Covariant actions for models with non-linear twisted self-duality

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Motivation

- Duality symmetry plays important role in many physical models of physical interest
- N=8 supergravity is invariant under $E_{7(7)}$ *(Cremmer & Julia '79)*

$$\mathsf{F}^{i}_{\mu\nu} = (\underset{\substack{\uparrow \\ \text{electric}}}{F^{A}_{\mu\nu}}, \underset{\substack{\uparrow \\ \text{magnetic}}}{G^{\bar{A}}_{\mu\nu}}) \quad i = 1,\dots,56 \ \text{ of } \ E_{7(7)} \ \text{ and } \ A, \bar{A} = 1,\dots,28 \ \text{ of } \ SU(8)$$

On-shell linear (twisted self-) duality:

$$G^{\bar{A}}_{\mu\nu} = \tfrac{1}{2}\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma A} \quad \Rightarrow \quad \mathsf{F}^{-i}_{\mu\nu} \equiv F^{i}_{\mu\nu} - \tfrac{1}{2}\Omega^{i}{}_{j}\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma j} = 0, \quad \Omega^{i}{}_{k}\Omega^{k}{}_{j} = -\delta^{i}_{j}$$

- N=8 supergravity is perturbatively finite at 3 and 4 loops *(Bern et. al.)*
- Assumption: SUSY + $E_{7(7)}$ may be in charge of the absence of divergences *(Kallosh)*
- $E_{7(7)}$-invariant counterterms can appear at 7 loops $\partial^{2k}F^4, \partial^{2k}R^4$

Motivation

- higher-order counterterms $\partial^{2k} F^4$ in the effective action will lead to a non-linear deformation of the twisted self-duality condition

$$L = \tfrac{1}{4} \mathrm{F}^2 + \partial^{2k} \mathrm{F}^4 + \cdots,$$

$$\tilde{G} = 2 \frac{\delta L(\mathrm{F})}{\delta \mathrm{F}} \quad \Rightarrow \quad \mathrm{F}_-{}^i = \frac{\delta \mathrm{I}\,(\mathrm{F}\,)}{\delta \mathrm{F}_+{}^i} \neq 0, \qquad \mathrm{F}_+{}^i = \mathrm{F}^{\,i} + \Omega^i{}_{\mathrm{j}} \tilde{\mathrm{F}}^{\,i}, \quad \tilde{G}_{\mu\nu} \equiv \tfrac{1}{2} \varepsilon_{\mu\nu\rho\lambda} G^{\rho\lambda}$$

$$\mathrm{F}^{\,i} = (\mathrm{F}^{\mathrm{A}}, G^{\bar{\mathrm{A}}})$$

$\mathrm{I}(\mathrm{F})$ - duality-invariant counterterm

- Questions to answer:
  - how, exactly, possible higher-order terms may deform the effective action and duality relation between ‘electric’ and ‘magnetic’ fields, while keeping duality symmetry?
  - check whether this deformation is compatible with supersymmetry

- in this talk we shall mainly concern with the first problem
- brief comments on supersymmetry in conclusion

Two ways of dealing with duality-symmetric theories

I. Lagrangian depends only on ‘electric’ fields *L(F)* and is not duality-invariant

- ○ **Duality symmetry** manifests itself **only on-shell:** $\tilde{G} = 2\frac{\delta L(F)}{\delta F} = F + \Delta(F)$

↑

in the linear case

$$\mathsf{F}^{\,i} = (F, G), \qquad \delta\mathsf{F}^{\,i} = M^i{}_j\,\mathsf{F}^{\,j} \text{ - linear duality transform } \; M^i{}_j \subset Sp(2N)$$

- ○ The variation of *L(F)* under duality transform should satisfy a condition *(Gaillard-Zumino ’81, ’97; Gibbons-Rasheed ‘95)*

$$\delta L = \tfrac{1}{4}\delta(F\tilde{G}) \quad \Rightarrow \quad F\tilde{F} + G\tilde{G} = 0$$

II. Lagrangian depends on both ‘electric’ and ‘magnetic’ fields $L(\mathsf{F}^{\,i})$.

It is manifestly duality invariant. Duality condition follows from e.o.m.

Subtleties with space-time covariance

Duality-invariant actions

- Space-time invariance is not manifest

(Zwanziger '71, Deser & Teitelboim '76, Henneaux & Teitelboim '87, ...)

