



Wave Processes Laboratory
Faculty of Mechanics & Mathematics
Moscow M. V. Lomonosov State University

**NON-LINEAR WAVES' EVOLUTION IN
META-STABLE MEDIA**

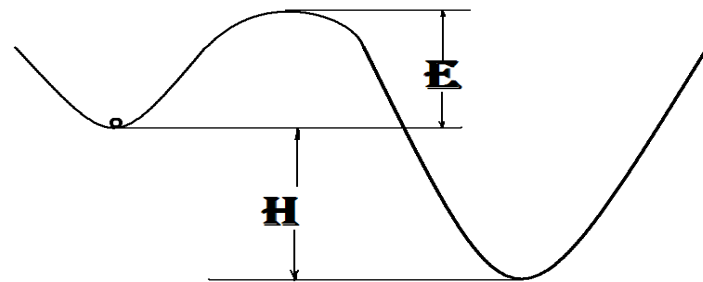
N.N. Smirnov



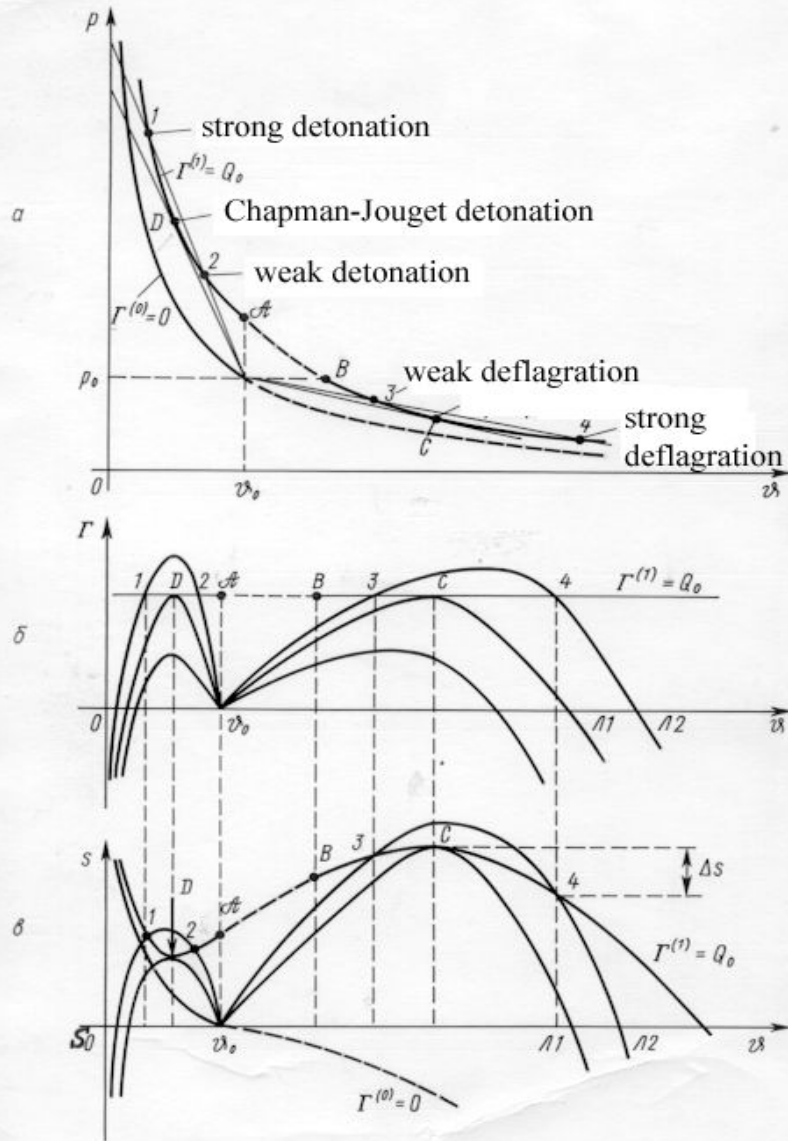
Ginzburg Conference on Physics, 2012

Self-sustaining waves in metastable media

- Energy needed to support such waves is released by the wave itself :
- waves of combustion,
- waves of boiling in overheated liquids,
- waves of thermonuclear fusion.



Two modes of combustion



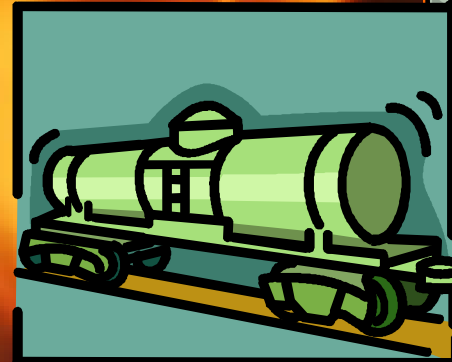
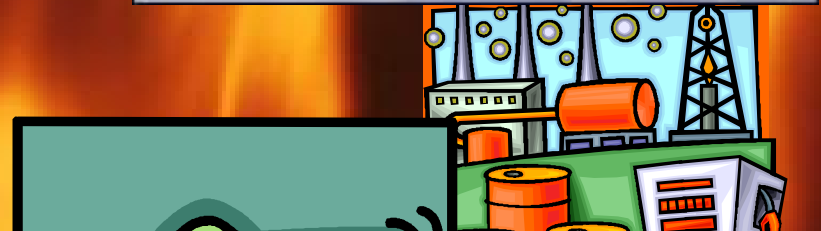
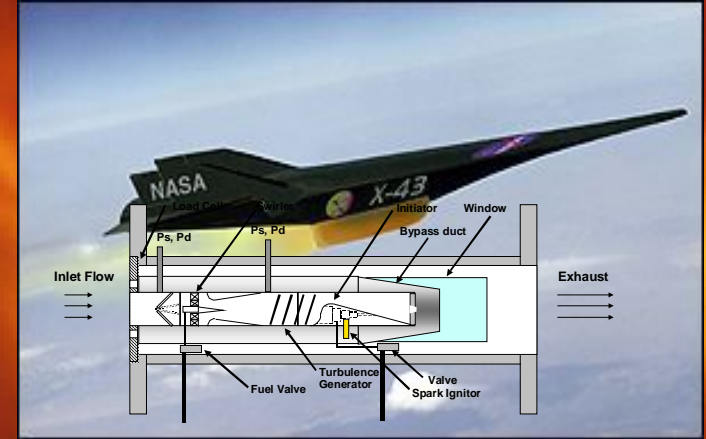
Advantages of detonation mode

- High thermodynamic efficiency of Chapman-Jouget detonation as compared to other combustion modes is due to the minimal entropy of the exhaust jet.
- CO emission reduction.
- High rate of energy conversion (10^3 times)
- Specific impulse increase

$$\frac{I_{PDE}}{I_{RAM}} \approx \sqrt{\gamma}$$

Practical importance of controlling detonation initiation

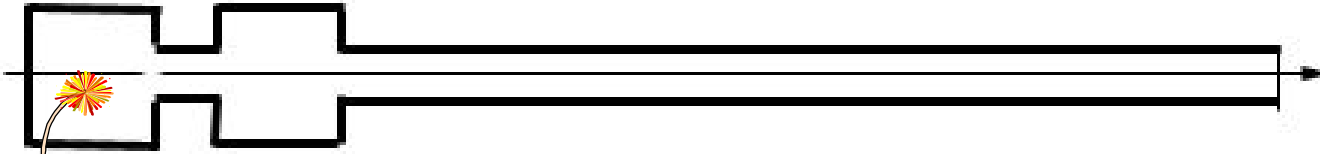
- working out effective preventive measures, such as suppressing deflagration to detonation transition (DDT) in case of combustible mixture ignition,
- the advantages of burning fuel in a detonation regime in comparison with slow burning at constant pressure attract increasing attention to pulse detonation engines.



Outline

- **Deflagration to detonation transition scenarios.**
- **Macro-kinetics mathematical model for DDT.**
- **Several new combustion modes in tubes incorporating cavities.**
- **The effect of volume ratio parameters on combustion modes.**
- **The effect of non-uniformity in cavities size distribution.**

Flame Acceleration Scenarios in Gases.



