CMB Imprints of a Pre-Inflationary Climbing Phase

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- E. Dudas, N. Kitazawa, AS, PLB 694 (2010) 80 [arXiv:1009.0874 [hep-th]].
- E. Dudas, N. Kitazawa, S. Patil, AS, arXiv:1202.6630 [hep-th], to appear in JCAP

SUPERFIELDS
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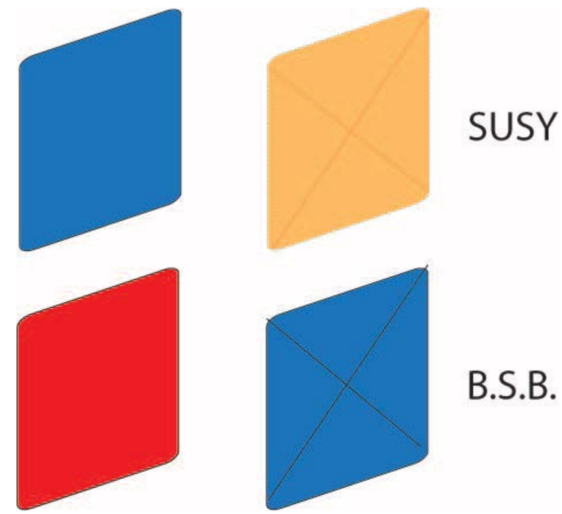
String Theory & SUSY Breaking

Brane SUSY Breaking :
(string-scale)

(Sugimoto, 1999)
(Antoniadis, Dudas, AS, 1999)
(Aldazabal, Uranga, 1999)

- **SUSY** : D9($T>0, Q>0$) + $O9_-$ ($T<0, Q<0$) → SO(32)
- **BSB** : anti-D9($T>0, Q<0$) + $O9_+$ ($T>0, Q>0$) → USp(32)



Tension unbalance → exponential potential

$$S_{10} = \frac{1}{2\kappa_{10}^2} \int d^{10}x\sqrt{-g}\,\left\{\left[R - \frac{1}{2}\,(\partial\phi)^2 \,-\, 2\,\alpha\, e^{\frac{3\phi}{2}}\right] - \frac{1}{12}\, e^{\phi}\, H^2\right\}$$

A climbing scalar in D dim's

- Consider the action for gravity and a scalar ϕ, in Einstein frame:

$$S \;=\; \frac{1}{2\kappa^2}\int d^D x\,\sqrt{-g}\,\left[R - \frac{1}{2}\,(\partial\phi)^2 \;-\; V(\phi)\;+\;\ldots\right]\;,$$

- Look for **cosmological solutions** of the type

$$ds^2 \;=\; -\,e^{\,2B(t)}\,dt^2 \;+\; e^{\,2A(t)}\,d\mathbf{x}\cdot d\mathbf{x}$$

(Halliwell, 1987)
....
(Dudas,,Mourad, 2000)
(Russo, 2004)

- Make the convenient **gauge choice** $V(\phi)\,e^{2B} = M^2$

- Let: $\beta \;=\; \sqrt{\frac{D-1}{D-2}}\;, \qquad \tau \;=\; M\,\beta\,t\;, \qquad \varphi = \frac{\beta\,\phi}{\sqrt{2}}\;, \qquad a \;=\; (D-1)\,A$

- In expanding phase :

$$\ddot{\varphi} \;+\; \dot{\varphi}\,\sqrt{1\;+\;\dot{\varphi}^{\,2}} \;+\; \left(1+\dot{\varphi}^{\,2}\right)\,\frac{1}{2V}\,\frac{\partial V}{\partial\varphi} \;=\; 0$$

damping → potential ←

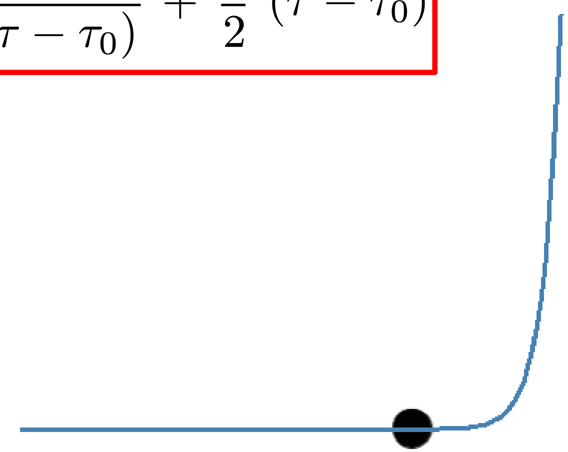
A climbing scalar in D dim's

- With exponential potentials $V(\varphi) = M^2\, e^{2\gamma\varphi}$

the equation for φ becomes $\ddot{\varphi} + \dot{\varphi}\sqrt{1+\dot{\varphi}^2} + \gamma\left(1+\dot{\varphi}^2\right) = 0$

- γ=1: $\dot{\varphi} = \frac{1}{2(\tau-\tau_0)} - \frac{1}{2}(\tau-\tau_0) \quad , \quad \dot{a} = \frac{1}{2(\tau-\tau_0)} + \frac{1}{2}(\tau-\tau_0)$

$$\varphi(\tau) = \varphi_0 + \frac{1}{2}\left[\log|\tau-\tau_0| - \frac{1}{2}(\tau-\tau_0)^2\right]$$
$$a(\tau) = a_0 + \frac{1}{2}\left[\log|\tau-\tau_0| + \frac{1}{2}(\tau-\tau_0)^2\right]$$



Integration constants:

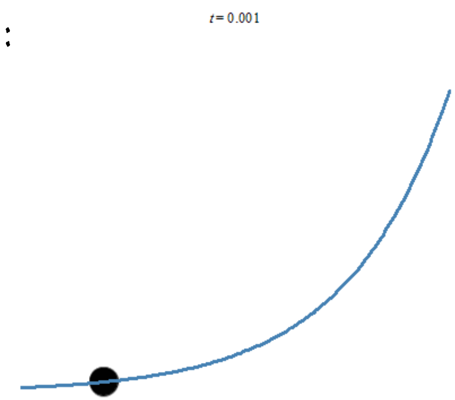
- "Big Bang time"
- φ(τ) and a(τ) at some (later) reference time.

