Aharonov-Bohm Oscillations of Resistance of Perforated Graphene

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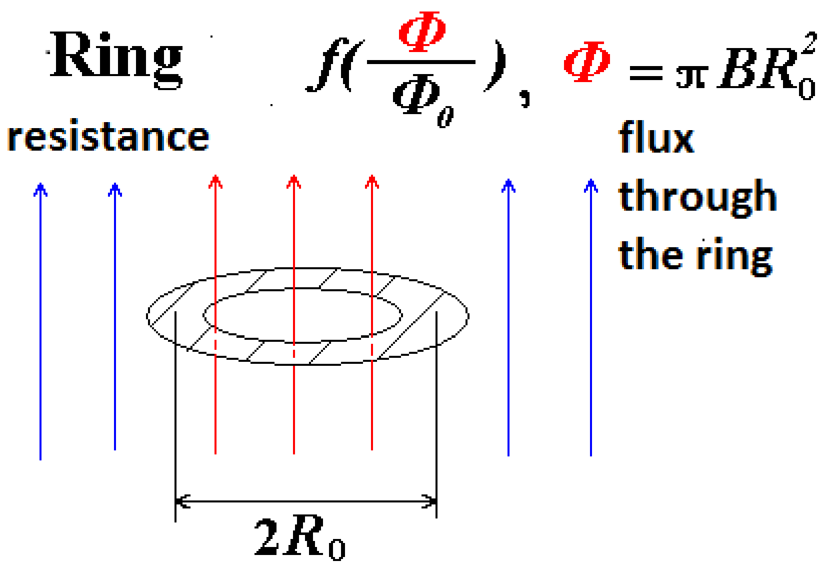
REMEMBER *the normal geometry* for the study of the Aharonov-Bohm effect in solid state transport:

the conducting ring in magnetic field *B* :



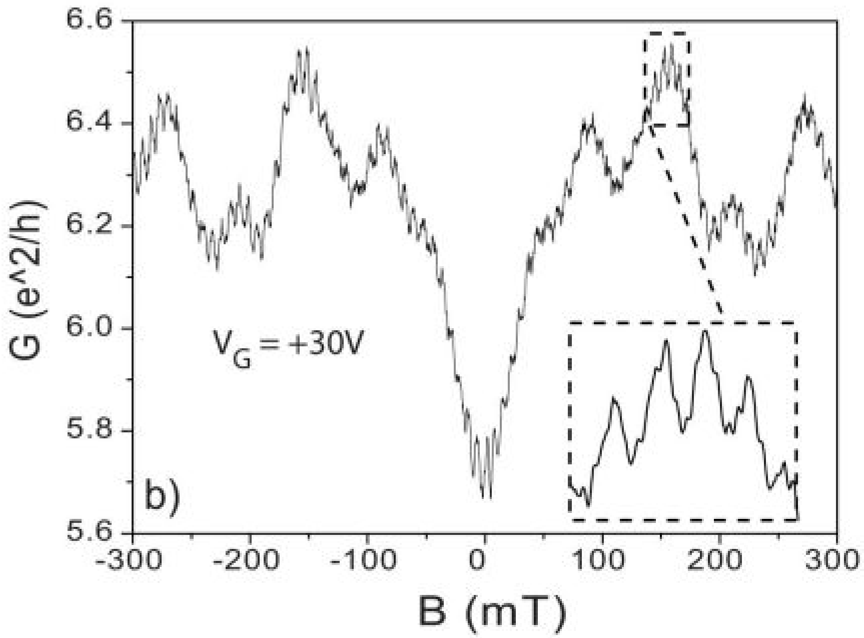
The AB effect:

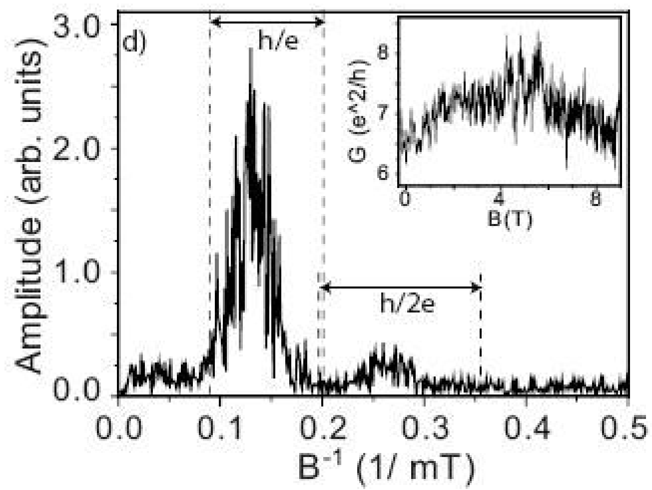
B-periodic (**Φ- periodic)** resistance oscillations $R=f(\Phi/\Phi_0)$ with the period $\Delta\Phi = \Phi_0$ (or $\Phi_0/2$)

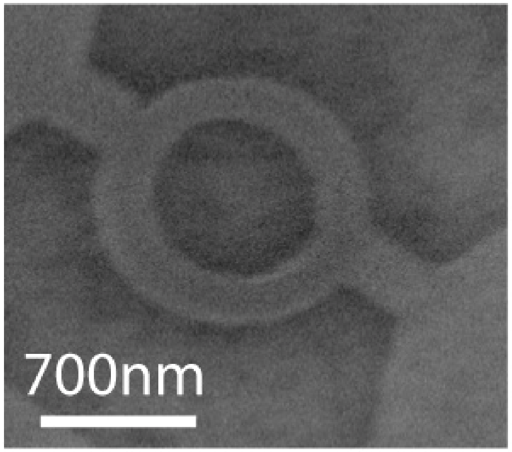
Φ is the flux through the electron orbit or through the ring (all orbits in narrow ring are the same),

$\Phi_0 = hc/e$ is the flux quantum

MOTIVATION: The Aharonov-Bohm effect in graphene ring. The first experiment.