Example: duality-symmetric Maxwell action for $\mathsf{F}^{\,i} = dA^i \quad (i = 1,2)$

$$L = \tfrac{1}{8}\,\mathsf{F}^{\,i}_{\mu\nu}\,\mathsf{F}^{\,i\mu\nu} + \tfrac{1}{4}(\mathsf{F}^{\,i}_{0a} - \varepsilon^{ij}\,\widetilde{\mathsf{F}}^{\,j}_{0a})(\mathsf{F}^{\,0ai} - \varepsilon^{ij}\,\widetilde{\mathsf{F}}^{\,0aj}) \qquad \mu = (0,a) \quad a = 1,2,3$$

breaks manifest Lorentz invariance

Modified Lorentz invariance: $\delta A^i_\mu = \delta_\Lambda A^i_\mu + x^a \Lambda_a{}^0(\mathsf{F}^{\,i}_{0\mu} - \varepsilon^{ij}\,\widetilde{\mathsf{F}}^{\,j}_{0\mu})$

Twisted self-duality condition is obtained by integrating the e.o.m. :

$$\frac{\delta L}{\delta A^i} = 0 \;\;\Rightarrow\;\; \mathsf{F}^{\,i}_{\mu\nu} - \varepsilon^{ij}\,\widetilde{\mathsf{F}}^{\,j}_{\mu\nu} = 0 \;\;\Rightarrow\;\; F^1_{\mu\nu} = \tilde{F}^2_{\mu\nu}$$

Space-time covariant and duality-invariant action

- Space-time covariance can be restored by introducing an auxiliary scalar field *a(x)* *(Pasti, D.S. & Tonin '95)*

$$L_{nonc} = \tfrac{1}{8} F^{i}_{\mu\nu} F^{i\mu\nu} + \tfrac{1}{4}(F^{i}_{0a} - \varepsilon^{ij} \tilde{F}^{j}_{0a})(F^{0ai} - \varepsilon^{ij} \tilde{F}^{0aj}) \quad \mu = (0, a) \quad a = 1,2,3$$

$$L_{cov} = \tfrac{1}{8} F^{i}_{\mu\nu} F^{i\mu\nu} + \tfrac{1}{4} v^{\mu} (F^{i}_{\mu\nu} - \varepsilon^{ij} \tilde{F}^{j}_{\mu\nu})(F^{\nu\lambda i} - \varepsilon^{ij} \tilde{F}^{\nu\lambda j}) v_{\lambda}(x)$$

$$\boxed{v_{\mu}(x) = \frac{\partial_{\mu} a(x)}{\sqrt{(\partial a)^2}}, \qquad v_{\mu} v^{\mu} = 1}$$

Local symmetries:

$$\delta A^{i}_{\mu} = \partial_{\mu} \lambda(x)$$

$$\delta_{I} A^{i}_{\mu} = \Phi(x) \partial_{\mu} a(x), \qquad \delta_{I} a(x) = 0 \quad \longrightarrow \quad v^{\mu} A^{i}_{\mu} \ \text{- is pure gauge}$$

$$\delta_{II} a(x) = \varphi(x), \qquad \delta_{II} A^{i}_{\mu} = \frac{\varphi(x)}{\sqrt{(\partial a)^2}} v^{\nu} (F^{i}_{\mu\nu} - \varepsilon^{ij} \tilde{F}^{j}_{\mu\nu}) \quad \longrightarrow$$

gauge fixing

$$a(x) = x^{0}, \quad v_{\mu} = \delta^{0}_{\mu}$$

Non-linear generalization

Another form of the Lagrangian:

$$L_{\rm cov} = \tfrac{1}{4}\Omega^{ij}(v^{\mu}F^{i}_{\mu\nu})\,(v_{\lambda}\tilde{F}^{\lambda\nu j}) - \tfrac{1}{4}(v^{\mu}\tilde{F}^{i}_{\mu\nu})(v_{\lambda}\tilde{F}^{\lambda\nu i}), \quad \Omega^{2} = -1, \quad i, j = 1,\dots, N$$

pure gauge component $v^{\mu}A^{i}_{\mu}$ enters only the 1st term under the total derivative

$$v^{\mu}\tilde{F}^{i}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\lambda}v^{\mu}F^{\rho\lambda i}$$ does not contain $v^{\mu}A^{i}_{\mu}$

Higher-order Lagrangian:

$$L = \tfrac{1}{4}\Omega^{ij}(v^{\mu}F^{i}_{\mu\nu})\,(v_{\lambda}\tilde{F}^{\lambda\nu j}) - \tfrac{1}{4}(v^{\mu}\tilde{F}^{i}_{\mu\nu})(v_{\lambda}\tilde{F}^{\lambda\nu i}) - \tfrac{1}{4}\mathcal{L}\,[i_{v}\tilde{F}, d i_{v}\tilde{F}, \phi]$$

where $(i_{v}\tilde{F})_{\nu} = v^{\mu}\tilde{F}_{\mu\nu}$

By construction *L* is invariant under $\delta_{I}A^{i}_{\mu} = \Phi(x)\partial_{\mu}a(x), \quad \delta_{I}a(x) = 0$