Macroscopical Kinetics for Modeling of DDT

- **Equations for multicomponent reacting compressible viscous flows.**
- **Three-equation turbulence model.**
- **Reduced chemistry modeling.**
- **Initial and boundary conditions.**
- **Confinement geometry: tubes with wider cavities.**

Governing equations for reacting turbulent flows

$$\partial_t(\rho) + \nabla \cdot (\rho \vec{u}) = 0$$

$$\partial_t(\rho Y_k) + \nabla \cdot (\rho \vec{u} Y_k) = -\nabla \cdot \vec{I}_k + \dot{\omega}_k$$

$$\partial_t(\rho \vec{u}) + \nabla \cdot (\rho \vec{u} \otimes \vec{u}) = \rho \vec{g} - \nabla p + \nabla \cdot \tau$$

$$\partial_t(\rho E) + \nabla \cdot (\rho \vec{u} E) = \rho \vec{u} \cdot \vec{g} - \nabla \cdot p \vec{u} - \nabla \cdot \vec{I}_q + \nabla \cdot (\tau \cdot \vec{u})$$

$$p = R_g \rho T \sum_k Y_k / W_k \quad E = \sum_k Y_k (c_{vk} T + h_{0k}) + \frac{\vec{u}^2}{2} + k$$

$$\vec{I}_q = \vec{J}_q + \sum_k (c_{pk} T + h_{0k}) \vec{I}_k \quad \vec{I}_k = -\rho (D + (v^t / \sigma_d)) \nabla \cdot Y_k \quad \vec{J}_q = -(\lambda + \sum_k c_{pk} Y_k \rho (v^t / \sigma_t)) \nabla \cdot T$$

$$\tau = (\mu + \rho v^t) (\nabla \vec{u} + \nabla \vec{u}^T - (2/3) (\nabla \cdot \vec{u}) U) - (2/3) \rho k U$$

$$v^t = C_\mu k^2 / \varepsilon.$$

Turbulence model.

$$\partial_t(\rho k) + \nabla \cdot (\rho \bar{u} k) = \nabla \cdot ((\mu + \rho(v^t / \sigma_k)) \nabla k) + \tau^t : \nabla \bar{u} - \rho \varepsilon$$

$$\partial_t(\rho \varepsilon) + \nabla \cdot (\rho \bar{u} \varepsilon) = \nabla \cdot ((\mu + \rho(v^t / \sigma_\varepsilon)) \nabla \varepsilon) + (\varepsilon / k)(C_{1\varepsilon} \tau^t : \nabla \bar{u} - C_{2\varepsilon} \rho \varepsilon)$$

$$\partial_t(\rho \tilde{c}_p \theta) + \nabla \cdot (\rho \bar{u} \tilde{c}_p \theta) = \nabla \cdot ((\lambda + \sum_k c_{pk} Y_k \rho(v^t / \sigma_k)) \nabla \theta) + P_\theta + W_\theta - D_\theta,$$

$$\theta = \overline{T'T'}, \quad T = \bar{T} + T'$$

$$P_\theta = 2\rho \sum_k c_{pk} Y_k (v^t / \sigma_k) (\nabla T)^2 \quad W_\theta = -\sum_k \overline{\dot{\omega}'_k T' h_{0k}}$$

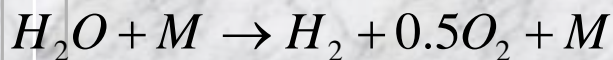
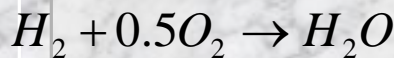
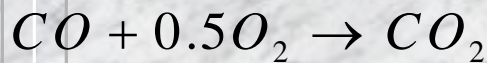
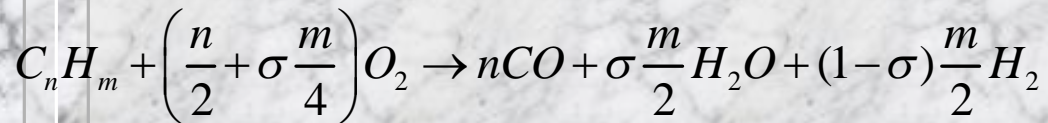
$$D_\theta = C_g \rho \sum_k c_{pk} Y_k \frac{\varepsilon}{k} \frac{\theta}{\theta_m - \theta}, \quad \tilde{c}_p = \sum_k c_{pk} Y_k$$

Model parameters:

$$k, \quad \varepsilon, \quad \theta \quad C_\mu = 0.09, \quad C_{1\varepsilon} = 1.45, \quad C_{2\varepsilon} = 1.92, \quad \theta_m = \bar{T}^2 / 4, \quad C_g = 2.8.$$

$$\sigma_d = 1, \quad \sigma_t = 0.9, \quad \sigma_k = 1, \quad \sigma_\varepsilon = 1.3.$$

Chemistry modeling.



$$\dot{\omega}_k = \sum_{j=1}^B \omega_{kj}$$

$$W_\theta = -T' \sum_{j=1}^B \sum_{k=1}^K h_k^0 \dot{\omega}_{kj}$$

$$A_j(T) = \begin{cases} K_j \exp\left(-\frac{T_{aj}}{T}\right), & T \geq T_{mj} \\ 0, & T < T_{mj} \end{cases}$$

$$\overline{T' A(T)} = \theta \frac{A(\bar{T} + \sqrt{3\theta}) - A(\bar{T} - \sqrt{3\theta})}{2\sqrt{3\theta}}$$

$$\overline{A(T)} = \frac{1}{6} A(\bar{T} + \sqrt{3\theta}) + \frac{2}{6} A(\bar{T}) + \frac{1}{6} A(\bar{T} - \sqrt{3\theta}).$$

Boundary conditions.

Mean flow parameters

$$\begin{aligned}
 &x = 0, \\
 &x = x_i, r_i \leq r \leq R_i: \quad u_x = u_r = 0, \frac{\partial T}{\partial x} = 0, \frac{\partial Y_k}{\partial x} = 0 \quad r = R_i, x_i \leq x \leq x_{i+1} \quad i = 1, \dots, N-1, : \quad u_x = u_r = 0, \frac{\partial T}{\partial r} = 0, \frac{\partial Y_k}{\partial r} = 0 \\
 &x = X, \quad r = r_i, x_{i-1} \leq x \leq x_i \\
 &r = 0, 0 \leq x \leq X: \quad u_r = 0, \frac{\partial u_x}{\partial r} = 0, \frac{\partial T}{\partial r} = 0, \frac{\partial Y_k}{\partial r} = 0
 \end{aligned}$$

Turbulent parameters k, ε, θ

$$k = 0, \quad \frac{\partial \varepsilon}{\partial \vec{n}} = 0, \quad \frac{\partial \theta}{\partial \vec{n}} = 0,$$

