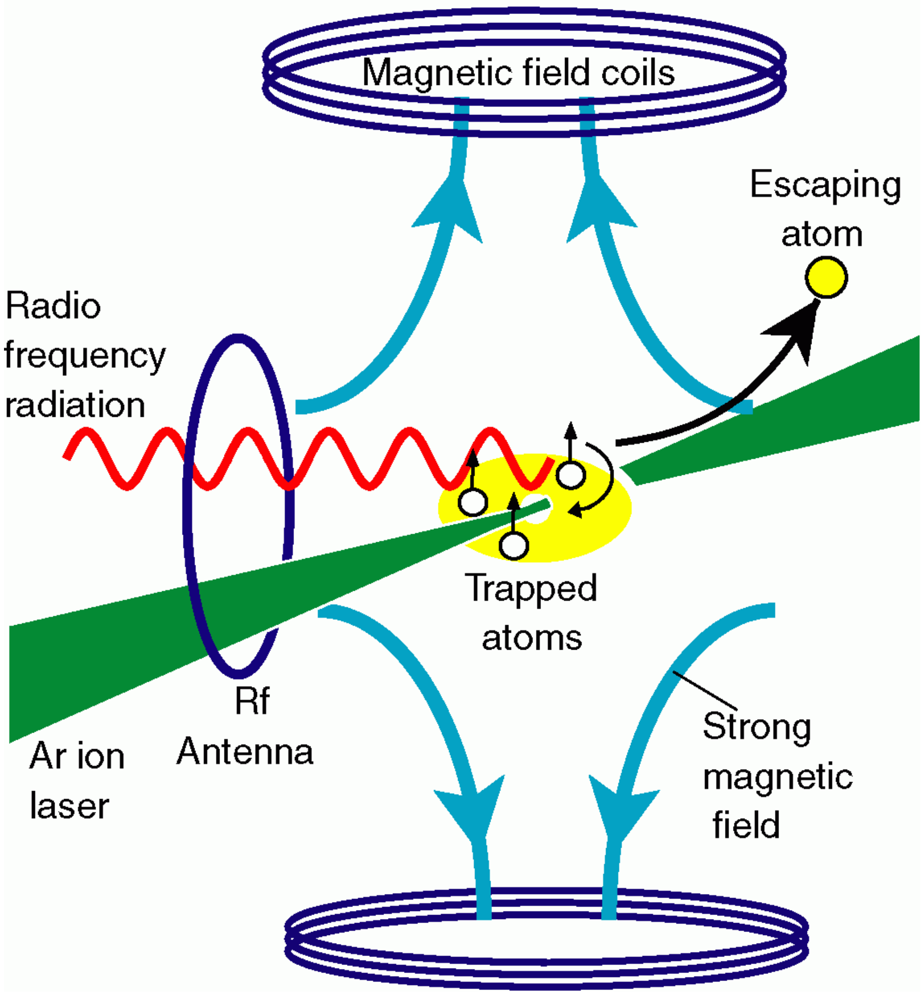
Near the absorption line

$$\alpha(\omega) = \frac{\left|\langle n|\mathbf{d}\bullet\mathbf{e}|0\rangle\right|^2}{\omega_n - \omega}$$

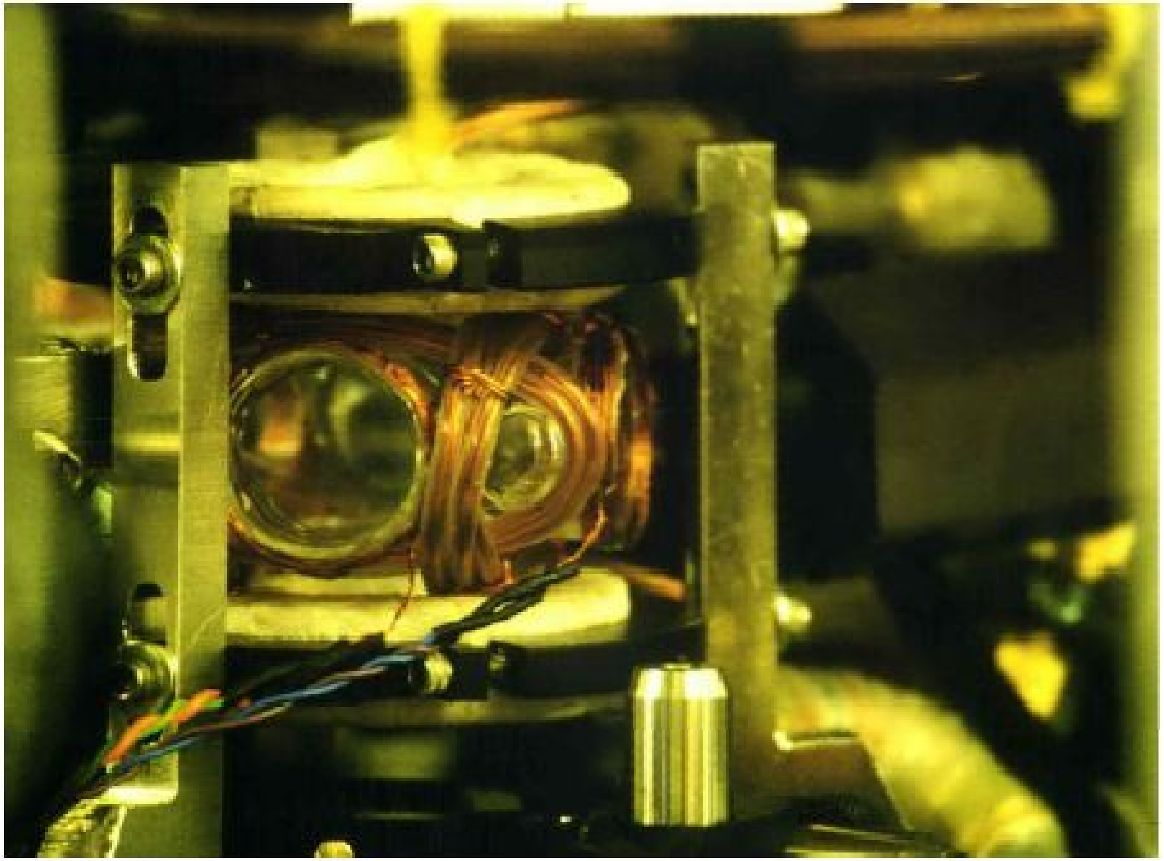
Trapping and cooling



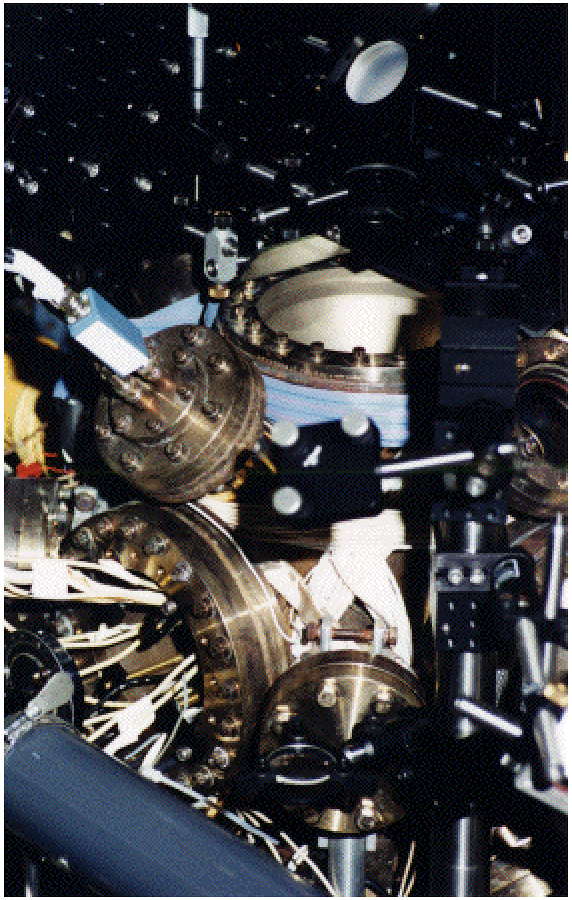
Magnetic trap with optical plug



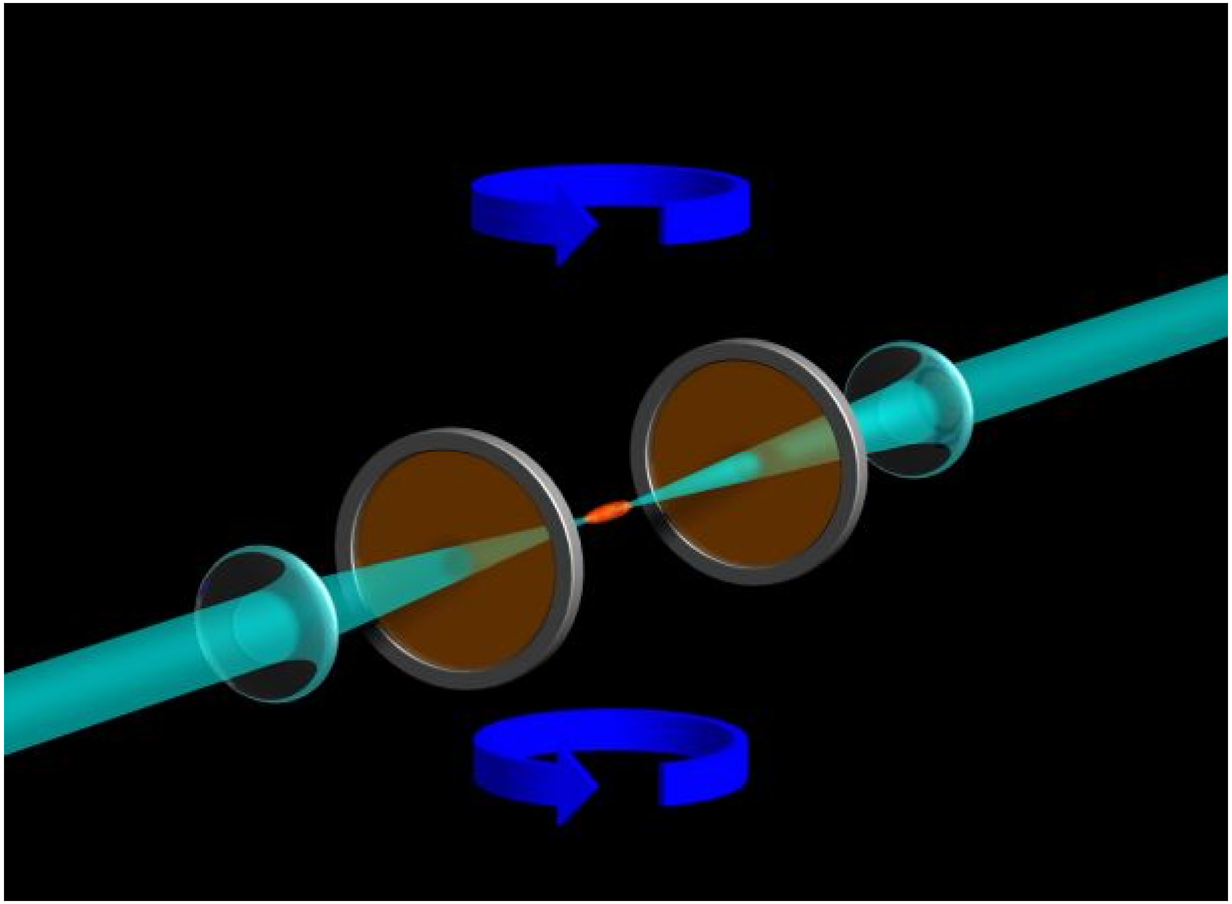
MOT (Boulder)