φ **SPEED** close to Big Bang **FIXED** : CLIMBING SCALAR!

A climbing scalar in D dim's

- $\gamma < 1$?   Both signs of speed should be allowed

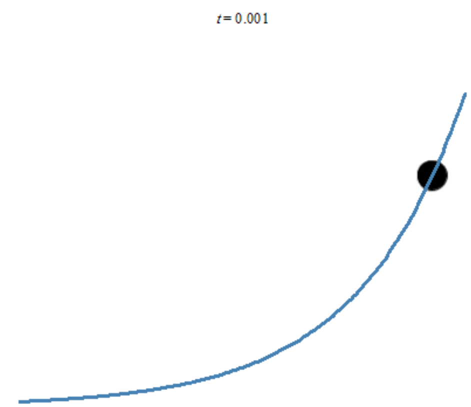
a. "Climbing" solution (φ climbs, descends → lim. τ-speed):



$$\dot{\varphi} = \frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}}\ \coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right) - \sqrt{\frac{1+\gamma}{1-\gamma}}\ \tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]$$

$$\dot{a} = \frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}}\ \coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right) + \sqrt{\frac{1+\gamma}{1-\gamma}}\ \tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]$$

b. "Descending" solution (φ descends → lim. τ- speed):



$$\dot{\varphi} = \frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}}\ \tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right) - \sqrt{\frac{1+\gamma}{1-\gamma}}\ \coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]$$

$$\dot{a} = \frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}}\ \tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right) + \sqrt{\frac{1+\gamma}{1-\gamma}}\ \coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]$$

Limiting τ- speed (LM attractor):

(Lucchin, Matarrese, 1985)

$$v_l = -\frac{\gamma}{\sqrt{1-\gamma^2}}$$

$\gamma \to 1$: v_l diverges & LM attractor and descending solution disappear

- Beyond $\gamma = 1$? → only climbing

Newtonian Analogy

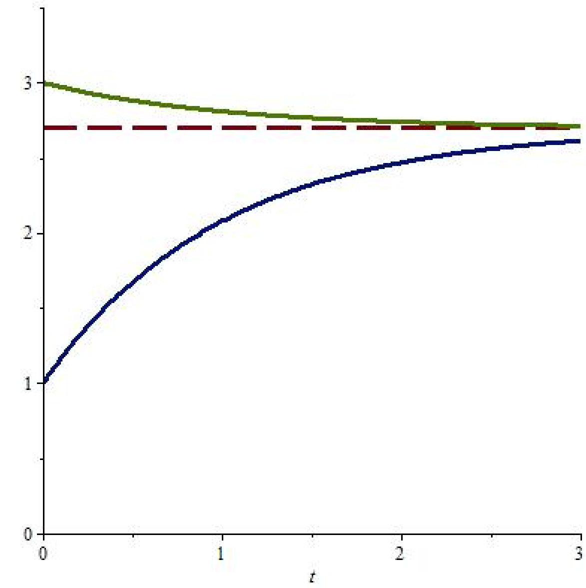
Particle subject to damping and constant force :

$$m \frac{d^2Y}{dt^2} + \beta \frac{dY}{dt} = f$$

- **Limiting speed:**

$$v_{lim} = \frac{f}{\beta}$$

$$\frac{dY}{dt} = v_{lim} + (v_0 - v_{lim})\, e^{-\frac{\beta t}{m}}$$



Two classes of solutions:

- approach v_{lim} from **below** or from **above**

As β is reduced, υ_{lim} increases **without limit**. The **"Upper branch"** is eventually **lost altogether**, leaving way to a single "undamped" solution.

String Theory Realizations

Correction to PLB 2010 :

- Other dimensions D : (γ = 1 for ALL D>10, up to volume stabilization)
- Non-Critical Strings : (from D>10 (super)critical + small exponent)

- 10D tadpole and KKLT : *(Kachru, Kallosh, Linde, Trivedi, 2003)*

  a) No-scale reduction: *(Cremmer, Ferrara, Kounnas, Nanopoulos, 1983)* *(Witten, 1985)*

$$g^{(10)}_{IJ} = e^{\sigma}\delta_{IJ}\ ,\ g^{(10)}_{\mu\nu} = e^{-3\sigma} g^{(4)}_{\mu\nu}\ ,\ C_{IJ} = \epsilon_{IJ}\ a_2$$

$$s = e^{3\sigma}\, e^{\frac{\phi}{2}}\ ,\ t = e^{\sigma} e^{-\frac{\phi}{2}}$$

  b) Two-form duality and "tadpole uplift":

$$s = e^{\Phi_s}\ ,\ t = e^{\frac{1}{\sqrt{3}}\ \Phi_t}$$

$$S_4 \ = \ \frac{1}{2\kappa_4^2}\int d^4x\,\sqrt{-g}\,\Big[R\ -\ \frac{1}{2}\,(\partial\Phi_s)^2\ -\ \frac{1}{2}\,(\partial\Phi_t)^2\ -\ 2\,\alpha\ e^{-\sqrt{3}\Phi_t}\ +\cdots\Big]$$

