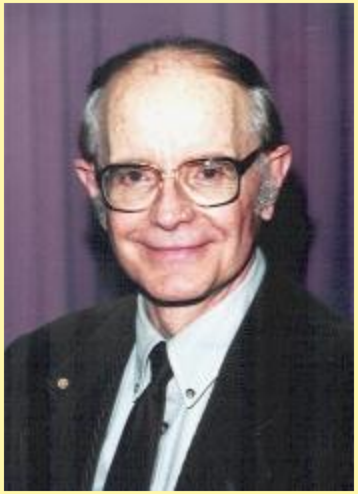
# Photon emission by atomic ensembles: collective, virtual and nonlocal effects


Anatoly A. Svidzinsky

Department of Physics & Astronomy, Texas A&M University


# Collaborator:

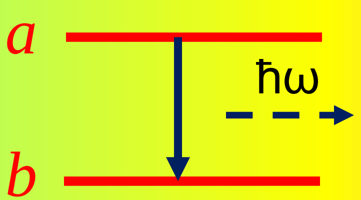


**Marlan Scully**

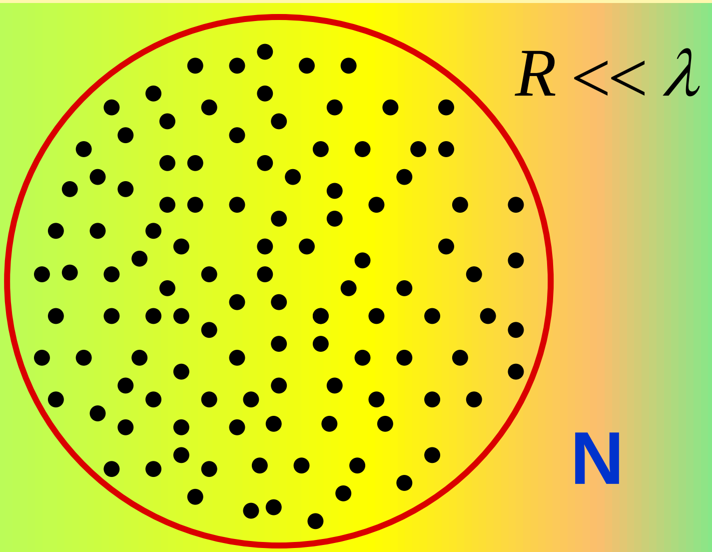
# What is Superradiance?

**Robert Dicke**
"Coherence in spontaneous emission processes"
Phys. Rev. 93, 99 (1954)

**One photon with energy $\hbar\omega$ is stored in the system**




**Single atom spontaneously decays during time $\tau$**



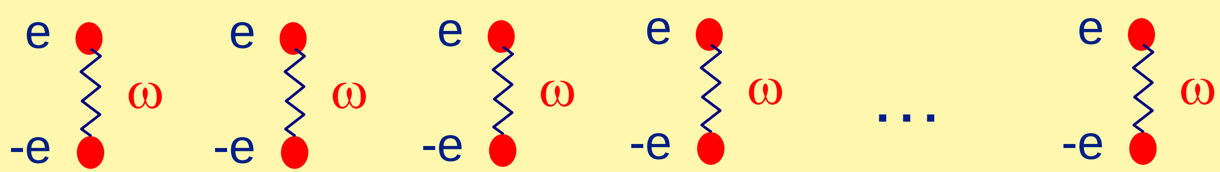

**N atoms can decay N-times faster**

**Decay time $\frac{\tau}{N}$**

# Simple example of superradiance

## Radiation of N classical harmonic oscillators

**N oscillators** $R << \lambda$




Assume that oscillators move with equal amplitude A

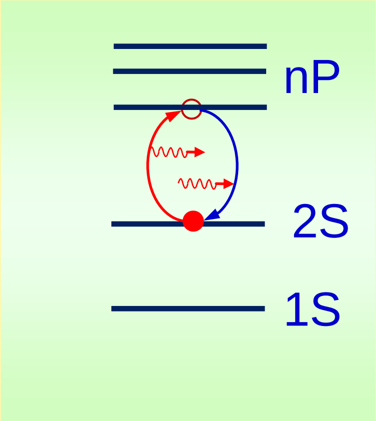
$$E = \frac{1}{2} m \omega^2 N A^2$$

**If they move in phase:** $d \propto NA$

**Radiation power:** $P \propto \ddot{d}^2 \propto N^2 A^2$ $\Rightarrow$ **Decay time:** $\tau \approx \frac{E}{P} = \frac{\tau_0}{N}$

# Virtual transitions

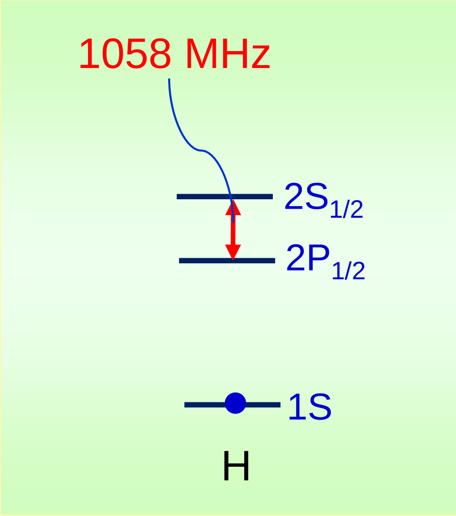
Atom can jump into an excited state and virtual photon is emitted, then atom quickly jumps back to the ground state and absorbs a photon.



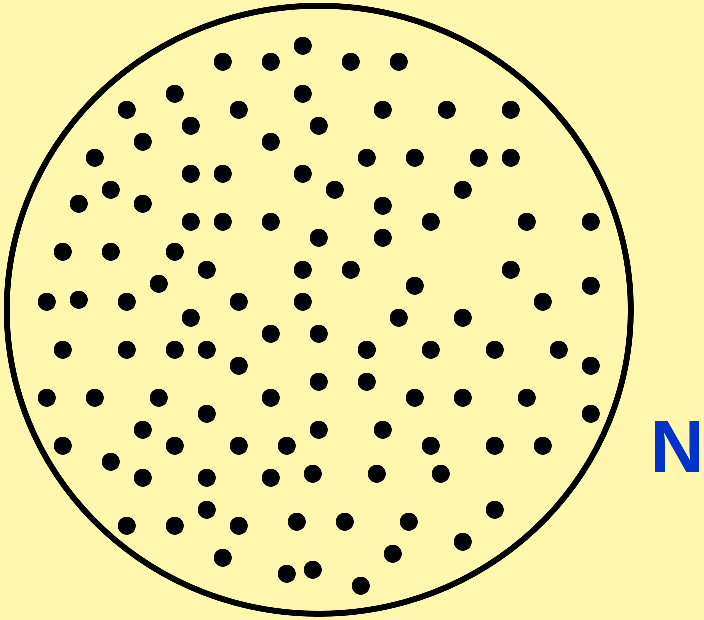

Virtual processes have real effects – they shift energy levels of emitting atoms (**Lamb shift**).