Lam-Bremhorst damping

$$f_\mu = [1 - \exp(-0.0165R_y)]^2 \left(1 + \frac{20.5}{R_t} \right),$$

$$C_\mu = C_\mu^0 f_\mu,$$

$$C_{1\varepsilon} = C_{1\varepsilon}^0 f_1,$$

$$C_{2\varepsilon} = C_{2\varepsilon}^0 f_2,$$

$$f_1 = 1 + \left(\frac{0.05}{f_\mu} \right)^3,$$

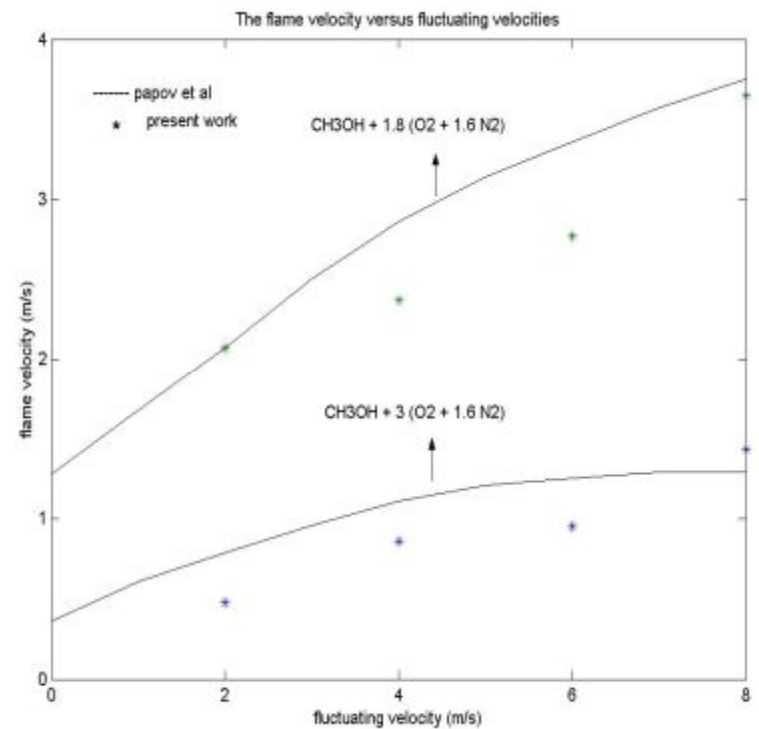
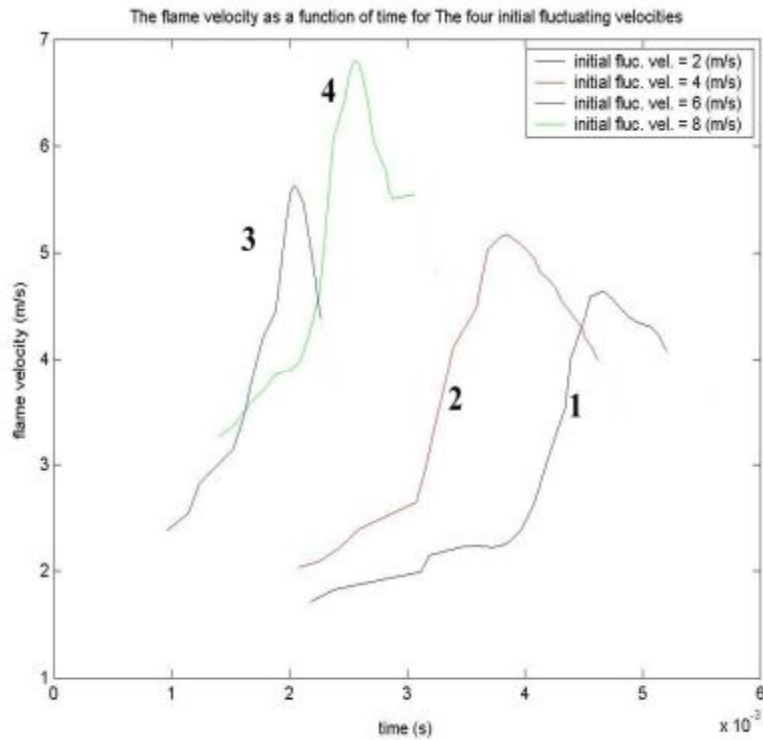
$$R_t = \frac{k^2}{\nu \varepsilon}, \quad R_y = \sqrt{k} \frac{y}{\nu},$$

$$f_2 = 1 - \exp(-R_t^2).$$

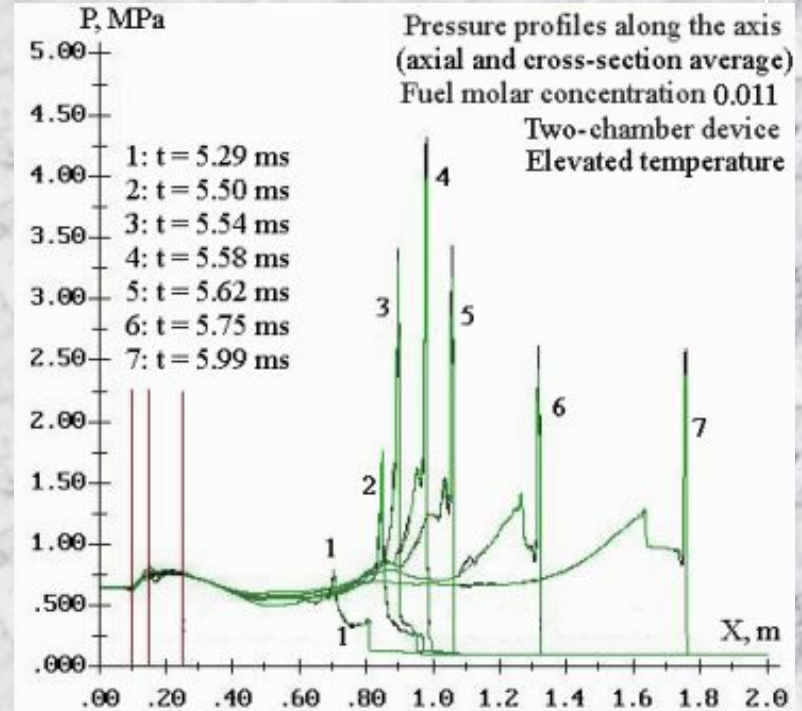
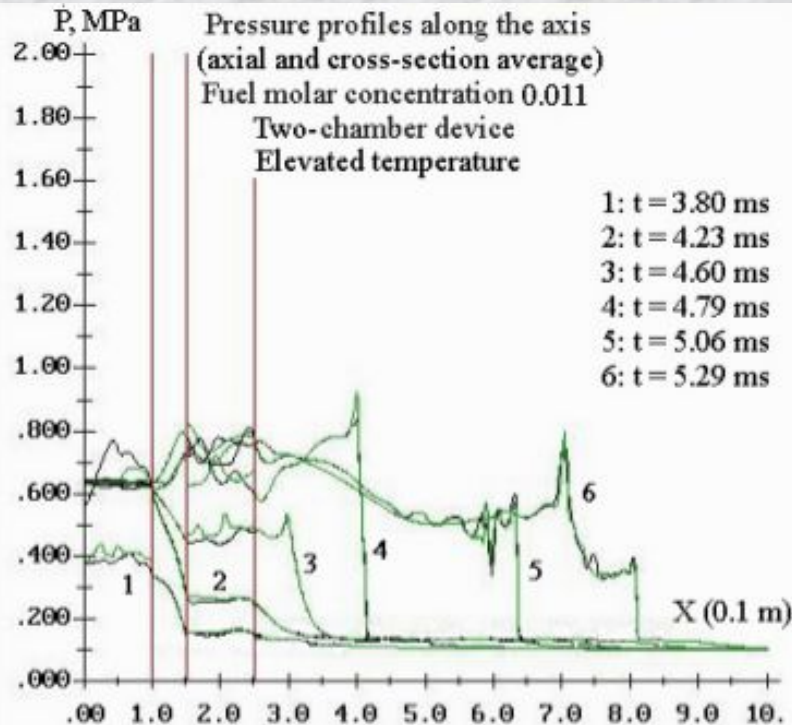
Turbulent combustion model validation

- A problem was regarded of flame propagation in a tube of constant cross-section filled in with $CH_3OH + 1.5\alpha(O_2 + 1.6N_2)$
- The kinetic mechanism was based on the one suggested by Marinov N.M. (1999) and incorporated 129 elementary stages.
- Results were compared with experiments by Karpov V.P. et al.(1986)

Turbulent combustion model validation II



Pressure profiles along the axis

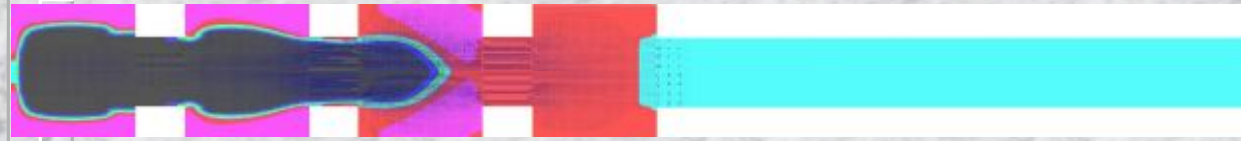


Onset of detonation via
an overdriven regime

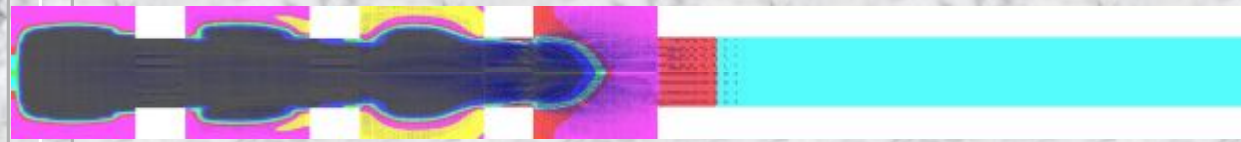
Deflagration to detonation transition simulation.



