Cold Dark Matter Cannot Consist of Relic Axions

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2 reasons for quick extintion

The primordial **axionic condensate**, still hypothetical, undergoes strongly enhanced self-induced electromagnetic decay into superradiant photon pairs and so **cannot survive on cosmological time scale**

This effect arises from the two physical reasons:

1. Quantum parametric resonance and subsequent nonlinear enhancement

2. Coherence of the photon pairs emitters, namely Bose-Einstein condensate particles

1. Quantum parametric resonance

B.R.Mollow and R.J.Glauber, *Quantum Theory of Parametric Amplification*, Phys. Rev. **160** (1967) 1076-1108

A frequency alignment

$$\omega_1 = \omega_2 \tag{1}$$

with an external force leads to linear in time growing of a donated amplitude.

A parametric frequency alignment

$$\omega_1 = 2\omega_2 \tag{2}$$

produces an exponential rate of grows.

This alignment automatically emerges in all two photon decays of any particle at rest (Brout-Englert-Higgs-Guralnik-Hagen-Kibble particle worth to mention) and acquires the form

$$mc^2 = 2h\nu \tag{3}$$

In an ensemble of rest particles quantum parametric resonance starts to work.

2. Coherence of the photon pairs emitters, namely Bose-Einstein condensate particles

V.Vanyashin, *Coherent decay of positronium Bose condensate*, Lett. in Math. Phys. **31** (1994) 143

A two photon decay of a pseudoscalar particle is determined by a phenomenological local electromagnetic interaction Lagrangian

$$L = g\vec{E}\vec{B} \tag{4}$$

that gives the particle lifetime in vacuum as

$$\frac{1}{\tau_{vac}} = \frac{g^2}{8\pi}\omega^3. \tag{5}$$

deceptive smallness

The smallness of the both input values – the dimensional coupling constant g (which is small due to the big value of the Peccei-Quinn symmetry breaking) and the photon frequency (half of the expected axion mass, that smallness enables easy Bose-Einstein condensation) – prompts the people, concerned with the dark matter problem, to consider an axion condensate survival on cosmological time scale as a triviality.

But this is not the case, and that is the main point of my talk.

coherent decay – nonevolutionary axion density

The formula above is not applicable to the decay of condensate particles. "The spooky action at a distance" drastically changes the decay picture through the coherent collective phenomena. From the same electromagnetic interaction Lagrangian the coherent lifetime is obtained as (V V, 1994)

$$\frac{1}{\tau_{coh}} = |g|\sqrt{\frac{\omega N}{V}} \tag{6}$$

We see that the both small input values change its power representation in a way favorable to the grows of the decay rate. Furthermore, they collaborate with a volume density of condensate particles – a new essential feature not present earlier (when the decay takes place in the vacuum).

This formula was obtained analyzing a feasibility of a parapositronium condensate gamma laser. For getting the necessary for lasing positronium condensate density I used a simplifying assumption of a constant in time condensate particle density what presupposes an appropriate pumping of positronium atoms into the condensate. For the coherent decay of axionic Bose condensate the assumption of a constant in time condensate particle density oversimplifies the problem. Now we have to analyze nonlinear oscillations when the system periodically swings between the initial state of a pure axionic condensate without real photon pairs and the state of entangled photon pairs created by the uttermost decay of all primordial axions.

The phenomenological Hamiltonian derived from the Lagrangian above contains the term responsible for the axion conversion into entangled photon pairs and vice versa:

$$\begin{aligned} H_{int} &= \frac{g}{\sqrt{2Vm_{axion}}} \sum_{k,|k|=m_{axion}/2} \frac{\omega}{2} \Bigg(\\ & -ib^*(0,t)\left(a_R(k,t)a_R(-k,t) - a^L(k,t)a^L(-k,t)\right) \\ & +ib(0,t)\left(a_R^*(k,t)a_R^*(-k,t) - a_L^*(k,t)a_L^*(-k,t)\right)\Bigg). \end{aligned} \quad (7)$$