## Hydrogen atom

**1947:** Experiment by Willis Lamb and Robert Retherford



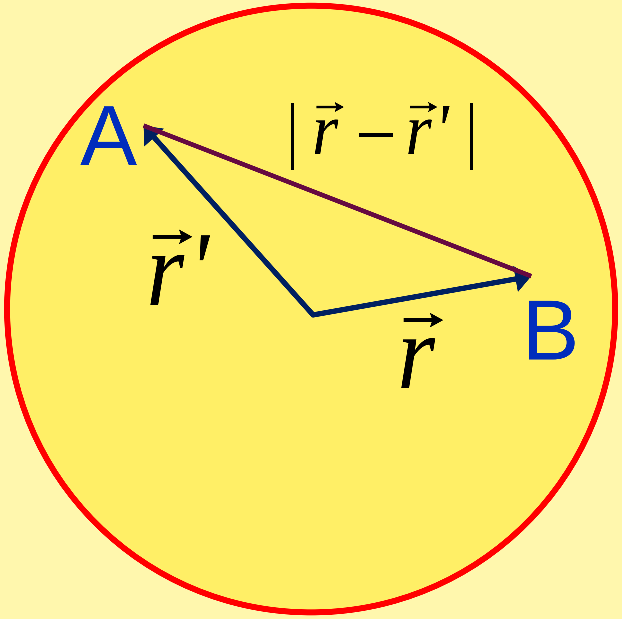

In **1955** W. Lamb won the Nobel Prize in Physics *"for his discoveries concerning the fine structure of the hydrogen spectrum"*

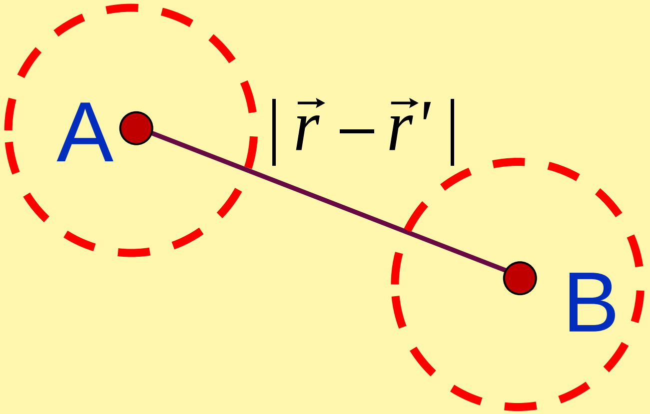



**What if there are N atoms?**

**How virtual photons modify system evolution?**

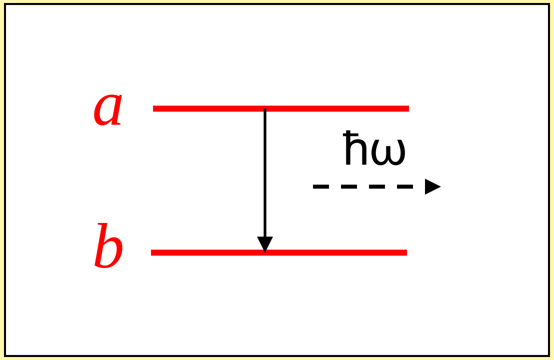
# What about retardation (nonlocal) effects?

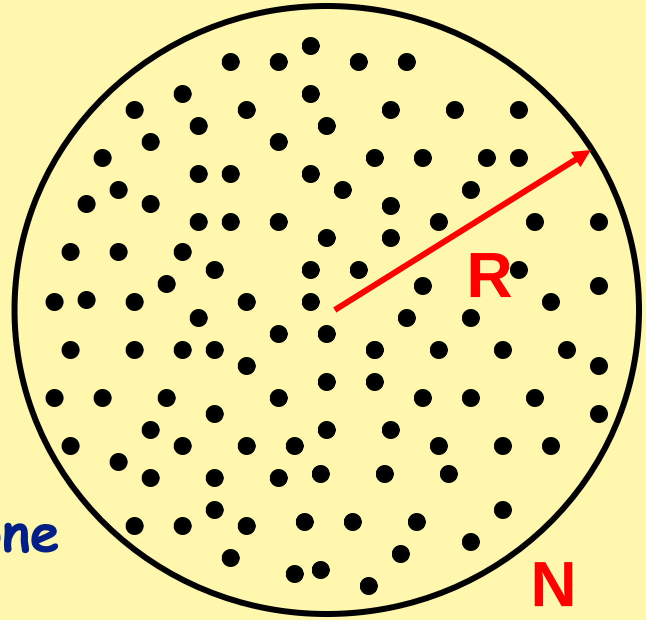



It takes time $t = \frac{|\vec{r}-\vec{r}\prime|}{c}$ for wave to travel from atom **A** to atom **B**

**System:** ***N* two-level atoms**, randomly distributed in a sample




**One atom is excited, but we don't know which one**

**Atoms interact only by photon exchange**

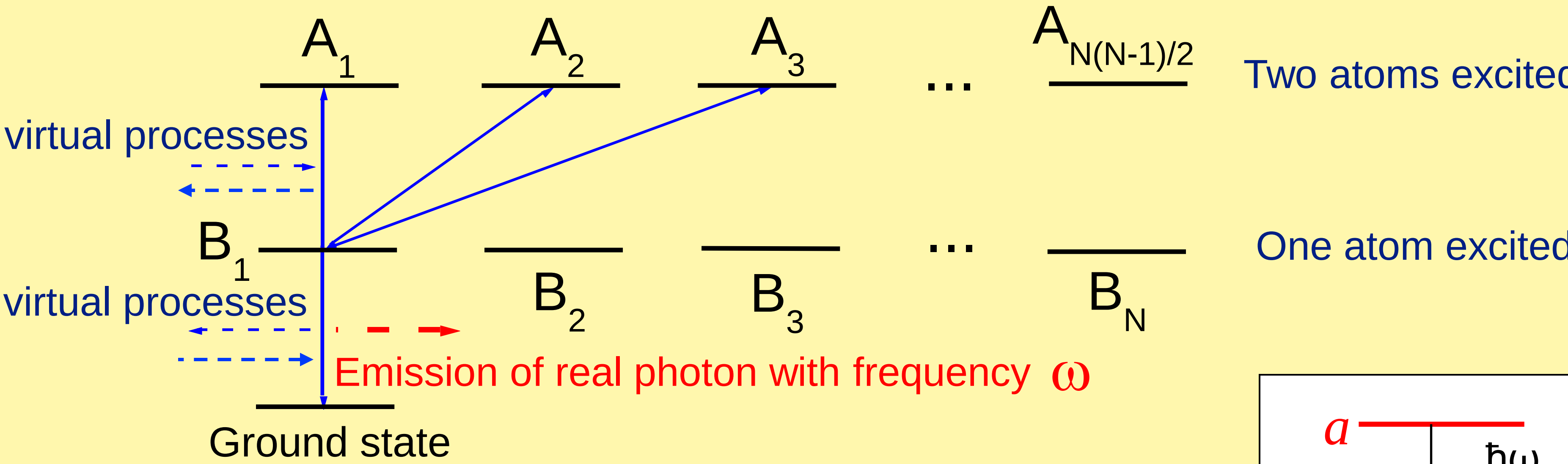
# Interaction Hamiltonian

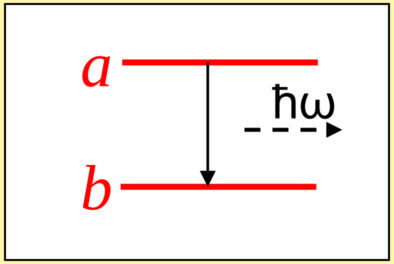
$$\hat{H}_{int} = \sum_{\vec{k}} \sum_{j=1}^{N} g_k \left( \hat{\sigma}_j e^{-i\omega t} + \hat{\sigma}_j^{+} e^{i\omega t} \right) \left( \hat{a}_{\vec{k}}^{+} e^{i\omega_k t - i\vec{k}\cdot\vec{r}_j} + \hat{a}_{\vec{k}} e^{-i\omega_k t + i\vec{k}\cdot\vec{r}_j} \right)$$