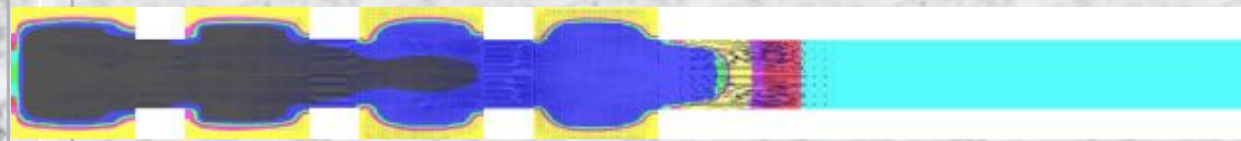
t = 0.42



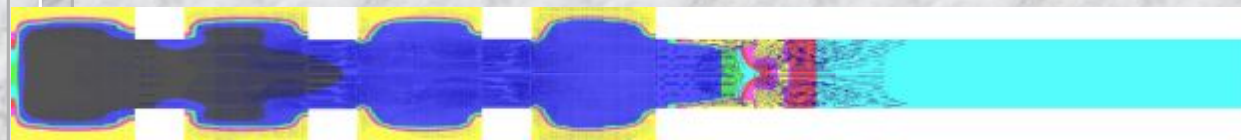
t = 0.82



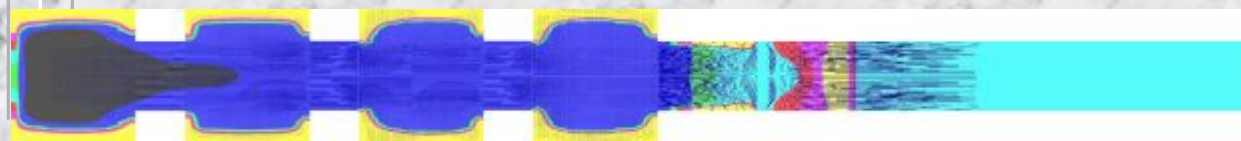
t = 1.01



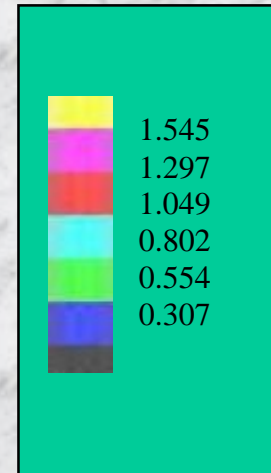
t = 1.25



t = 1.29

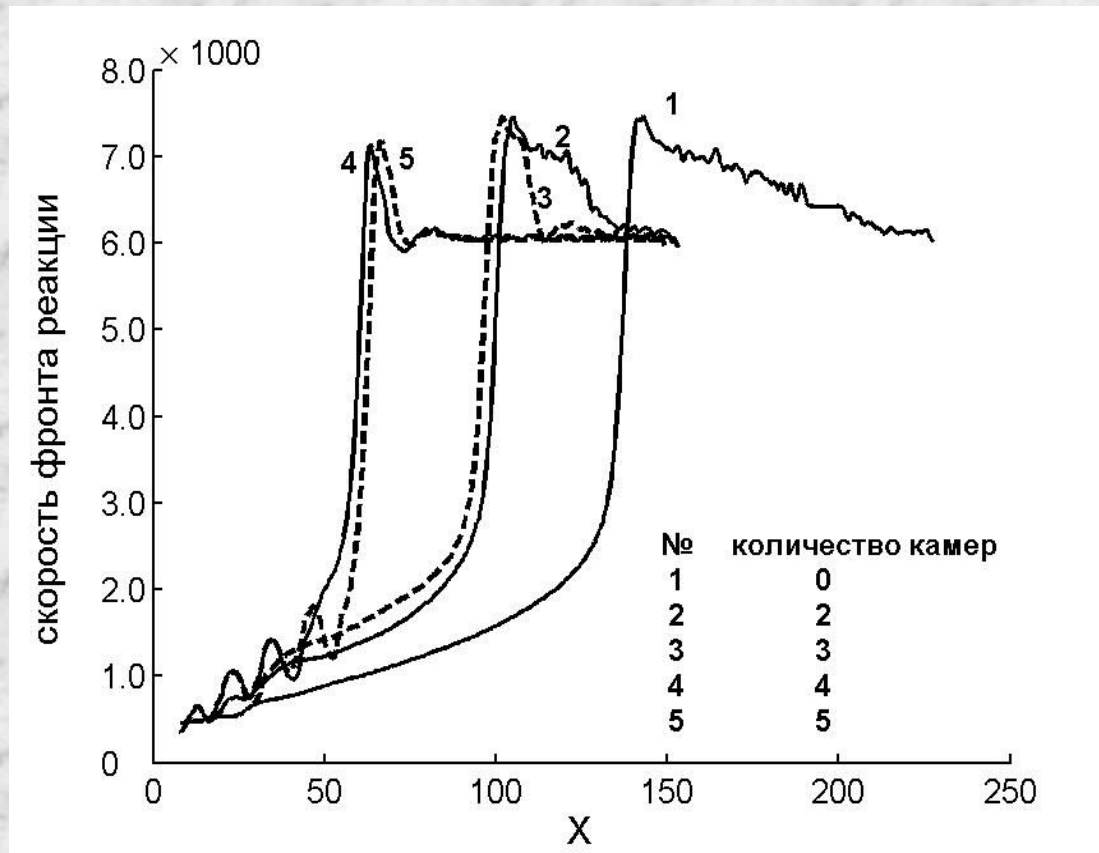


t = 1.33

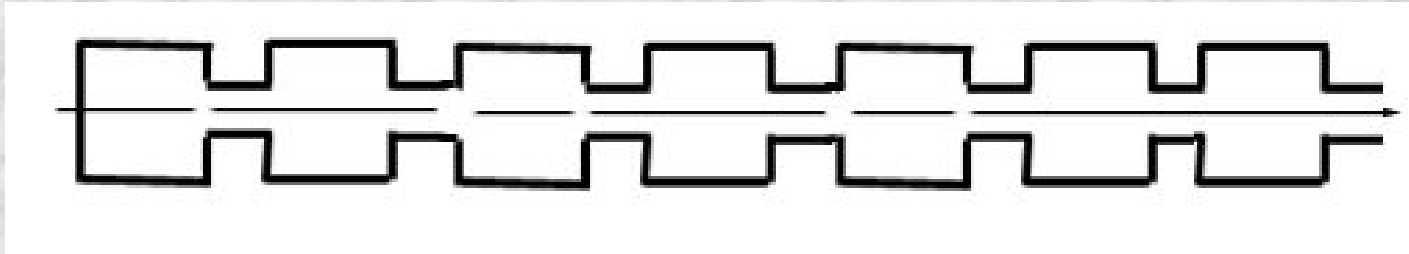


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Reaction front velocity

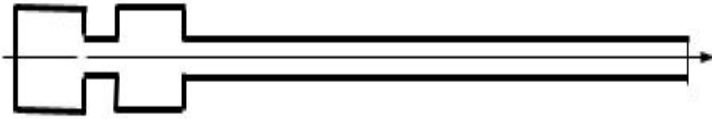


Studying the geometrical effect on the onset of detonation



$$\beta_{ER} = \frac{S_{chamb} - S_{tube}}{S_{chamb}} ; \alpha_{ER} = \frac{S_{chamb} L_{chamb} + S_{tube} L_{tube}}{S_{chamb} (L_{chamb} + L_{tube})} ; \alpha_{ER} = 1 - \frac{\beta_{ER}}{1 + A} ; A = \frac{L_{chamb}}{L_{tube}}$$