String Theory Realizations

Assume : s stabilized by fluxes

$$T = e^{\frac{\Phi_t}{\sqrt{3}}} + i\,\frac{\theta}{\sqrt{3}}$$

$$W = W_0 + a\,e^{-bT}\,, \qquad K = -3\,\ln(T+\bar{T})$$

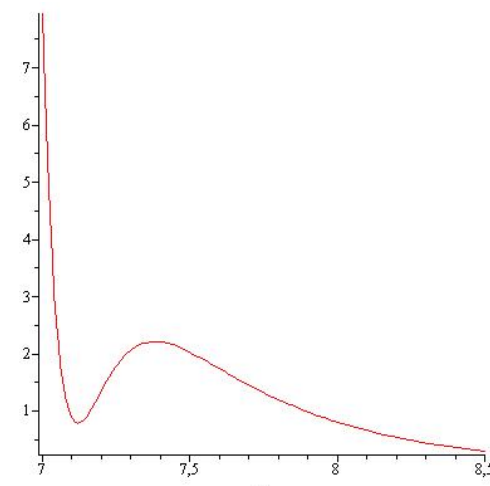
$$V_F = e^K\,\left[|D_T W|^2 - 3|W|^2\right] =$$

(Cremmer, Ferrara, Girardello, vanProeyen, 1983)

$$\frac{b}{(T+\bar{T})^2}\left\{a\,\bar{W}_0 e^{-bT} + \bar{a}\,W_0 e^{-b\bar{T}} + \frac{|a|^2}{3}\,[6+b(T+\bar{T})]\,e^{-b(T+\bar{T})}\right\}$$

Uplift :

$$V = V_F + \frac{c}{(T+\bar{T})^3}$$



$$S = \frac{1}{2\kappa_4^2}\int d^4x\sqrt{-g}\left[R - \frac{1}{2}\,(\partial\Phi_t)^2 - \frac{1}{2}\,e^{-\frac{2}{\sqrt{3}}\Phi_t}\,(\partial\theta)^2 - V(\Phi_T,\theta)\right]$$

Climbing in the KKLT System

Performing the redefinitions $\Phi_t = \frac{2}{\sqrt{3}}\, x \;,\; \theta = \frac{2}{\sqrt{3}}\, y \;,\; \tau = M\sqrt{\frac{3}{2}}\, t$

Working in the metric

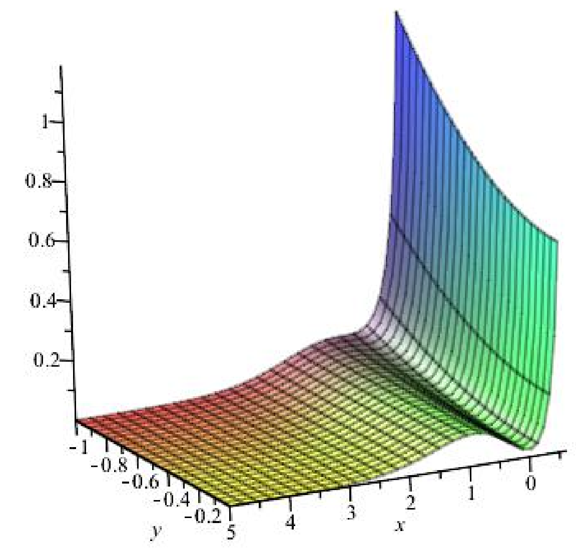
$$ds^2 = -\, e^{2B(t)}\, dt^2 + e^{2A(t)}\, d\mathbf{x}\cdot d\mathbf{x}$$

Choosing the gauge

$$V(\Phi_t, \theta)\, e^{2B} = M^2$$

$$\frac{d^2x}{d\tau^2} + \frac{dx}{d\tau}\sqrt{1+\left(\frac{dx}{d\tau}\right)^2 + e^{-\frac{4x}{3}}\left(\frac{dy}{d\tau}\right)^2} + \frac{1}{2V}\frac{\partial V}{\partial x}\left[1+\left(\frac{dx}{d\tau}\right)^2\right] + \frac{1}{2V}\frac{\partial V}{\partial y}\frac{dx}{d\tau}\frac{dy}{d\tau} + \frac{2}{3}\, e^{-\frac{4x}{3}}\left(\frac{dy}{d\tau}\right)^2 = 0\,,$$

$$\frac{d^2y}{d\tau^2} + \frac{dy}{d\tau}\sqrt{1+\left(\frac{dx}{d\tau}\right)^2 + e^{-\frac{4x}{3}}\left(\frac{dy}{d\tau}\right)^2} + \left(\frac{1}{2V}\frac{\partial V}{\partial x} - \frac{4}{3}\right)\frac{dx}{d\tau}\frac{dy}{d\tau} + \frac{1}{2V}\frac{\partial V}{\partial y}\left[e^{\frac{4x}{3}} + \left(\frac{dy}{d\tau}\right)^2\right] = 0$$



$$V(x,y) = \frac{c}{8}\, e^{-2x} - \frac{b}{2}\, e^{-\frac{4x}{3} - b\, e^{\frac{2x}{3}}}\left[(Re\; a\overline{W_0})\cos\frac{2by}{3} + (Im\; a\overline{W_0})\sin\frac{2by}{3} + \frac{|a|^2}{3}\left(3 + b\, e^{\frac{2x}{3}}\right) e^{-b\, e^{\frac{2x}{3}}}\right]$$

Climbing in the KKLT System

Leave aside momentarily the potential well → only uplift (γ =1 in KKLT)

- "Polar" variables :

$$\frac{dx}{d\tau} = r\,w\,, \qquad e^{-\frac{2x}{3}}\,\frac{dy}{d\tau} = r\,\sqrt{1-w^2}$$

- Dynamical γ_{eff} (= γw) :

$$\frac{dr}{d\tau} + r\,\sqrt{1+r^2} \;-\; \gamma\,w\,\left(1+r^2\right) \;=\; 0$$

$$\frac{dw}{d\tau} + \left(1-w^2\right)\,\left(\frac{2}{3}\,r \;-\; \frac{\gamma}{r}\right) \;=\; 0$$

1) Late-time attractors ("TOO FAST" FOR INFLATION)

2) Right after Big Bang :

$$\frac{dr}{d\tau} + (\epsilon - w)\,r^2 \approx 0\,, \qquad \frac{dw}{d\tau} + \frac{2}{3}\,r(1-w^2) \approx 0$$