MOT (MIT)



Fermions in optical trap
(Duke University)



Typical parameters

$$N \sim 3\times10^{6} - 10^{7}$$

$$n \sim 2\times10^{12}\,cm^{-3}, n^{-1/3} \sim 0.3\mu m$$

$$\nu_{\perp} \sim 60 - 300Hz, \nu_{z} \sim 20Hz$$

$$T_{Deg} \sim 200 - 500nK - 1.7\mu K$$

$$T / T_{Deg} < 0.06, T_{Min} \sim 20\,pK$$

Two BEC on atom chip. (MIT)

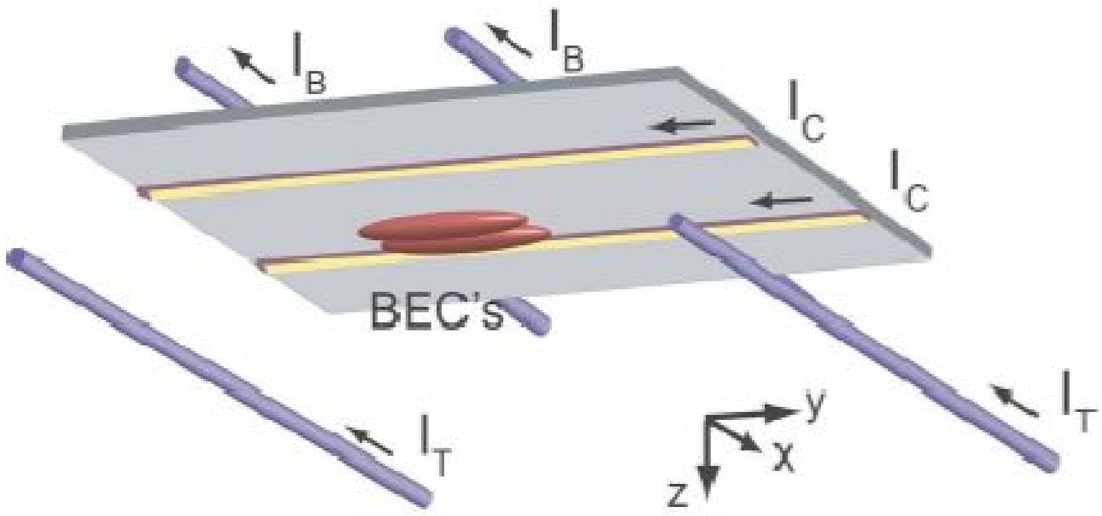


FIG. 1. (Color online) Schematic diagram of the atom chip. A magnetic double-well potential was created by two chip wires with a current I_C in conjunction with an external magnetic field. The distance between the two chip wires was 300 μm.

Possibility to change interaction.

$$f = -\frac{1}{a^{-1} + ik}$$

a - scattering length

1) $a > 0 : a = \hbar / \sqrt{m|\varepsilon|}$.

Weakly bound state

with energy $\varepsilon < 0$.

2) $a < 0$ - "virtual level"