$\hat{\sigma}_j$ - atom operator, $\hat{a}_k$ - photon operator, $g_k$ - atom-photon coupling constant

$$g_k = \frac{d_{ab}}{\hbar} \sqrt{\frac{\hbar \omega^2}{\varepsilon_0 \nu_k V_{ph}}}$$

# States of N atoms




## State vector

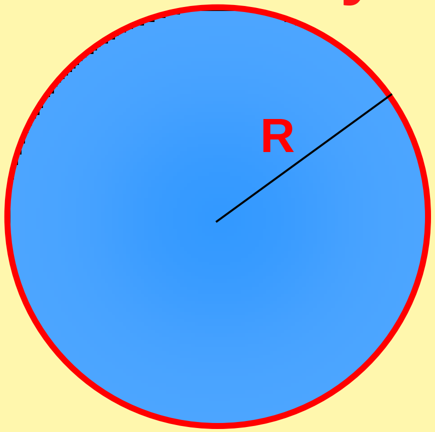
$$\Psi = \sum_{j=1}^{N} \beta_j(t)\,|\,b_1 b_2 \ldots a_j \ldots b_N 0 > + \sum_{\vec{k}} \gamma_{\vec{k}}(t)\,|b_1 b_2 \ldots b_N 1_{\vec{k}} > + \ldots$$

**Initial condition:** $\gamma_{\vec{k}}(0) = 0, \quad \beta_j(0)$ State has zero dipole moment

$\beta_j(t)$ - probability amplitude to find atom $j$ excited at time t

# For dense atomic cloud state evolution is described by:

$$\frac{\partial \beta(t,\vec{r})}{\partial t} = i\gamma \frac{N}{V} \int d\vec{r}\,' \frac{\exp(ik_0 |\vec{r}-\vec{r}\,'|)}{k_0 |\vec{r}-\vec{r}\,'|} \beta\left(t - \frac{|\vec{r}-\vec{r}\,'|}{c}, \vec{r}\,'\right)$$




Includes retardation effects and virtual photon contribution

$$k_0 = \frac{\omega}{c}$$

**Local approximation (slow decay):**

$$\frac{\partial \beta(t,\vec{r})}{\partial t} = i\gamma \frac{N}{V} \int d\vec{r}\,' \frac{\exp(ik_0 |\vec{r}-\vec{r}\,'|)}{k_0 |\vec{r}-\vec{r}\,'|} \beta(t, \vec{r}\,')$$

**Omitting virtual photons:**

$$\frac{\partial \beta(t,\vec{r})}{\partial t} = -\gamma \frac{N}{V} \int d\vec{r}\,' \frac{\sin(k_0 |\vec{r}-\vec{r}\,'|)}{k_0 |\vec{r}-\vec{r}\,'|} \beta(t, \vec{r}\,')$$

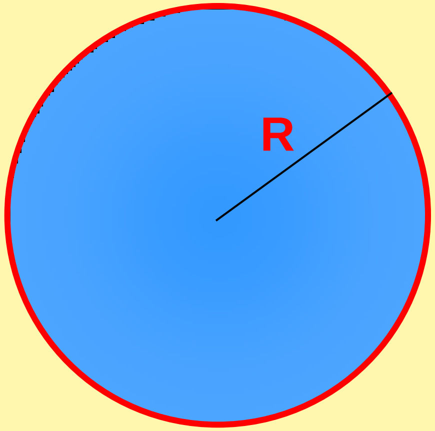
$\gamma$ - **Spontaneous decay rate of single atom**

$\beta(t,\vec{r})$ - probability amplitude to find atom at $\vec{r}$ excited at time t

# Effect of virtual transitions

# Evolution of eigenstates is given by

$$\beta(t,\vec{r}) = \beta(\vec{r})\, e^{-\lambda_n t}$$




$\mathrm{Re}[\lambda_n]$ **- gives collective decay rate**

$\mathrm{Im}[\lambda_n]$ **- frequency shift (Lamb shift)**

**Sum rule:**

$$\sum_{n=1}^{N} \lambda_n = N\gamma$$

If there are states which decay faster than single atom decay rate $\gamma$ then inevitably there are states which decay more slowly.

# Eigenstates of spherical atomic system

## Exp kernel:

$$\beta_{nm}(\vec{r}) = j_n(ak_0r)Y_{nm}(\theta,\varphi)$$

Eigenvalues are determined from

$$a = \frac{j_n(ak_0R)}{j_{n-1}(ak_0R)}\frac{h^{(1)}_{n-1}(k_0R)}{h^{(1)}_n(k_0R)}$$

$$\lambda_n = \frac{3iN}{k_0^3R^3(1-a^2)}$$

## Sin kernel:

$$\beta_{nm}(\vec{r}) = j_n(k_0r)Y_{nm}(\theta,\varphi)$$

$$\lambda_n = \frac{3N}{2}\left[j_n^2(k_0R) - j_{n-1}(k_0R)\,j_{n+1}(k_0R)\right]$$

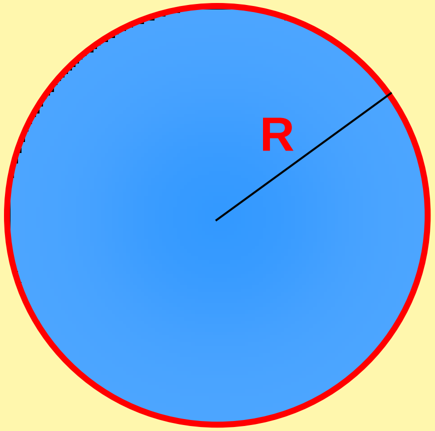
**(Ernst 1969)**

$Y_{nm}$ - spherical harmonics, $j_n(z)$, $h_n^{(1)}(z)$ – spherical Bessel functions

