Spontaneous magnetization of the vacuum and the strength of the magnetic field in the hot universe

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ABSTRACT

Intergalactic magnetic fields are assumed to have been spontaneously generated at the reheating stage of the early Universe, due to vacuum polarization of non-Abelian gauge fields at high temperature. The fact that the screening mass of this type of fields has zero value was discovered recently. A procedure to estimate their field strengths, $B(T)$, at different temperatures is here developed, and the value $B \sim 10^{14} G$, at the electroweak phase transition temperature, is derived by taking into consideration the present value of the intergalactic magnetic field strength, $B \sim 10^{-15} G$, coherent on the ~ 1 Mpc scale. As a particular case, the standard model is considered and the field scale at high temperature is estimated in this case.

OUTLINE

- INTERGALACTIC MAGNETIC FIELDS
- MECHANISMS FOR GENERATION of MAGNETIC FIELDS IN HOT UNIVERSE
- SPONTANEOUS VACUUM MAGNETIZATION at HIGH TEMPERATURE
- MAGNETIC FIELD CHARACTERISTICS
- QUALITATIVE CONSIDERATION of PHENOMENA
- EFFECTIVE POTENTIAL at HIGH TEMPERATURE
- MAGNETIC FIELD at T_{ew}
- MAGNETIC FIELD SCALE
- CONCLUSION

INTERGALACTIC MAGNETIC FIELDS

Magnetic fields $B \sim \mu G$ presence everywhere – in galaxies, clusters of galaxies
Determination of intergalactic magnetic fields $B_0 \sim 10^{-15} G$:

[S. Ando, A. Kusenko, Astrophys. J. Lett. **722 (2010) L 39**][arXiv:1005.1924] looked at the source morphology (hallo, γ cascades: $\gamma \to e^+ e^- \to \gamma^*, \gamma^*, ...$);

[S. Ando, A. Kusenko, [arXiv:1012.5313]] looked at blazer spectra.

Complementary and independent methods.

[W. Essey, S. Ando, A. Kusenko,Astropart. Phys. **35 (2011) 1351**] Model independent 2σ CL interval $1 \times 10^{-17} \leq B \leq 3 \times 10^{-14}$ G was estimated

This value was estimated either as lower or upper limit. So, it is actual value at $3.5\mathrm{CL}$ accuracy.

[A. Neronov, E. Vovk. Science **328 (2010) 73.**] $B_0 \sim 10^{-16} G$.

ASTROPHYSICAL CONSTRAINTS

- Bing Bang Nucleosynthesis (BBN) limit $B \leq 10^{11} G$ or $B \leq 7 \cdot 10^{-7} G$ at galaxy formation;

- Cosmic microwave background (CMB) limit $B \leq 10^{-9} G$.

MECHANISMS for GENERATION of B

Popular mechanisms for generation of seed magnetic fields at high temperature in the early universe:

- metric perturbations
- strong first order EW phase transition [Hogan (1983), Vachaspaty (1991)]
- stochastic electric currents
- paramagnetic resonances in scalar (or axion) - electromagnetic field system
- Born-Infield electrodynamics, HE effective Lagrangian
- inflation
- cosmic strings
- trace anomaly
- extradimensions
- gravitational couplings of gauge field potentials

In all these considerations it is **ASSUMED**

magnetic flux is conserved

and therefore the dependence of $B \sim T^2$

takes place at cooling of the universe.

OUR MAIN IDEA:

primordial magnetic field is *spontaneously generated* due to

vacuum polarization of non-Abelian gauge fields.

These magnetic fields are temperature dependent

[Starinets, Vshivtsev, Zhukovsky(1994)],

[Skalozub(1996)], [Bordag, Skalozub (2000)],

[Demchik, Skalozub (2008)] **(in lattice simulations)**:

$$B(T) \sim \frac{g^3 T^2}{\log \frac{T}{\tau}}. \quad (1)$$

So, **there is no magnetic flux conservation at high temperature!**

Spontaneous vacuum magnetization

At zero temperature, [Savvidy (1978)]. The magnetized vacuum state is unstable because of the mode $p_0^2 = p_{||}^2 - gB$ in the gluon spectrum,

$$p_0^2 = p_{||}^2 + (2n+1)gB, \; n = -1, 0, 1, ..., \tag{2}$$

that results in a condensate. Because of instability, the Abelian constant magnetic field $B = const$ is completely screened.