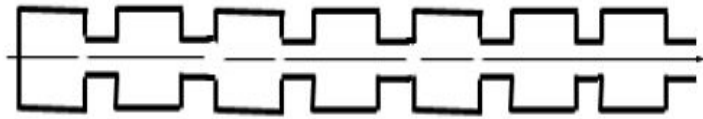
Different geometries



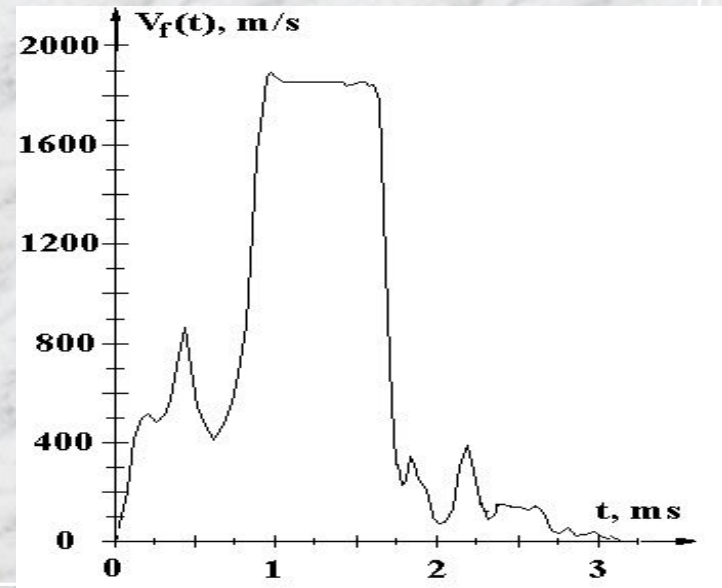
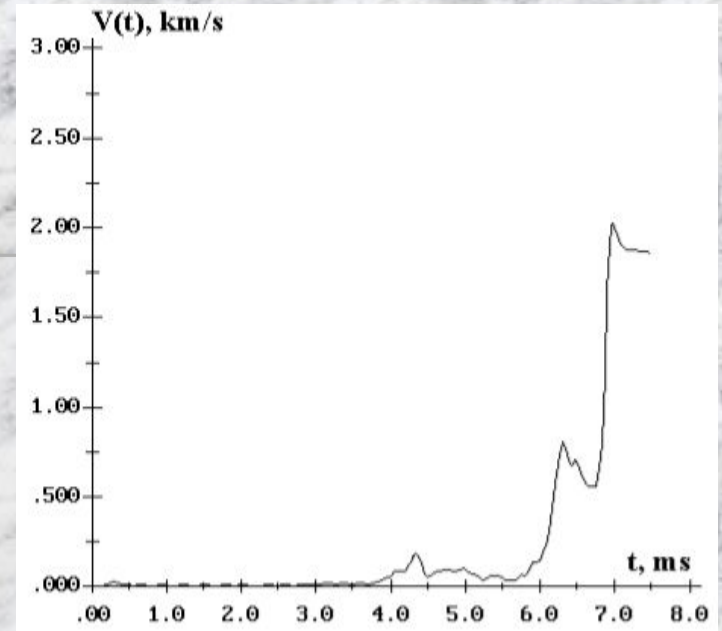
a) two chambers in the ignition section;



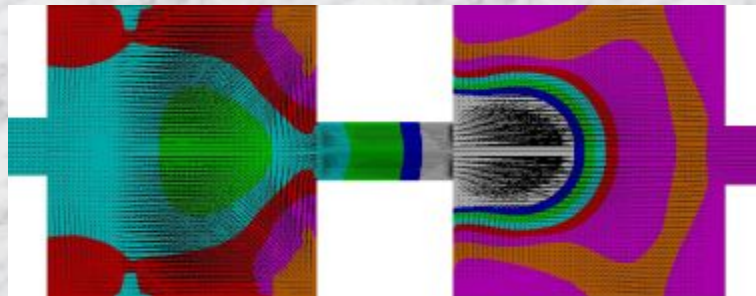
b) two chambers in the far end section of the tube;



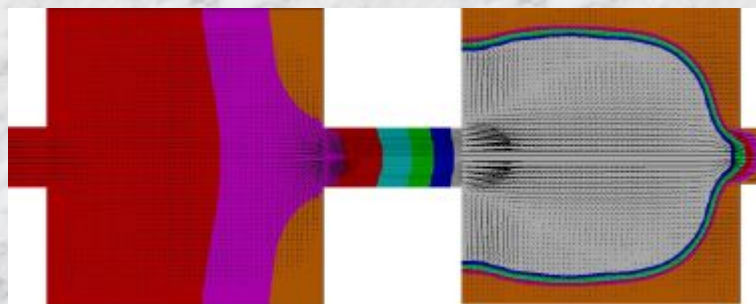
c) chambers incorporated into the tube along the whole length.



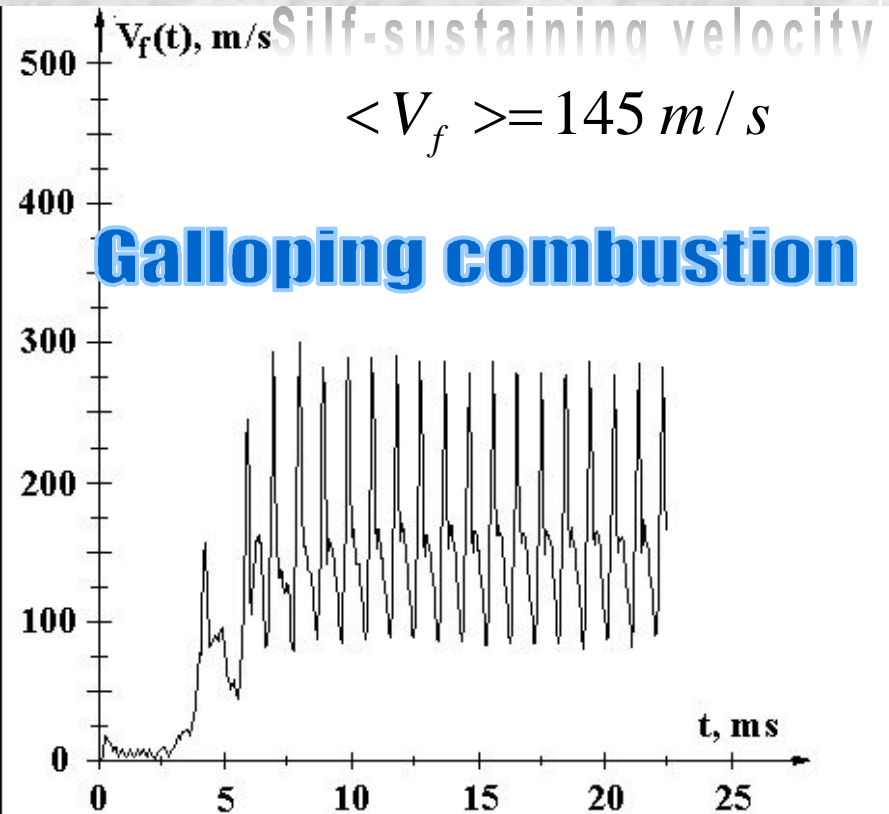
The influence of cavities incorporated into the tube along the whole length



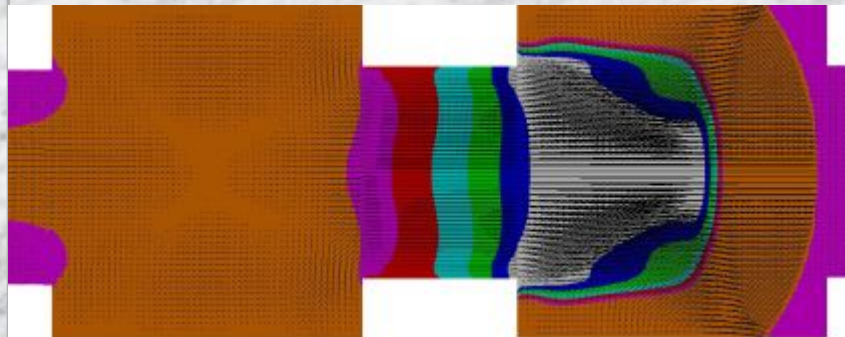
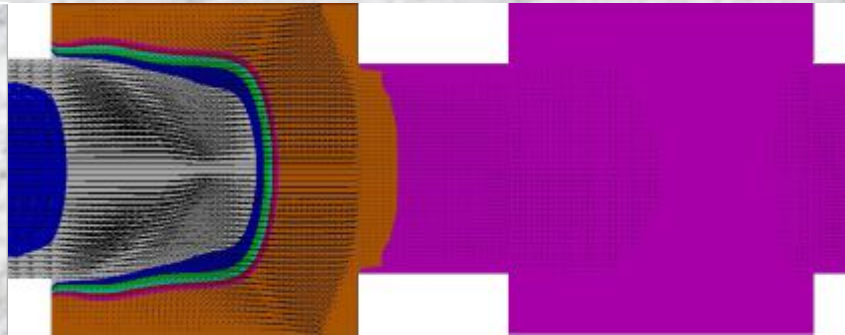
$$A = \frac{L_{chamb}}{L_{tube}}$$



$$\beta_{ER} = \frac{S_{chamb} - S_{tube}}{S_{chamb}} ; \quad \alpha_{ER} = \frac{S_{chamb} L_{chamb} + S_{tube} L_{tube}}{S_{chamb} (L_{chamb} + L_{tube})} \quad \beta_{ER} = 0.96, \quad \alpha_{ER} = 1 - \frac{\beta_{ER}}{1 + A}$$