# Small sample limit $R \ll \lambda \quad (k_0 R \ll 1)$

## Fastest decaying eigenstate for spherical sample

$$\beta(t,\vec{r}) = \beta(\vec{r})\, e^{-\lambda_n t}$$




**Sin kernel:**

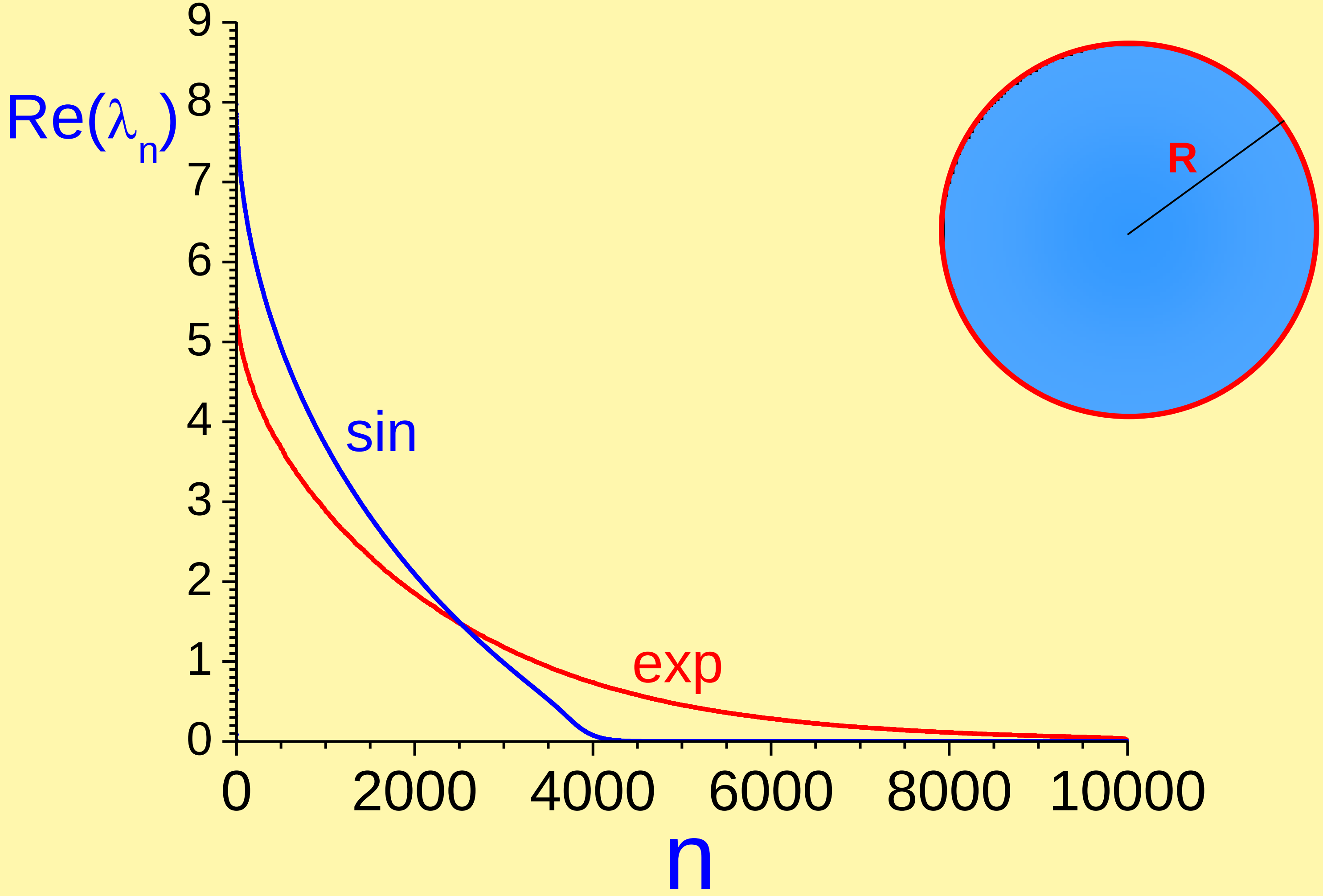
$$\beta_0(\vec{r}) = 1$$
$$\lambda_0 = N\gamma$$

**Exp kernel:**

$$\beta_0(\vec{r}) = \frac{1}{r}\sin\left(\frac{\pi r}{2R}\right)$$
$$\lambda_0 = -1.22i\,\frac{N\gamma}{k_0 R} + 0.986 N\gamma$$

# Large sample limit  $k_0 R >> 1 \quad (R >> \lambda)$

## Distribution of eigenstate decay rates for $R = 10\lambda,\ N = 10^4$

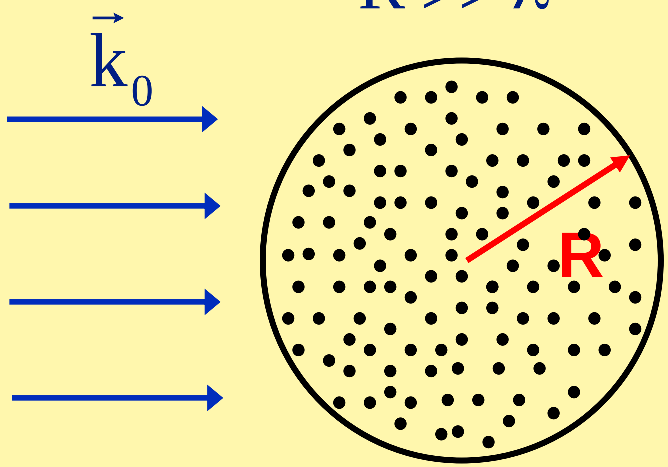


# Effect of virtual photons on evolution of N atoms

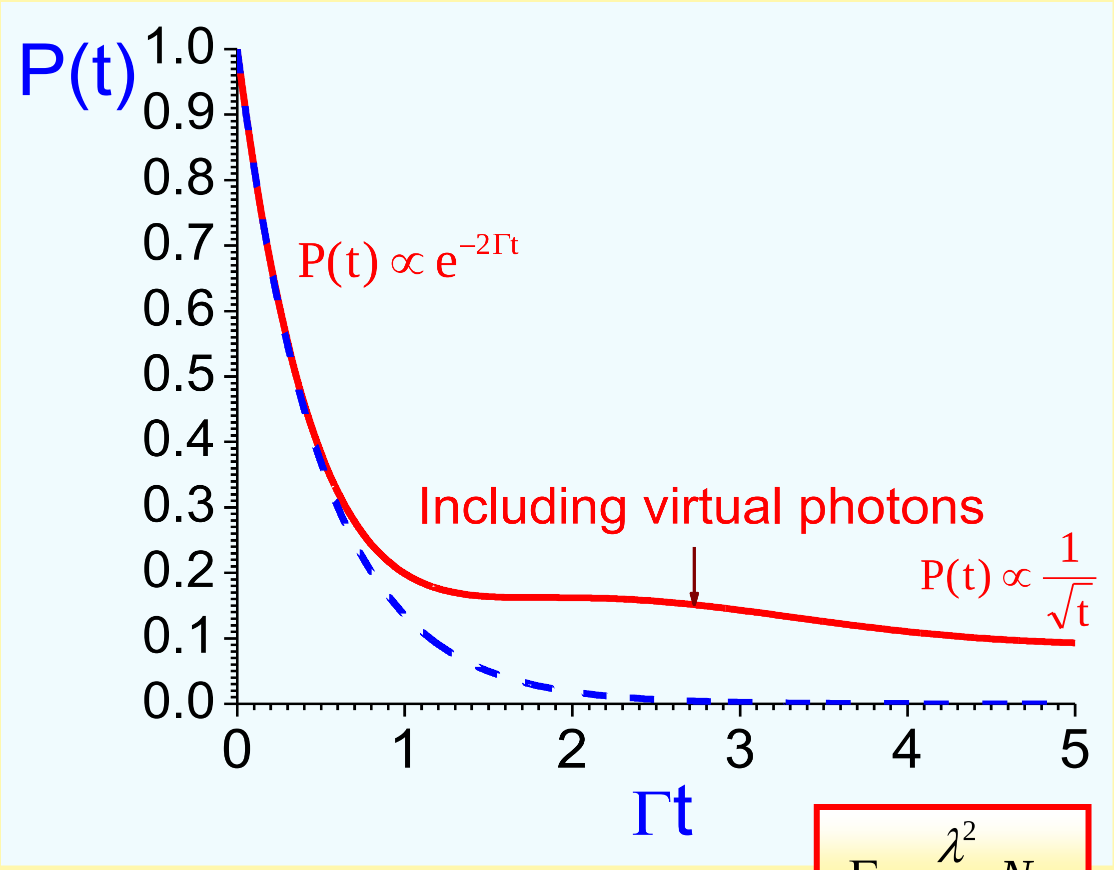
## Probability P(t) that atoms are excited