At $T \neq 0$ the spectrum stabilization happens due to either a gluon magnetic mass of charged gluons [Bordag, Skalozub (2000)] or a so-called A_0-condensate which is proportional to the Polyakov loop [Starinets, Vshivtsev, Zhukovskii (1996)].

Hence, stabilization of magnetized vacuum takes place.

SPONTANEOUS VACUUM MAGNETIZATION at HIGH TEMPERATURE

On a lattice, the main continuous object is a magnetic flux. We relate the free energy density of the flux to the effective action [Demchik, Skalozub (2008)],

$$F(\varphi) = \bar{S}(\varphi) - \bar{S}(0), \tag{3}$$

where $\bar{S}(\varphi)$ and $\bar{S}(0)$ are the effective lattice actions with and without chromomagnetic field, φ is the field flux.

The spontaneous creation of the field follows if free energy has a global minimum at non-zero flux, $\varphi_{min} \neq 0$.

The hypercubic lattice $L_t \times L_s^3$ $(L_t < L_s)$ with the hypertorus geometry was used; L_t and L_s are the temporal and the spatial sizes of the lattice, respectively. In the limit of $L_s \to \infty$ the temporal size L_t is related to physical temperature.

The Wilson action of the $SU(2)$ lattice gauge theory is

$$S_W = \beta \sum_x \sum_{\mu>\nu} \left[1 - \frac{1}{2} \, \mathrm{Tr} \, U_{\mu\nu}(x) \right] ; \tag{4}$$

$$U_{\mu\nu}(x) = U_\mu(x) U_\nu(x + a\hat{\mu}) U_\mu^\dagger(x + a\hat{\nu}) U_\nu^\dagger(x), \tag{5}$$

where $\beta = 4/g^2$ is lattice coupling, g is the bare coupling, $U_\mu(x)$ is the link variable located on the link leaving the lattice site x in the μ direction, $U_{\mu\nu}(x)$ is the ordered product of the link variables. The effective action $\bar{S}$ in (3) is the Wilson action S_W averaged over the Boltzmann configurations produced in the MC simulations.

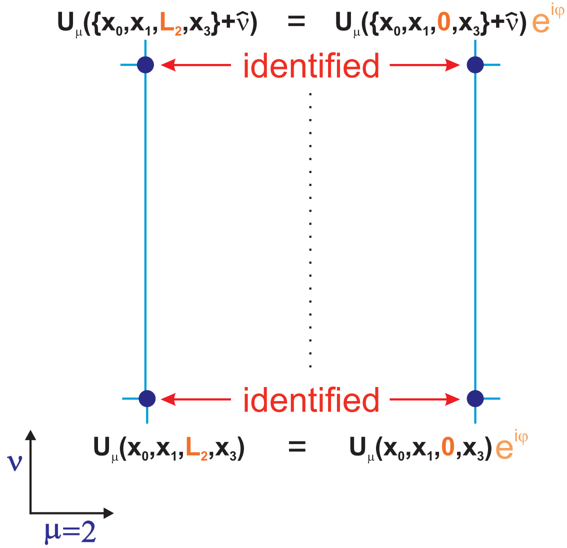

Fig. 3: The plaquette presentation of the twisted boundary conditions

The chromomagnetic flux φ through the whole lattice was introduced by applying the *twisted boundary conditions*. In this approach, the edge links in all directions are identified as usual periodic boundary conditions except for the links in the second spatial direction, for which the additional phase φ is added. The magnetic flux φ is measured in angular units, $\varphi \in [0; 2\pi)$.

The MC simulations are carried out by means of the heat bath method. The lattices 2×8^3, 2×16^3 and 4×8^3 at $\beta = 3.0,\ 5.0$ are considered. These values of the coupling constant correspond to the deconfinement phase and perturbative regime.