- Eqs. combine to:

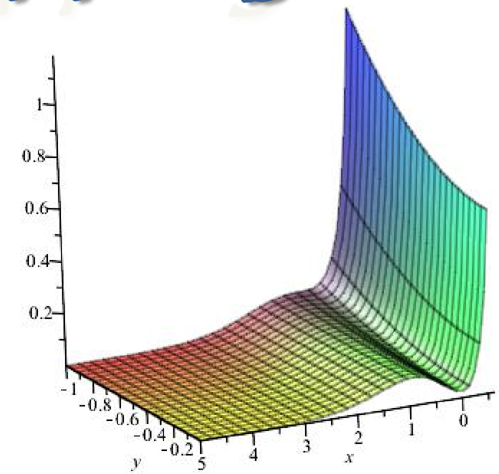
$$\dot{r} \;\approx\; -\,2\,\epsilon\,r^2$$

$$r \;\approx\; \frac{1}{2\,\epsilon\,\tau}\,, \quad w \approx -\,\epsilon \qquad \Rightarrow \qquad r\,w \;\approx\; -\,\frac{1}{2\,\tau}$$

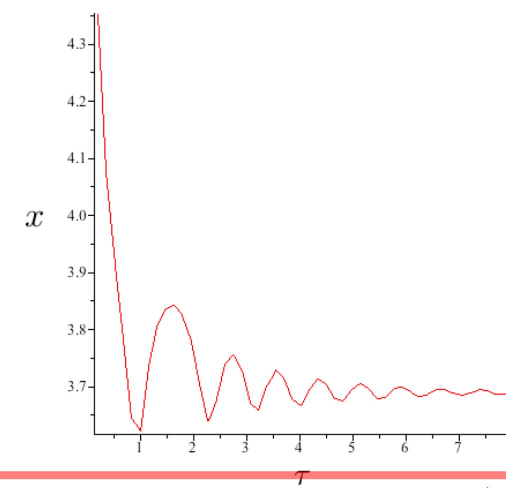
Climbing scalar !

KKLT Moduli Trapping

Climbing can yield **moduli trapping**



For a **range of parameters** the cosmological evolution can **trap** a climbing scalar inside the tiny **KKLT potential well**



Analytic approach: approximate wells by piece-wise exponentials

Climbing + Inflation

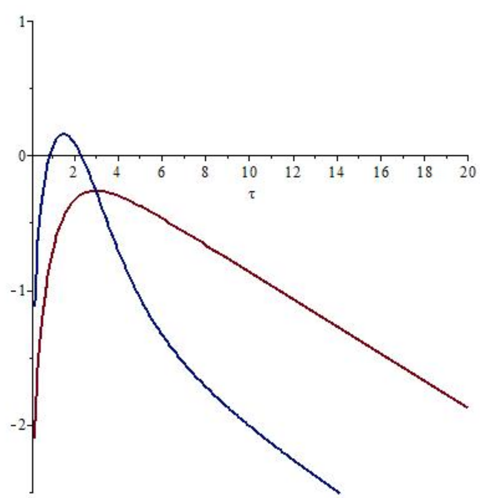
Combine two exponentials :

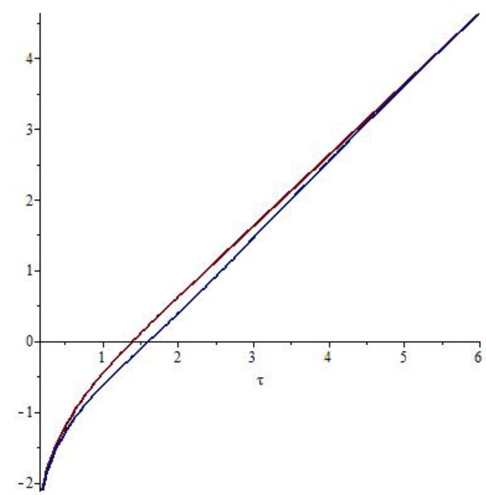
a. "Hard" exponential of BSB
b. "Soft" exponential (e.g. from non-BPS branes)

$$V(\phi) = \frac{M^2}{2\kappa^2}\left(e^{2\varphi} + e^{2\gamma\varphi}\right)$$

(Sen , 1998)
(Dudas, Mourad, AS 2001)

- "Hard" exponential: makes initial climbing phase inevitable
- "Soft" exponential: drives inflation during subsequent descent





The Mukhanov-Sasaki Equation

Schroedinger-like equation for scalar (or tensor) fluctuations :

$$\frac{d^2 v_k(\eta)}{d\eta^2} + \left[k^2 - W_s(\eta)\right] v_k(\eta) = 0$$

$$ds^2 = a^2(\eta)\left(-d\eta^2 + d\mathbf{x}\cdot d\mathbf{x}\right)$$

$$Scalar : z(\eta) = a^2(\eta)\,\frac{\phi_0'(\eta)}{a'(\eta)}$$

$$Tensor : z(\eta) = a$$

$$W_s = \frac{1}{z}\,\frac{d^2 z}{d\eta^2}$$

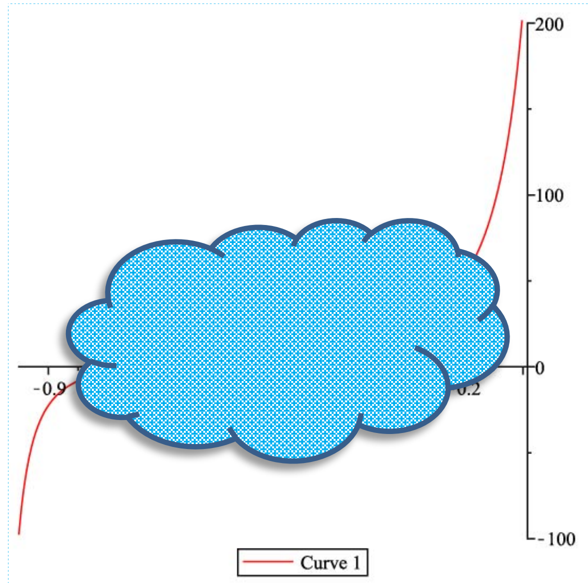
"MS Potential" : determined by the background

$$\text{Initial Singularity} : W_s \underset{\eta\to-\eta_0}{\sim} -\frac{1}{4}\,\frac{1}{(\eta+\eta_0)^2}$$