### Large cloud

$R >> \lambda$




$$\beta_j = \frac{1}{\sqrt{N}} e^{i\vec{k}_0 \cdot \vec{r}_j}$$



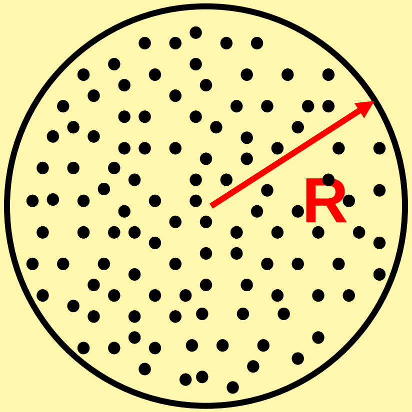

$$\Gamma \approx \frac{\lambda^2}{R^2} N\gamma$$

**Virtual processes transform population into slowly decaying states**

# Effect of virtual photons on evolution of N atoms

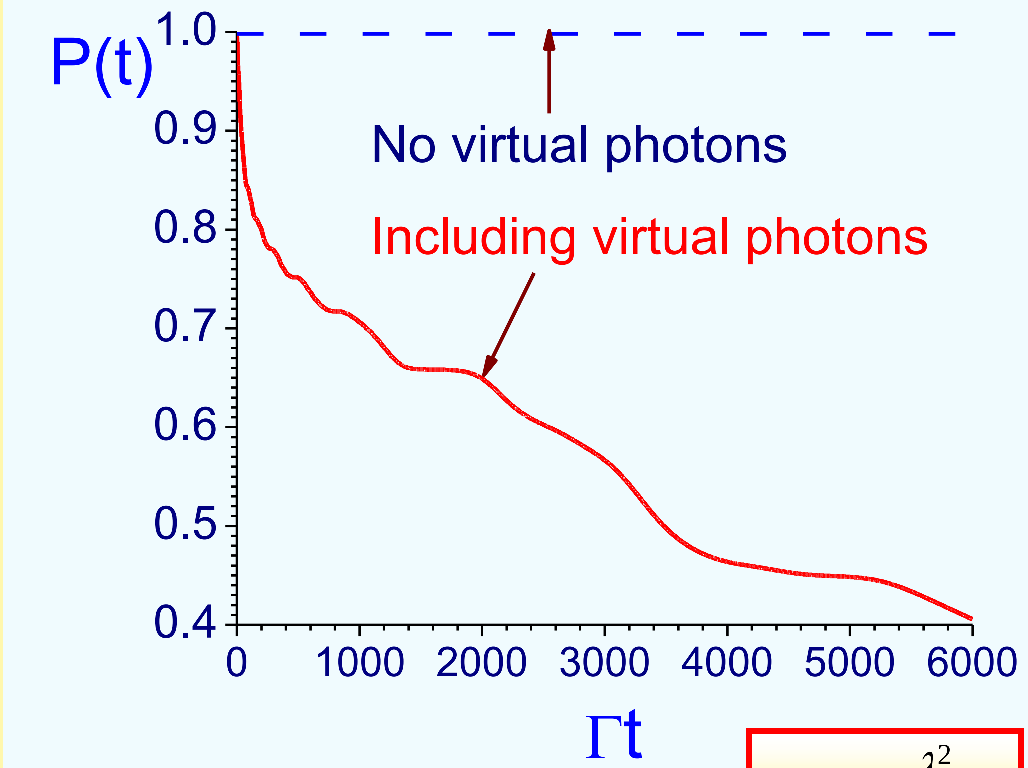
## Uniform excitation

$R >> \lambda$




$$\beta_j = \frac{1}{\sqrt{N}}$$

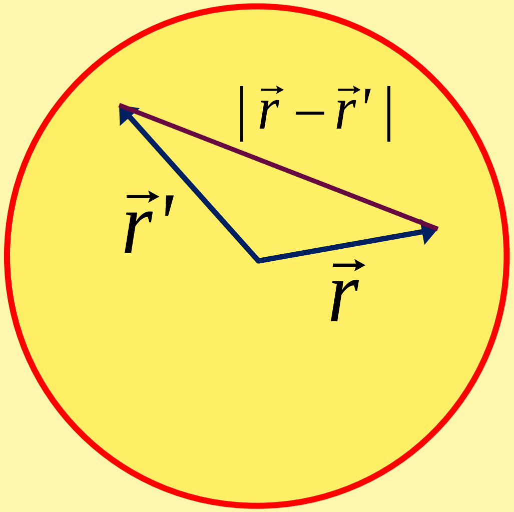
## Probability P(t) that atoms are excited




$$\Gamma \approx N \frac{\lambda^2}{R^2} \gamma$$

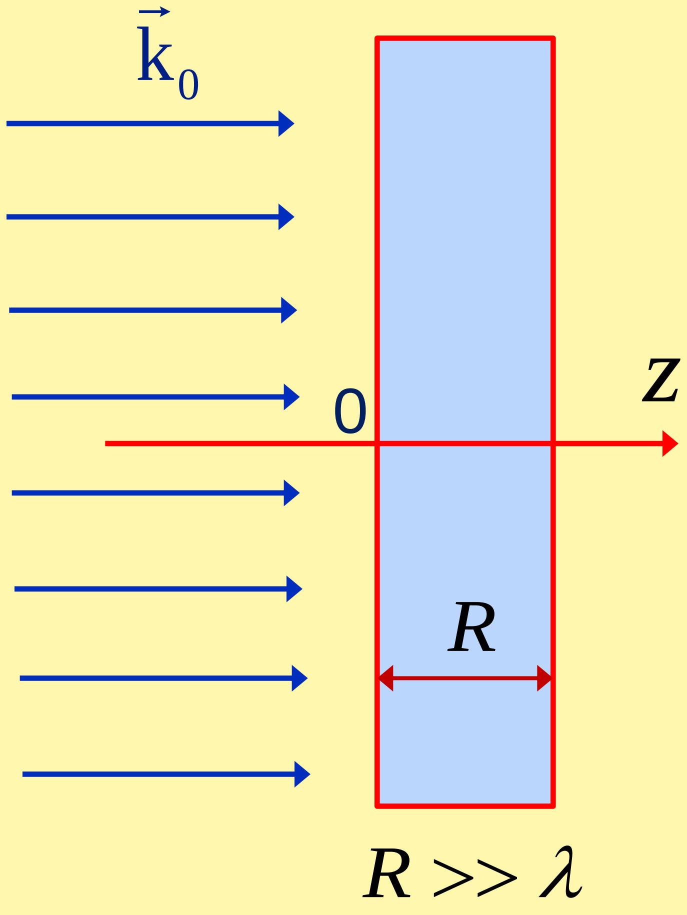
**Virtual photons result in slow decay of trapped states**

# Non-local (retardation) effects




$$\frac{\partial\beta(t,\vec{r})}{\partial t} = i\gamma\frac{N}{V}\int d\vec{r}\,'\,\frac{\exp(ik_0|\vec{r}-\vec{r}\,'|)}{k_0|\vec{r}-\vec{r}\,'|}\,\beta\left(t-\frac{|\vec{r}-\vec{r}\,'|}{c},\vec{r}\,'\right)$$