They found existence of weak oscillations of magnetoresistance **with a flux periodicity of hc/e, while a contribution of hc/2e has been highly suppressed.**

S. Russo et al. Phys. Rev. B 77, 085413 (2008).

Processing of perforated graphene (Latyshev et al)

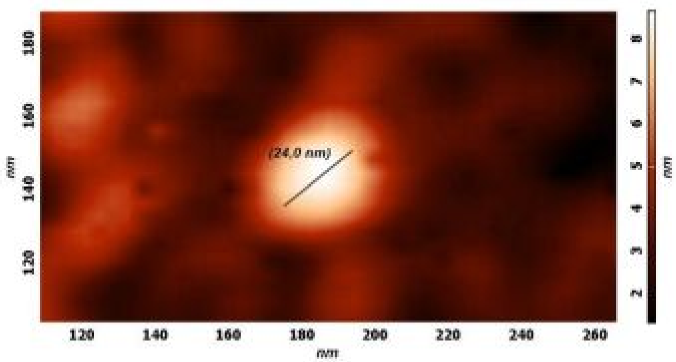
Nano-holes in nano-thin graphite (a, b) and graphene (c) produced:

by heavy ion irradiation (a, 24 nm, AFM image),

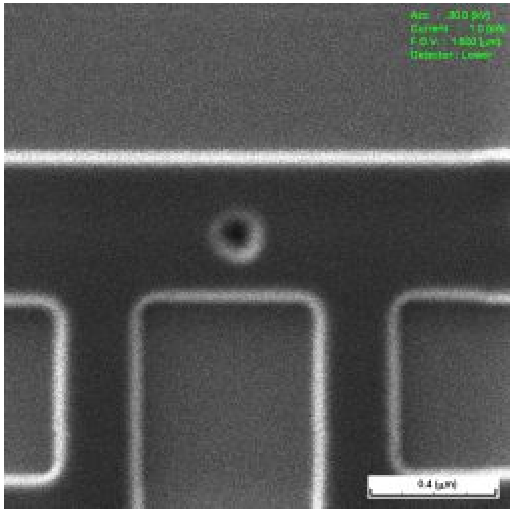
by FIB (b, 35 nm, SEM image) and

by helium ions (c, 20 nm, SHIM image)

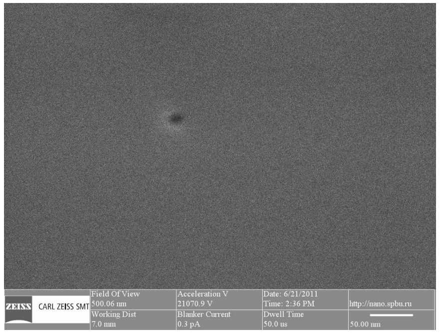
a)



b)

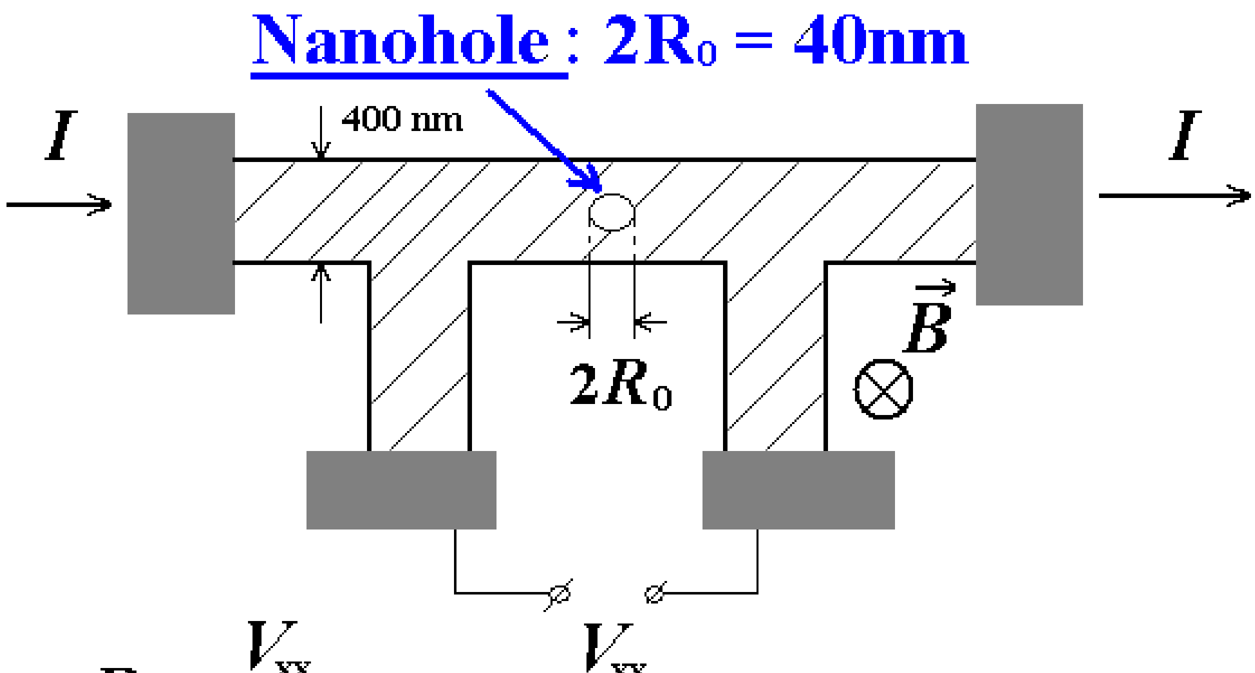


c)