$$|\varepsilon| << \hbar^2 / m r_0^2, |a| >> r_0.$$

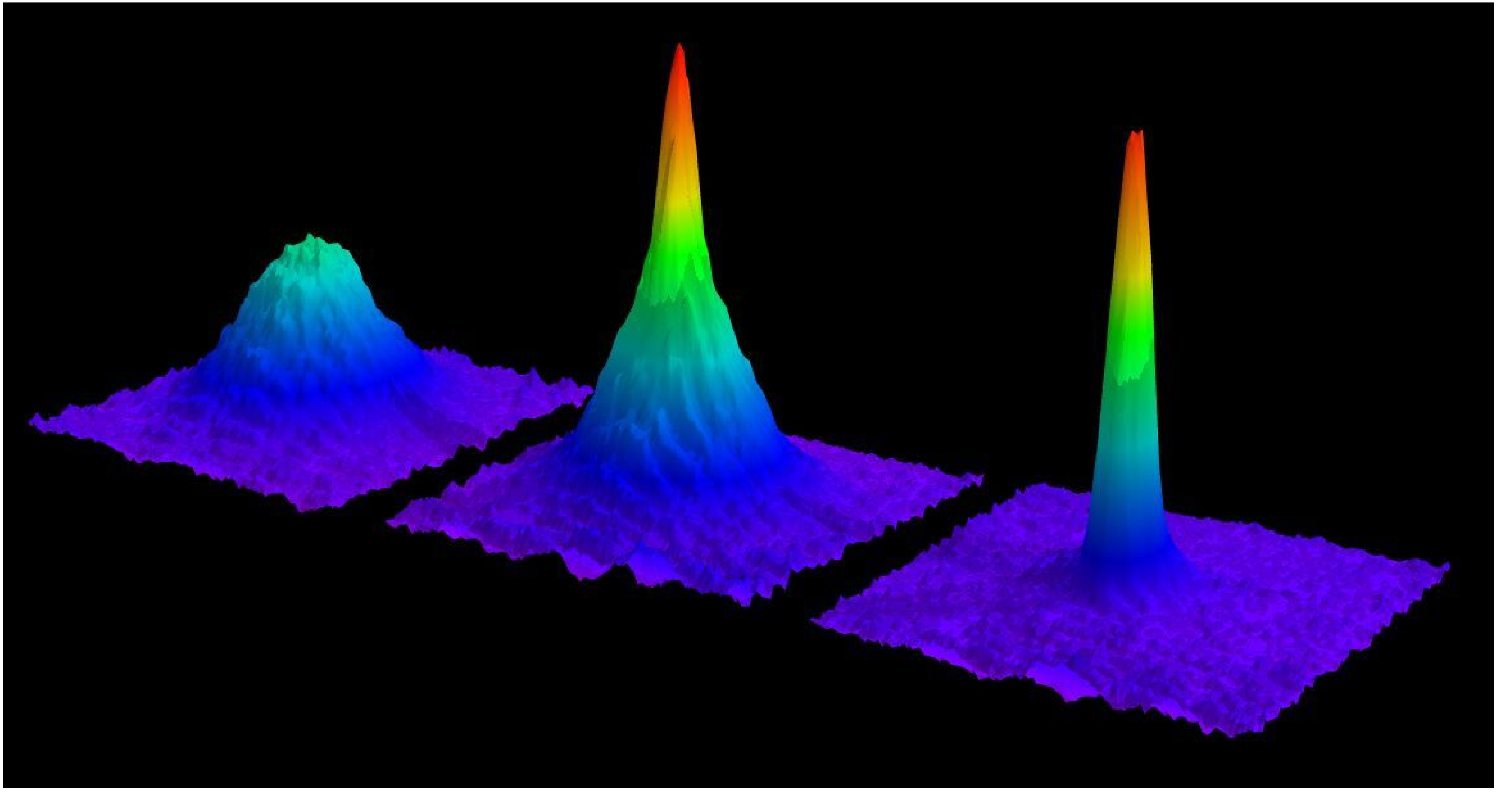
Feshbach resonance

$$a = a_{bg}\left(1 - \frac{\Delta_B}{B - B_0}\right)$$

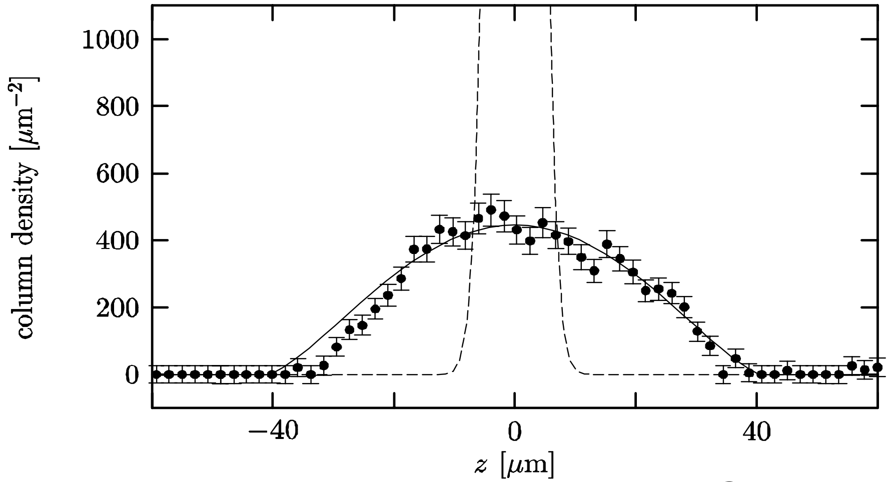
$$B \to B_0, a \to \pm\infty$$

$$f = 1/ik$$

Bose-Einstein Condensation



Density distribution

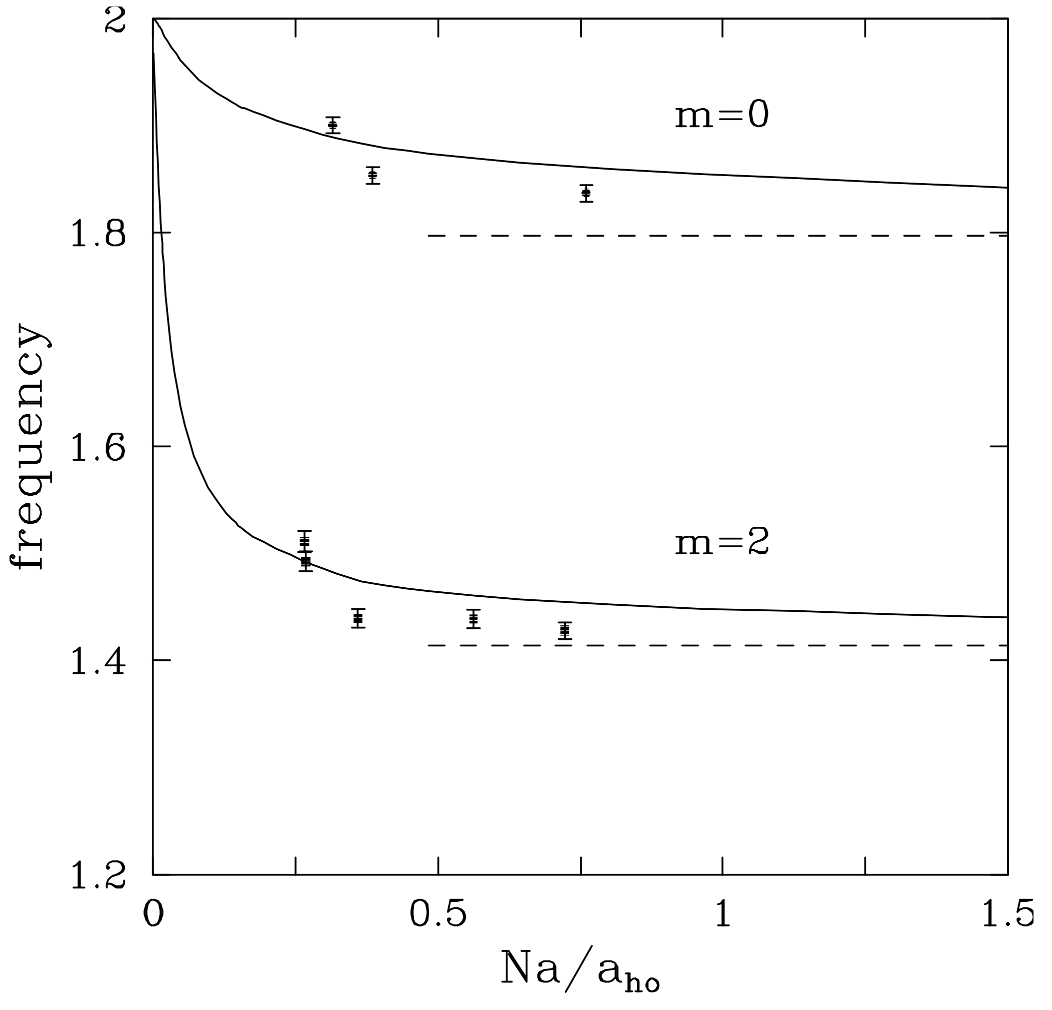


$$\mu = gn,\; g = \frac{4\pi\hbar^2}{m} a$$

N.N. Bogoljubov, 1947

$$gn + \frac{m\omega^2 r^2}{2} = \mu_0$$

Collective oscillations. Stringari prediction and experiment



Lee-Huang-Yang correction

$$\delta E \sim \sum_{p}^{mu} \frac{\hbar\omega}{2}$$

$$\delta\mu / \mu \propto \sqrt{na^3}$$

Lee, Huang, Yang, 1957

LHY correction. N. Navon et. all, 2011

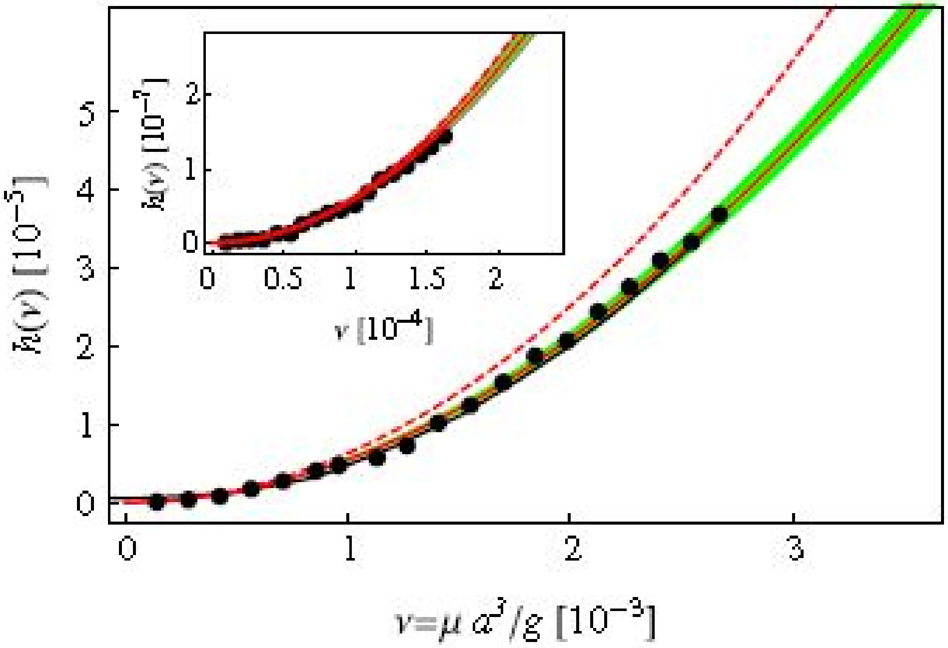


FIG. 2 (color online). Equation of state of the homogeneous Bose gas expressed as the normalized pressure h as a function of the gas parameter ν. The gas samples for the data shown in the main panel (inset) have been prepared at scattering lengths of $a/a_0 = 1450$ and 2150 ($a/a_0 = 700$). The gray (red online) solid line corresponds to the LHY prediction, and the gray (red online) dashed line to the mean-field EOS $h(\nu) = 2\pi\nu^2$.

Rotation. Vortex lines

Ordinary liquid :

$$v_\varphi = \Omega r, \quad \mathbf{curl}\ \mathbf{v} = 2\Omega \neq 0$$

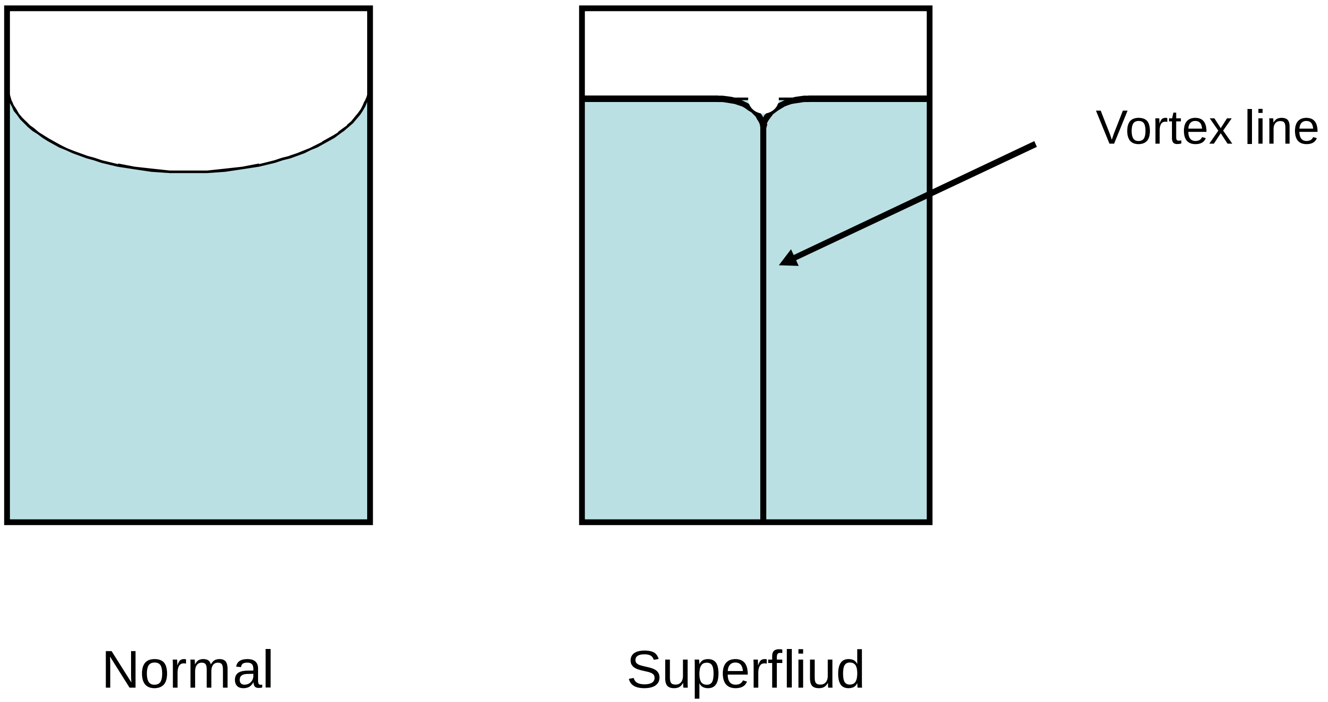
Condensate :

$$\Psi = |\Psi| e^{i\varphi}, v_\varphi = \frac{\hbar}{m}\frac{1}{r}, n_v = \frac{m\Omega}{\pi\hbar}$$

Vortex line.

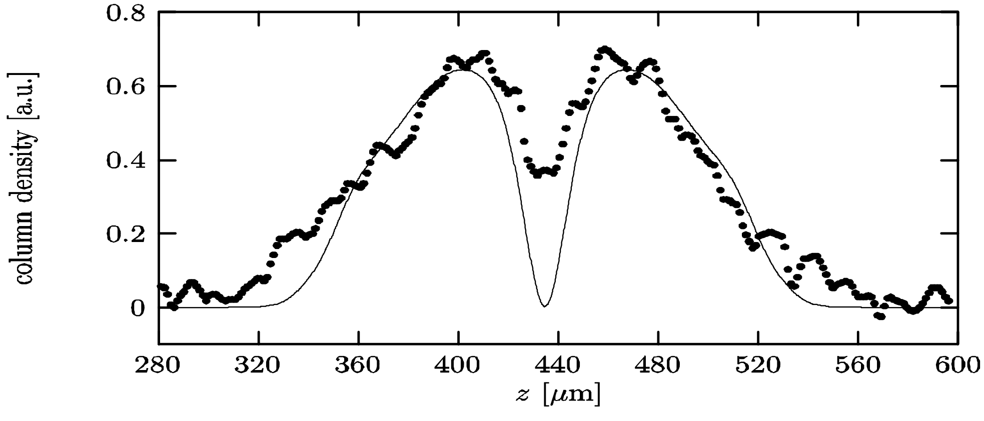
L. Onsager, 1949; R. Feynman, 1954.

