Boundary state from Ellwood invariants

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Outline

- Introduction
- Boundary state from Ellwood invariants
- Analytic solutions: Rolling tachyon
- Numerical solutions: Lumps in Siegel gauge
- Conclusion

Boundary states

- The nicest problems in theoretical physics are ones which are: easy to state, difficult to solve and with many relations to other branches to physics

- One such problem is classification of the boundary states in a given CFT or equivalently admissible open string vacua or D-branes.

Boundary states

- Describe possible boundary conditions from the closed string channel point of view.

- Conformal boundary states obey:

1) the gluing condition $(L_n - \bar{L}_{-n})|B\rangle = 0$

2) Cardy condition (modular invariance)

3) sewing relations (factorization constraints)

See e.g. reviews by Gaberdiel or by Cardy

Boundary states

- The gluing condition is easy to solve:

  For any spin-less primary $|V_\alpha\rangle$ we can define

$$||V_\alpha\rangle\rangle = \sum_{IJ} M^{IJ}(h_\alpha) L_{-I} \bar{L}_{-J} |V_\alpha\rangle$$

where M^{IJ} is the inverse of the real symmetric Gramm matrix

$$M_{IJ}(h_\alpha) = \langle V^\alpha | L_I L_{-J} | V_\alpha \rangle$$

where $L_{-X} \equiv L_{-n_k} \ldots L_{-n_1}$,
(with possible null states projected out).

Ishibashi 1989

Boundary states

- Explicitly:

$$||V_\alpha\rangle\rangle = \Bigg[1 + \frac{1}{2h_\alpha} L_{-1}\bar{L}_{-1} + B(h_\alpha, c)\left(2(1+2h_\alpha)L_{-2}\bar{L}_{-2} - 3(L_{-2}\bar{L}_{-1}^2 + L_{-1}^2\bar{L}_{-2}) + \frac{8h_\alpha + c}{4h_\alpha} L_{-1}^2\bar{L}_{-1}^2\right) + \cdots \Bigg] |V_\alpha\rangle$$

$$B(h_\alpha, c) = \frac{1}{2h_\alpha(8h_\alpha - 5) + c(2h_\alpha + 1)}.$$

- The other conditions are much harder to deal with however. Probably not even the full set of necessary conditions is known.
- We will now show how to construct boundary states (appropriate linear combinations of Ishibashi states) from OSFT solutions.

Lightning minireview of OSFT

Open string field theory uses the following data

$$\mathcal{H}_{BCFT}, \quad *, \quad Q_B, \quad \langle . \rangle .$$

All of the string degrees of freedom are assembled in

$$|\Psi\rangle = \sum_i \int d^{p+1}k \, \phi_i(k)|i,k\rangle,$$

Witten (1986) proposed the following action

$$S = -\frac{1}{g_o^2}\left[\frac{1}{2}\langle \Psi * Q_B \Psi \rangle + \frac{1}{3}\langle \Psi * \Psi * \Psi \rangle\right],$$

Lightning minireview of OSFT

- The classical equation of motion take the form

$$Q_B\Psi + \Psi * \Psi = 0$$

- Analytic solutions can be found in the K,B,c basis

$$c^2 = 0, \quad B^2 = 0, \quad \{c, B\} = 1$$

$$[K, B] = 0, \quad [K, c] = \partial c$$

$$Q_B K = 0, \quad Q_B B = K, \quad Q_B c = cKc.$$

- For example

$$\Psi = Fc\frac{KB}{1-F^2}cF,$$

Here *F=F(K)* is arbitrary
M.S. (2005), Okawa 2006

Ellwood conjecture

- Solutions to OSFT e.o.m. are believed to be in 1-1 correspondence with consistent boundary conditions.

- The widely believed (and tested, but unproven) Ellwood conjecture states that for every on-shell $\mathcal{V}_{cl}$:

$$\langle \mathcal{V}_{cl} | c_0^- | B_\Psi \rangle = -4\pi i \, \langle I | \mathcal{V}_{cl}(i) | \Psi - \Psi_{\rm TV} \rangle \,,$$

Here Ψ is a solution of the e.o.m., $\Psi_{\rm TV}$ is the tachyon vacuum and $|B_\Psi\rangle$ is the boundary state we are looking for.

Ellwood (2008)

Generalized Ellwood invariants

- The restriction to on-shell state can be bypassed. Any solution built using reference $BCFT_0$ can be written as

$$\Psi = \sum_j \sum_{\substack{I=\{n_1,n_2,\ldots\} \\ J=\{m_1,m_2,\ldots\}}} a^j_{IJ}\, L^{\text{matter}}_{-I} |\mathcal{V}_j\rangle \otimes L^{\text{ghost}}_{-J} c_1 |0\rangle$$

and uplifted to $BCFT_0 \otimes BCFT_{aux}$, where $BCFT_{aux}$ has c=0 and contains free boson Y with Dirichlet b.c. One can then compute Ellwood invariant with $\tilde{\mathcal{V}}^\alpha = c\bar{c}V^\alpha e^{2i\sqrt{1-h}\,Y} w$