# Slab geometry: exact solution in nonlocal regime




## Initial condition:

$$\beta(0,\vec{r}) = e^{ik_0 \cdot z}$$

$$k_0 = \frac{\omega}{c}$$

## State evolution is described by:

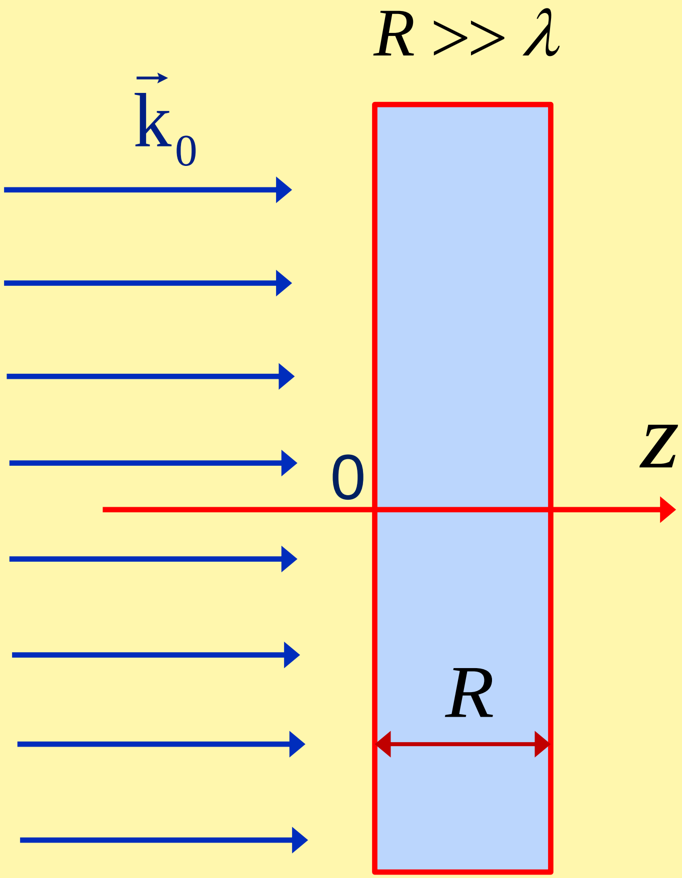
$$\frac{\partial \beta(t,\vec{r})}{\partial t} = i\gamma \frac{N}{V} \int d\vec{r}\,' \frac{\exp(ik_0 |\vec{r} - \vec{r}\,'|)}{k_0 |\vec{r} - \vec{r}\,'|} \beta\left(t - \frac{|\vec{r} - \vec{r}\,'|}{c}, \vec{r}\,'\right)$$

# Exact solution for any slab thickness:

$$\beta(t,z) = e^{ik_0 z}\left(\cos(\Omega t) + \Theta(ct-z)\frac{\Omega}{c}\int_z^{ct}\sqrt{\frac{ct-z'}{z'}}J_1\left(\frac{2\Omega}{c}\sqrt{z'(ct-z')}\right)dz'\right)$$

$$\Omega = \gamma\sqrt{\frac{3}{4\pi}n\lambda^2\frac{c}{\gamma}}$$ – collective Rabi frequency

n – atomic density




## Local approximation:

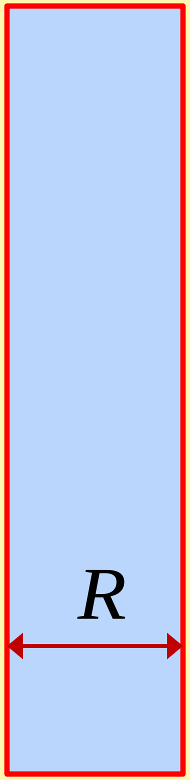
$$\beta(t,\vec{r}) = J_0\left(\frac{2\Omega}{c}\sqrt{ctz}\right)e^{ik_0 z}$$

## Crossover between local and nonlocal dynamics occurs at

$$R_0 = \frac{c}{\Omega}$$

$$R \ll \frac{c}{\Omega}$$ - **Local regime**

$$R \gg \frac{c}{\Omega}$$ - **Nonlocal regime**

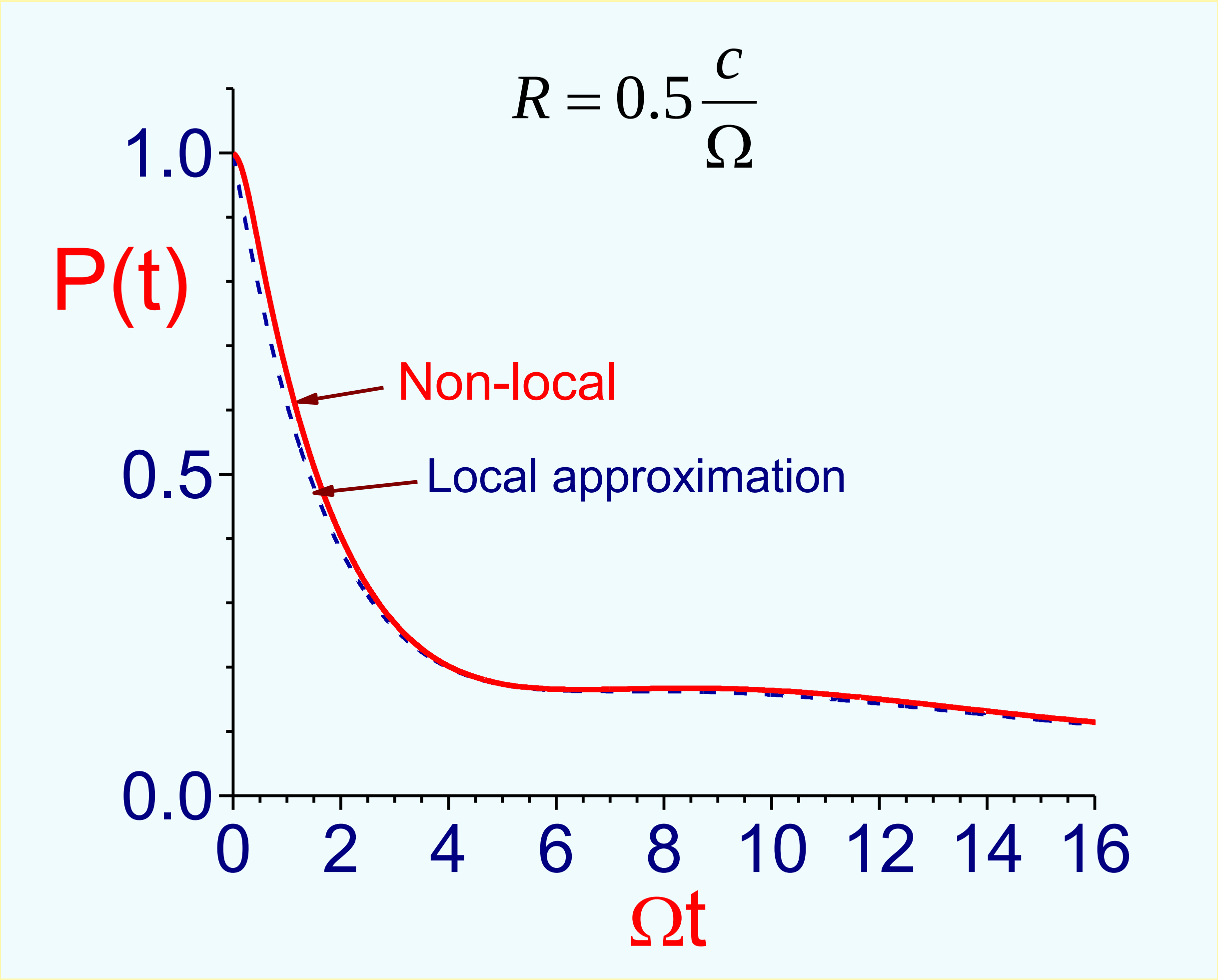


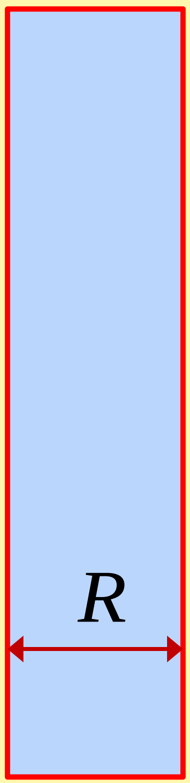

For $\gamma = 10^7 s^{-1},\ \lambda = 0.5\mu\text{m},\ n = 10^{15} cm^{-3}$

Collective Rabi frequency $\Omega = \gamma\sqrt{\frac{3}{4\pi} n \lambda^2 \frac{c}{\gamma}} = 4.2\times 10^{11} s^{-1}$

