Hairy black holes and self-accelerating cosmologies in the ghost-free bigravity

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M.S.V. arXiv:1205.5713

M.S.V., arXiv:1202.6682, to appear in Phys.Rev.D;

M.S.V., JHEP 1201 (2012) 035;

A.Chamseddine, M.S.V., Phys.Lett. B704 (2011) 652.

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Massive Gravity

- A deformation of GR that allows to explain the observed universe acceleration $\Rightarrow m \sim 1/$(cosm. horizon size).
- Problems: does not reduce to GR in the weak field when $m \to 0$ (VdVZ discontinuity), has a ghost, no uniqueness.
- Remedies seem to exist for some of these problems (Vainstein mechanism). Very recently a class of models has been discovered that seem to be free of the ghost.
- We wish to study black holes and cosmologies in these models.

I. Massive gravity in D=4

Non-linear Pauli-Fierz

4D manifold with two metrics

$$g_{\mu\nu}(x) \qquad \text{and} \qquad f_{\mu\nu}(x) = \eta_{AB}\partial_\mu X^A(x)\partial_\nu X^B(x)$$

and the action

$$S = \frac{1}{8\pi G}\int\left(-\frac{1}{2}R + m^2\mathcal{L}_{\rm int}\right)\sqrt{-g}\,d^4x + S_{\rm (mat)}$$

where $\mathcal{L}_{\rm int}$ is a scalar function of $H^\alpha_{\ \beta} = g^{\alpha\sigma}f_{\sigma\beta} - \delta^\alpha_\beta$

$$\mathcal{L}_{\rm int} = \frac{1}{8}((H^\alpha_{\ \alpha})^2 - H^\alpha_{\ \beta}H^\beta_{\ \alpha}) + O((H^\alpha_{\ \beta})^3)$$

Theory is not unique, but has a unique weak field limit.

EOM for $g_{\mu\nu}, X^A$

$$G_{\mu\nu} = m^2 T_{\mu\nu} + 8\pi G T_{\mu\nu}^{(\mathrm{mat})}$$

with

$$T_{\mu\nu} = 2\frac{\partial \mathcal{L}_{\mathrm{int}}}{\partial g^{\mu\nu}} - g_{\mu\nu}\mathcal{L}_{\mathrm{int}},$$

varying with respect to X^A gives

$$\nabla^\mu T_{\mu\nu} = 0.$$

The matter equations imply

$$\nabla^\mu T_{\mu\nu}^{(\mathrm{mat})} = 0.$$

In the unitary gauge, $X^\alpha = x^\alpha$ and $f_{\mu\nu} = \eta_{\mu\nu}$, in the weak field limit $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ one recovers the

Pauli-Fierz equations

$$\frac{1}{2}\{-\Box h_{\mu\nu} + \ldots\} = \frac{1}{2} m^2 (h_{\mu\nu} - h\eta_{\mu\nu}) + 8\pi G T^{(\mathrm{mat})}_{\mu\nu}$$

which imply 4 constraints

$$\partial^\mu h_{\mu\nu} - \partial_\nu h = 0.$$

Taking the trace gives the fifth constraint

$$3m^2 h = 16\pi G T^{(\mathrm{mat})}$$

$\Rightarrow$ there remain 5 degrees of freeedom of massive graviton.

For generic $g_{\mu\nu}$ there are 5 degrees + 1 extra state with negative norm – Boulevard-Deser ghost.

II. Ghost free theories

The RGT massive gravity

$$\mathcal{L}_{\rm int} = \frac{m^2}{2}(K^2 - K^\nu_\mu K^\mu_\nu) \quad \text{with} \quad \boxed{K^\mu_\nu = \delta^\mu_\nu - \sqrt{g^{\mu\sigma} f_{\sigma\nu}}}$$

is claimed to be ghost-free and unique up to 2-parameter deformations /de Rham, Gabadadze, Tolley '10/.

For: Hassan, Rosen (2011); Mirbabayi (2011); Golovnev (2011); Hassan, Rosen (2012); Kluson (2012); Hassan, Schmidt-May, von Strauss (2012)

Against: Creminelli, Nicolis, Papucci, Trincherini (2005); Chamseddine, Mukhanov (2010-2011)

No asymptotically flat black holes. No Schwarzschild.

The ghost-free bigravity

$$S = -\frac{1}{16\pi G}\int R\sqrt{-g}\,d^4x - \frac{1}{16\pi\mathcal{G}}\int \mathcal{R}\,\sqrt{-f}d^4x$$
$$+ \frac{\sigma}{8\pi G}\int \mathcal{L}_{\rm int}\sqrt{-g}\,d^4x + S_{\rm m}[g_{\mu\nu}, \text{matter}]\,,$$

$$\mathcal{L}_{\rm int} = \frac{1}{2}(K^2 - K^\nu_\mu K^\mu_\nu) + \frac{c_3}{3!}\,\epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\sigma}K^\mu_\alpha K^\nu_\beta K^\rho_\gamma$$
$$+ \frac{c_4}{4!}\,\epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\delta}K^\mu_\alpha K^\nu_\beta K^\rho_\gamma K^\sigma_\delta\,,$$

$\sigma = m^2\cos^2\eta$ and $\mathcal{G} = G\tan^2\eta$ (massive gravity for $\eta \to 0$)

$K^\mu_\nu = \delta^\mu_\nu - \gamma^\mu{}_\nu\,,$ $\boxed{\gamma^\mu{}_\sigma\gamma^\sigma{}_\nu = g^{\mu\sigma}f_{\sigma\nu}}$ /Hassan, Rosen '11/

Taking the square root

Two tetrads e^{ν}_{B} and ω^{A}_{μ}

$$g^{\mu\nu} = \eta^{AB} e^{\mu}_{A} e^{\nu}_{B}\,, \qquad\qquad f_{\mu\nu} = \eta_{AB}\omega^{A}_{\mu}\omega^{B}_{\nu}\,,$$

the local $SL(1,3) \times SL(1,3) \Rightarrow$

$$\boxed{e^{\mu}_{A}\omega_{B\mu} = e^{\mu}_{B}\omega_{A\mu}} \qquad\qquad (\bullet)$$

Then

$$\gamma^{\mu}_{\ \nu} \equiv \sqrt{g^{\mu\sigma} f_{\sigma\nu}} = e^{\mu}_{A}\omega^{A}_{\nu}$$