Trick inspired by Kawano, Kishimoto and Takahashi (2008)

Generalized Ellwood invariants

- Since $|B_\Psi\rangle^{\mathrm{CFT}_0 \otimes \mathrm{CFT}_{\mathrm{aux}}} = |B_\Psi\rangle^{\mathrm{CFT}_0} \otimes |B_0\rangle^{\mathrm{CFT}_{\mathrm{aux}}}$
  we find

$$\langle\, c\bar{c}V^\alpha |\, c_0^- \,|B_\Psi\,\rangle = -4\pi i \langle\, E[\tilde{\mathcal{V}}^\alpha] | \tilde{\Psi} - \tilde{\Psi}_{TV}\,\rangle$$

- This is gauge invariant even w.r.t. the gauge symmetry of the original OSFT based on BCFT_0

$$\mathrm{Lift} \circ (\mathrm{Gauge\ Transf})_\Lambda = (\mathrm{Gauge\ Transf})_{\mathrm{Lift}(\Lambda)} \circ \mathrm{Lift}$$

Boundary state from Ellwood invariants

- Hence the coefficients of the boundary state

$$|B_\Psi\rangle = \sum_\alpha n^\alpha_\Psi ||V_\alpha\rangle\rangle$$

are given by

$$n^\alpha_\Psi = -\frac{4\pi i}{\mathcal{N}_{gh}} \left\langle I|\tilde{\mathcal{V}}^\alpha(i)|\tilde{\Psi} - \tilde{\Psi}_{\rm TV} \right\rangle$$

where $\tilde{\mathcal{V}}^\alpha = c\bar{c}V^\alpha e^{2i\sqrt{1-h}\,Y}w$

$$\mathcal{N}_{gh} = \left\langle c\bar{c}|c_0^-|B_{bc} \right\rangle$$

For alternative attempt see:
Kiermaier, Okawa, Zwiebach (2008)

Translating back to BCFT?

- The coefficients n_Ψ^α of the Ishibashi states can be related by the above conjecture to more BCFT-like quantities

$$n_\Psi^\beta = -\frac{\pi}{2} \sum_j 4^{h_j} A_\Psi^j \left\langle V^\beta(0)\phi_j(1) \right\rangle_{\text{disk}}^{\text{BCFT}_0^{\text{matter}}}$$

where A_Ψ^j are gauge-invariant linear combinations of coefficients in the ϕ_j sector of the OSFT solution Ψ

Example: Rolling tachyon

- OSFT solution known analytically

$$\Psi_\lambda = Fc\frac{B}{1+\lambda e^{X^0}\frac{1-F^2}{K}}\lambda c e^{X^0} F$$

M.S.; Kiermaier et al. 2007

- Computing the Ellwood invariant one finds

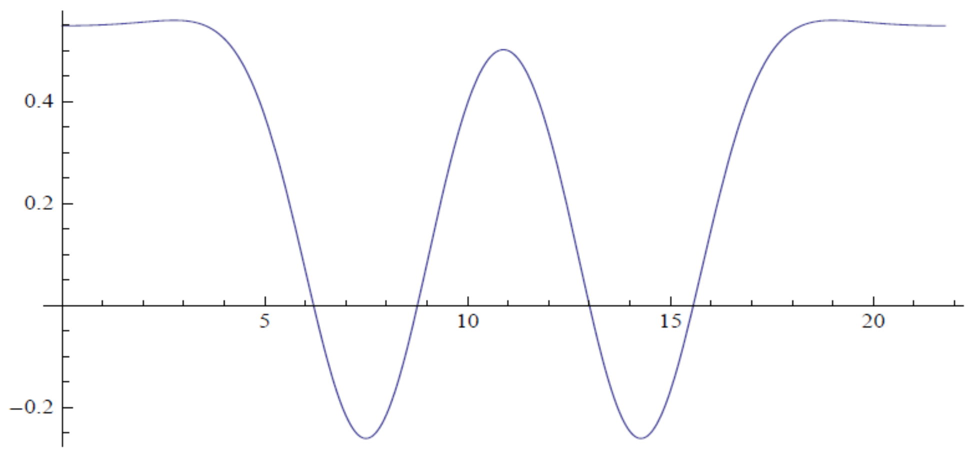
$$|B_\Psi\rangle = e^{-\lambda\int_0^{2\pi} d\theta\, e^{X^0}}|B_0\rangle$$

- From this Sen's tachyon matter conjecture follows (by repeating computation of Larsen et al.)

$$\begin{aligned}\frac{T^{ij}(x^0)}{\mathrm{Vol}_{25}} &= -\frac{1}{1+\lambda e^{x^0}}\delta^{ij}\\ \frac{T^{00}(x^0)}{\mathrm{Vol}_{25}} &= 1.\end{aligned}$$

Example: double lump

- At radius $R = 2\sqrt{3}$ we find several double lump solutions. One of them looks like this