The effective action depends smoothly on the flux φ in the region $\varphi \sim 0$. So, the free energy density can be fitted by a quadratic function of φ,

$$F(\varphi) = F_{min} + b(\varphi - \varphi_{min})^2. \tag{6}$$

In Eq.(6), there are three unknown parameters, F_{min}, b and φ_{min}. φ_{min} denotes the minimum position of free energy, whereas the F_{min} and b are the free energy density at the minimum and the curvature of the free energy function, correspondingly. They have been fitted by a standard χ^2 method.

Table 1: The values of the generated fluxes φ_{min} for different lattices (at the 95% confidence level).

	2×8^3	2×16^3	4×8^3
$\beta = 3.0$	$0.019^{+0.013}_{-0.012}$	$0.0069^{+0.0022}_{-0.0057}$	$0.005^{+0.005}_{-0.003}$
$\beta = 5.0$	$0.020^{+0.011}_{-0.010}$		

The fit results are given in the Table 1. As one can see, φ_{min} demonstrates the 2σ-deviation from zero.

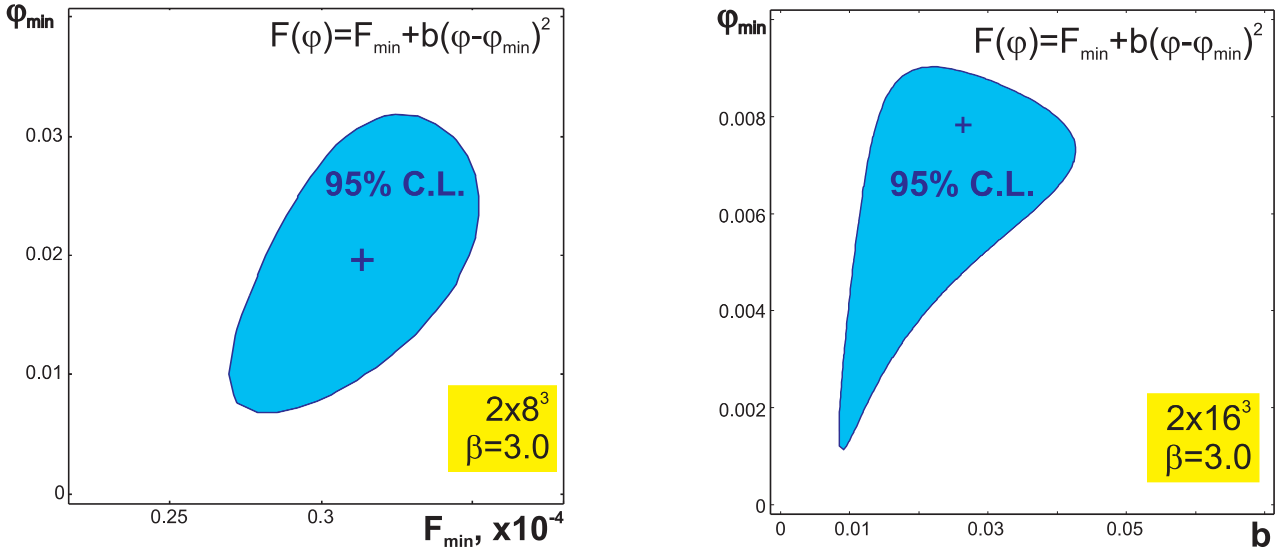

Fig. 4: **The 95% confidence level area for the parameters F_{min} and φ_{min} (b for right fig.).**

The flux φ_{min} is positively determined!

MAGNETIC FIELD CHARACTERISTICS

The most essential for what follows characteristics of the field:

Stability

To verify stability we substituted the value of $B_{min}(T)$ in the one-loop EP, the imaginary part was of the order 10^{-12} of the real one.

This means the stable state!