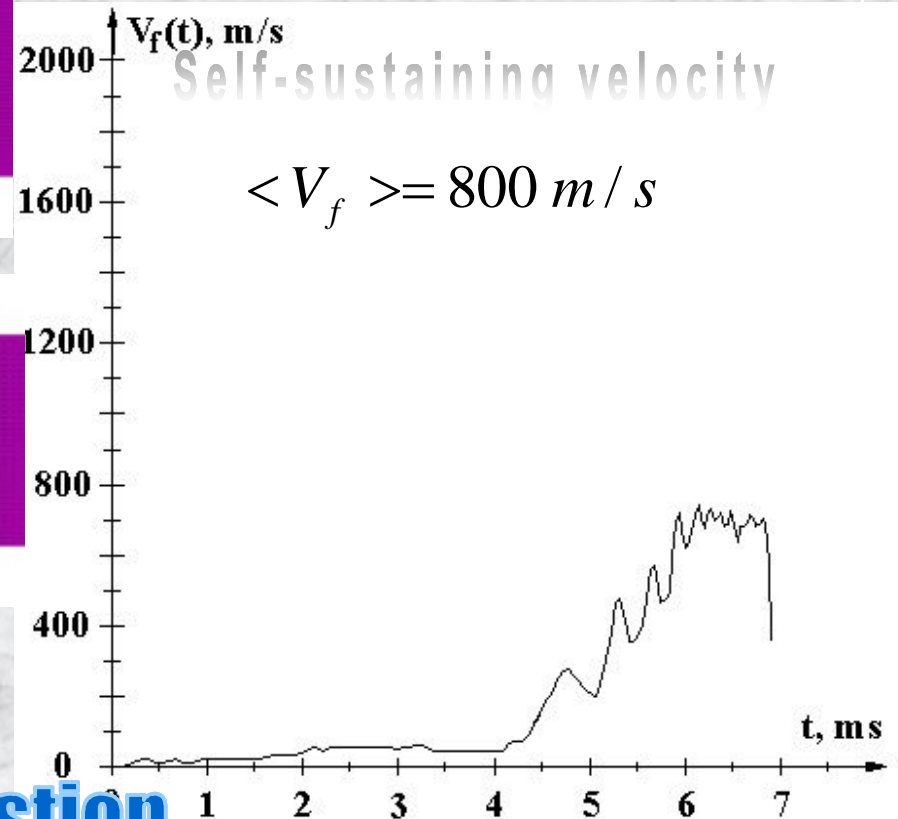


The effect of expansion ratio



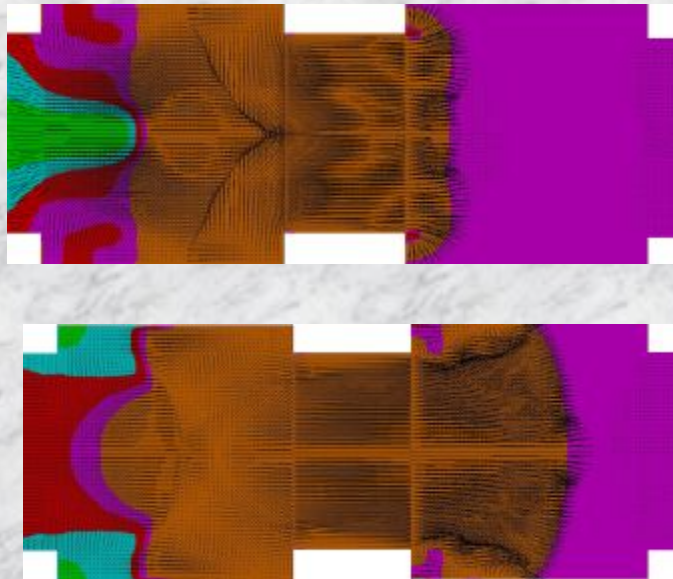
$$\beta_{ER} = 0.60$$

**High speed
galloping combustion**



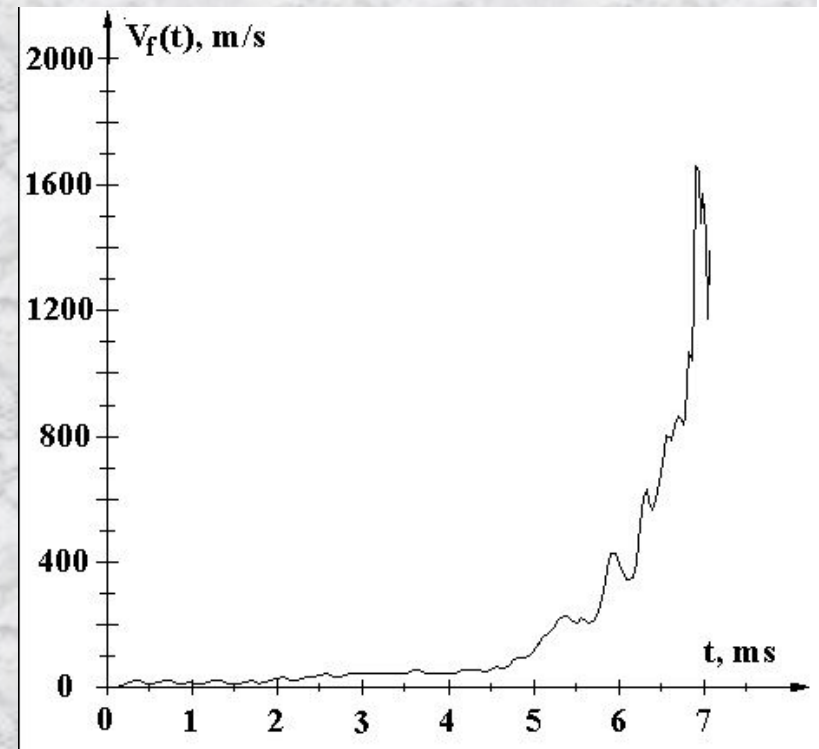
The effect of expansion ratio

$$\beta_{ER} = 0.40$$



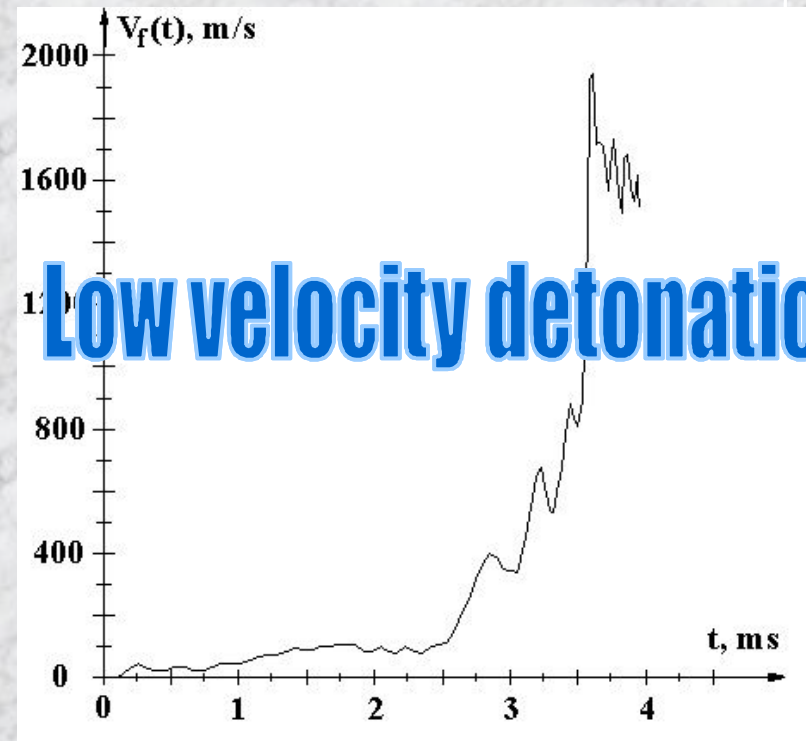
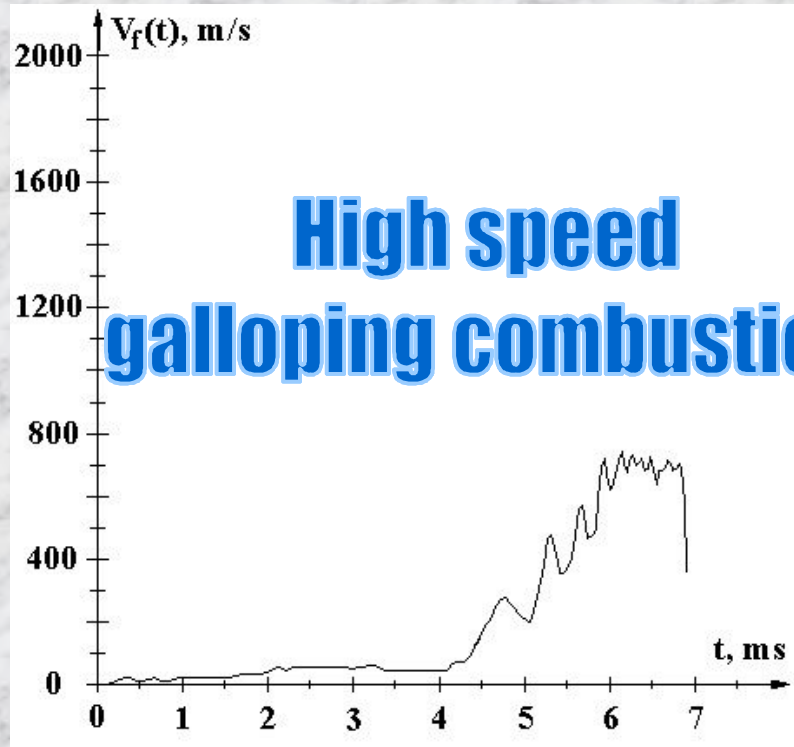
Self-sustaining velocity