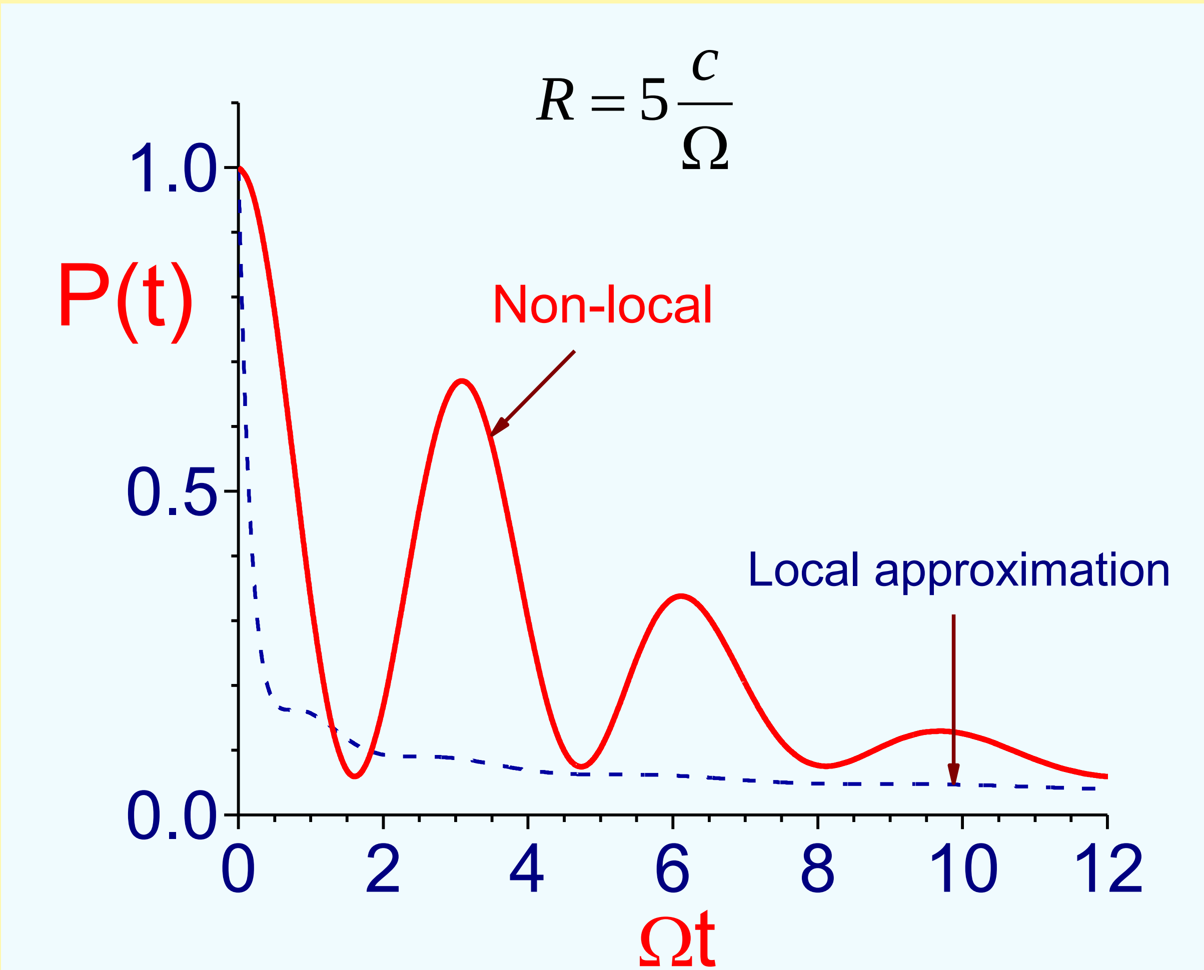
Characteristic slab thickness: $R_0 = \frac{c}{\Omega} = 0.7\text{mm}$

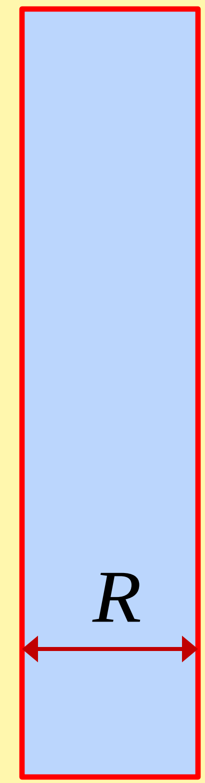
# Probability P(t) that atoms are excited