Non-linear Lagrangian & local *a(x)*-symmetry

$$L = \tfrac{1}{4}\Omega^{ij}(v^{\mu}\mathrm{F}_{\mu\nu}^{\;i})\,(v_{\lambda}\tilde{\mathrm{F}}^{\lambda\nu j}) - \tfrac{1}{4}(v^{\mu}\tilde{\mathrm{F}}_{\mu\nu}^{\;i})(v_{\lambda}\tilde{\mathrm{F}}^{\lambda\nu i}) - \tfrac{1}{4}\mathcal{L}\,[i_v\tilde{\mathrm{F}}, d i_v\tilde{\mathrm{F}}, \ldots]$$

2nd local symmetry in the linear case:

$$\delta_{II}a(x) = \varphi(x), \qquad \delta_{II}A_{\mu}^{i} = \frac{\varphi(x)}{\sqrt{(\partial a)^2}}v^{\nu}(\mathrm{F}_{\mu\nu}^{\;i} - \Omega^{ij}\tilde{\mathrm{F}}_{\mu\nu}^{\;j}) \;=0 \text{ on shell}$$

A^i equation of motion:

$$\frac{\delta L}{\delta A^i} = d\left(v(i_v\mathrm{F}^{\,i} - \Omega^{ij}i_v\tilde{\mathrm{F}}^{\,j} - \Omega^{ij}\frac{\delta\mathcal{L}}{\delta(i_v\tilde{\mathrm{F}}_{\mathrm{j}})})\right) = 0 \;\Rightarrow\; \boxed{v^{\nu}(\mathrm{F}_{\mu\nu}^{\;i} - \Omega^{ij}\tilde{\mathrm{F}}_{\mu\nu}^{\;j}) - \Omega^{ij}\frac{\delta\mathcal{L}}{\delta(v_{\nu}\tilde{\mathrm{F}}_{\mathrm{j}}^{\;\mu\nu})} = 0}$$

2nd local symmetry in non-linear case:

$$\delta_{II}a(x) = \varphi(x), \qquad \delta_{II}A_{\mu}^{i} = \frac{\varphi(x)}{\sqrt{(\partial a)^2}}\left(v^{\nu}(\mathrm{F}_{\mu\nu}^{\;i} - \Omega^{ij}\tilde{\mathrm{F}}_{\mu\nu}^{\;j}) - \Omega^{ij}\frac{\delta\mathcal{L}}{\delta(v_{\nu}\tilde{\mathrm{F}}^{\mu\nu j})}\right)$$

Consistency condition on non-linear deformation $L(F)$

$$\delta_{II} L = 0 \quad \Rightarrow \quad \Omega^{ij} d\left[\frac{v}{\sqrt{(\partial a)^2}}\left(i_v \tilde{F}^{i} + \frac{\delta L}{2\delta(i_v \tilde{F}^{i})}\right)\frac{\delta L}{\delta(i_v \tilde{F}^{j})}\right] = 0$$

The condition on L ensures the auxiliary nature of the scalar $a(x)$

upon gauge fixing $a(x)$ it ensures non-manifest space-time invariance

Known examples:

- Born-Infeld-like form of the M5-brane action
  (Perry & Schwarz '96; Pasti, D.S. and Tonin '97)
- Born-Infeld-like form of the duality-symmetric D3-brane action
  (Berman '97; Nurmagambetov '98)
- New Born-Infeld-like deformations *(Kuzenko et. al, Bossard & Nicolai; Kallosh et. al '11)*

a(x)-independence of twisted self-duality condition

$$v^{\nu}(F^{i}_{\mu\nu} - \Omega^{ij}\tilde{F}^{j}_{\mu\nu}) - \Omega^{ij}\frac{\delta \mathcal{L}}{\delta(v_{\nu}\tilde{F}_{j}^{\ \mu\nu})} = 0$$

$$\Downarrow$$

$$F^{i} - \Omega^{ij} * F^{j} = \underbrace{v\frac{\delta \mathcal{L}}{\delta(i_{v}\tilde{F}_{i})} - \Omega^{ij} * v\frac{\delta \mathcal{L}}{\delta(i_{v}\tilde{F}_{j})}}_{\text{should not depend on } v(x)\sim da(x) \text{ independently of gauge fixing}} = \frac{\delta I(F)}{\delta F_{+}^{\ i}}, \qquad F_{+}^{\ i} = F^{i} + \Omega^{ij} * F^{j}$$

This establishes on-shell relation between manifestly duality-symmetric and Gaillard-Zumino approach to the construction of non-linear self-dual theories

Main issues: Whether counterterms of N=8,4 sugra can provide the form of $I(F)$?
If yes, whether this deformation is consistent with supersymmetry ?