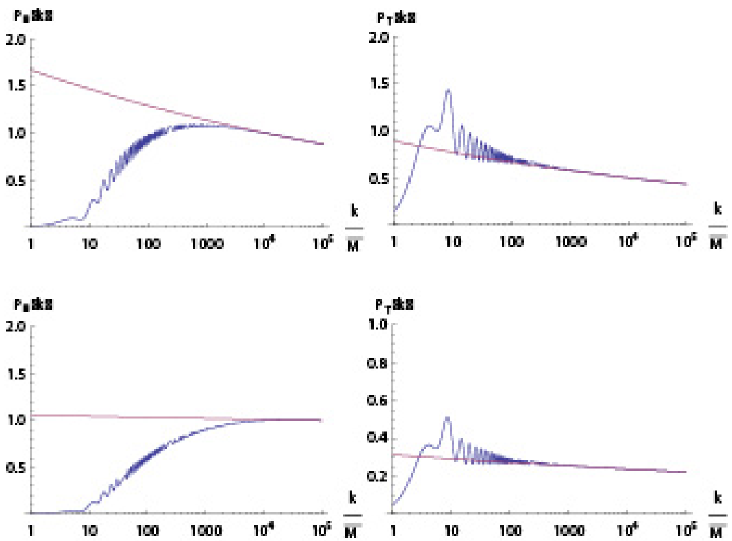
$$\text{LM Inflation} : W_s \underset{\eta\to 0}{\sim} -\frac{\nu^2-\frac{1}{4}}{\eta^2}$$

$$\left[\nu = \frac{3}{2}\,\frac{1-\gamma^2}{1-3\gamma^2}\right]$$

$$P(k) \sim k^3 \left|\frac{v(-\epsilon)}{z(-\epsilon)}\right|^2$$



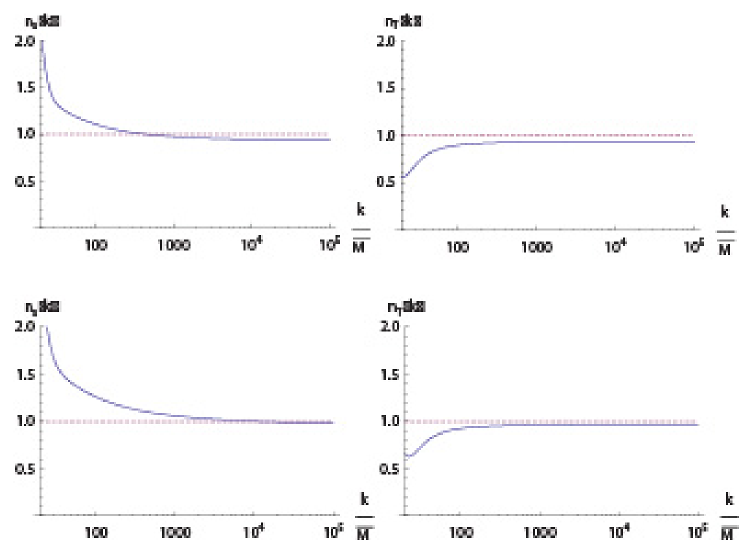
Numerical Power Spectra



$$\epsilon_\phi(k) \equiv -\frac{\dot{H}}{H^2}\,, \qquad \eta_\phi(k) \equiv \frac{V_{\phi\phi}}{V}$$

$$\begin{aligned} n_S(k) - 1 &= 2\left[\eta_\phi(k) - 3\,\epsilon_\phi(k)\right] \\ n_T(k) - 1 &= -\,2\,\epsilon_\phi(k) \end{aligned}$$

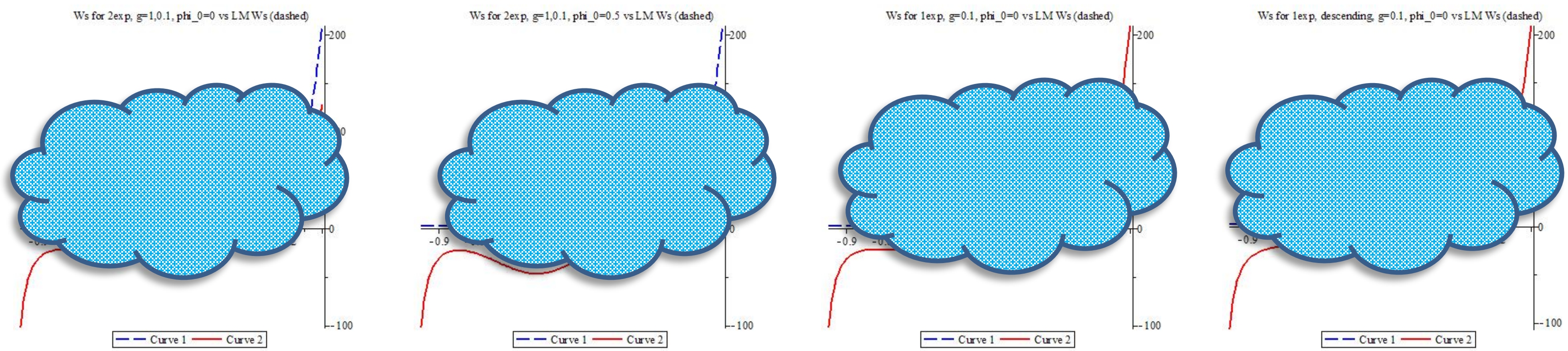
$$P_{S,T} \sim \int \frac{dk}{k}\, k^{n_{S,T}-1}$$