Temperature dependence

In $SU(2)$ gluodynamics, from the EP
$V(B,T) = V^{(1)}(B,T) + V^{(ring)}(B,T)$ it was determined

$$(gH)^{1/2} = \frac{g^2}{2\pi} \frac{T}{1 + \frac{11 g^2}{12\pi^2} \log(T/\tau)}. \qquad (7)$$

Masslessness (long-range magnetic fields)

In $SU(2)$ lattice gauge theory in the presence of Abelian magnetic fields [Antropov, Bordag, Demchik, Skalozub (2011)].

We use the **General Purpose computation on Graphics Processing Units (GPGPU)** technology allowing to study the large lattices up to 32×64^3.

The magnetic flux is introduced by applying the twisted boundary conditions.

For each lattice geometry $L_t \times L_s^3$, we have fitted the effect of magnetic field with the lattice plaquette average by means of different functions:

$$\langle U_{untwisted} \rangle - \langle U_{twisted} \rangle = f(m, L_s). \tag{8}$$

The best fit function for Abelian magnetic field **is** $C/r \exp(-mr)$ **with** a small value of **the magnetic mass** $m = 0.0000125$. This case corresponds to the magnetic tube with increasing field strength. Actually, the magnetic mass is equal to zero within the statistical uncertainties appeared.

Spontaneously generated chromomagnetic field is temperature dependent, stable and massless (long range).

	Abelian field		
Fit function	χ^2	C	m
$C\exp(-mr)$	901.8	0.063	$m = (2.44^{+0.06}_{-0.06}) \times 10^{-2}$
$C\exp(-m^2r^2)$	1924.4	0.035	$m = (1.57^{+0.02}_{-0.02}) \times 10^{-2}$
C/r	7.090	0.911	
$C/r\exp(-mr)$	7.086	0.912	$m = (1.25^{+52}_{-54}) \times 10^{-6}$
$C/r\exp(-m^2r^2)$	7.090	0.911	$m^2 = (2.4^{+5951.2}_{-5784}) \times 10^{-10}$
C/r^2	31400	28.13	
$C/r^2\exp(-m^2r^2)$	7550	18.26	$m^2 = -3.3 \times 10^{-5}$
C/r^4	159500	248.9	
$C/r^4\exp(-m^4r^4)$	161000	10.0	$m = 0.0$

Table 3: **Fit results for magnetic mass of Abelian magnetic field.**

QUALITATIVE CONSIDERATION

The most relevant aspects of the phenomena of interest are *consequences of asymptotic freedom and spontaneous symmetry breaking* at finite temperature – the basic principles of modern QFT.

Our main assumption is that the intergalactic magnetic field had been spontaneously created at reheating stage of the universe evolution.

First, in non-Abelian gauge theories magnetic at high temperatures flux conservation does not hold. The vacuum acts as a specific source generating classical fields.

Second, the spontaneous vacuum magnetization takes place for small scalar field $\phi \neq 0$, only. For the values of ϕ corresponding to any first order phase transition it does not happen.

After the electroweak phase transition, the vacuum polarization ceases to generate magnetic fields and magnetic flux conservation holds. As a result, the familiar dependence on the temperature $B \sim T^2$ is restored.

Composite structure of electromagnetic field A_μ. The potentials

$$A_\mu = \frac{1}{\sqrt{g^2 + g'^2}}(g' A^3_\mu + g b_\mu), \quad (9)$$

$$Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}}(g A^3_\mu - g' b_\mu),$$

Only the component $A_\mu = \frac{1}{\sqrt{g^2+g'^2}} g' A^3_\mu = \sin\theta_w A^3_\mu$ is present at high temperature. Here θ_w is the Weinberg angle, $\tan\theta_w = \frac{g'}{g}$.

This is the only component responsible for the intergalactic magnetic field at low temperature.

In restored phase, $b_\mu = 0$, and $A_\mu^{(3)}$ is unscreened. This is because the magnetic mass of this field is zero [S. Antropov, M. Bordag, V. Demchik, V. Skalozub (2011)].

The field is a long range. Its coherence length is to be sufficiently large.

The constituent of the weak isospin field corresponding to the magnetic one is

$$B(T) = \sin\theta_w(T) B^{(3)}(T), \tag{10}$$

where $B^{(3)}(T)$ is the strength of the field generated spontaneously.