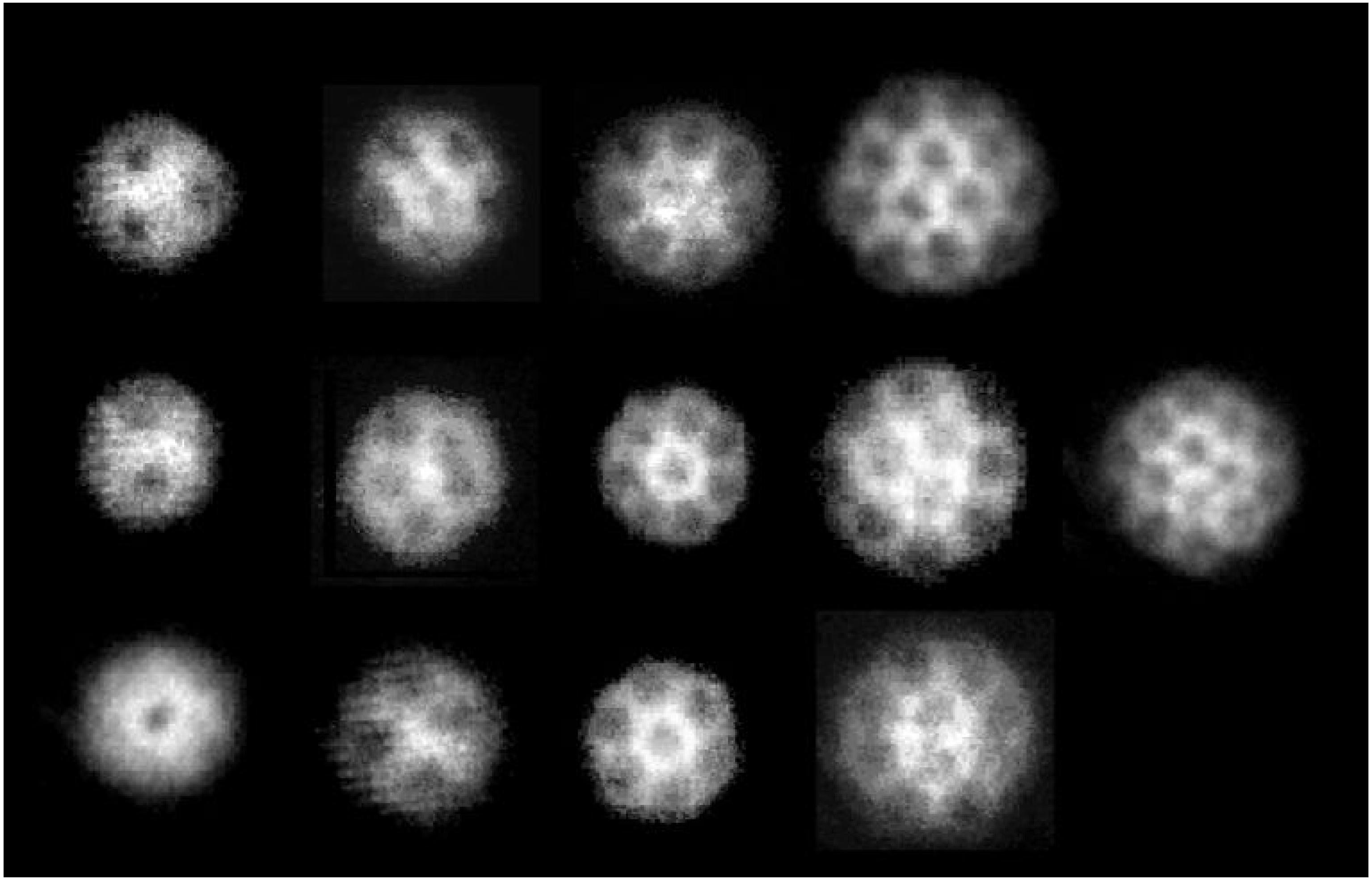
Rotation of normal and superfluid liquids

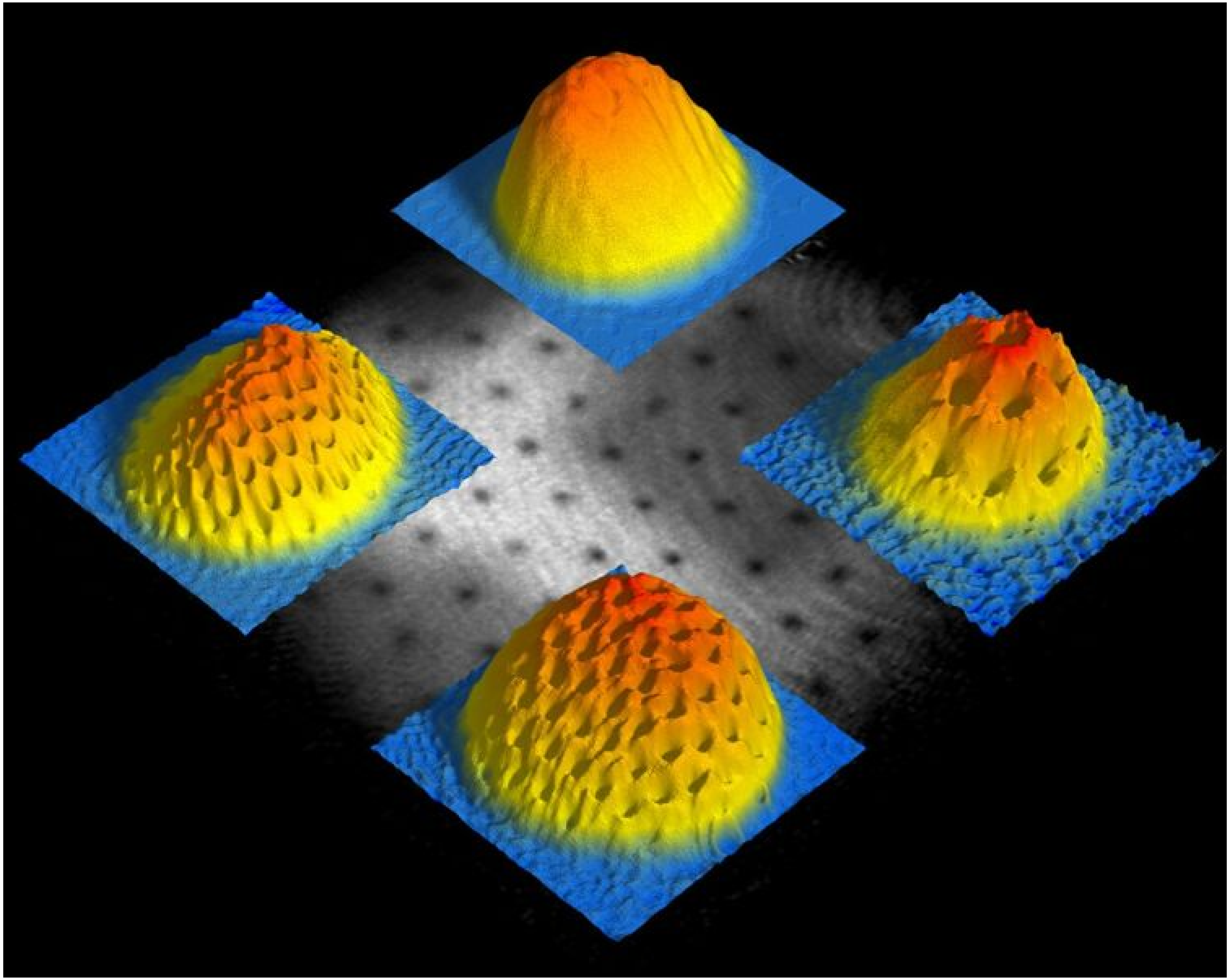


Structure of a vortex line

$$|\Psi| = f(r/\xi), \xi = \hbar/\sqrt{2ngm}, n = |\Psi|^2$$







Two classical limits of QM

$$1.\ \text{Classical body}:\ m \to \infty$$

$$2.\ \text{Classical electromagnetic waves}:$$

$$\text{Number of photons}\ N_{ph} = \frac{E}{\hbar\omega} \to \infty$$

From quantum electrodynamics to classical Maxwell equations

Commutation relations for the vector - potential :

$$\left[\hat{A}_i, \frac{\partial \hat{A}_k}{\partial t}\right] \sim \hbar, \quad \hat{A}_i(\mathbf{r},t) \to A_i(\mathbf{r},t)$$

$$\hat{\mathbf{E}}(\mathbf{r},t) \to \mathbf{E}(\mathbf{r},t), \hat{\mathbf{B}}(\mathbf{r},t) \to \mathbf{B}(\mathbf{r},t)$$

The Maxwell equations :

$$\text{curl } \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \quad \text{etc.}$$

Why Maxwell equations do not contain *h* ?

Energy - momentum relation for photons :

$$\varepsilon = cp$$

Transition from particles to waves :

$$\varepsilon , \ p \to \omega , k : \quad \varepsilon = \hbar \omega , \ p = \hbar k$$

Frequency – wave vector relation

$$\omega = ck$$

does not contain $\hbar$.

Bose-Einstein Condensation - classical limit for the Broglie waves

$$\left[\hat{\Psi}(\mathbf{r},t),\hat{\Psi}^{+}(\mathbf{r}',t)\right]=\hbar\delta(\mathbf{r}-\mathbf{r}')$$

$$\Psi\sim\sqrt{N}\ ,\quad \hat{\Psi}(\mathbf{r},t)\rightarrow\Psi(\mathbf{r},t)$$

In an uniform condensate $\hat{\Psi}\rightarrow\sqrt{N}$

N.N. Bogoliubov, 1947

From particles to classical waves

$$\text{Energy - momentum relation for atoms}: \varepsilon = \frac{p^2}{2m}$$

$$\text{Transition from particles to waves}:$$

$$\varepsilon,\ p \to \omega, k: \quad \varepsilon = \hbar\omega,\ p = \hbar k$$

$$\text{Frequency – wave vector relation}: \omega = \frac{\hbar k^2}{2m}$$

This relation does contain $\hbar$.

Equation for the classical function $\Psi(\mathbf{r}, t)$

will contain $\hbar$.