Field equations

$$G^{\rho}_{\lambda} = m^2 \cos^2\eta\, T^{\rho}_{\lambda} + 8\pi G T^{(\mathrm{mat})\,\rho}{}_{\lambda}\,, \qquad \mathcal{G}^{\rho}_{\lambda} = m^2 \sin^2\eta\, \mathcal{T}^{\rho}_{\lambda}\,,$$

with

$$T^{\rho}_{\lambda} = \tau^{\rho}_{\lambda} - \delta^{\rho}_{\lambda}\mathcal{L}_{\mathrm{int}}\,, \qquad \mathcal{T}^{\rho}_{\lambda} = -\frac{\sqrt{-g}}{\sqrt{-f}}\,\tau^{\rho}_{\lambda}\,,$$

$$\tau^{\rho}_{\lambda} = (\gamma^{\sigma}_{\sigma} - 3)\gamma^{\rho}_{\lambda} - \gamma^{\rho}_{\sigma}\gamma^{\sigma}_{\lambda} - \frac{c_3}{2}\,\epsilon_{\lambda\mu\nu\sigma}\epsilon^{\alpha\beta\gamma\sigma}\gamma^{\rho}_{\alpha}K^{\mu}_{\beta}K^{\nu}_{\gamma} - \frac{c_4}{6}\,\epsilon_{\lambda\mu\nu\sigma}\epsilon^{\alpha\beta\gamma\delta}\gamma^{\rho}_{\alpha}K^{\mu}_{\beta}K^{\nu}_{\gamma}K^{\sigma}_{\delta}\,.$$

- Reduces to the RGT massive gravity for $\eta \to 0$ if $f_{\mu\nu}$ becomes flat.
- $g_{\mu\nu} = f_{\mu\nu} \Rightarrow T^{\mu}_{\nu} = \mathcal{T}^{\mu}_{\nu} = 0 \Rightarrow G^{\mu}_{\nu} = 0 \Rightarrow$ vacuum GR

Spherical symmetry

Most general case

$$e^0 = \frac{1}{Q}\,dt, \quad e^1 = \frac{1}{N}\,dr, \quad e^2 = R\,d\vartheta, \quad e^3 = R\sin\vartheta\,d\varphi$$

$$\omega^0 = aQ\,dt + cN\,dr, \quad \omega^1 = -cQ\,dt + bN\,dr,$$

$$\omega^2 = uRd\vartheta, \qquad \omega^3 = uR\sin\vartheta d\varphi$$

where a, b, c, Q, N, u, R functions of t, r. Two different cases:

- $c = f_{0r} \neq 0 \Rightarrow$ metrics are not simultaneously diagonal
- $c = f_{0r} = 0 \Rightarrow$ metrics are simultaneously diagonal

III. Self-accelerating cosmologies

A.H. Chamseddine, M.S. Volkov arXiv:1107.5504

G.D'Amico, C. de Rham, S. Dubovsky, G. Gabadadze, D. Pirtskhalava, A.J. Tolley arXiv:1108.5231

A.E. Gumrukcuoglu, C. Lin, S. Mukohyama arXiv:1109.3845

M.S. Volkov arXiv:1110.6153

M.S. Volkov arXiv:1205.5713

Non-diagonal $f_{\mu\nu}$

$$ds^2 = Q^2dt^2 - N^2dr^2 - R^2d\Omega^2,$$

$$df^2 = (aQ\,dt + cN\,dr)^2 - (bN\,dr - cQ\,dt)^2 - u^2R^2d\Omega^2.$$

FRW $\Rightarrow G^0_r = T^0_r = 0 \Rightarrow$

$$u = \frac{1}{c_3 + c_4}\left(2c_3 + c_4 - 1 \pm \sqrt{1 - c_3 + c_4 + c_3^2}\right)$$

$\Rightarrow T^0_0 = T^r_r = const. \Rightarrow 0 = \overset{(g)}{\nabla}_\mu T^\mu_\nu = 2(\dot{Q}/Q)(T^r_r - T^\theta_\theta)$ with

$$T^r_r - T^\theta_\theta = (c_3u - u - c_3 + 2)((u - a)(u - b) + c^2)$$

$$\Rightarrow \boxed{T^\mu_\nu = const \times \delta^\mu_\nu}$$

Equations

$$(A) \qquad G^\mu_\nu = \Lambda\delta^\mu_\nu + 8\pi G T^{(\mathrm{mat})\mu}{}_\nu$$

$$(B) \qquad \mathcal{G}^\mu_\nu = \tilde{\Lambda}\delta^\mu_\nu$$

$$(C) \qquad (u-a)(u-b)+c^2 = 0$$

$$\Lambda = m^2 \cos^2\eta\,(u-1)(c_3 u - u - c_3 + 3),$$

$$\tilde{\Lambda} = m^2 \sin^2\eta\,\frac{1-u}{u^2}\,(c_3 u - c_3 + 2)$$

$$8\pi G T^{(\mathrm{mat})\mu}{}_\nu = \mathrm{diag}[\rho(t), -P(t), -P(t), -P(t)]$$

Equations (A) decouple from (B)+(C)

Solutions for $g_{\mu\nu}$

FRW, cosmological term + matter $\Rightarrow$

$$ds^2 = \mathbf{a}(t)^2 \left(dt^2 - \frac{dr^2}{1-kr^2} - r^2 d\Omega^2 \right), \qquad k = 0, \pm 1$$

$$3\frac{\dot{\mathbf{a}}^2 + \mathrm{k}\mathbf{a}^2}{\mathbf{a}^4} = \Lambda + \rho, \qquad \Rightarrow \quad \text{self-acceleration}$$

Solution for $f_{\mu\nu}$

$$\mathcal{G}^{\mu}_{\nu} = \tilde{\Lambda}\delta^{\mu}_{\nu} \quad \text{and} \quad (u-a)(u-b)+c^2=0 \qquad (\star)$$

should be fulfilled by

$$df^2 = \mathbf{a}^2(a\,dt + c\,dr)^2 - \mathbf{a}^2(b\,dr - c\,dt)^2 - U^2 d\Omega^2$$

where $\mathbf{a}, U = uR$ are already fixed.