QUALITATIVE ANALYSIS

- Dynamics yields $\epsilon_\phi(k)$, $\eta_\phi(k)$
- Slow-roll relations estimate $n_{S,T}(k)$

Analytic Power Spectra



WKB:

$$v_k(-\epsilon) \sim \frac{1}{\sqrt[4]{|W_s(-\epsilon) - k^2|}} \exp\left(\int_{-\eta^\star}^{-\epsilon} \sqrt{|W_s(y) - k^2|}\, dy\right)$$

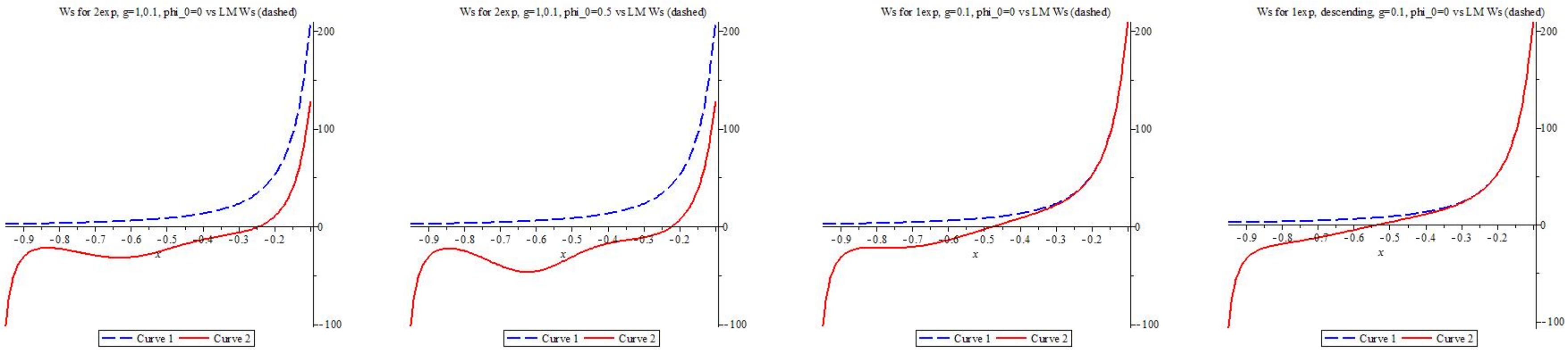
Analytically:

- **Bessel** functions (LM attractor)
- **Coulomb** (or Whittaker) functions
- [**Heun** functions]

$$P(k) \sim k^{3-2\nu}$$

$$\frac{d^2 v_k}{d\eta^2} + \left[k^2 - \frac{(1-a)\left(\nu^2 - \frac{1}{4}\right)}{\eta_0^2} - \frac{(2-a)\left(\nu^2 - \frac{1}{4}\right)}{\eta_0\, \eta} - \frac{\nu^2 - \frac{1}{4}}{\eta^2}\right] v_k = 0$$

Analytic Power Spectra



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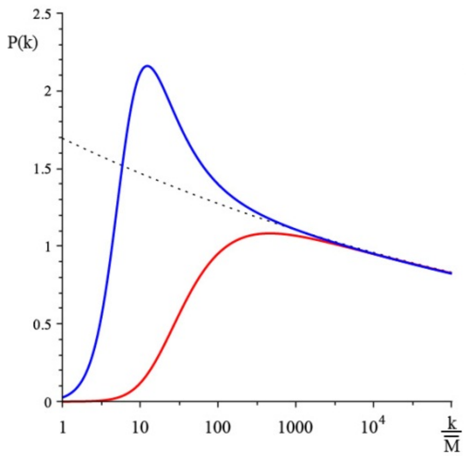
Analytically:

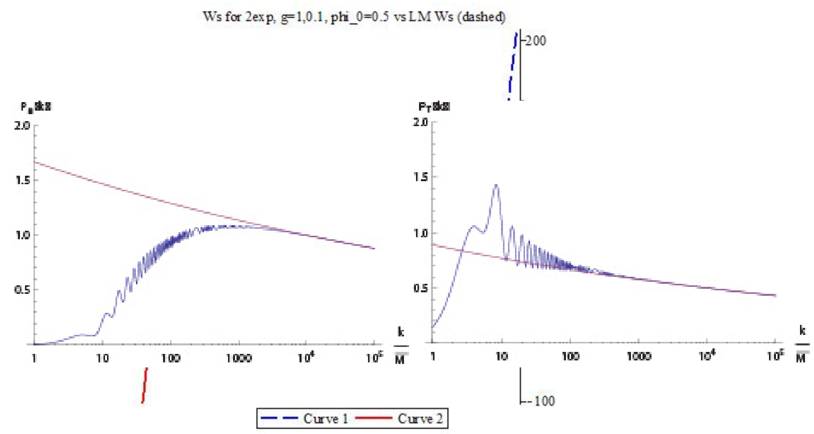
- Bessel functions (LM attractor)
- Coulomb (or Whittaker) functions
- [Heun functions]

$$P(k) \sim k^{3-2\nu}$$

$$\frac{d^2 v_k}{d\eta^2} + \left[k^2 - \frac{(1-a)\left(\nu^2-\frac{1}{4}\right)}{\eta_0^2} - \frac{(2-a)\left(\nu^2-\frac{1}{4}\right)}{\eta_0\,\eta} - \frac{\nu^2-\frac{1}{4}}{\eta^2}\right] v_k = 0$$

Analytic Power Spectra





Letting :

$$\Delta = \sqrt{(k\eta_0)^2 - (1-a)\left(\nu^2 - \frac{1}{4}\right)}\,, \quad \rho = -\frac{\eta\Delta}{\eta_0}$$

$$L = \nu - \frac{1}{2}\,, \quad \alpha = -\frac{(2-a)\left(\nu^2 - \frac{1}{4}\right)}{2\,\Delta}$$