Equation for $\Psi(\mathbf{r}, t)$

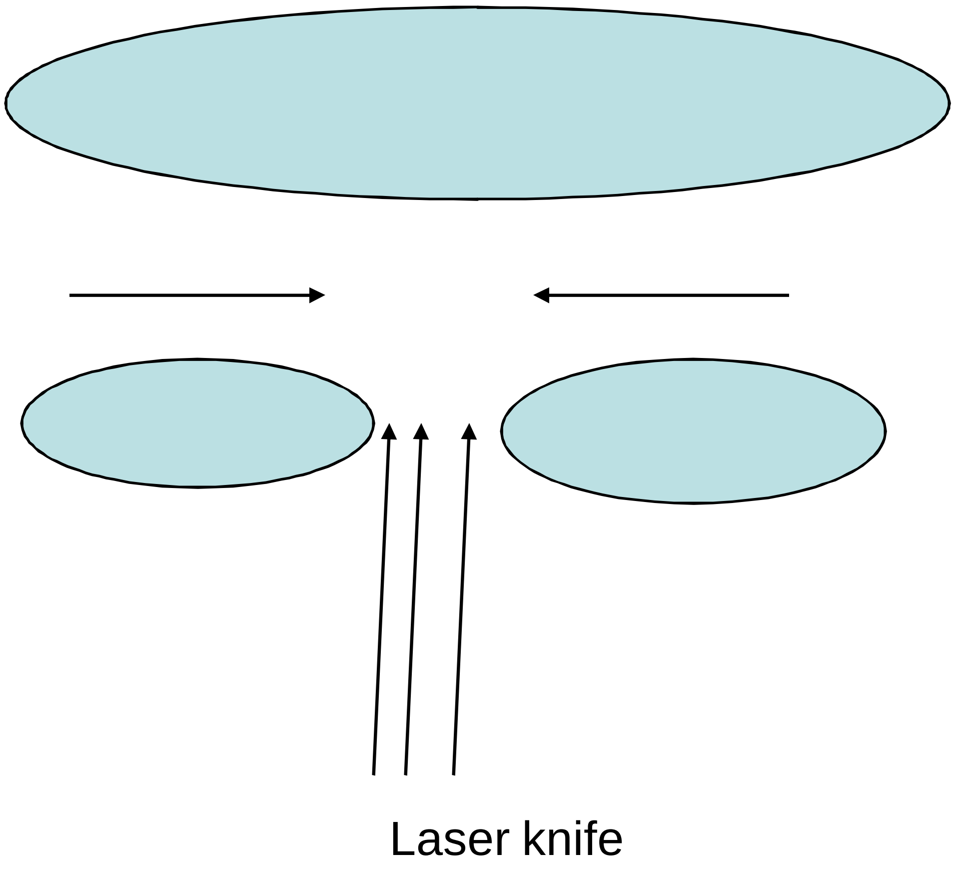
$$i\hbar \frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2m}\Delta\Psi + g\Psi\left|\Psi\right|^2$$

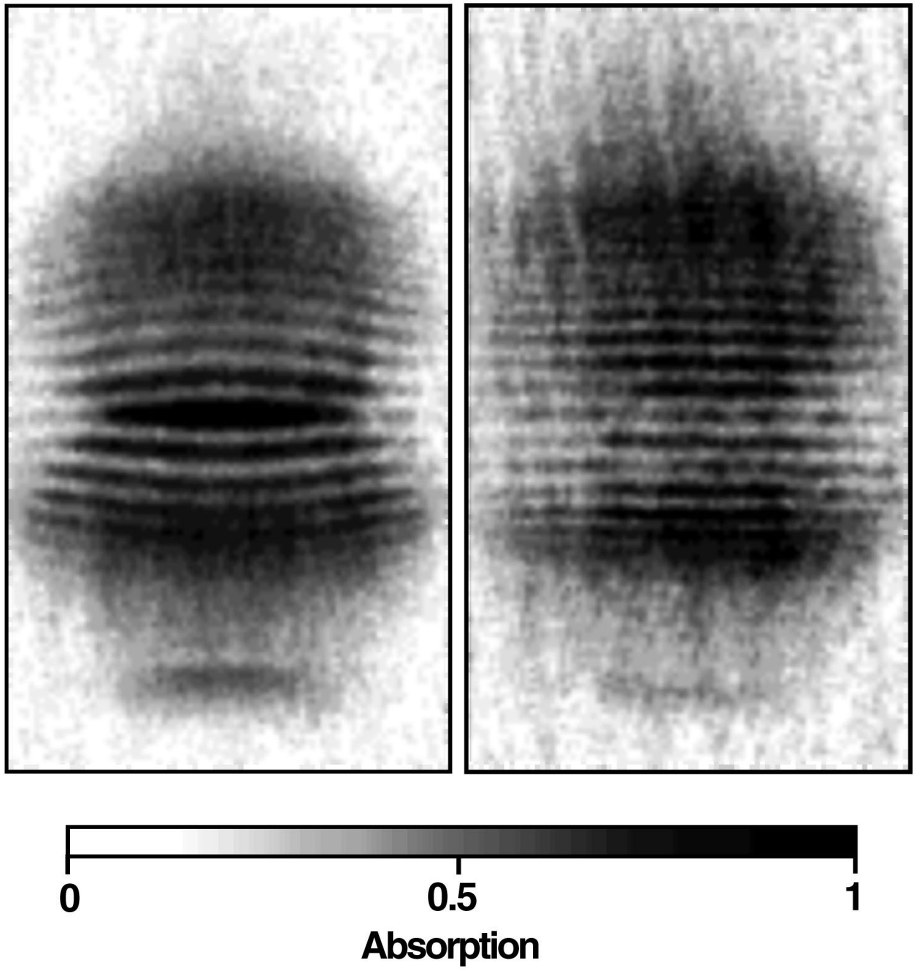
E.P. Gross, 1961; L.P. Pitaevskii, 1961

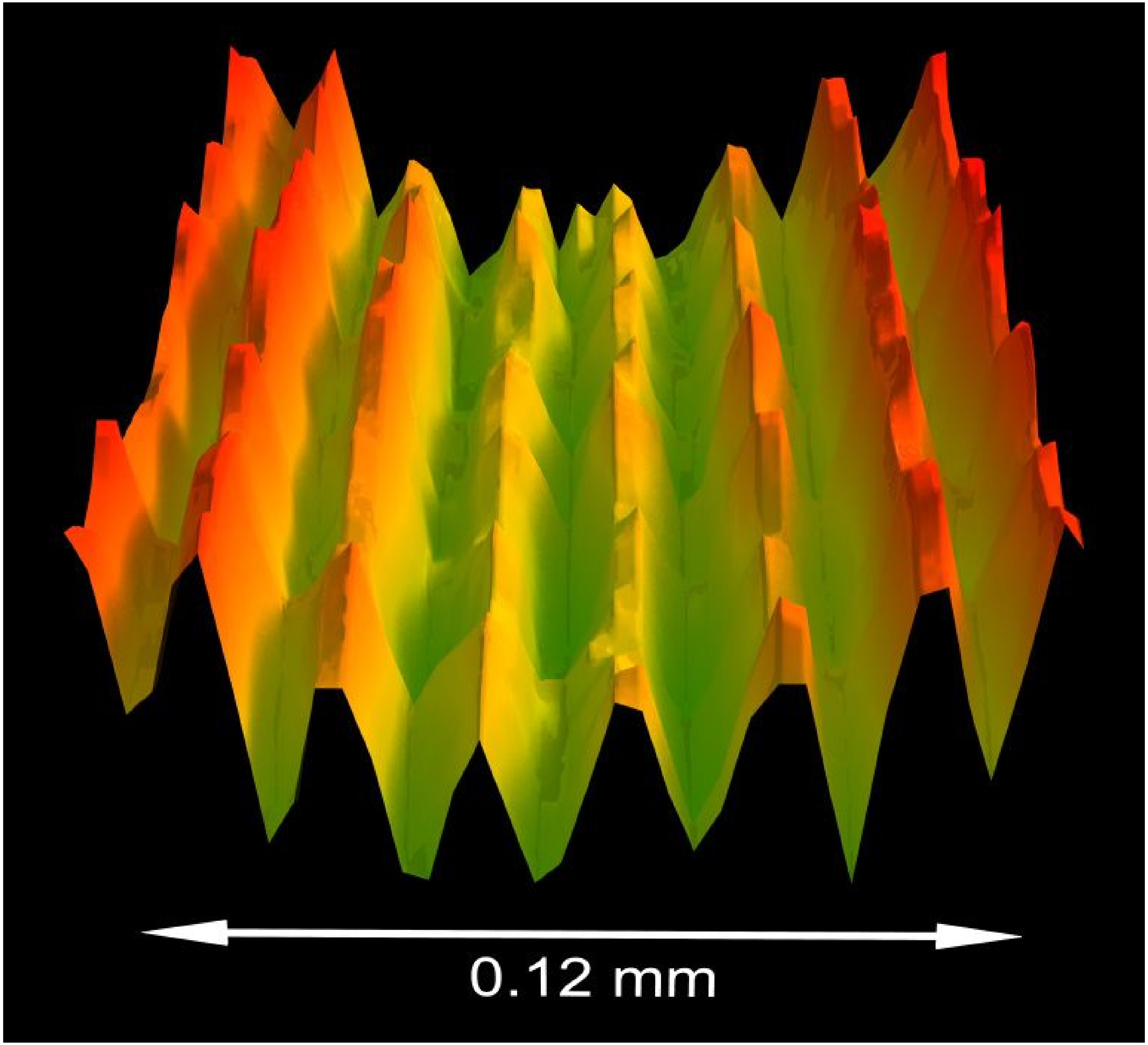
$$g = \frac{4\pi\hbar^2}{m}a\,, \quad a \text{ is } s\text{-wave scattering length}$$