$$\langle V_f \rangle = 1450 \text{ m/s}, \quad V_{\max} = 1700 \text{ m/s}$$



Low velocity detonation

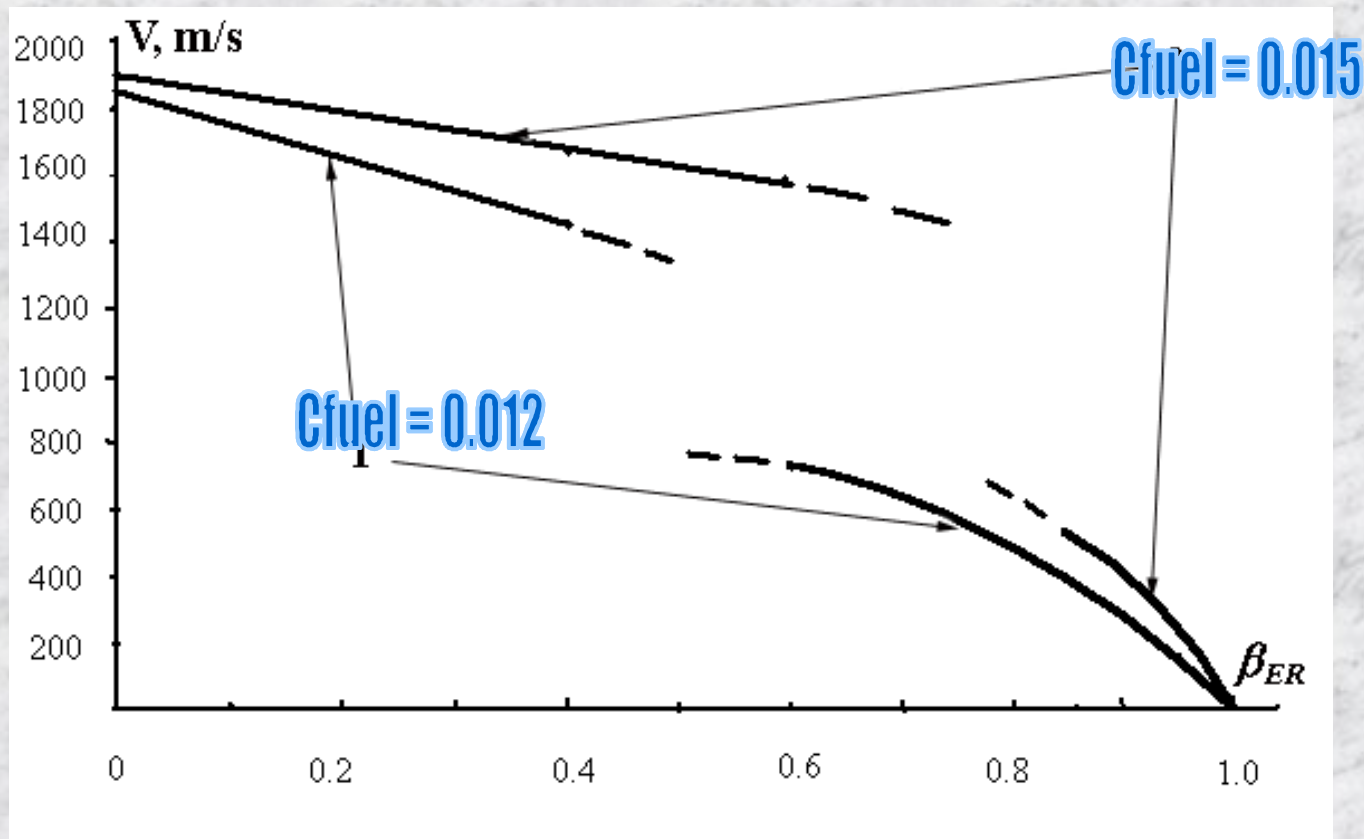
The joint effect of expansion ratio and fuel concentration.



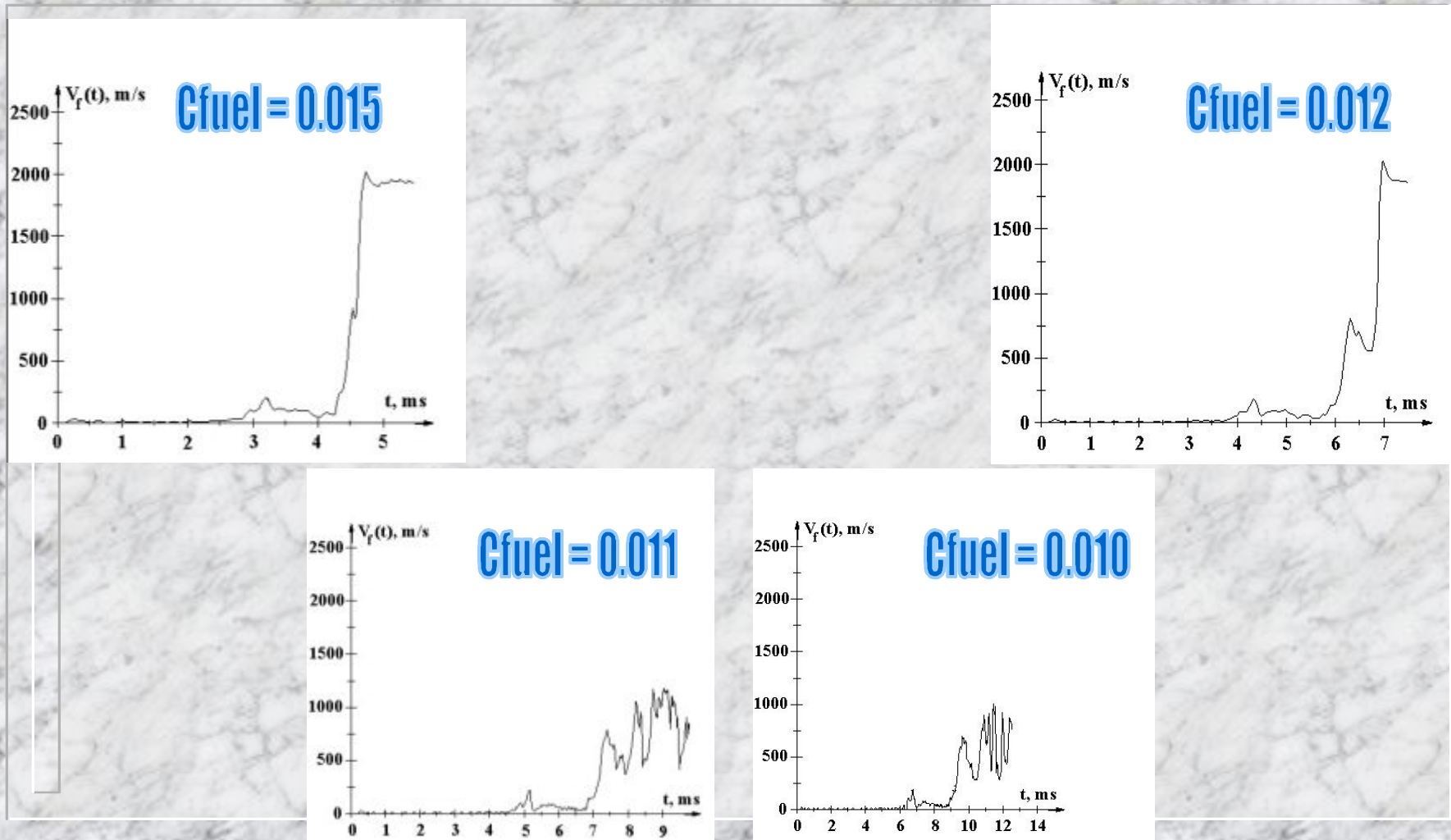
$$\beta_{ER} = 0.60, \quad C_{\text{fuel}} = 0.012$$