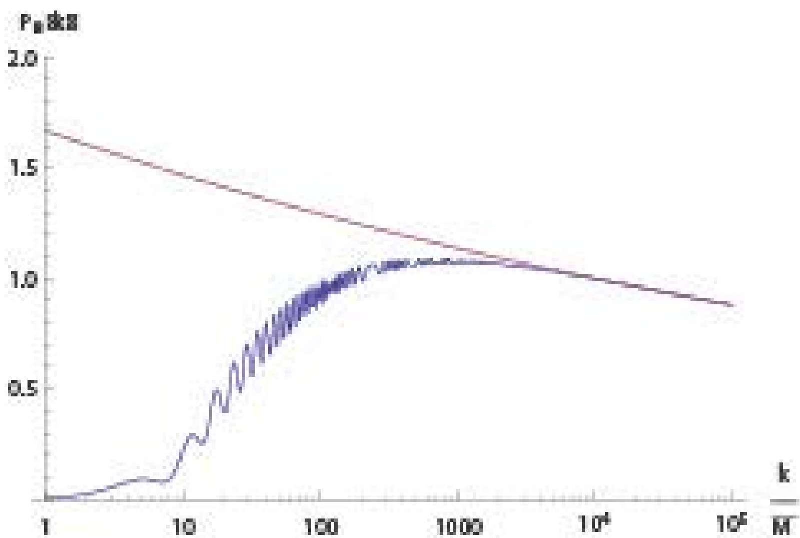
MS → Coulomb :

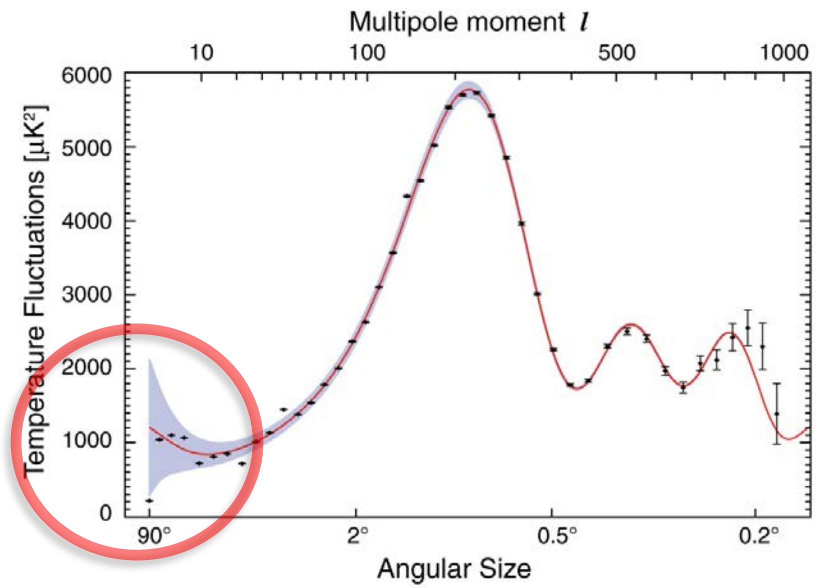
$$\frac{d^2 v_k}{d\rho^2} + \left[1 - \frac{2\,\alpha}{\rho} - \frac{L(L+1)}{\rho^2}\right] v_k = 0$$

$$P(k) \sim \frac{k^3 \exp\left(-\frac{(2-a)\left(\frac{2-3\,\gamma^2}{(1-3\,\gamma^2)^2}\right)}{2\sqrt{(k\eta_0)^2 + (a-1)\left(\frac{2-3\,\gamma^2}{(1-3\,\gamma^2)^2}\right)}}\right)}{\left|\Gamma\left(L+1+\frac{i\,(2-a)\left(\frac{2-3\,\gamma^2}{(1-3\,\gamma^2)^2}\right)}{2\sqrt{(k\eta_0)^2 + (a-1)\left(\frac{2-3\,\gamma^2}{(1-3\,\gamma^2)^2}\right)}}\right)\right|^2 \left[(k\eta_0)^2 + (a-1)\left(\frac{2-3\,\gamma^2}{(1-3\,\gamma^2)^2}\right)\right]^{\nu}}$$

An Observable Window ?

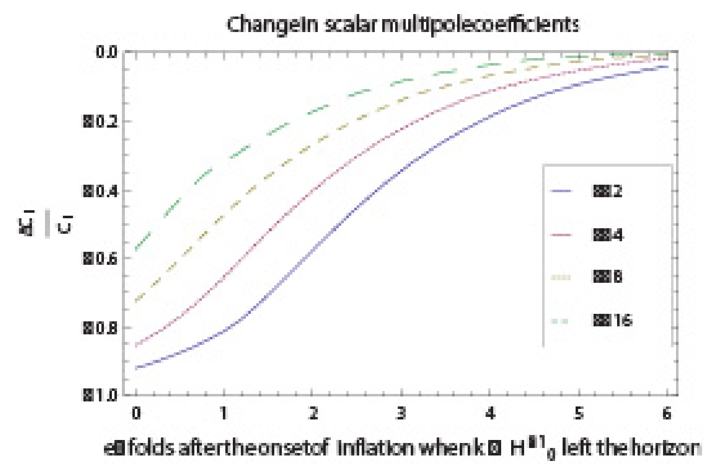
- WMAP7 power spectrum :





- Depression of scalar multipoles :

$$\left(vs \quad \left| \frac{\Delta C_\ell}{C_\ell} \right| = \sqrt{\frac{2}{2\ell+1}} \right)$$



Summary & Outlook

- String Theory → **logarithmic slope** of certain potentials that accompany the (string-scale) **Brane SUSY Breaking** mechanism