Supersymmetry issue

- Counterterms $\partial^{2k} F^4$ that can appear at 7 loops in N=8 sugra are supersymmetric and $E_{7(7)}$- invariant on the mass-shell, i.e.

modulo linear twisted self-duality $F_-^{\,i} = F^{\,i} - \Omega^i{}_j \tilde{F}^{\,i} = 0$

$$\longrightarrow \quad I_0(F^{\,i},\phi) = I_0(F_+^{\,i},\phi) = I_0(F^A,\phi)$$

- When included into the effective action, $I_0(F)$ **deforms** duality condition

$$F_-^{\,i} = \frac{\delta I_0(F,\phi)}{\delta F_+^{\,i}} \neq 0 \quad \longrightarrow \quad I(F_+^{\,i}, F_-^{\,i}, \phi), \quad I(F_+^{\,i}, F_-^{\,i}, \phi)\Big|_{F_-^{\,i}=0} = I_0$$

whose form is determined by Gaillard-Zumino condition or space-time invariance of the deformed action

Supersymmetry of $I(F_+^{\,i}, F_-^{\,i}, \phi)$ should be checked

Standard N=8 superspace methods are not applicable. Use component formalism

Supersymmetry of duality-symmetric actions

- Example: duality-symmetric $N=1$ Maxwell action $\mathrm{F}^{i} = dA^{i} \quad (i=1,2)$

$$L_{N=1} = \tfrac{1}{8}\mathrm{F}^{i}_{\mu\nu}\mathrm{F}^{i\mu\nu} + \tfrac{1}{4}v^{\mu}(\mathrm{F}^{i}_{\mu\nu} - \varepsilon^{ij}\tilde{\mathrm{F}}^{j}_{\mu\nu})(\mathrm{F}^{\nu\lambda i} - \varepsilon^{ij}\tilde{\mathrm{F}}^{\nu\lambda j})v_{\lambda} + \tfrac{i}{2}\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi$$

Susy transformations *(Schwarz & Sen '93, Pasti, D.S. & Tonin '95)*

$$\delta A^{i}_{\mu} = i\overline{\psi}\gamma_{\mu}\varsigma^{i}, \qquad \varsigma^{i} = \varepsilon^{ij}\gamma_{5}\varsigma^{j}, \qquad \delta_{\varsigma}a(x) = 0$$

$$\delta\psi = \tfrac{1}{8}K^{i}_{\mu\nu}\gamma^{\mu\nu}\varsigma^{i}, \qquad K^{i}_{\mu\nu} = \mathrm{F}^{i}_{\mu\nu} + v_{[\mu}(\mathrm{F}^{i}_{\nu]\rho} - \varepsilon^{ij}\tilde{\mathrm{F}}^{i}_{\nu]\rho})v^{\rho}$$

On shell $(F^{2} = -\tilde{F}^{1})$: $\delta A^{1}_{\mu} = i\overline{\psi}\gamma_{\mu}\varsigma^{1}, \quad \delta\psi = \tfrac{1}{4}F^{1}_{\mu\nu}\gamma^{\mu\nu}\varsigma^{1}$

- In the non-linear case: $K^{i} = \mathrm{F}^{i} + v(i_{v}\mathrm{F}_{-}^{i} - \dfrac{\delta \mathrm{I}(\mathrm{F},\psi)}{\delta \mathrm{F}_{+}^{i}})$

Non-linear duality, supersymmetry and UV

Examples:

- N=1,2,3,4, D=4 Born-Infeld theories (D3-branes) *(known since '95)*
- Abelian N=(2,0) D=6 self-dual theory on the worldvolume of the M5-brane
- BI models (including higher-order derivatives) coupled to N=1,2 D=4 sugra *(Kuzenko and McCarthy '02, Kuzenko '12)*

In most of the known examples non-linear deformation of duality is related to a partial spontaneous breaking of supersymmetry

Issues:

- Whether non-linear deformations are possible for vector fields inside supergravity multiplets, in particular, in N=4,8 supergravities?
- Whether this interplay between dualities and supersymmetry may shed light on the UV behavior of N=4,8 supergravities?