$$\beta_{ER} = 0.60, \quad C_{\text{fuel}} = 0.015$$

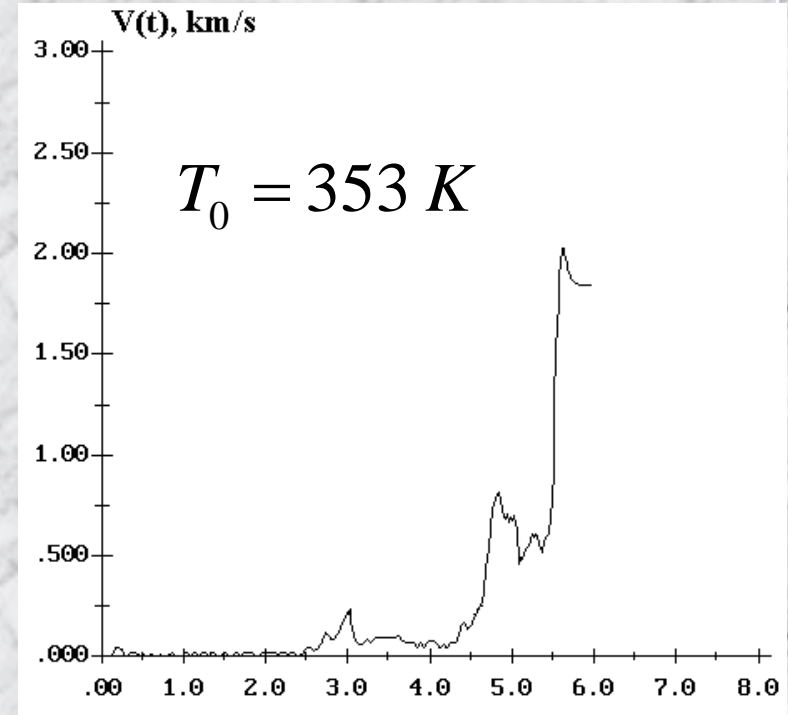
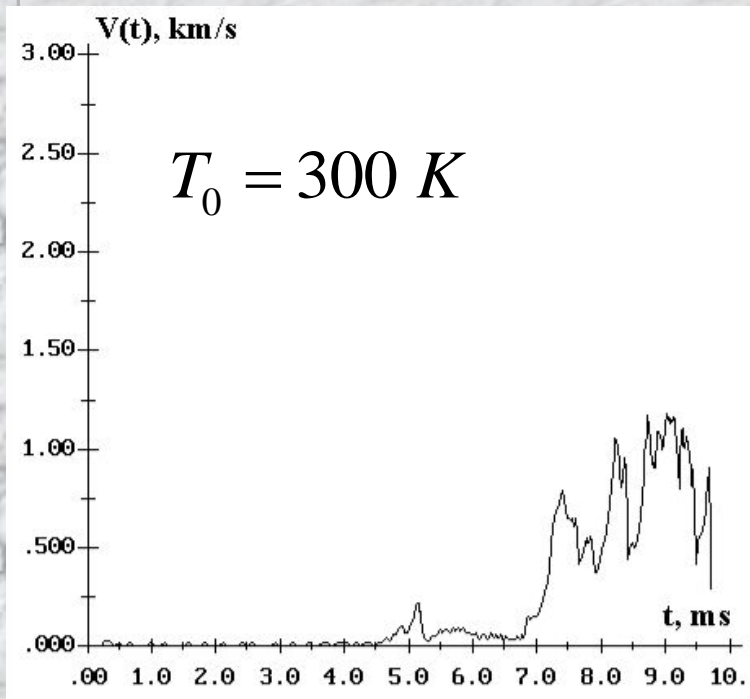
Self-sustaining velocity versus expansion ratio



The effect of fuel concentration (two-chambers in ignition section)



The influence of initial gas temperature on the DDT in tubes with 2 fore-chambers

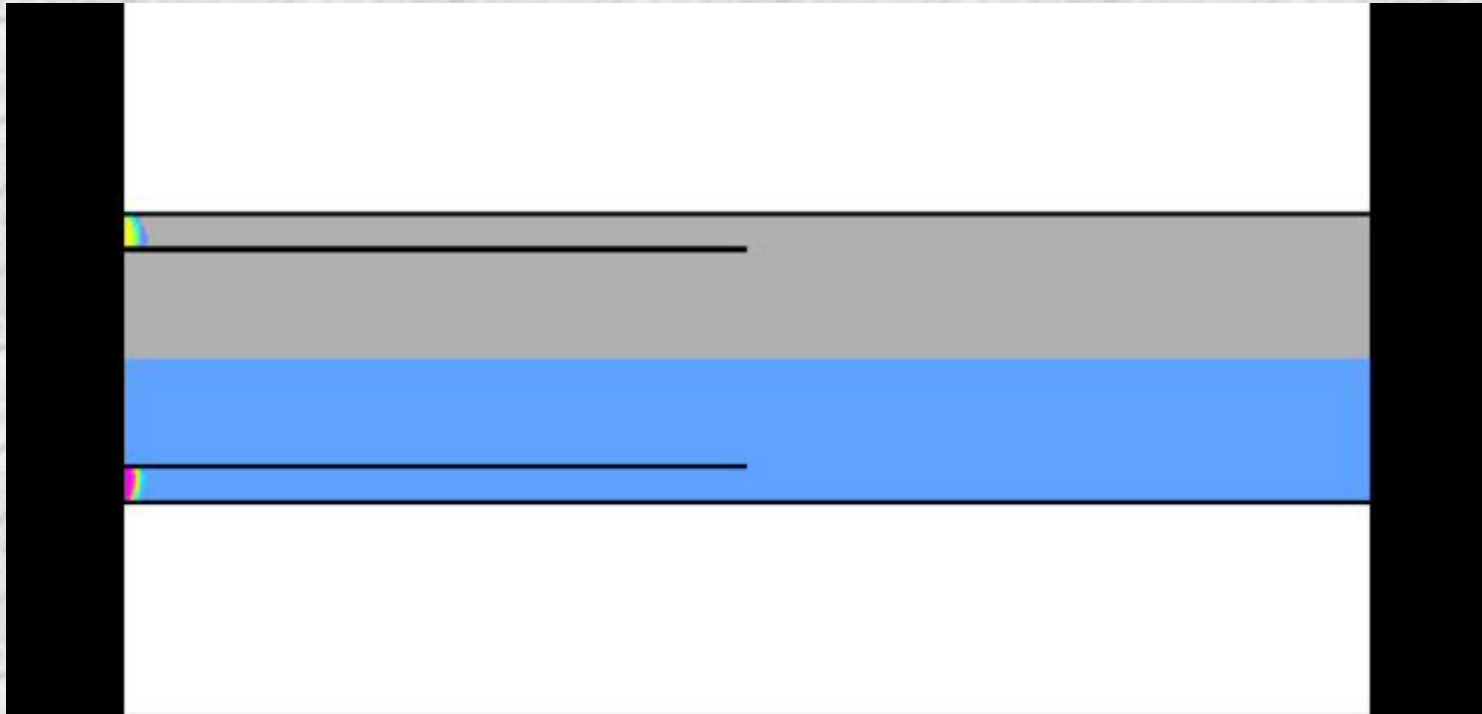


Galloping combustion

Detonation onset

$$C_{fuel} = 0.011; \quad \beta_{ER} = 0.96$$

Combustion in detonation engine



Detonation wave transmission from thin gap into a cylinder. Upper part – pressure maps, lower part – temperature maps

Conclusions

- **Transition processes between two modes of self-sustaining waves propagation in meta-stable media are essentially controlled by:**
 - **geometry of the channel,**
 - **fuel concentration,**
 - **mixture temperature.**