To put aside all nonprincipal details we further proceed with a dimensionless time variable

$$\tau = \frac{4}{g}\sqrt{2 \text{ Volume } m_{axion}}t \tag{8}$$

and collectivize creation and annihilation operators into some shift operators A and A^* , which change the half difference between axion particles and photon pairs on one unit step. So we get the simplest expression for the time dependence of all kinds of operators in the form:

$$O(\tau) = e^{\tau(A-A^*)}O(0)e^{-\tau(A-A^*)}. \tag{9}$$

algebra of shift (collectivized) operators

Algebra of shift operators is shown below

$$
\begin{aligned}
[A, n_{photon\ pairs}] &= -[A, n_{axions}] = A, \\
[A^*, n_{photon\ pairs}] &= -[A^*, n_{axions}] = -A^*, \\
[A, A^*] &= 2(n_{photon\ pairs} + N_{vac})(N_{total} - n_{photon\ pairs}) \\
&\quad - n_{photon\ pairs}(n_{photon\ pairs} + N_{vac} - 1)
\end{aligned}
$$

where N_{vac} is a number of available degrees of freedom of photon pairs, divided by two.

The both N_{vac} and N_{total} are positive integers, infinite with infinite volume, but a volume density should be finite.

For given N_{vac} and N_{total} a matrix representation of introduced new shift operators can be constructed. It appears that the matrix A consists only of upper second diagonal but square root arguments are not the well known customary natural numbers but a special cubic polynomial of them:

$$\begin{aligned} A_{n+1,n} \;\; &= \;\; \Big(-2(n^3 - n^2(1/2 - 2N_{vac} + N_{total}) \\ &\quad - n(2N_{vac} - 1/2) - N_{vac}N_{total})\Big)^{1/2}, \\ &\quad n = 0, 1, 2, ..., N_{total} - 2, N_{total} - 1. \end{aligned}$$

The matrix A^* is a transposed matrix A.

Computer simulations with matrix exponents of Hamiltonian's $(N_{total}+1)*(N_{total}+1)$ matrices are expected to give the picture of the system time evolution in the physical limit $N_{total}\to\infty$.

It appears, that the probability for an axionic condensate to remain in initial state quickly drops to almost zero in a time of order

$$\tau_{dropping}\approx\sqrt{\frac{2}{N_{total}+N_{vac}}}\mathrm{EllipticK}\left(\frac{N_{vac}}{N_{total}+N_{vac}}\right),$$

then stays at almost zero value for the time

$$\tau_{period}\approx\sqrt{\frac{2}{N_{total}+N_{vac}}}\mathrm{EllipticK}\left(\frac{N_{total}}{N_{total}+N_{vac}}\right),$$

nonlinear oscillatory regime

and at last surges to almost one in the same dropping time τ (which now is the raising time), entering into a highly nonlinear oscillatory regime. The nonlinearity is so high that the shown elliptic functions rather good for the evaluation of the time points $\tau_{dropping}$ and τ_{period}, have nothing in common with the computer simulations in intermediate time points.

A very weak axion – photon coupling leads to ratio $N_{vac}/N_{total} \approx 0$, and we can get for the extinction time of an axion Bose-Einstein condensate the formula

$$t_{extinction} = \frac{2}{\pi}\sqrt{\frac{2\mathrm{Volume}}{g^2 m_{axion} N_{total}}}.$$

extinction time evaluated

Due to disentangling of photon pairs in real environment any resurrection after the inevitable extinction will not be allowed. The mentioned oscillatory regime does not survive in reality. With a typical input

$$g = 10^{-15}\mathrm{GeV}^{-1},$$
$$\frac{m_{axion} N_{total}}{V} = 0.45 \frac{\mathrm{GeV}}{\mathrm{cm}^3}$$

the resulting extinction time is $1.25 * 10^4$ years.

Thank you for your attention!

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