Processing of perforated graphene

Typical sample made by FIB:
nano-hole in nano-thin graphite flake

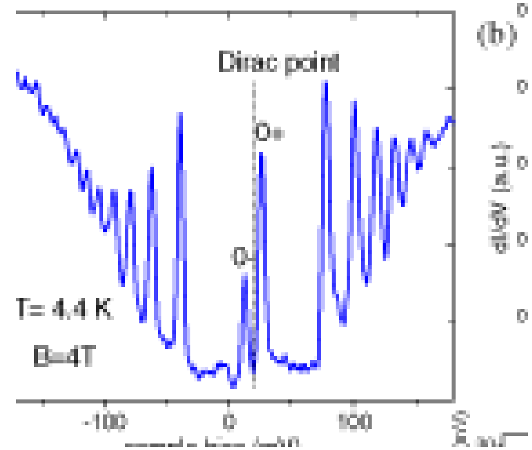


$$R_{xx} = \frac{V_{xx}}{I}$$

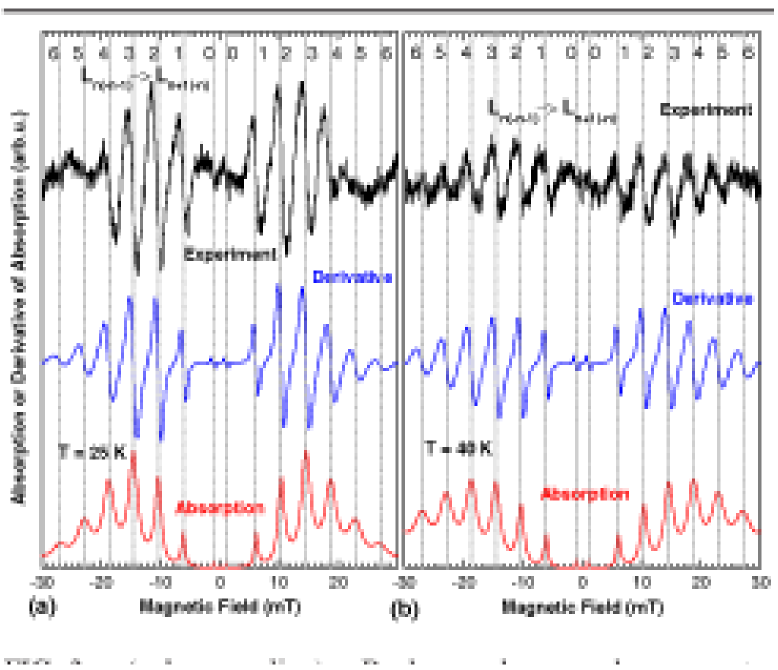
Graphene and graphite:

There is a graphene sheet on the natural graphite surface

G. Li, A. Luican, E. Andrei. PRL 2009



STM data



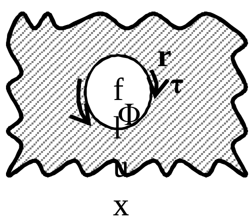
Cyclotron resonanse data P. Nuegebauer *et al*. PRL 2009

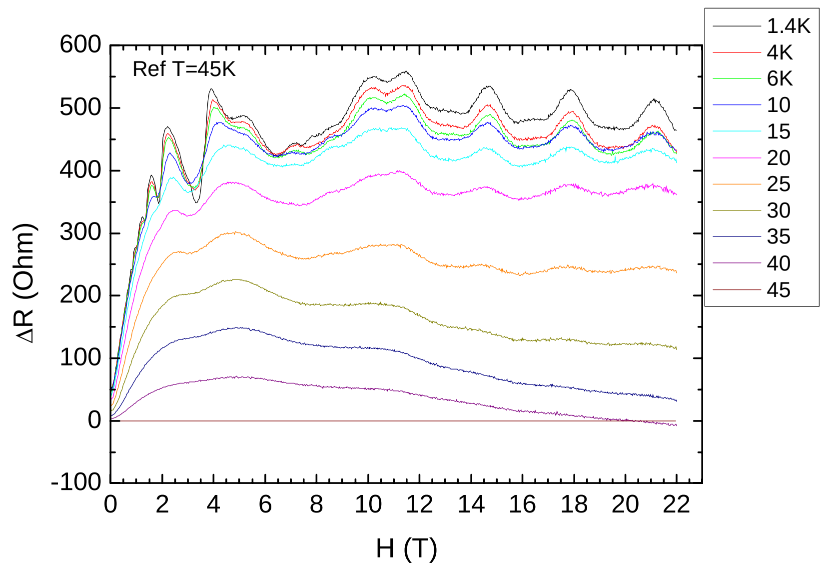
Landau quantization was observed down to 1 mT: estimate of mobility gives μ >10^7 cm^2/(V s)

Thin natural graphite is one of the most prominent system for studying of graphene

EXPERIMENT: B-periodic oscillations $R_{xx}(B)$ in nano-thin graphite single-hole sample b at B> 8 T: the Aharonov-Bohm effect in abnormal geometry?