It has always been a problem to understand the point-like nature of D-branes

Example: double lump

- Energy density profile from our boundary state should look like

$$E_{(a)}(x) = \delta\Big(x - \pi R(1-a)\Big) + \delta\Big(x - \pi R(1+a)\Big) = \frac{1}{\pi R}\left(\frac{1}{2}E_0 + \sum_{n=1}^{\infty} E_n \cos\frac{nx}{R}\right)$$

where the Ellwood invariants E_n should take values

$$E_n = 2(-1)^n \cos(n\pi a)$$

Example: double lump

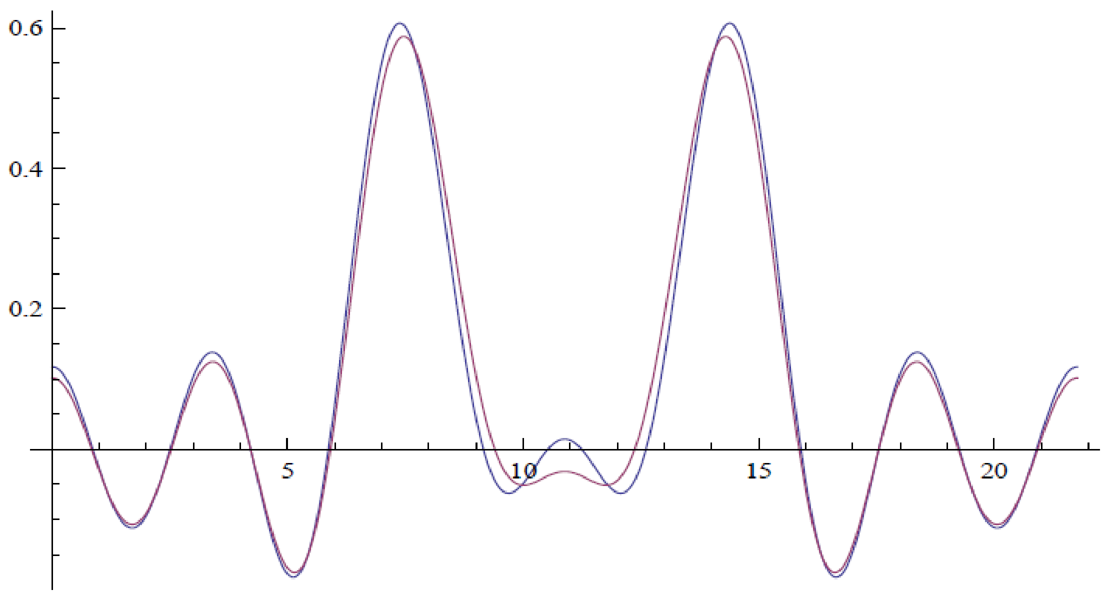
- With the help of computer cluster we were able to go to level (12,36) and found the following invariants

L	Action	D	E_0	E_1	E_2	E_3	E_4	E_5
1	2.57014	2.4209	2.4209	– 0.816955	– 0.54184	1.3133	–	–
2*	2.21165	−1.69337	2.1897	– 0.848747	– 0.60583	1.89707	– 1.62092	–
3	2.19355	−2.50001	2.11767	– 0.908501	– 0.838798	1.84278	– 1.24372	– 0.987367
4*	2.06874	−1.39183	2.08709	– 0.919667	– 0.850043	1.88425	– 1.0523	– 1.02488
5	2.05531	−1.37542	2.07382	– 0.983959	– 0.812633	1.91245	– 1.15202	– 0.57724
6	2.03894	−2.09185	2.05368	– 1.00138	– 0.788653	1.92175	– 1.30591	– 0.518028
7	2.03494	−2.1419	2.04912	– 1.03283	– 0.765547	1.90846	– 1.35827	– 0.488344
8	2.0269	−1.71527	2.04119	– 1.04599	– 0.743696	1.90879	– 1.35485	– 0.42022
9	2.02525	−1.70495	2.03899	– 1.06273	– 0.734362	1.91644	– 1.37781	– 0.37505
10	2.02052	−2.07063	2.03154	– 1.07229	– 0.717661	1.91526	– 1.44161	– 0.329759
11	2.01969	−2.08504	2.03029	– 1.08369	– 0.709787	1.90937	– 1.45664	– 0.295048
12	2.01658	– 1.81655	2.02687	– 1.09091	– 0.696749	1.90744	– 1.45907	– 0.256288
Expected	2	– 2	2	– 1.18	– 0.61	1.90	– 1.63	0.03

The expected value is obtained for the best fit value a=0.3

Example: double lump

- For a visual comparison with the delta function profile we plot both curves where we keep only 6 harmonics



Conclusions

- We have described a very nice new tool to extract physics out of OSFT. It works with both analytic and numerical solutions. One should also try to prove the Ellwood conjecture, however.
- OSFT, a grand Sudoku, is a potentially systematic approach to find all consistent boundary states.
- We are coming to an era of possible computer exploration of the OSFT landscape - stay tuned!