1. Choose U as new Schwarzschild coordinate.
2. Change $t \to T$ to get diagonal metric
3. Solve $\Rightarrow$ solution is AdS

$$df^2 = \Delta dT^2 - \frac{dU^2}{\Delta} - U^2 d\Omega^2, \qquad \Delta = 1 - \frac{\tilde{\Lambda}}{3}U^2$$

4. Choose $T(t,r)$ such that$(\star)$ is fulfilled

Determining $T(t, r)$

$$df^2 = (\theta^0)^2 - (\theta^1)^2 - U^2 d\Omega^2 = (\omega^0)^2 - (\omega^1)^2 - U^2 d\Omega^2$$ with

$$\theta^0 = \sqrt{\Delta} dT, \quad \theta^1 = \frac{dU}{\sqrt{\Delta}}, \quad \omega^0 = \mathbf{a}(a\, dt + c\, dr), \; \omega^1 = \mathbf{a}(-c\, dt + b\, dr).$$

$U = u\mathbf{a}(t) f_{\mathrm{k}}$. One has to have

$$\omega^0 = \sqrt{1+\alpha^2}\theta^0 + \alpha\theta^1, \qquad \omega^1 = \sqrt{1+\alpha^2}\theta^1 + \alpha\theta^0,$$

Collecting coefficients in front of dt, dr expresses a, b, c, α in terms of U and $T(t, r)$. Imposing

$$(u-a)(u-b) + c^2 = 0$$

then gives

Solution for $T(t,r)$

$$\frac{A_+A_-(\dot{U}T' - \dot{T}U' - u^2\mathbf{a}^2 + u\mathbf{a}\sqrt{A_+A_-}/\Delta)}{(\Delta^2\dot{T} + U')^2} = 0,$$

with $A_\pm = \Delta^2\dot{T} + U' \pm (\Delta^2 T' + \dot{U})$.

$$A_+ = 0$$

$$T = -\int \frac{\partial_+ U(x_+ + x_-, x_+ - x_-)}{\Delta^2(x_+ + x_-, x_+ - x_-)}\, dx_+ + F(x_-),$$

where

$$U(t,r) = u\mathbf{a}(t) f_k(r), \qquad \Delta^2 = 1 - (\tilde{\Lambda}/3)U^2$$

Boost between t, r and T, U is lightlike.

Properties of the solution

- Exists for any m, η, c_3, c_4.
- $g_{\mu\nu}$: FRW with open, closed or flat sections. Matter-dominated at early times, Λ-dominated at late time $\Rightarrow$ self-acceleration at late time.
- $f_{\mu\nu}$: AdS. When $\eta \to 0$, $\tilde{\Lambda} \sim \sin^2 \eta \to 0 \Rightarrow f_{\mu\nu}$ is flat.

$$df^2 = dT^2 - dU^2 - U^2 d\Omega^2 = dF^2 - 2dU dF - U^2 d\Omega^2,$$

where the Stueckelberg scalars $T = -U + F(t - r)$ where $U = u\mathbf{a}(t) f_{\rm k}(r) \Rightarrow$ massive gravity soluiton

- The two metrics are separately diagonal in two coordinate systems related by a lightlike boost.
- Static Schwarzschild-de Sitter with non-diagonal $f_{\mu\nu}$.

IV. More exotic cosmologies

M.S.V. JHEP 1201 (2012) 035

Diagonal metrics

$$ds^2 = dt^2 - \mathbf{a}^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right), \qquad k = 0, \pm 1$$

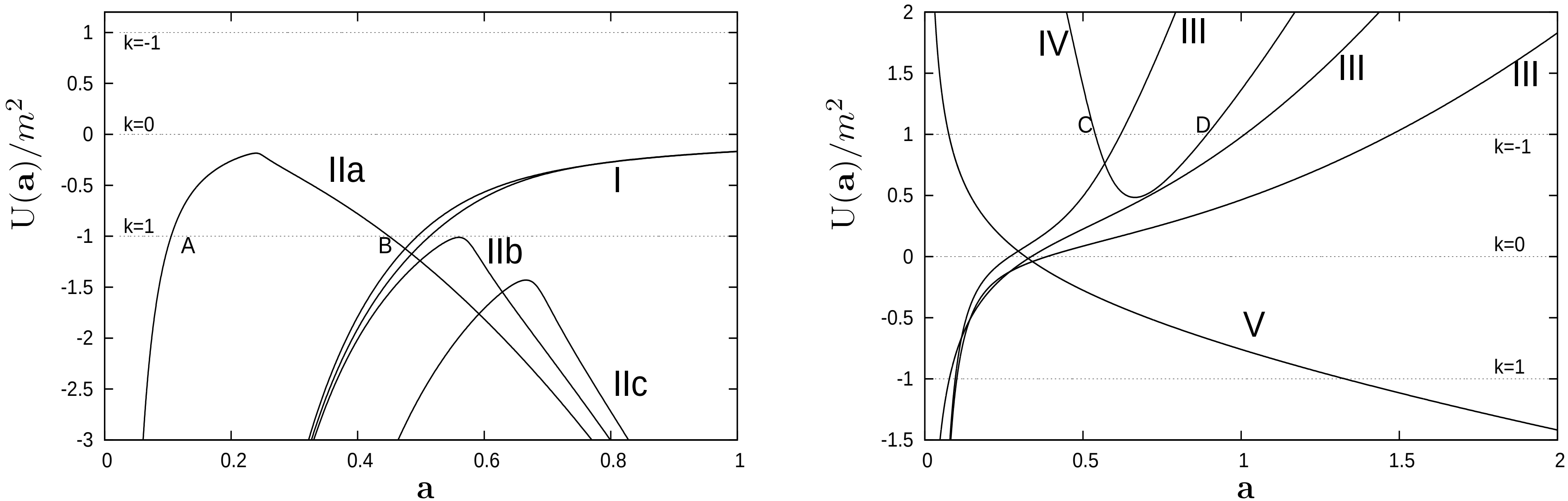
$$df^2 = \alpha^2(t)dt^2 - \sigma^2(t)\mathbf{a}^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right).$$

Equations reduce to the mechanical problem

$$\boxed{\dot{\mathbf{a}}^2 + \mathrm{U}(\mathbf{a}) = -k}$$

$\mathrm{U}(\mathbf{a})$ is expressed in terms of roots of an algebraic equations $\Rightarrow$ several solution branches.

Physical and exotic cosmologies



- physical: total energy $\rho_{\rm tot} = m^2 \cos^2\eta\, T^0_0 + \rho \approx \rho$ as $\mathbf{a} \to 0$.
- exotic: $m^2 \cos^2\eta\, T^0_0 \approx -\rho$, $\rho_{\rm tot} \sim \rho^{2/3}$ can be negative $\Rightarrow$ solutions can be non-singular.
- $f_{\mu\nu}$ is not flat for $\eta \to 0 \Rightarrow$ no massive gravity limit

V. Hairy black holes

M.S.V. arXiv:1202.6682

Static, diagonal metrics

$$ds^2 = Q^2 dt^2 - \frac{dr^2}{N^2} - r^2 d\Omega^2, \quad df^2 = a^2 dt^2 - \frac{U'^2}{Y^2} dr^2 - U^2 d\Omega^2$$