3.3 T periodicity corresponds to flux quantum *hc/e* per hole.
Oscillations were observed up to 45K.

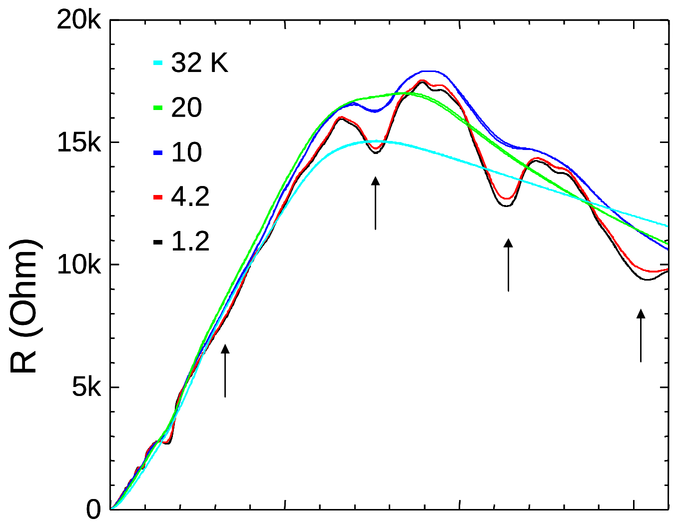


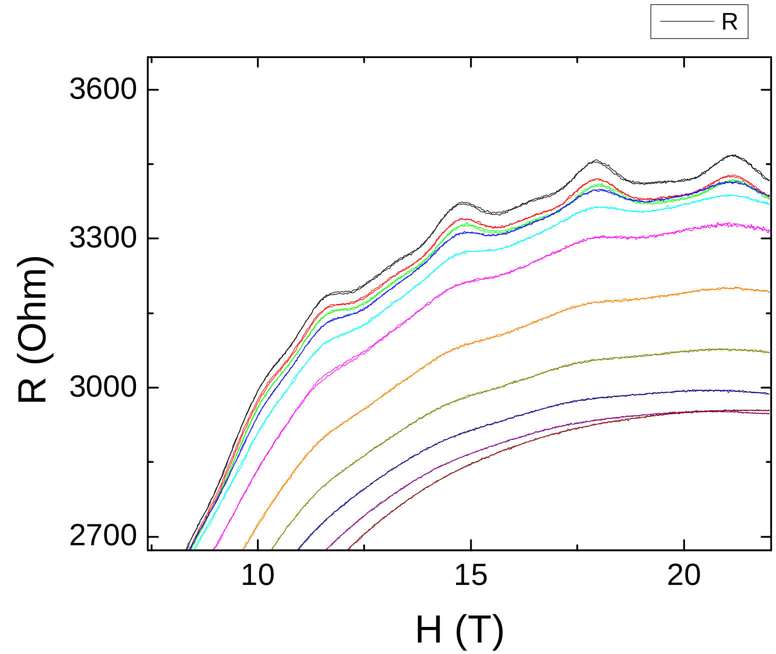


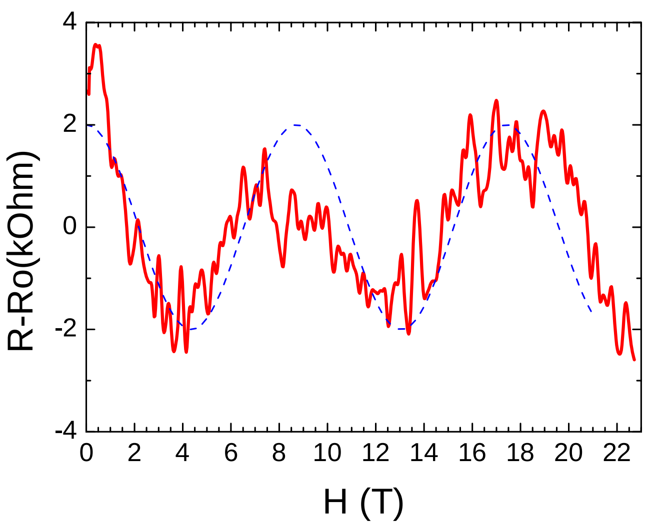
$$\Delta H \, \pi D^2 / 4 = \Phi_0$$

A ∝ exp (-kT/ Eo)
with ***Eo*** = 2 mV

AB effect in magnetoresistance for three types of nanohole structures



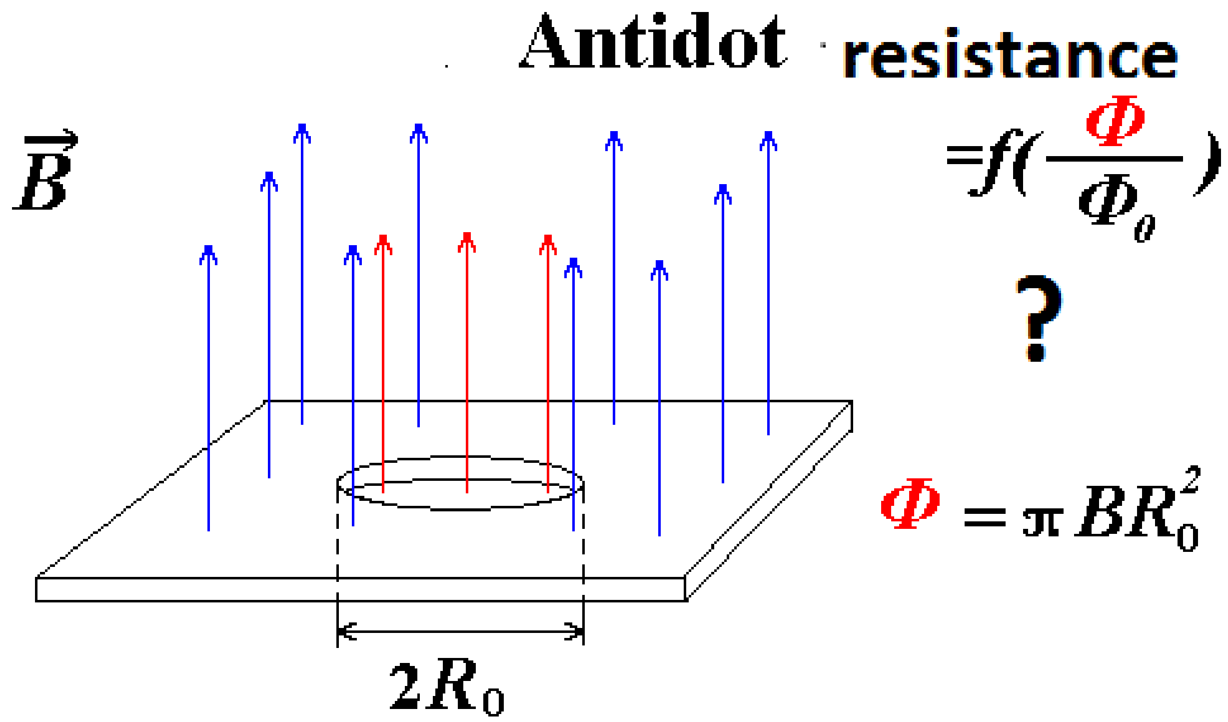




a) columnar defects with diameter D =24 nm
b) FIB made nanohole, D=37 nm
c) nanohole made using helium ion microscope, D=20 nm

Graphene antidot in magnetic field ***B:***

what is a possible mechanism of the AB effect ?

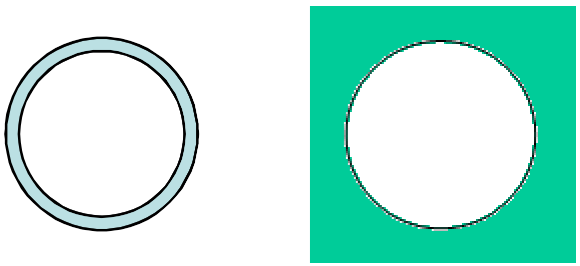


WHY ???

The magnetic edge states due to skipping orbits?

The Tamm-like edge states ?!

The Tamm-Dirac edge states in graphene

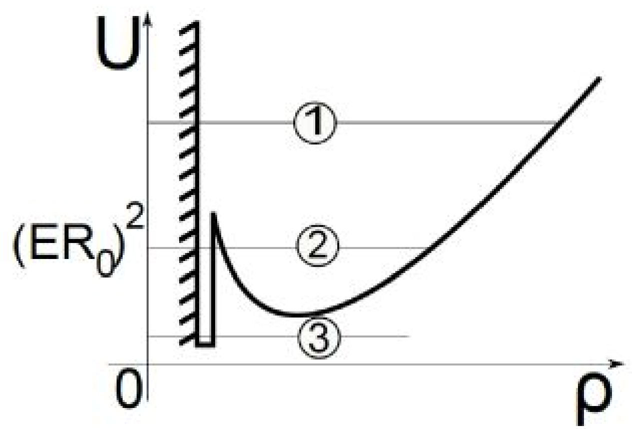


AB effect can exist **only in ring shape geometry** for trajectories localized inside the ring. For a hole geometry the averaged contribution of continuum of trajectories beyond the hole should smear out AB oscillations.