After the phase transition, part of the field is screened.

For EWPT temperature T_{ew}:

$$\frac{B(T_{ew})}{B_0} = \frac{T_{ew}^2}{T_0^2} = \frac{\sin\theta_w(T_{ew})B^{(3)}(T_{ew})}{B_0}, \tag{11}$$

$B_0 \sim 10^{-15}G$. Parameter τ can be fixed for given temperature and B_0. After that, the field strength values at various temperatures can be calculated.

Conclusion:

This is the lower bound on the magnetic field strength in the hot Universe.

EFFECTIVE POTENTIAL at HIGH T

The spontaneous vacuum magnetization and zero magnetic mass for the Abelian magnetic fields were determined in lattice simulations [V. Demchik, V. Skalozub (2008)], [S. Antropov, M. Bordag, V. Demchik, V. Skalozub (2011)].

The value of $B(T)$ is close to that calculated with EP

$V(B,T) = V^{(1)}(B,T) + V^{ring}(B,T)$.

We present analytic results, considering the W-boson contributions as an example.

Consider two limits,

1. weak magnetic field and large scalar field condensate, $h = eB/M_w^2 < \phi^2$, $\phi = \phi_c/\phi_0$, $\beta = 1/T$,

2. the case of the restored symmetry, $\phi = 0$, $gB \neq 0$, $T \neq 0$.

For the former, we show the absence of spontaneous vacuum magnetization at finite temperature.

For the latter , we estimate $B(T)$. Here M_w is the W-boson mass at zero temperature, ϕ_c is a scalar field condensate, and ϕ_0 its value at zero temperature.

1. Contribution of W-bosons,

$$V_w^{(1)}(T,h,\phi) = \frac{h}{\pi^2\beta^2}\sum_{n=1}^{\infty}\Big[\frac{(\phi^2-h)^{1/2}\beta}{n}K_1(n\beta(\phi^2-h)^{1/2}) - \frac{(\phi^2+h)^{1/2}\beta}{n}K_1(n\beta(\phi^2+h)^{1/2})\Big]. \tag{12}$$

Here n labels discrete energy values and $K_1(z)$ is the MacDonald function.

The high temperature limit is the pure Yang-Mills part $(\tilde{B} \equiv B^{(3)})$,

$$V_w^{(1)}(\tilde{B},T) = \frac{\tilde{B}^2}{2} + \frac{11}{48}\frac{g^2}{\pi^2}\tilde{B}^2\log\frac{T^2}{\tau^2} - \frac{1}{3}\frac{(g\tilde{B})^{3/2}T}{\pi} - i\frac{(g\tilde{B})^{3/2}T}{2\pi} + O(g^2\tilde{B}^2), \tag{13}$$

where τ is a temperature normalization point.

2. The charged scalar contribution

$$V_{sc}^{(1)}(\tilde{B},T) = -\frac{1}{96}\frac{g^2}{\pi^2}\tilde{B}^2\log\frac{T^2}{\tau^2} + \frac{1}{12}\frac{(g\tilde{B})^{3/2}T}{\pi} + O(g^2\tilde{B}^2), \qquad (14)$$

describing the contribution of longitudinal vector components.

The imaginary part is generated because of the unstable mode in the spectrum (2). It is canceled by the term in daisy diagrams for the unstable mode

$$V_{unstable} = \frac{g\tilde{B}T}{2\pi}[\Pi(\tilde{B},T,n=-1) - g\tilde{B}]^{1/2} + i\frac{(g\tilde{B})^{3/2}T}{2\pi}. \qquad (15)$$

Here $\Pi(\tilde{B},T,n=-1)$ is the mean value in the ground state $n=-1$ of the spectrum (2). *If this value is large*, spectrum stabilization takes place.

In the review [V. Demchik, V. Skalozub arXiv:hep-th/9912071 (1999)] the complete EP is present. The mean value of one-loop PT in the spectrum ground state reads,

$$\Pi(\tilde{B}, T, n = -1) = \alpha[12.33 \frac{(g \sin\theta_w B)^{1/2}}{\beta} + i4 \frac{(g \sin\theta_w B)^{1/2}}{\beta}]. \qquad (16)$$

Here $\beta = 1/T$. Sufficiently large real part stabilizes the spectrum due to radiation corrections included.