- Naturally weak coupling : **climbing** can induce **moduli trapping** &/or **inject inflation**

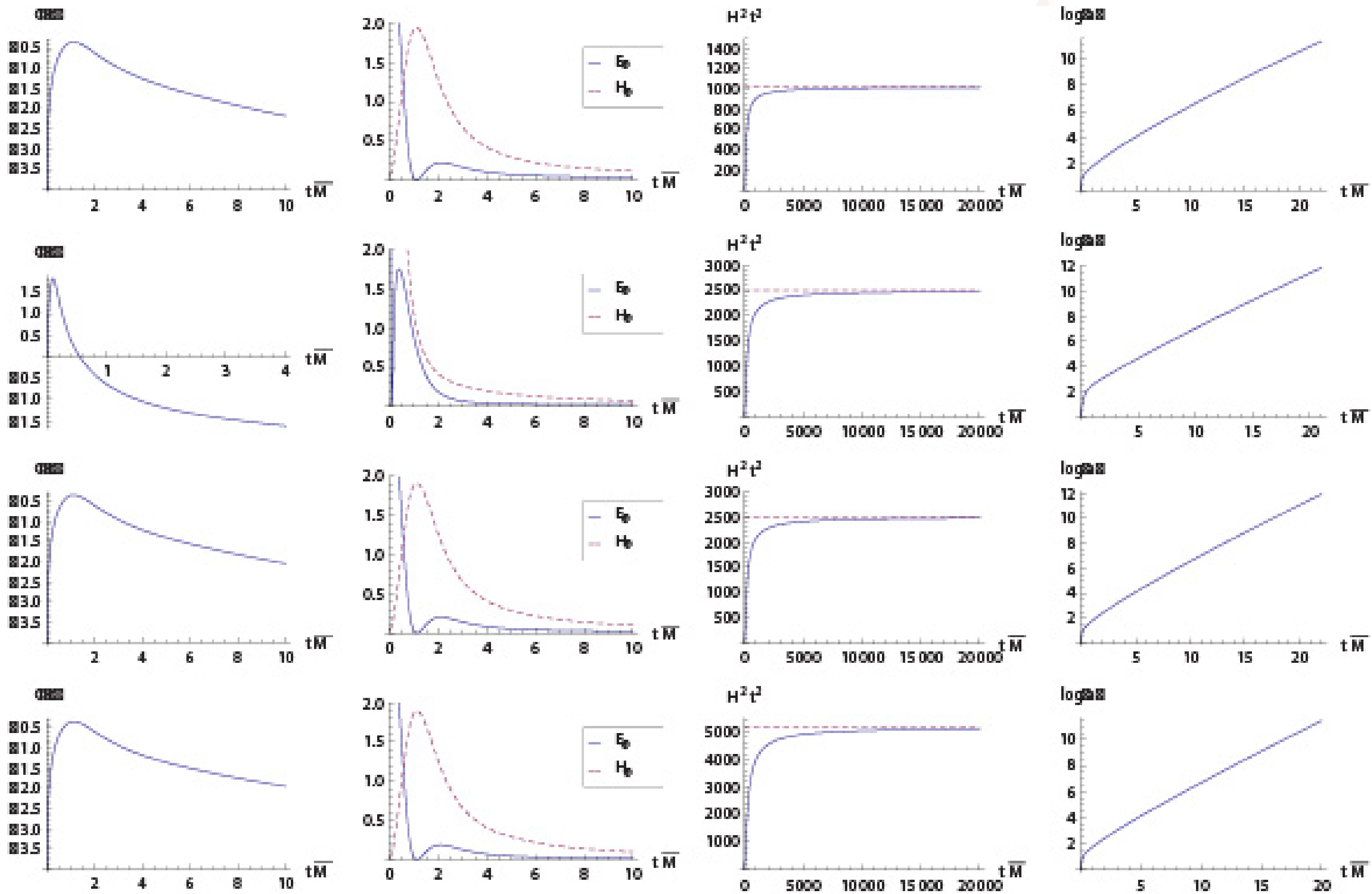
COMBINING "mild exponential" for **inflation** and "hard exponential" of **BSB** :

- STRONG IR depression of **scalar** power spectrum (over ~ 6 e-folds)
- [MILDER IR enhancement of **tensor** spectrum]
- Models where inflation begins **well off** the attractor (new CMB codes?)

Signs in the sky of pre-inflationary climbing ?

IF inflationary epoch was "short enough"

Numerical Power Spectra



$$\epsilon_\phi \equiv -\,\frac{\dot{H}}{H^2}\ , \qquad \eta_\phi \equiv \frac{V_{\phi\phi}}{V}$$

$$\begin{aligned} n_S \;-\; 1 \;&=\; 2(\,\eta_\phi \;-\; 3\,\epsilon_\phi)\ , \\ n_T \;-\; 1 \;&=\; -\,2\,\epsilon_\phi \end{aligned}$$

$$P_{S,T} \;\sim\; \int \frac{dk}{k}\; k^{n_{S,T}-1}$$

A non-BPS D3 brane : Inflation

The Sugimoto model has a **stable** non-BPS D3 brane

(Sen , 1998)
(Dudas, Mourad, AS 2001)

$$V \sim e^{-\sqrt{3}\Phi_t} + e^{-\frac{\sqrt{3}\Phi_t}{2}}$$

IF Φ_s is somehow stabilized (as assumed in the KKLT setting) → for Φ_t :

- "critical" exponential (**climbing**);
- "subcritical" exponential (eventual **inflation**)

- INFLATION:

$$\mathcal{I} = \frac{d^2A}{dt^2} + \frac{dA}{dt}\left(\frac{dA}{dt} - \frac{dB}{dt}\right) > 0$$

- One-field :

$$\dot{\varphi}^2 < \frac{1}{D-2} \rightarrow \gamma < \frac{1}{\sqrt{D-1}}$$

- KKLT: NO inflation by itself