Q, N, Y, U, a are 5 functions of r, they fulfill 5 equations

$$\begin{aligned}
G^0_0 &= m^2 \cos^2\eta\, T^0_0, \\
G^r_r &= m^2 \cos^2\eta\, T^r_r, \\
\mathcal{G}^0_0 &= m^2 \sin^2\eta\, \mathcal{T}^0_0, \\
\mathcal{G}^r_r &= m^2 \sin^2\eta\, \mathcal{T}^r_r, \\
T^{r\,\prime}_r &+ \frac{Q'}{Q}(T^r_r - T^0_0) + \frac{2}{r}(T^\vartheta_\vartheta - T^r_r) = 0.
\end{aligned}$$

Equations

$$\frac{2NN'}{r} + \frac{N^2-1}{r^2} + m^2\cos^2\eta\left(\alpha_1\frac{N}{Y}U' + \alpha_2\right) + \rho = 0,$$

$$\frac{2N^2Q'}{Qr} + \frac{N^2-1}{r^2} + m^2\cos^2\eta\left(\alpha_1\frac{a}{Q} + \alpha_2\right) - P = 0,$$

$$\{Y^2 - 1 + m^2\sin^2\eta\,\alpha_3\}NU' + 2UNYY' + m^2\sin^2\eta\,Y\alpha_4 = 0,$$

$$\{a(Y^2-1) + m^2\sin^2\eta\,\alpha_5\}U' + 2UY^2a' = 0,$$

$$\alpha_6U' + \alpha_7a' = 0,$$

where $\alpha_1 \ldots \alpha_7$ are

$$
\begin{aligned}
\alpha_1 &= 3 - 3c_3 - c_4 + \frac{2(c_4 + 2c_3 - 1)U}{r} - \frac{(c_4 + c_3)U^2}{r^2}, \\
\alpha_2 &= 4c_3 + c_4 - 6 + \frac{2(3 - c_4 - 3c_3)U}{r} + \frac{(c_4 + 2c_3 - 1)U^2}{r^2}, \\
\alpha_3 &= c_4 U^2 - 2(c_3 + c_4)rU + (c_4 + 2c_3 - 1)r^2, \\
\alpha_4 &= (3 - c_4 - 3c_3)r^2 - (c_4 + c_3)U^2 + (4c_3 + 2c_4 - 2)rU, \\
\alpha_5 &= [(a - Q)c_4 - Qc_3]U^2 + [2(2Q - a)c_3 + (Q - a)c_4 - Q]rU, \\
&+ [(2a - 3Q)c_3 + (a - Q)c_4 + 3Q - a]r^2, \\
\alpha_6 &= Q' N[(3c_3 + c_4 - 3)r^2 + (2(1 - c_4 - 2c_3))Ur + (c_4 + c_3)U^2], \\
&+ 2Q(Y - N)[(3 - c_4 - 3c_3)r + (c_4 + 2c_3 - 1)U], \\
&+ 2a(N - Y)[(1 - c_4 - 2c_3)r + (c_4 + c_3)U], \\
\alpha_7 &= Y[(3 - c_4 - 3c_3)r^2 + 2(c_4 + 2c3 - 1)Ur - (c_4 + c_3)U^2].
\end{aligned}
$$

Background black holes

$$f_{\mu\nu} = C^2 g_{\mu\nu}, \qquad ds^2 = N^2 dt^2 - \frac{dr^2}{N^2} - r^2 d\Omega^2,$$

$$N^2 = 1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}, \qquad \Lambda = m^2 (C-1)(a_2 C^2 + a_1 C + a_0),$$

where C is a root of

$$(C-1)(b_3 C^3 + b_2 C^2 + b_1 C + b_0) = 0,$$

and a_k, b_s depend on c_3, c_4, η. If $\eta = 1$, $c_3 = 0.1$, $c_4 = 0.3$,

$$\begin{aligned} \{C_1, C_2, C_3, C_4\} &= \{1;\ -0.6458\,; 2.6333\,; -8.5566\}, \\ \frac{\Lambda(C_k)}{m^2} &= \{0;\ -3.0559;\ -1.1812;\ +21.5625\}. \end{aligned}$$

$\Rightarrow$ Schwarzschild, SdS, SAdS

U, a backgrounds

$$N^2 = 1 + m^2\cos^2\eta((1-2c_3-c_4)U^2 - \frac{2M}{r}$$
$$+ (3c_3+c_4-3)Ur + (2 - \frac{4}{3}c_3 - \frac{1}{3}c_4)r^2),$$
$$\frac{Q}{N} = a\frac{m^2\cos^2\eta}{2}\int_{r_1}^{r}\frac{dr}{xN^3}\mathcal{F}, \quad Y = \frac{m^2\sin^2\eta}{2U}\int_{r_2}^{r}\frac{dr}{N}\mathcal{F},$$
$$\mathcal{F} = (c_4 - 3 + 3c_3)x^2 + 2(1-2c_3-c_4)Ux + (c_3+c_4)U^2$$

U, a, M, r_1, r_2 constants.

$g_{\mu\nu}$ approaches AdS as $r \to \infty$ in the leading order.

$f_{rr} = 0 \Rightarrow f_{\mu\nu}$ is degenerate. If $U \to const$ as $r \to \infty$ then the proper volume is finite – spontaneous compactification.

Event horizon at $r = r_h$

$$N^2 = \sum_{n\geq 1} a_n (r - r_h)^n, \quad Y^2 = \sum_{n\geq 1} b_n (r - r_h)^n, \quad U = u r_h + \sum_{n\geq 1} c_n (r - r_h)^n$$

a_n, b_n, c_n depend on one free parameter u (and $\epsilon = \pm 1$).

- Horizon is common for both metrics
- Set of all black holes is one-dimensional and labeled by $u = U(r_h)/r_h$ = ratio of the even horizon radius measured by $f_{\mu\nu}$ to that measured by $g_{\mu\nu}$.