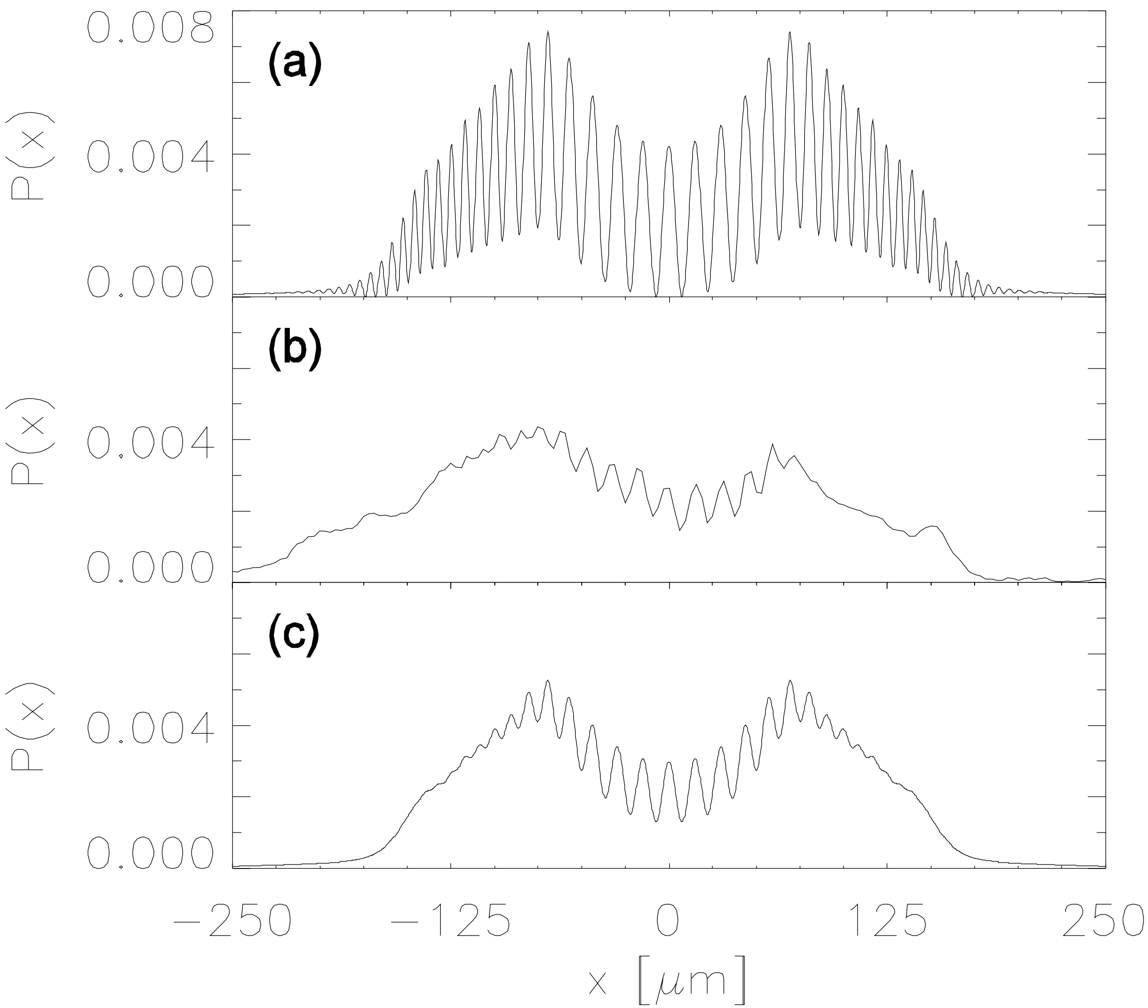
Plays role of the Maxwell equations in this problem.

Interference experiment



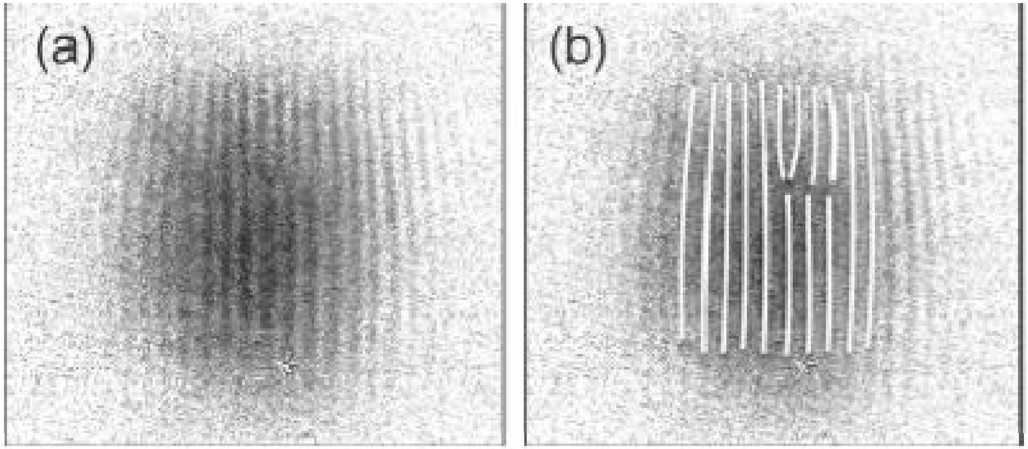

0
0.5
1
Absorption


0.12 mm



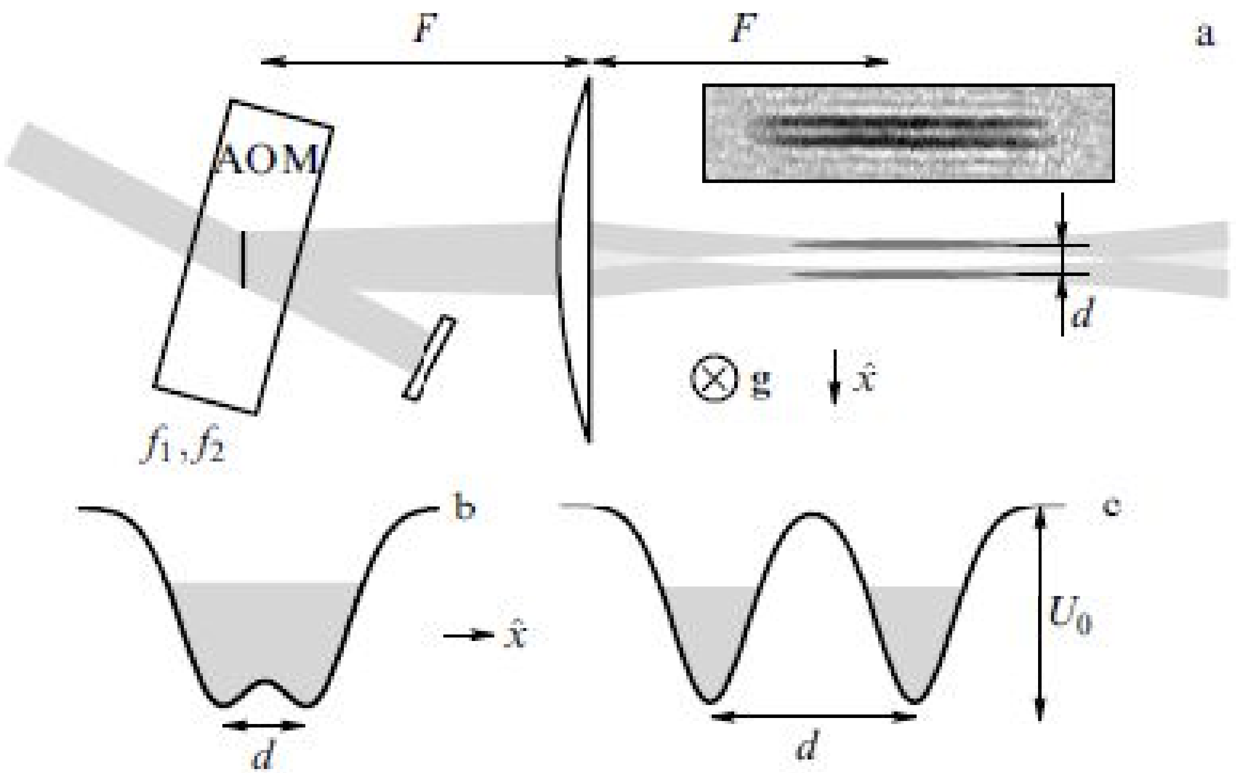
(a)
(b)
(c)
0.008
0.004
0.000
P(x)
x [μm]
−250
−125
0
125
250

Interference in presence of vortex

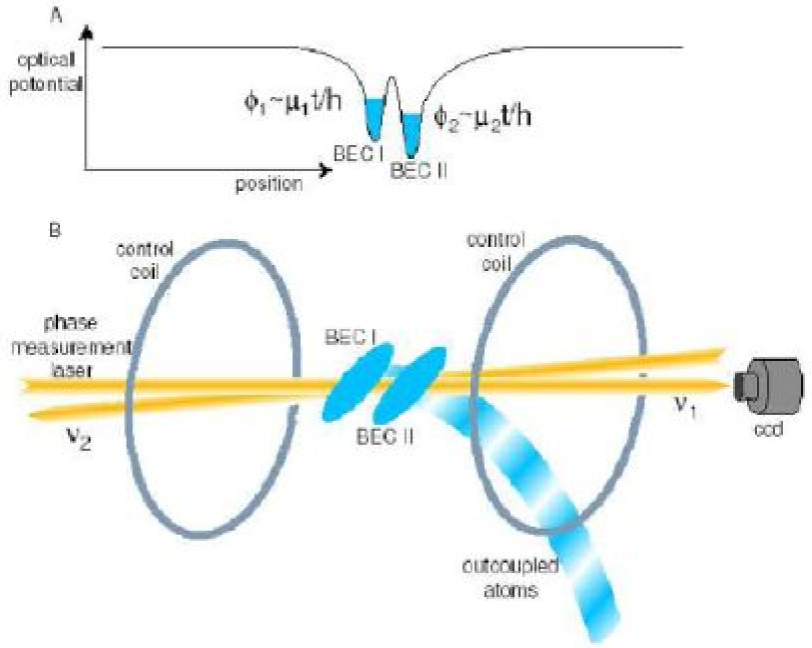


Vortex interference.

Interference of two condensates



Continuous phase measurement



Setup for continuous phase measurement.
M. Saba, T. Pasquini, C. Sanner, Y. Shin, W. Ketterle, and D. Pritchard, Science (2005).