However, **the unique possibility for existence of AB effect on hole-type geometry is related with existence of edge states**. They can play a role of the ring.

$$\tau c(\vec{\sigma}\vec{p})\psi_\tau = E\psi_\tau \qquad [\psi_{1\tau} + i a^\tau e^{-i\alpha}\psi_{2\tau}]_\Gamma = 0$$

a is an edge parameter



Effective potential energy of electron in graphene antidot. Boundary condition is equivalent to δ-like potential well pinned to the boundary.

V.A. Volkov and I.V. Zagorodnev, 2009

The aims

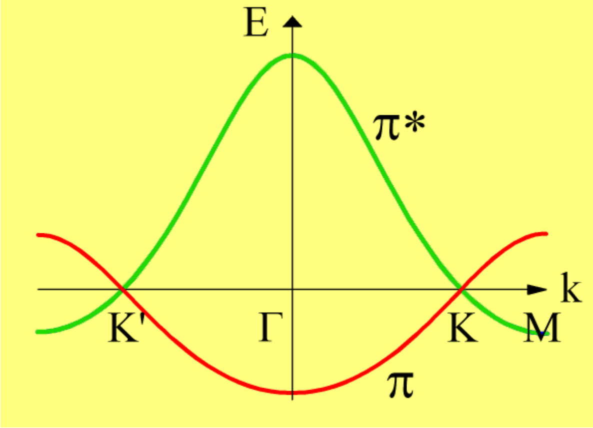
- AB effect in single antidot (nano-hole) samples: **new experiment results for nano-structured graphite and graphene**
- **The theory** of Tamm-Dirac states –

  the Tamm-type edge states in graphene semi-plane and antidot ***without magnetic field***.
- The same ***in magnetic field***.
- The AB-type effect taking the Tamm-Dirac states in graphene antidot into account.