Horizon temperatures

$$g_{00} = Q^2 = q^2\{r - r_h + \sum_{n\geq 2} c_n (r-r_h)^n\}, \qquad f_{00} = a^2 = q^2 \sum_{n\geq 1} d_n (r-r_h)$$

ξ – timelike Killing. Surface gravities $(T = \kappa/2\pi)$

$$\kappa_g^2 = -\frac{1}{2}\, g^{\mu\alpha} g_{\nu\beta} \overset{(g)}{\nabla}_\mu \xi^\nu \overset{(g)}{\nabla}_\alpha \xi^\beta = \lim_{r\to r_h} Q^2 N'^2 = \frac{1}{4}\, q^2 a_1\,,$$

$$\kappa_f^2 = -\frac{1}{2}\, f^{\mu\alpha} f_{\nu\beta} \overset{(f)}{\nabla}_\mu \xi^\nu \overset{(f)}{\nabla}_\alpha \xi^\beta = \lim_{r\to r_h} a^2 \left(\frac{Y}{U'}\right)^{\prime 2} = \frac{1}{4}\, q^2 \frac{d_1 b_1}{(c_1)^2}\,.$$

$$\boxed{\frac{\kappa_g^2}{\kappa_f^2} = \frac{T_g^2}{T_f^2} = \frac{a_1 (c_1)^2}{d_1 b_1} = 1}$$

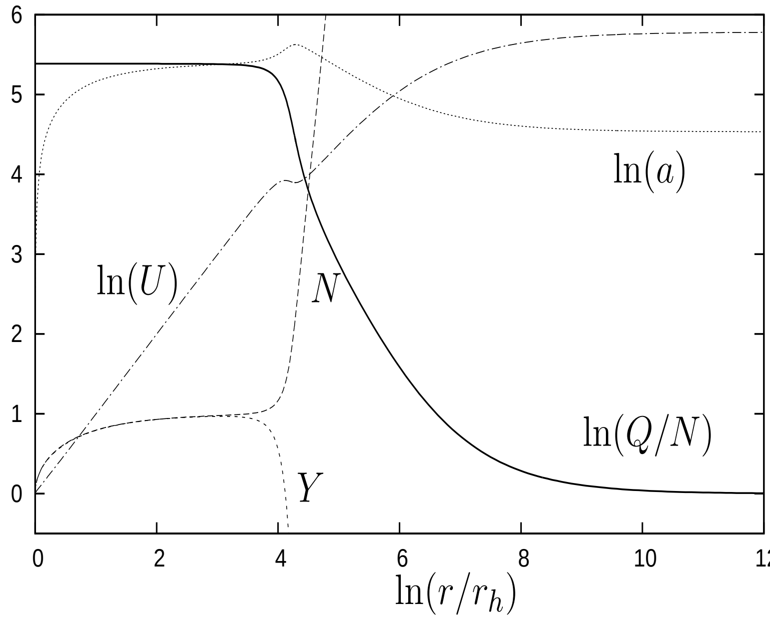
Deffayet, Jacobson ’11

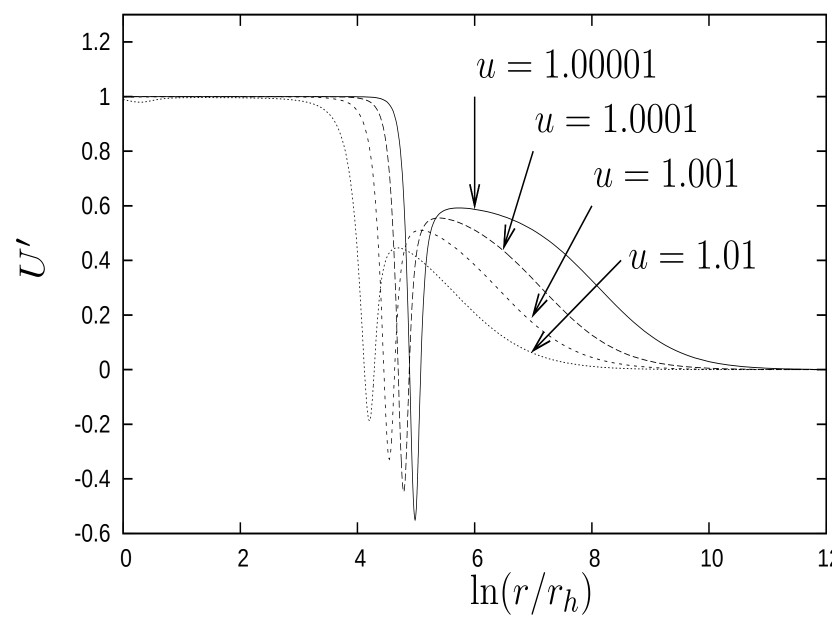
Strategy

- Solutions are obtained by integrating from the horizon for a given value of $u = U(r_h)$ towards large r.
- For $u = C_k$ they are the background black holes.
- For $u = C_k + \delta u$ they describe hairy deformations of the background black holes.

For $u = 1 + \delta u$ they describe hairy deformations of the Schwarzschild black hole.

Deforming Schwarzschild

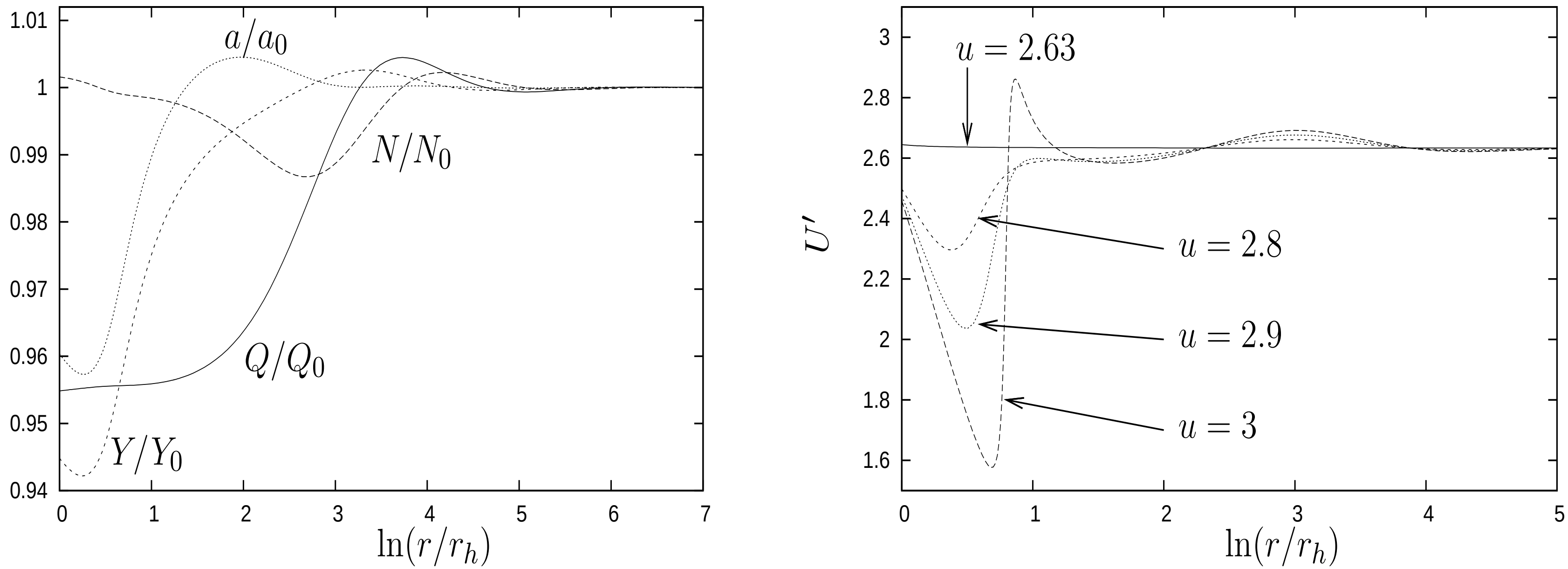




- Close to Schwarzschild for $r < r_{\max}(u)$ but approaches U, a for $r \to \infty$. Deformations stay small close to horizon but are always large at infinity.