MAGNETIC FIELD at T_{ew}

Spontaneous vacuum magnetization at $T \neq 0$ and non-small $\phi \neq 0$.

Notice, the magnetization is produced by the gauge field contribution given in Eq. (12). We consider the limit of $\frac{gB}{T^2} \ll 1$ and $\phi^2 > h$. We use the asymptotic expansion of $K_1(z)$,

$$K_1(z) \sim \sqrt{\frac{\pi}{2z}} e^{-z} \big(1 + \frac{3}{8z} - \frac{15}{128z^2} + \cdots\big), \qquad (17)$$

where $z = n\beta(\phi^2 \pm h)^{1/2}$.

Let us investigate the limit of $\beta \to \infty, \frac{T}{\phi} \ll 1$ and substitute $(\phi^2 \pm h)^{1/2} = \phi(1 \pm \frac{h}{2\phi^2})$.

The sum of the tree level energy and (12) reads

$$V = \frac{h^2}{2} - \frac{h^2}{\pi^{3/2}} \frac{T^{1/2}}{\phi^{1/2}} \left(1 - \frac{T}{2\phi}\right) e^{-\frac{\phi}{T}}. \tag{18}$$

The second term is exponentially small and the stationary equation $\frac{\partial V}{\partial h} = 0$ has the trivial solution $h = 0$.

We conclude:

after symmetry breaking the spontaneous vacuum magnetization does not take place.

At the EWPT temperature the total EP must be used. This can be best done numerically.

To explain the procedure, we consider the part of this EP accounting for the one-loop W-boson contributions.

The high temperature expansion for the EP coming from charged vector fields is given in (13).

The value of chromomagnetic weak isospin field coming from (13) and (14) is

$$\tilde{B}(T) = \frac{1}{16}\frac{g^3}{\pi^2}\frac{T^2}{(1+\frac{5}{12}\frac{g^2}{\pi^2}log\frac{T}{\tau})^2}. \qquad (19)$$

We relate this expression with the intergalactic magnetic field B_0.

Let us introduce the notations, $\frac{g^2}{4\pi} = \alpha_s, \alpha = \alpha_s \sin\theta_w^2, \frac{(g')^2}{4\pi} = \alpha_Y$ and $\tan^2\theta_w(T) = \frac{\alpha_Y(T)}{\alpha_s(T)}$, where α is the fine structure constant.

For a rough estimate, we substitute: $\sin^2\theta_w(T) = \sin^2\theta_w(0) = 0.23$.

For the given temperature EWPT, T_{ew}, the field strength is

$$B(T_{ew}) = B_0 \frac{T_{ew}^2}{T_0^2} = \sin\theta_w(T_{ew})\tilde{B}(T_{ew}). \tag{20}$$

Assuming $T_{ew} = 100GeV = 10^{11}eV$ and $T_0 = 2.7K = 2.3267 \cdot 10^{-4}eV$, we obtain

$$B(T_{ew}) \sim 1.85\ 10^{14}G. \tag{21}$$

This value can serve as a lower bound on $B(T)$ at the EWPT. Hence, for the value of $X = \log \frac{T_{ew}}{\tau}$, we have the equation

$$B_0 = \frac{1}{2}\frac{\alpha^{3/2}}{\pi^{1/2}\sin^2\theta_w}\frac{T_0^2}{(1+\frac{5\alpha}{3\pi\sin^2\theta_w}X)^2}, \tag{22}$$

and $\log\tau$ can be estimated.

To guess the value of τ we take $B_0 \sim 10^{-9}G$, usually used in cosmology. From Eq. (22) we obtain $\tau \sim 300eV$.

For the lower bound value $B \sim 10^{-15}G$ this parameter is much smaller.