Mott transition in optical lattice

$$U(z) = sE_r \cos^2(qz),\ E_r = \hbar^2 q^2 / 2m$$

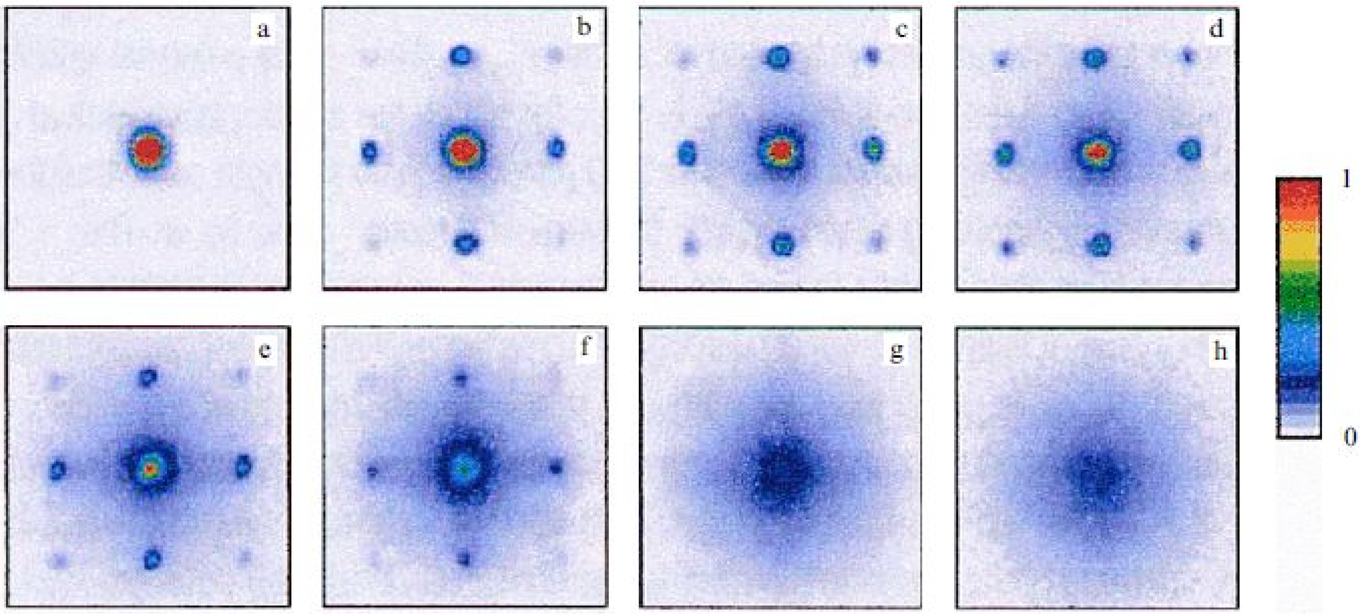


Figure 16. Interference pattern upon Bose-gas expansion from a three-dimensional lattice for different values of the parameter s: (a) $s = 0$, (b) 3, (c) 7, (d) 10, (e) 13, (f) 14, (g) 16, (h) 20. The disappearance of diffraction spots for $s > 13$ signifies the Mott transition to the dielectric phase [48].

Strongly interacting dilute liquid

$$r_0 << n^{-1/3}$$

BUT:

$$|f| \sim n^{-1/3}$$

Universal liquid

$$r_0 << n^{-1/3}, |a| \sim n^{-1/3}$$

Properties of the liquid are defined by a unique parameter a.

$|a| = \infty$ - "universal liquid"

Weakly bound dimers of fermions

$$a > 0 \;\; \text{Binding energy} \left|\varepsilon\right| = \hbar^2 / ma^2.$$

Dimer-dimer scatering length:

$$a_{dd} = 0.6a > 0. \quad \text{(A)}$$

Recombination rate

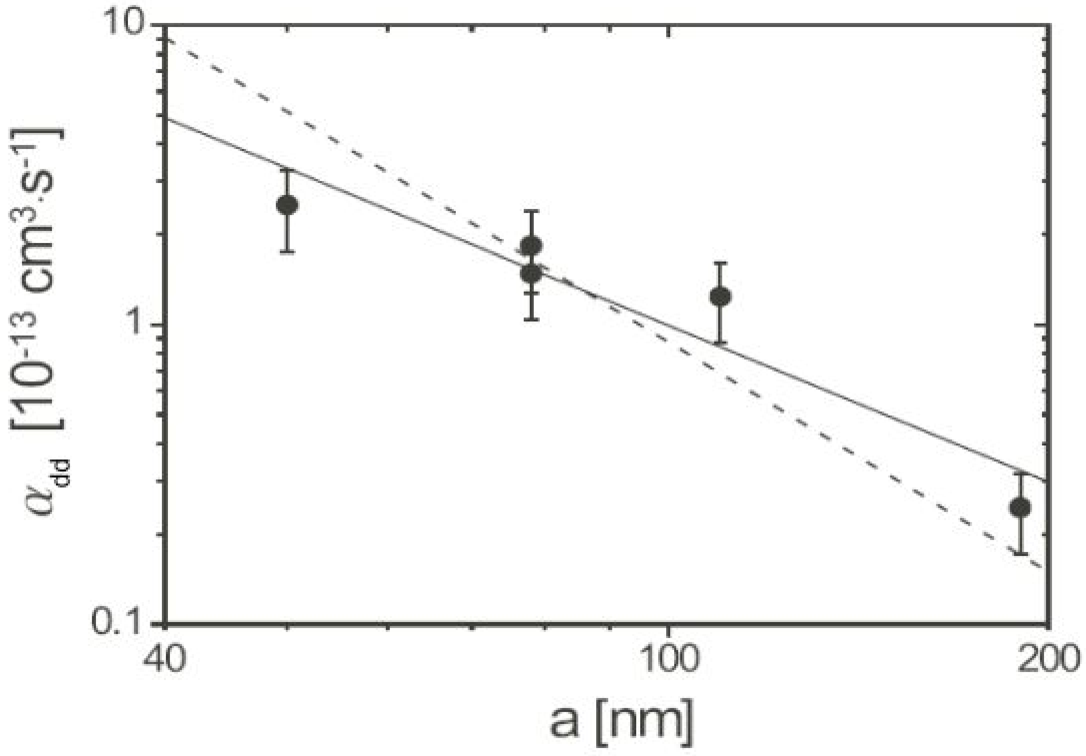
$$dn_d / dt = -\alpha_{dd} n_d^2, \alpha_{dd} \propto a^{-2.25} \quad \text{(B)}$$

$$a \to \infty, \alpha_{dd} \to 0\,!!!$$

(A) и (B):

Petrov, Salomon and Shlyapnikov, 2004

Recombination rate against scattering length



Limiting cases at *T=0*

$$n^{-1/3} \sim k_F$$

$$1) \quad a > 0, ak_F << 1$$

Superfluid dilute gas of dimers.

Bogoliubov theory.

$$a_{dd} = 0.6a > 0$$

$$2) \; a < 0, |a| k_F << 1$$

Superfluid dilute BCS - gas

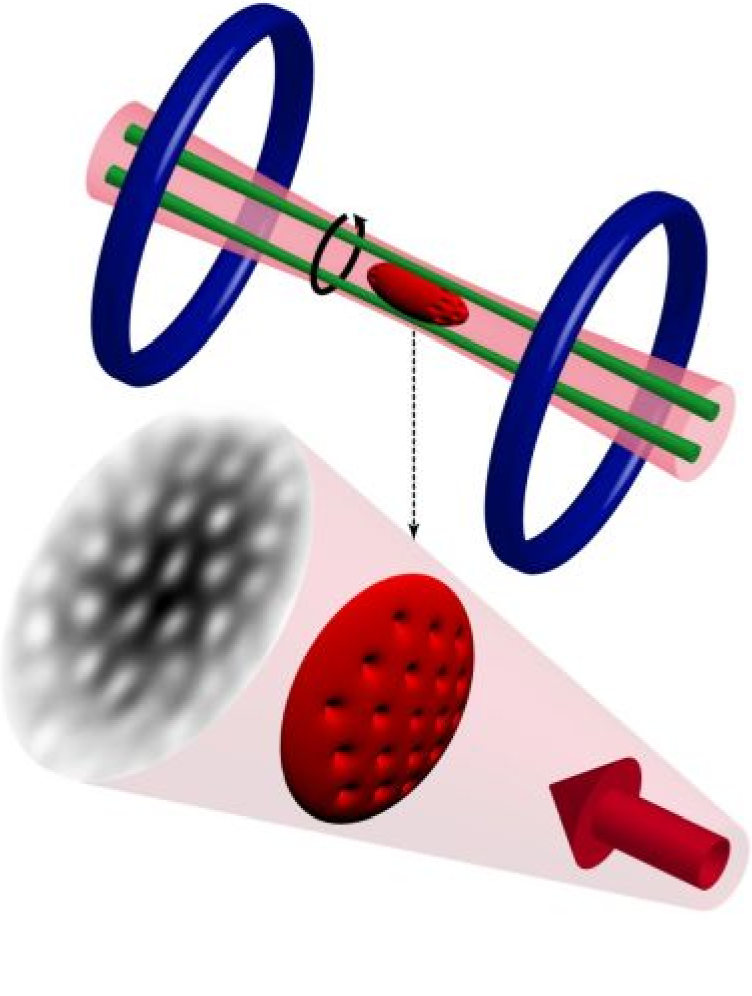
$$3) \; a = \infty - \text{ universal liquid}$$