Deforming Schwarzschild-AdS

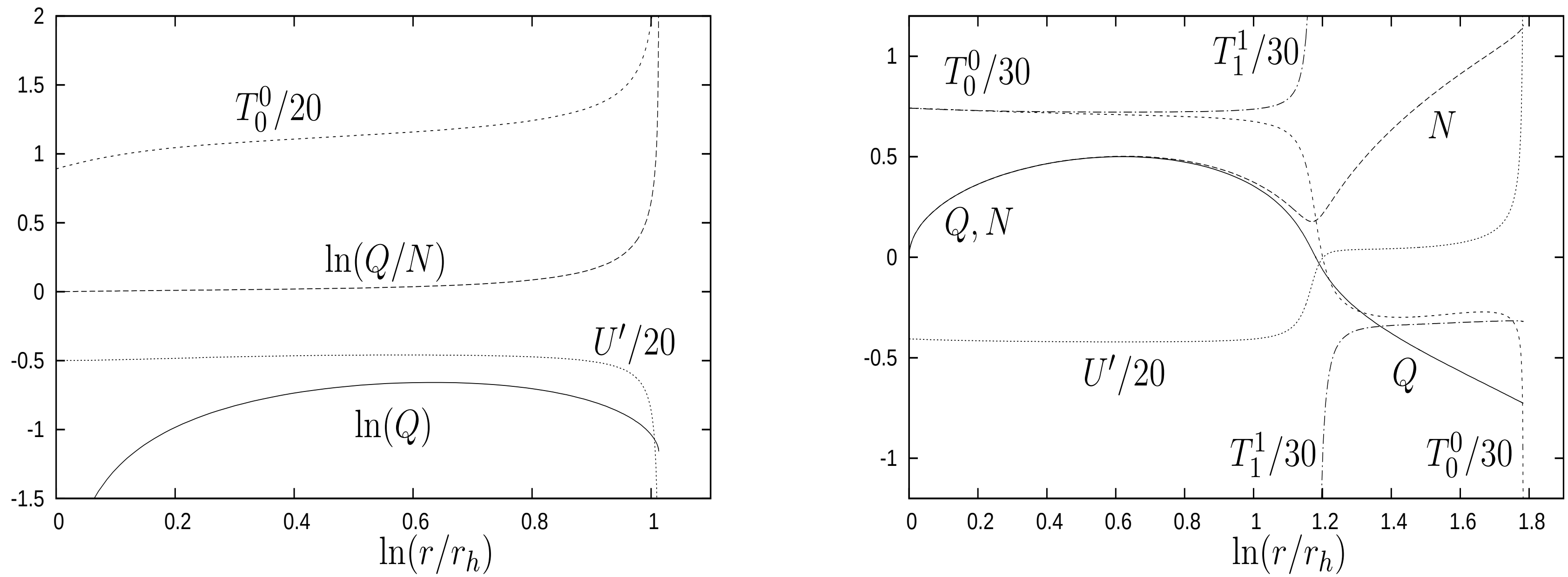
$u = C_k + \delta u$ $(k = 2, 3)$, deformations stay close to the horizon



N_0, Q_0, Y_0, a_0 correspond to the background AdS.
Hair is localized close to horizon.

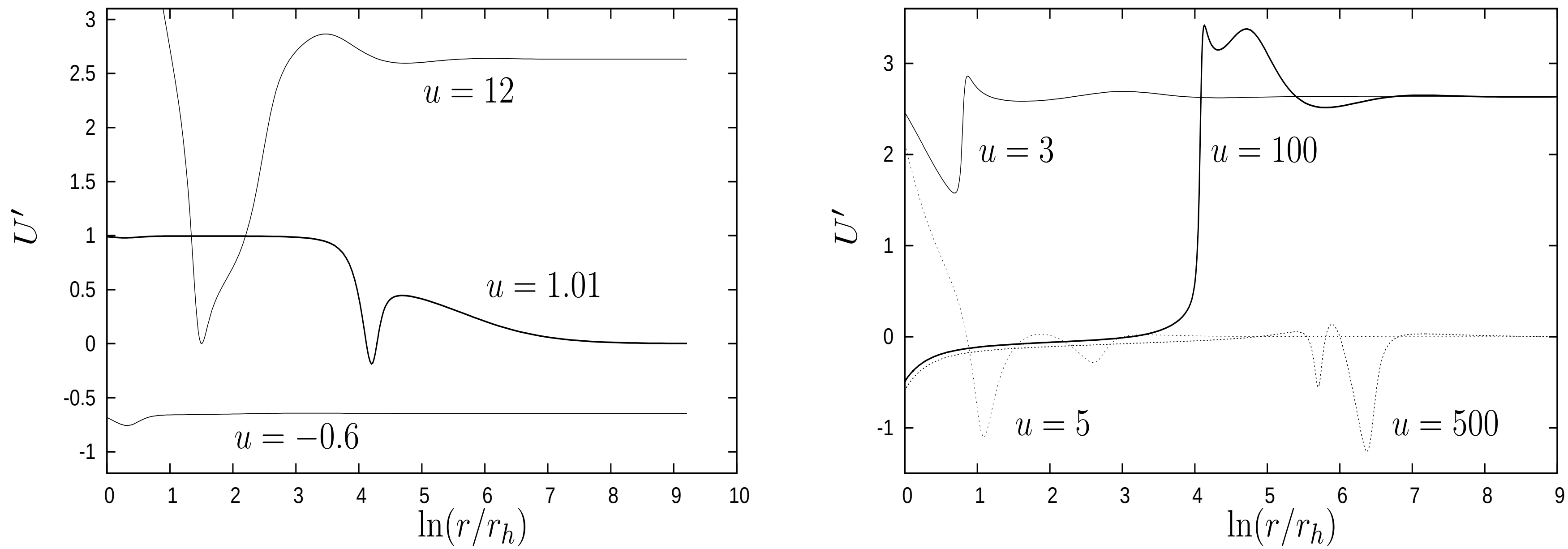
Deforming Schwarzschild-dS

$u = C_4 + \delta u$ with $\delta u < 0$ (left) and $\delta u > 0$ (right).