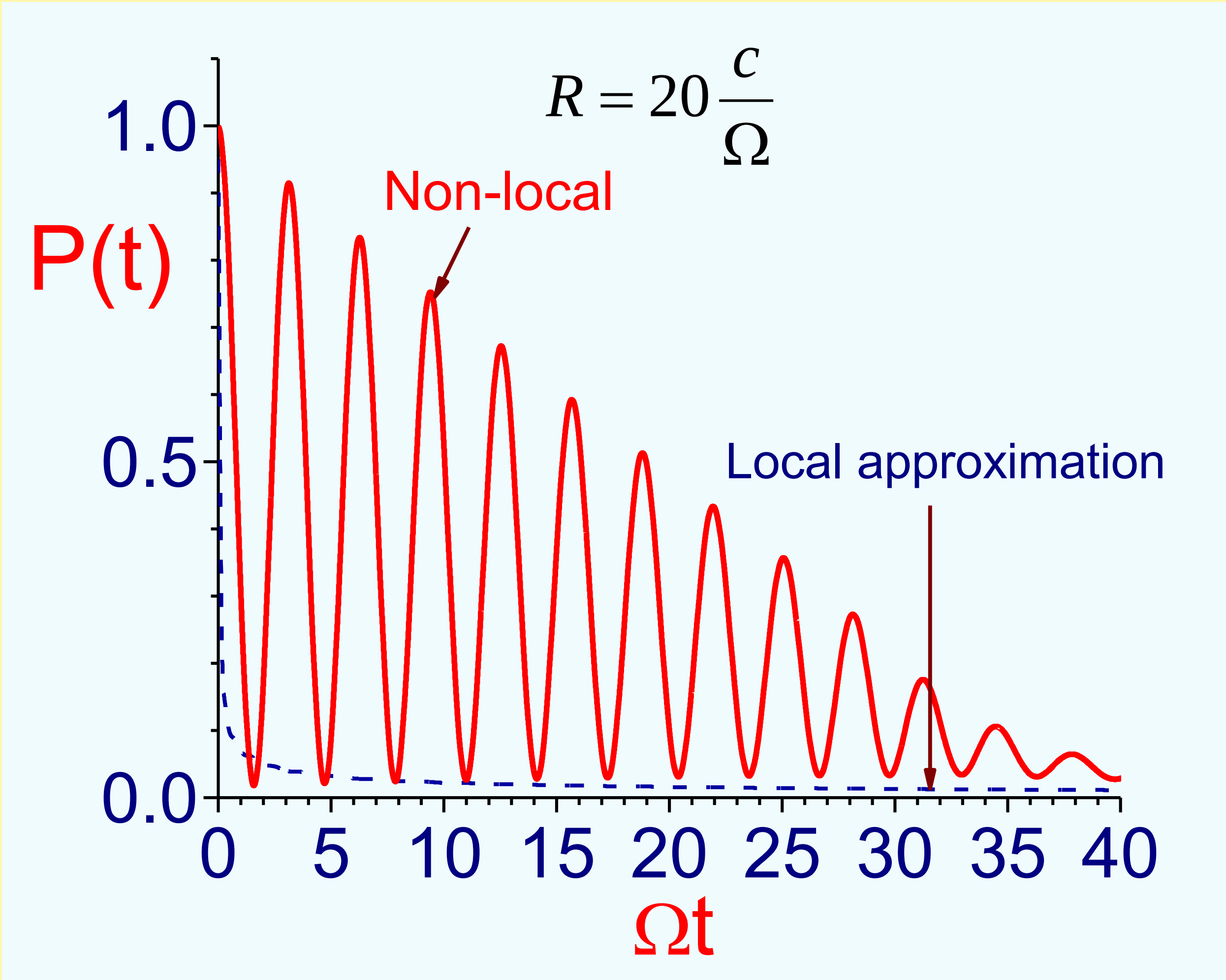



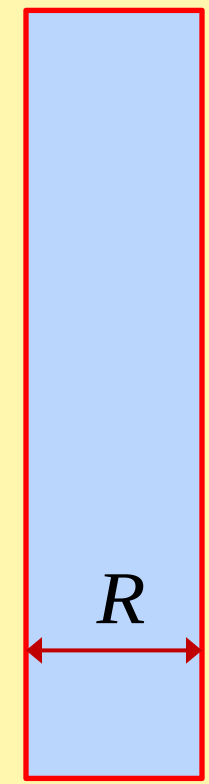
# Probability P(t) that atoms are excited



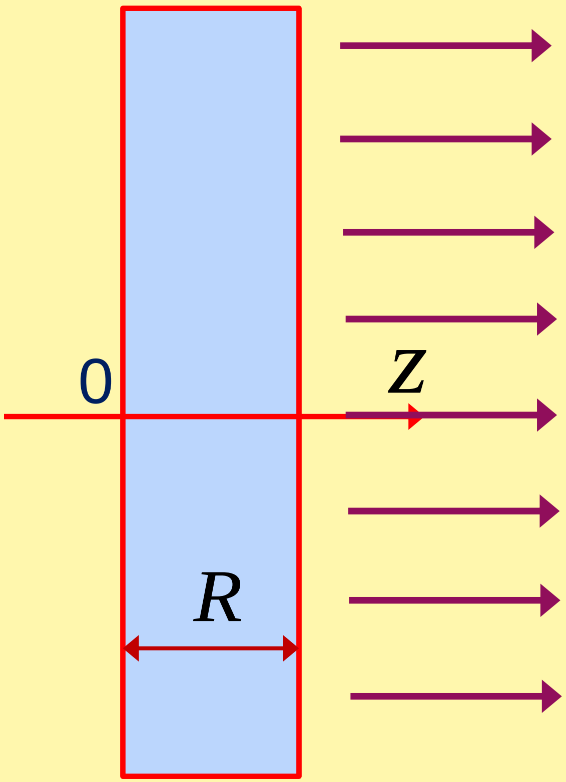
# Probability P(t) that atoms are excited

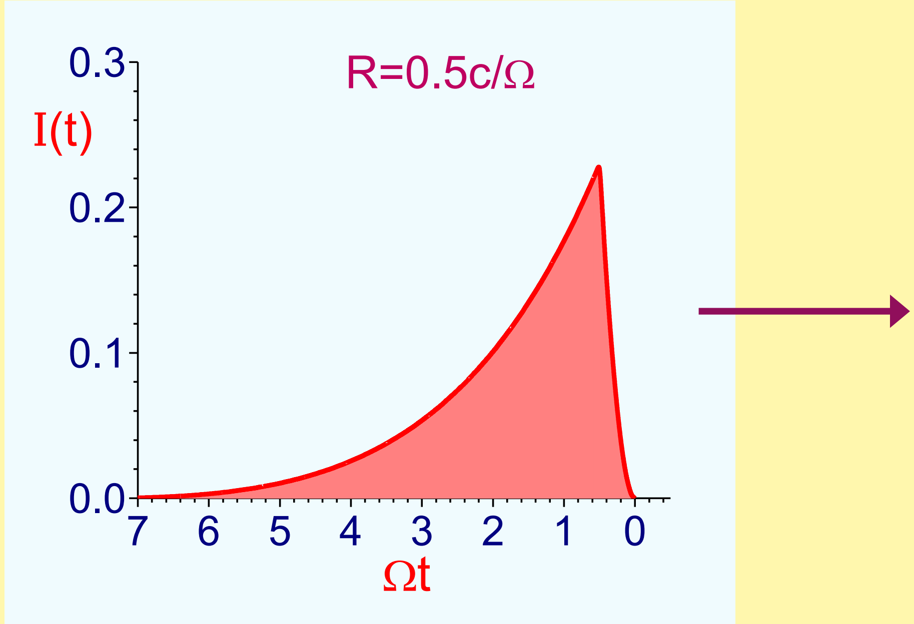


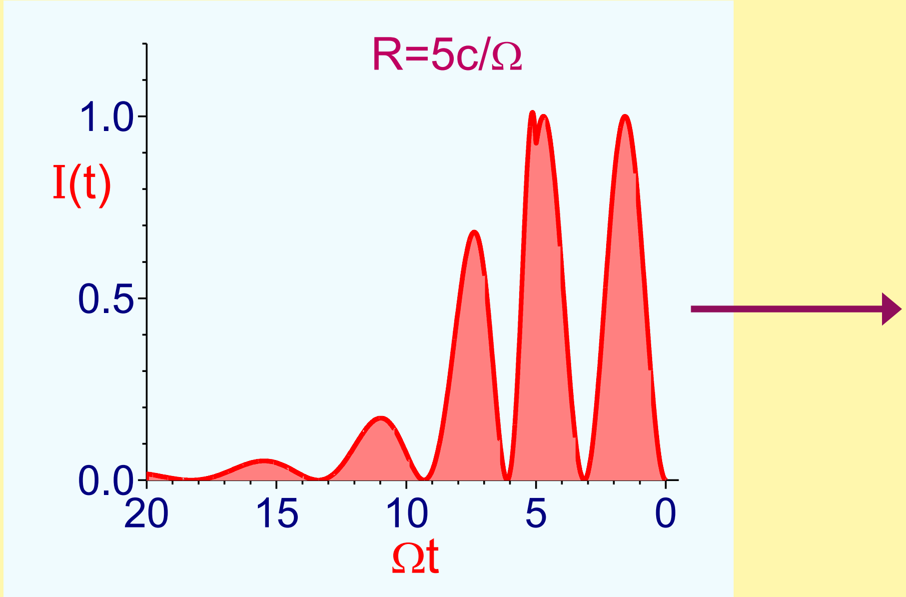



# Shape of the emitted pulse

**Pulse intensity** $I(t)$




$$\beta(0,\vec{r}) = e^{ik_0 \cdot z}$$

# Summary

Superradiance of atomic ensembles is a fascinating phenomenon which still offers interesting directions of exploration.

Influence of virtual transitions on collective emission and nonlocal effects are among intriguing subjects of future theoretical and experimental investigation.

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