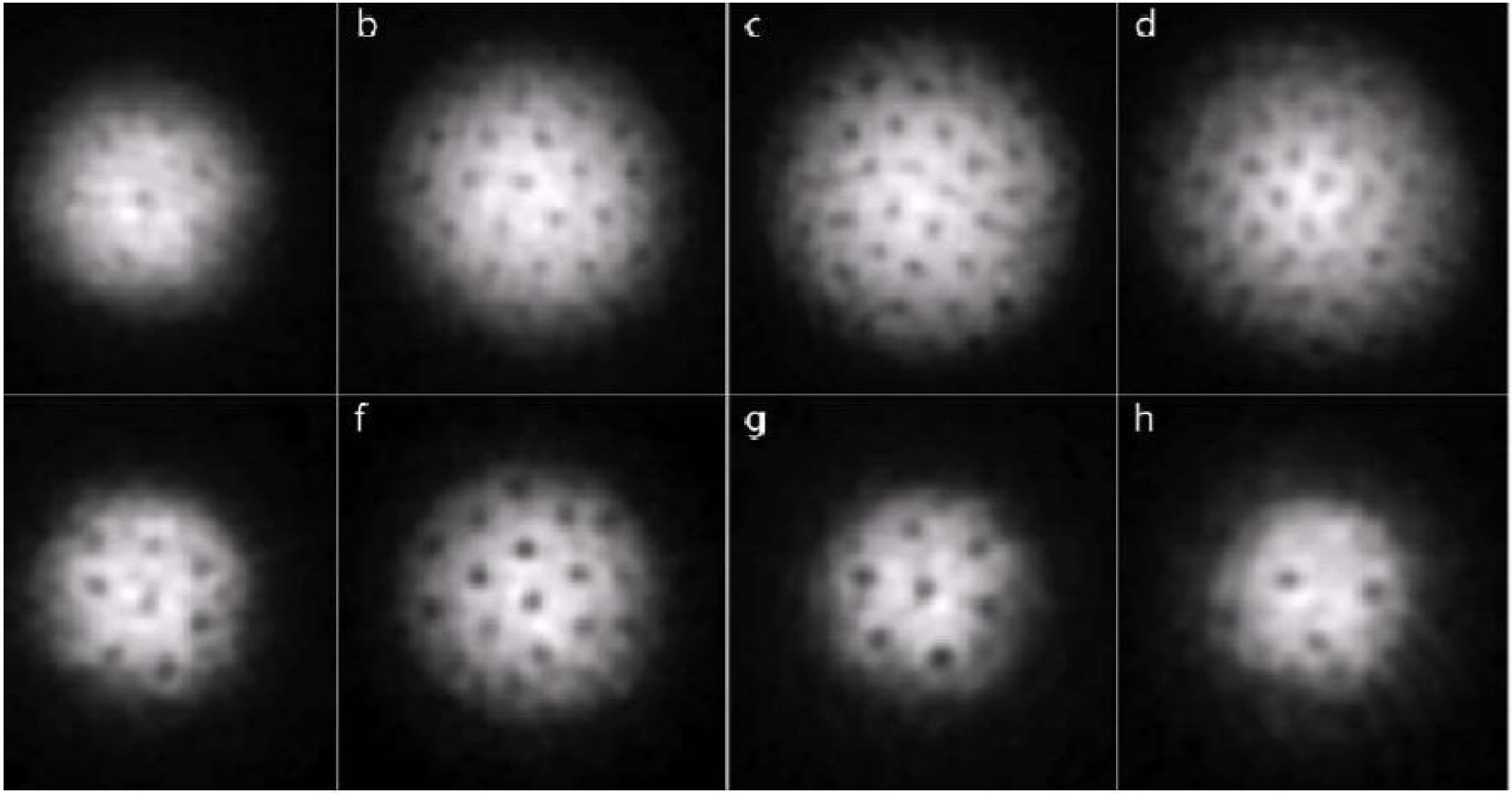
Vortexes in superfluid Fermi-liquid

Velocity circulation $v = \frac{\hbar}{2m}\frac{1}{r}$

Density of vortexes $n_v = \frac{2m\Omega}{\pi\hbar}$

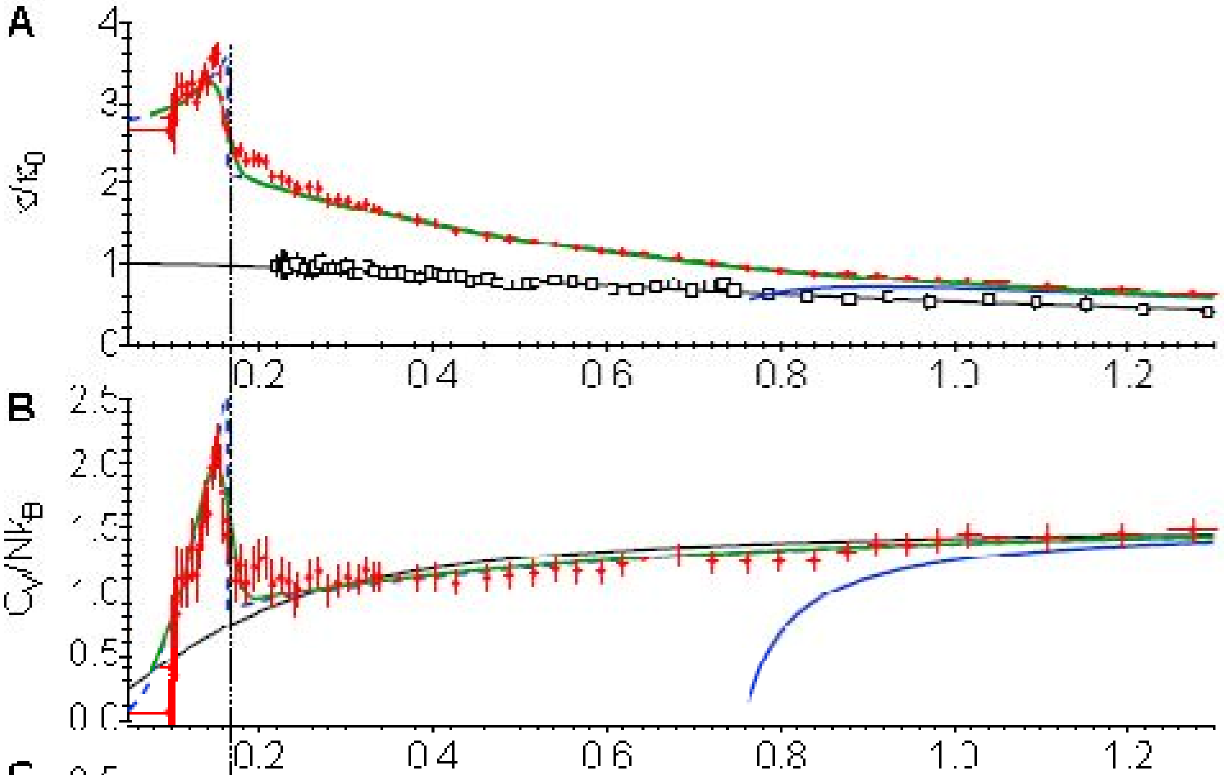
Experiment at MIT





2: Vortices in a strongly interacting gas of fermionic atoms on the BEC- and the BCS-side of Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms to 500 ms (b-h) followed by an equilibration time of 500 ms. After 2 ms of ballistic ansion, the magnetic field was ramped to 735 G for imaging (see text for details). The gnetic fields were (a) 740 G, (b) 766 G, (c) 792 G, (d) 812 G, (e) 833 G, (f) 843 G, (g) 853 G (h) 863 G. The field of view of each image is $880\ \mu\text{m} \times 880\ \mu\text{m}$.

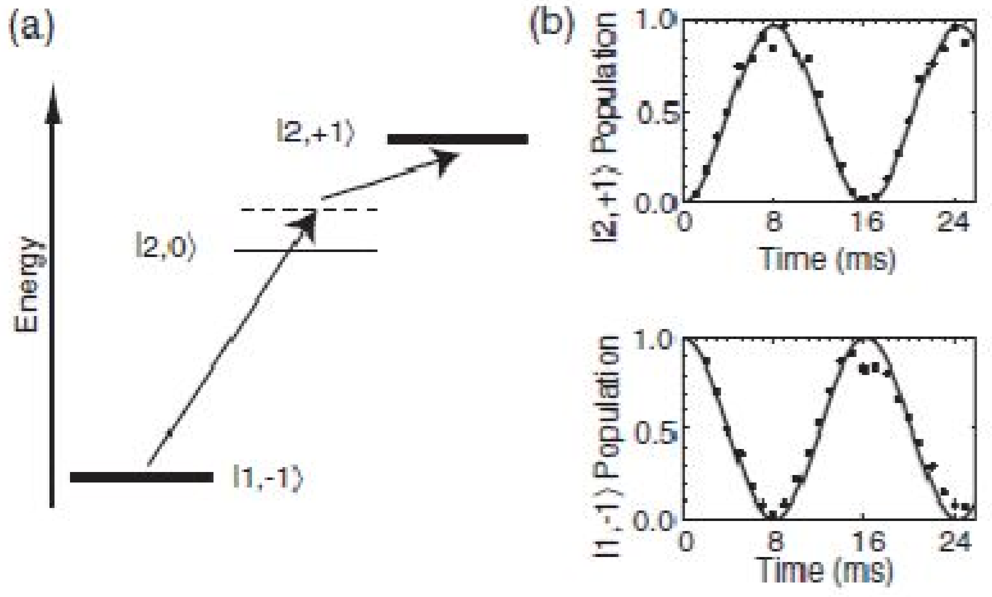
Transition point singularity in the unitary Fermi gas.



T/T_F

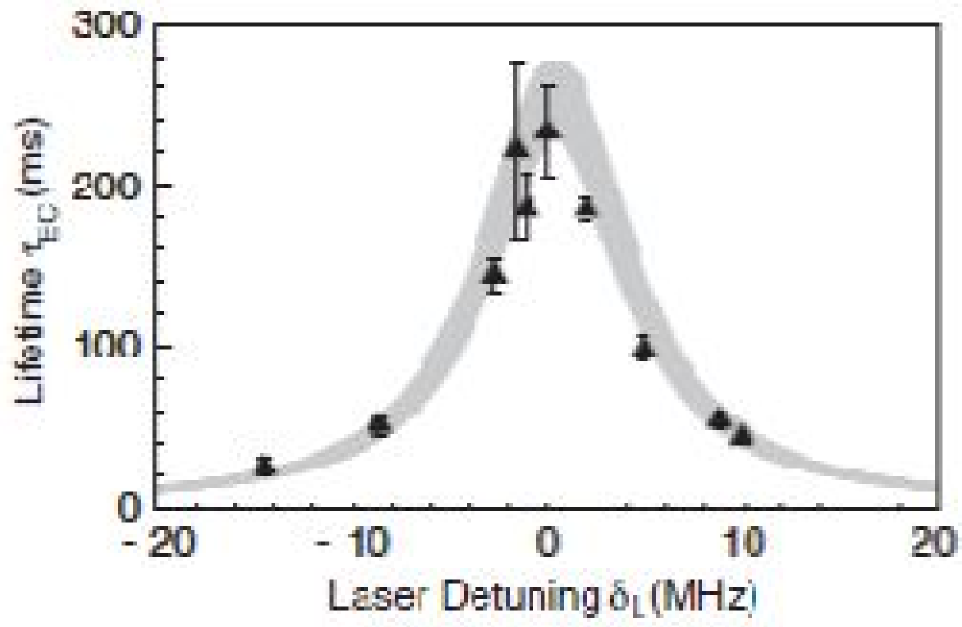
M. Ku et. Al., 2012

Quantum Zeno effect. E. Streed, 2006



Two-level Rabi oscillation. The two-level quantum system consisted of the $|1, -1\rangle$ and $|2, +1\rangle$ ground hyperfine states of ^{87}Rb. (a) Energy level diagram for relevant ^{87}Rb ground hyperfine states. Arrows depict the components of the two-photon transition between the $|1, -1\rangle$ and $|2, +1\rangle$ states.

Continuous Zeno effect



Continuous quantum Zeno lifetime as a function of the measurement laser detuning δ_L. Gray band indicates range of expected lifetimes