Deformations become singular at a finite distance from the horizon – solutions are compact and singular.

Generic solutions – arbitrary u



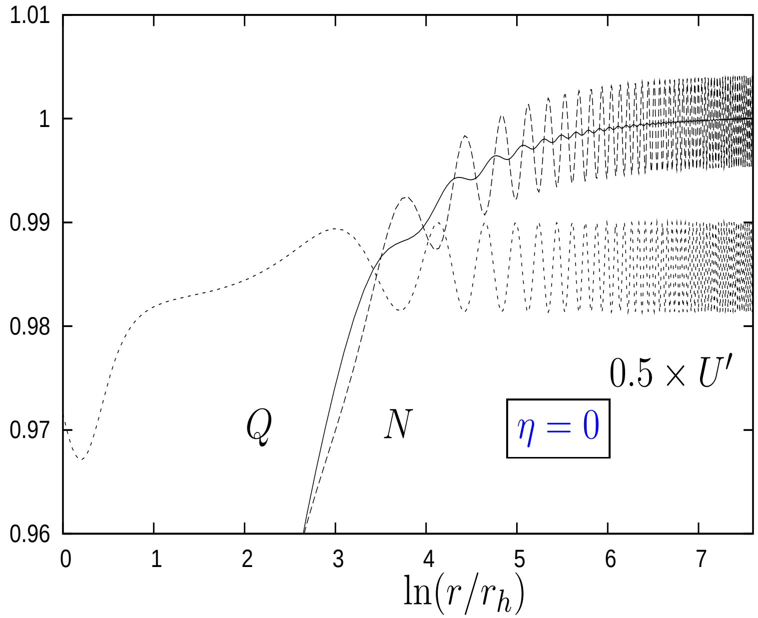
Generic solutions are either asymptotically AdS, or U, a, or they are compact and singular.

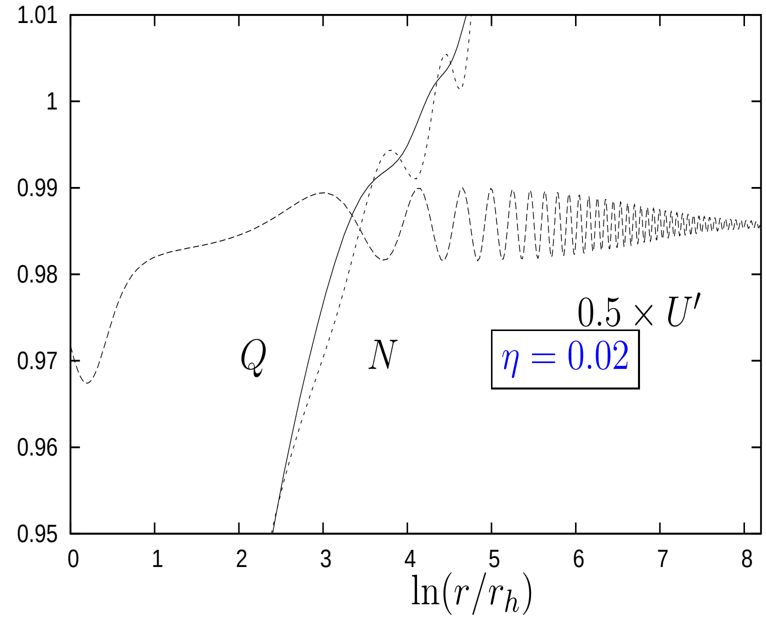
The only asymptotically flat is pure Schwarzschild.

The only asymptotically dS is pure dS.

Special solutions for $\eta = 0$

$f_{\mu\nu}$ is fixed and Schwarzschild





Tachyonic oscillations around flat metric at infinity

$$N = 1 + \delta N, \quad Q = 1 + \delta Q/r, \quad U = x + \delta U$$

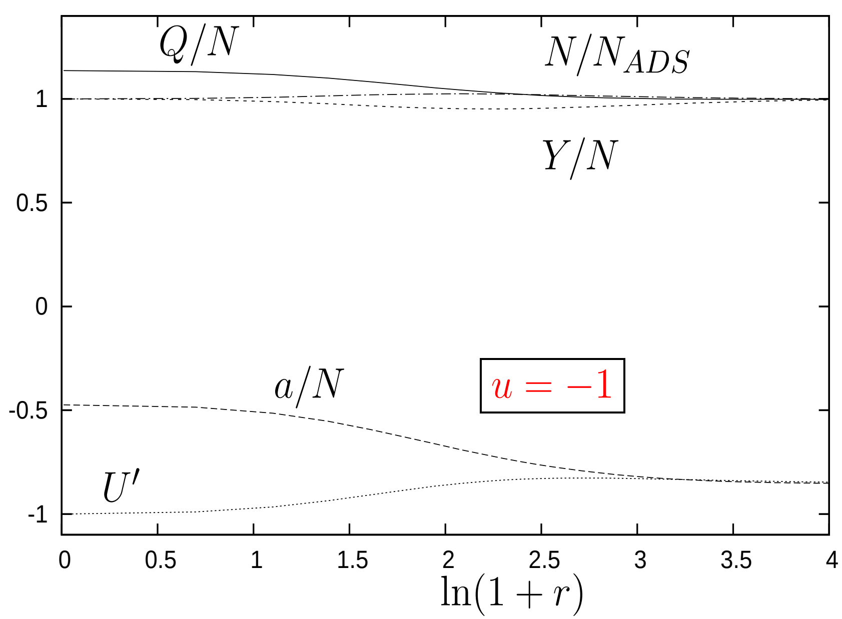
$$\delta N \sim \delta Q \sim \delta U = \exp\{i\sqrt{2}m(r + \frac{1}{2}\ln(r))\}$$