The strong suppression of $B(T)$ **is difficult to explain within the SM!**

Let us compare the above results with the field strength $B(T_{ew})$ calculate directly in the Standard model [V. Demchik, V. Skalozub (2002)]:

$$B(T_{ew}) \sim 10^{20} G. \tag{23}$$

This is 10^6 times larger than in Eq.(21), and corresponds to the comoving magnetic field 10^{-9} G.

This huge discrepancy requires relevant explanations!

MAGNETIC FIELD SCALE

Consider the scale of the field generated in the restored phase.

We used the "frozen in" conditioins. Let us discuss its applicability.

If one assumes that after the EWPT the field $B(T_{ew})$ was frozen in the plasma at the Hubble scale, $R_H(T_{ew})$, then its comoving coherence scale at present will be $\lambda_B(T_0) = 6 \cdot 10^{-4}$ **pc.**

This is much smaller than is needed.

One possible scenario is based on the stochastic processes considered already by [Hogan (1983)].

It was pointed out that magnetic fields correlated on large scales can be produced not only through causal processes but also by a stochastic random walk mechanism, if the magnetic lines generated in some domain of space "forget" about their origin.

The field strength developed on large scales by this process

can be estimated as $B_N \sim B/\sqrt{N}$, where N counts the number of domains, with the field B of a given size, crossed by a magnetic line. The correlation length λ_B in this case can be estimated as $\lambda_B(T) \sim N R_H(T)$.

[If spontaneous vacuum magnetization occurs.]

At a given temperature, each uncorrelated domain of space having a Hubble radius $R_H(T)$ is filled up with a constant magnetic field $B(T)$.

Its orientation in both external and internal spaces is arbitrary. Hence, a stochastic behavior of the field lines and the appearance of magnetic fields having large correlation lengths $\lambda_B(T) \geq R_H(T)$ are expected.

After the EWPT, these fields evolve as $B \sim T^2$.

[Let us check this possibility].

Make use the usual relation

$$\frac{a(T_{ew})}{a(T_0)} = \frac{T_0}{T_{ew}}, \tag{24}$$

taken at the EWPT epoch, and the present-day parameters,

$T_{ew} = 100\ GeV = 10^{11}\ eV, T_0 = 2.3267 \cdot 10^{-4}$ **eV.**

If $\lambda_B(T) \sim a(T)$, then from (24) it follows: $\lambda_B(T_0) = 6 \cdot 10^{-4}$ **ps** .

On the other hand, if one takes $\lambda_B(T_0) = 1$ **Mpc,**

the value $\lambda_B(T_{ew}) = 2.33 \cdot 10^{-15}$ **Mpc** is obtained.

The horizon size is $a(T_{ew}) = 1.27 \cdot 10^{-24}$ **Mpc,**

thus, $\lambda_B(T_{ew}) >> a(T_{ew})$.

Such long-range magnetic fields are not affected by turbulence.

Now, we relate the size of the correlated field with the random walk process.
At T_{ew}, $\lambda_B(T_{ew}) = Na(T_{ew})$. Hence, we get $\sqrt{N} = 3 \cdot 10^4$**,**
and for $B(T)$ "straightened" on the N-domain scale, $B_N \sim \frac{B(T_{ew})}{\sqrt{N}}$
Accounting for $B(T_{ew}) \sim 10^{20} G$, we obtain $B_{ls}(T_{ew}) \sim 3 \cdot 10^{15}$ **G**
This value is close to $B_{ls}(T_{ew}) \sim 2 \cdot 10^{14}$ **G.**
The discrepancy can be explained:
1) due to the roughness of our estimate.
2) by the necessity of substituting the standard model.

CONCLUSION

- At the T_{ew}, long range magnetic fields of the order $B(T_{ew}) \sim 10^{14}G$ did exist. They are not affected by turbulence processes.

- Vacuum polarization is responsible for the value of $B(T)$ at each temperature and serves as a source of it.

- After symmetry breaking, ϕ-condensate suppresses the magnetization and usual dependence $B \sim T^2$ recovers.

- Due to stability and zero magnetic mass of the fields, there are no problem with creating of long-range magnetic fields at high temperature.

- **Intergalactic magnetic field has been generated at reheating stage of the universe evolution**.

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