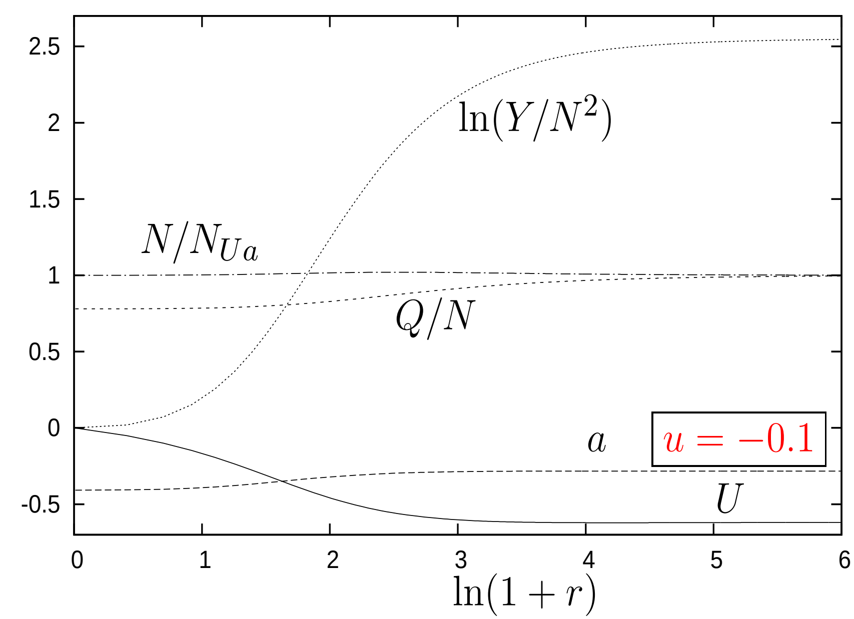
VI. Globally regular solutions

M.S.V. arXiv:1202.6682

Lumps of pure gravity

Solutions with a regular center at $r = 0$, curvature is bounded. At $r \to \infty$ the same asymptotic behavior as for black holes. Can be viewed as black hole remnants for $r_h \to 0$ – globally regular soliton deformations of AdS or U, a by the graviton massive modes.





Asymptotically flat stars

One adds $T^{(\mathrm{mat})\mu}_{\ \nu} = \mathrm{diag}(\rho, -P, -P, -P), \quad \rho(r) = \rho_\star(r - r_\star)$

Diagonal metrics

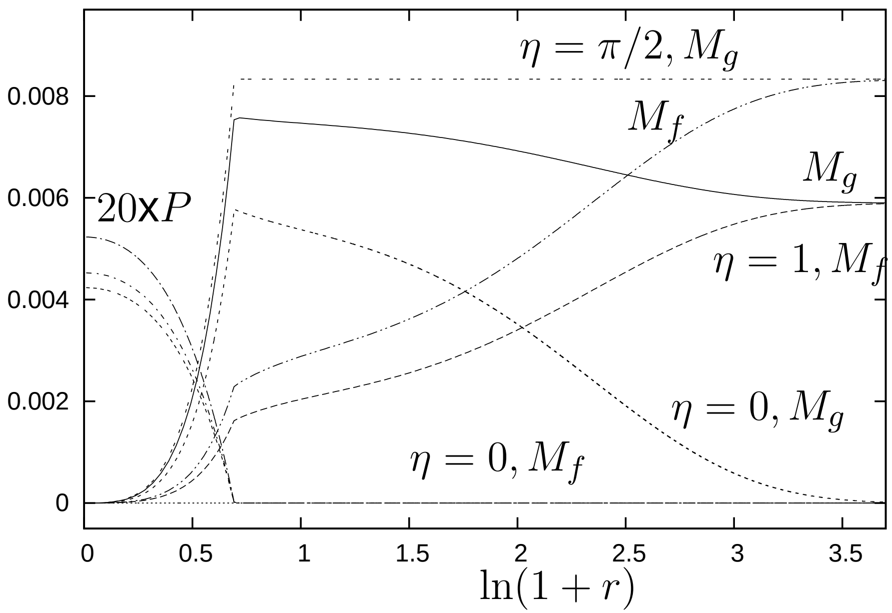
$$ds^2 \;=\; Q^2dt^2 - \frac{dr^2}{N^2} - r^2d\Omega^2, \quad df^2 = a^2dt^2 - \frac{U'^2}{Y^2}dr^2 - U^2d\Omega^2$$

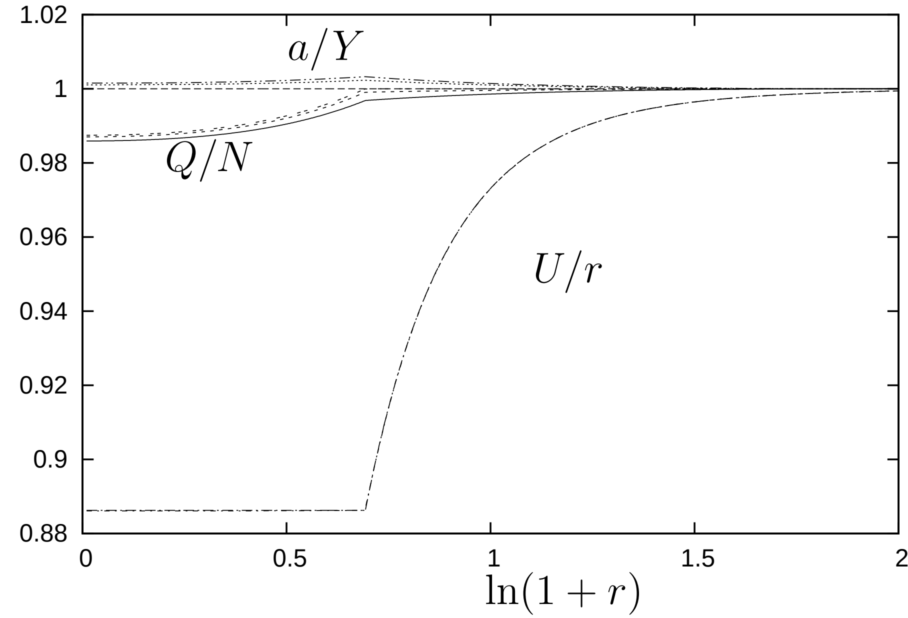
Regular origin: curvature is finite

Spatial infinity: metrics are flat + massless Newtonian + massive Yukawa corrections.

$\exists$ Globally regular solutions with such boundary conditions

Solutions





$$g^{rr} = N^2 = 1 - 2M_g(r)/r, \quad f^{rr} = Y^2/U'^2 = 1 - 2M_f(r)/r$$

$0 \leftarrow M_g, M_f \rightarrow A \sin^2 \eta$. If $m \rightarrow 0$ then $M_g \approx const$ near the horizon $\Rightarrow$ Vainstein mechanism in the ghost-free theory (with ghost – Deffayet, Babichev, Ziour 2010)

Summary of results

- Most general self-accelerating cosmologies in bigravity and massive gravity. Physical metrics – FRW, second metric – AdS.
- More exotic bigravity cosmologies for which the graviton contribution to the energy can be large and negative. Can be non-singular at $t = 0$.
- Hairy black holes of several different types. Not asymptotically flat (apart from pure Schwarzschild), reduce to the lumps of pure gravity when $r_h \to 0$.
- Static asymptotically flat solutions with matter (stars) exhibiting the Vainstein mechanism.