The Weyl-Dirac fermions in graphene

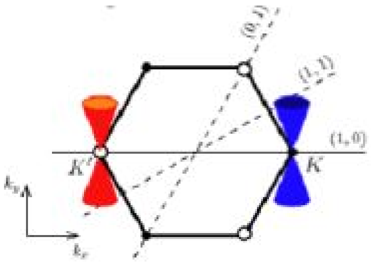
Near K-points:

$$\tilde{\mathrm{H}}_D = \begin{pmatrix} c\vec{\sigma}\vec{p} & 0 \\ 0 & -c\vec{\sigma}\vec{p} \end{pmatrix} = \begin{pmatrix} \mathrm{H}_W & 0 \\ 0 & -\mathrm{H}_W \end{pmatrix}$$



$$E_{\pm} = \pm cp$$

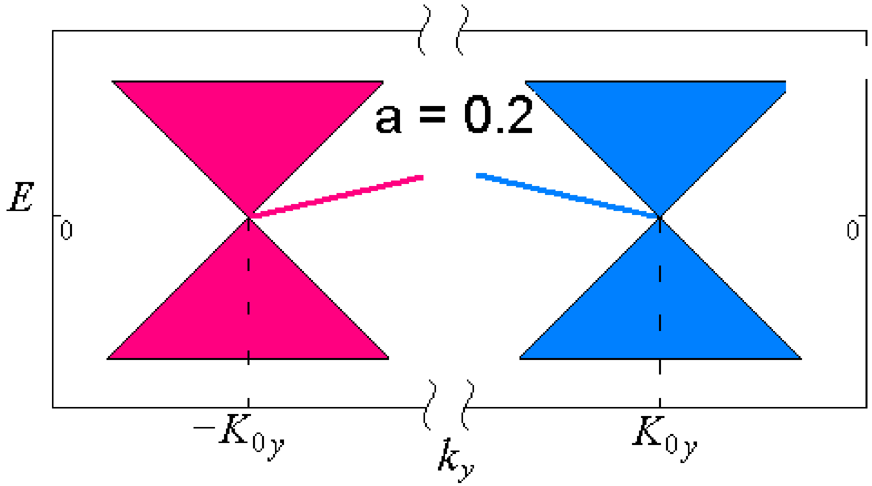
Weyl 2x2 Eq. ⇔ $\mathrm{H}_W = c\vec{\sigma}\vec{p}$



But: spin ⇒ pseudospin ("isospin")

$$c \approx 10^{6}\, м / c$$

The Tamm-Dirac states on semi-plane



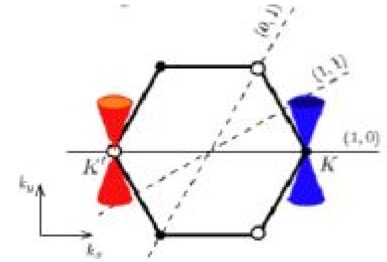
$$E_\tau(k_\parallel) = \frac{2a}{1+a^2}\tau v k_\parallel, \qquad \tau k_\parallel \geq 0$$

$$\tau = \pm 1$$

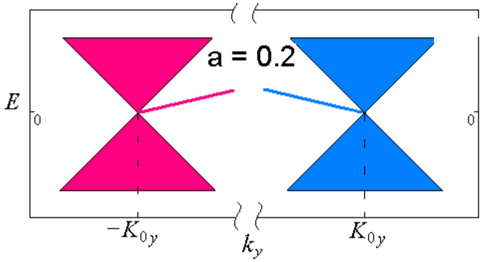
B=0: Tamm-Dirac states on graphene semi-plane

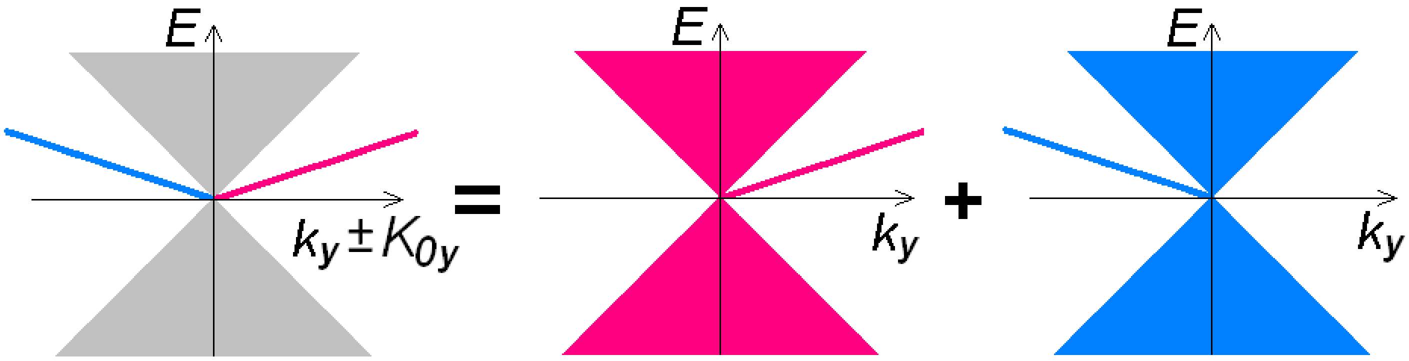
V.Volkov, I. Zagorodnev (2009)

Left and right valleys in bulk graphene:



The Tamm-type states on graphene semi-plane
In the left valley and the right valley
(a is an edge parameter):





The Tamm-Dirac states on semi-plane
In valley-reduced scheme:

B=0: the antidot as an quantum object

$$\tau c(\vec{\sigma}\vec{p})\psi_\tau = E\psi_\tau$$

$$[\psi_{1\tau} + ia^\tau e^{-i\alpha}\psi_{2\tau}]_\Gamma = 0$$

Envelope functions $\psi_\tau = \begin{pmatrix}\psi_{1\tau}\\ \psi_{2\tau}\end{pmatrix}$

Γ - edge

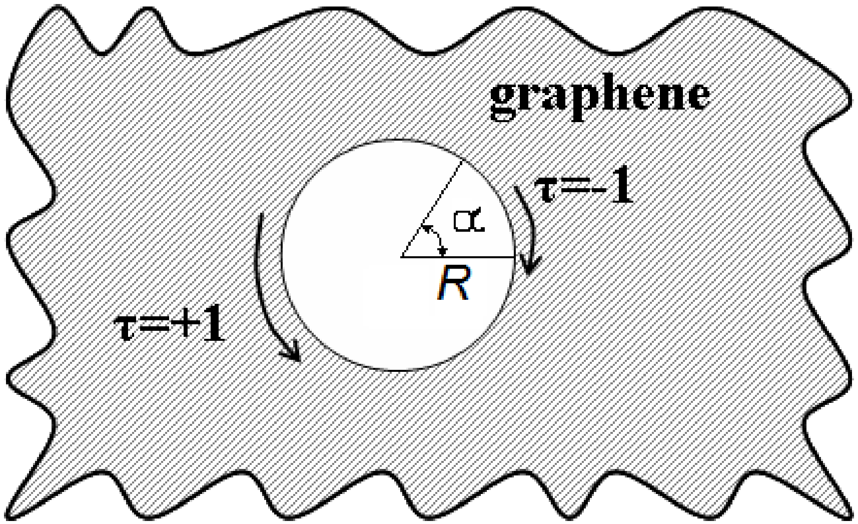
$\tau = 1$ - red valley

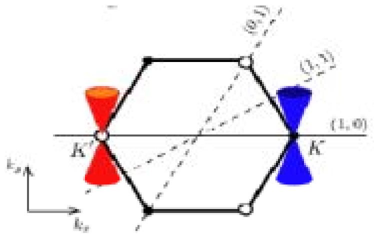
$\tau = -1$ - blue valley

a – edge an parameter

$\vec{\sigma} = (\sigma_x, \sigma_y)$ – Pauli matrixes

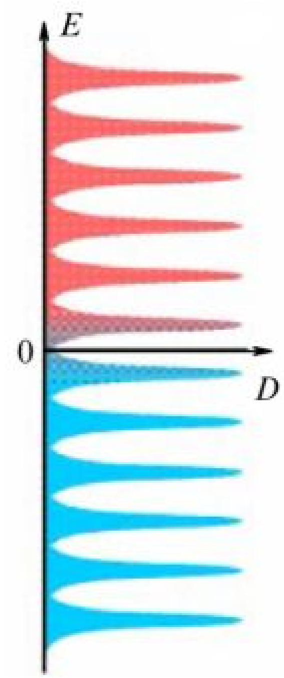
$c \approx 10^6 м/с$



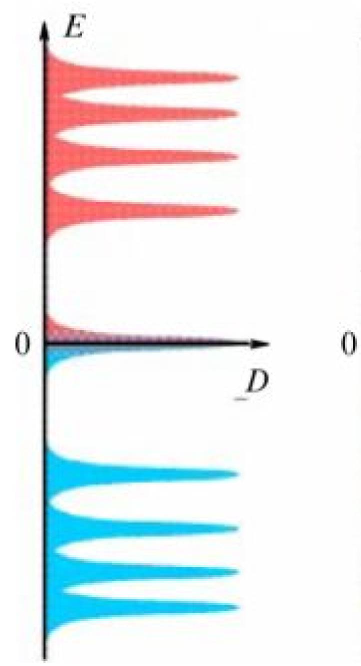


The Tamm-Dirac edge states from different valleys (τ =±1) are occupied by electrons rotating clockwise or counter-clockwise around the antidot.
The energies of these states are quantized

Remember Landau quantization for usual and Dirac fermions



$E_n = \hbar\omega_c\ (n+1/2)$



$$E_n = \mathrm{sgn}\, n\, v_F \sqrt{2e\hbar\,|\,n\,|\,H}$$

$$E_n \propto \sqrt{nH}$$

Usual fermions

Dirac fermions:

there is zero’th level at the Dirac point

Graphene semi-plane in magnetic field: edge states spectra

Skipping orbits

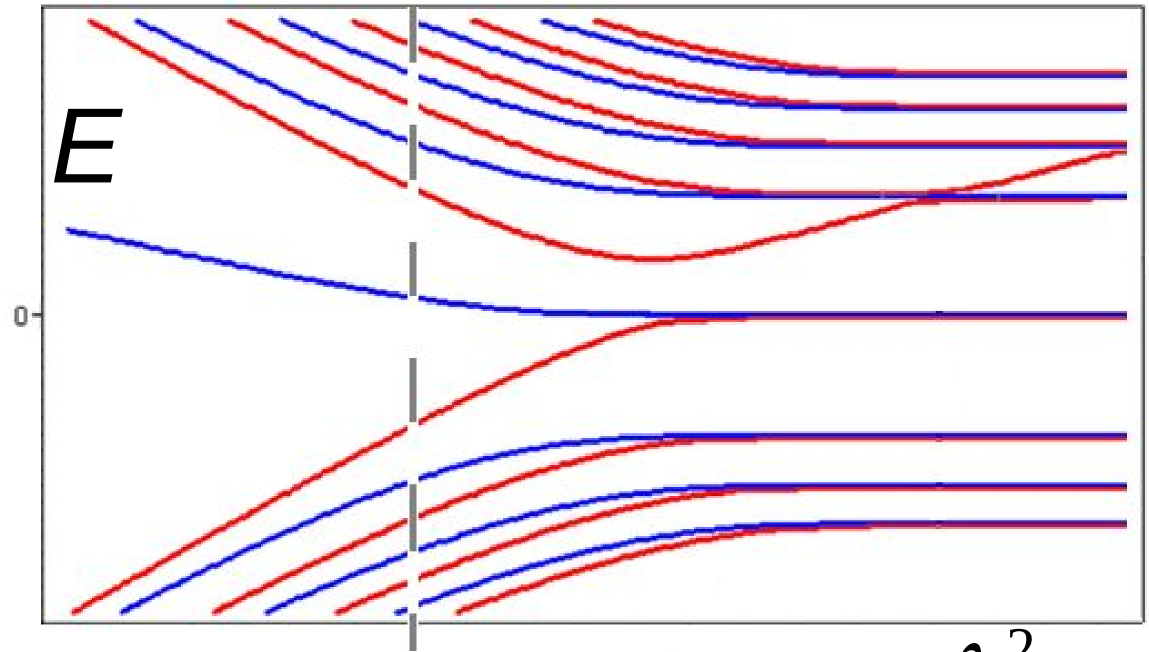
+

Tamm-type states

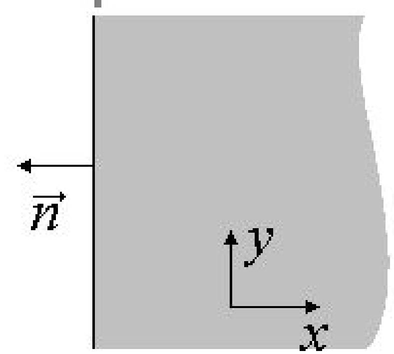
+

Landau levels
of massless Dirac electrons

left valley: red
right valley: blue

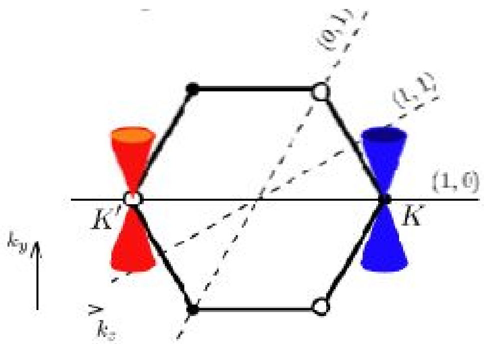


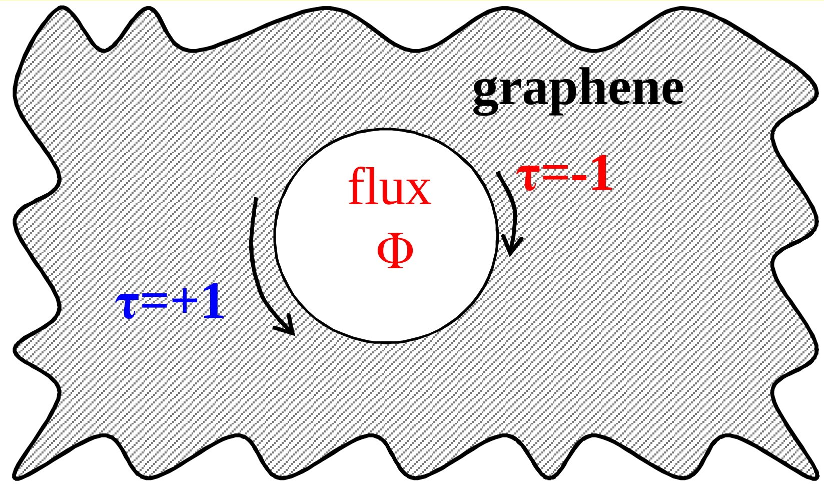
$$x_c = \lambda^2 p_y$$



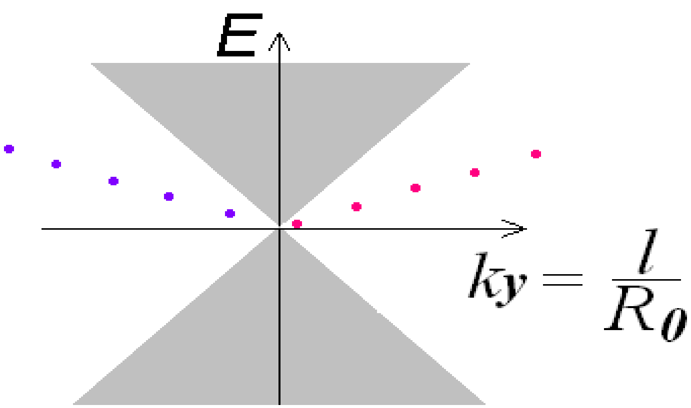
K_y is measured from the projection of the valley center on the direction of the sample edge

Magnetic field effect on the Tamm-Dirac edge states in graphene antidot





Magnetic field ***B***



$$E_\tau R = \tau 2va(j + \Phi/\Phi_0 - \tau/2)$$

$$j = \pm 1/2, \pm 3/2, \pm 5/2, \ldots$$

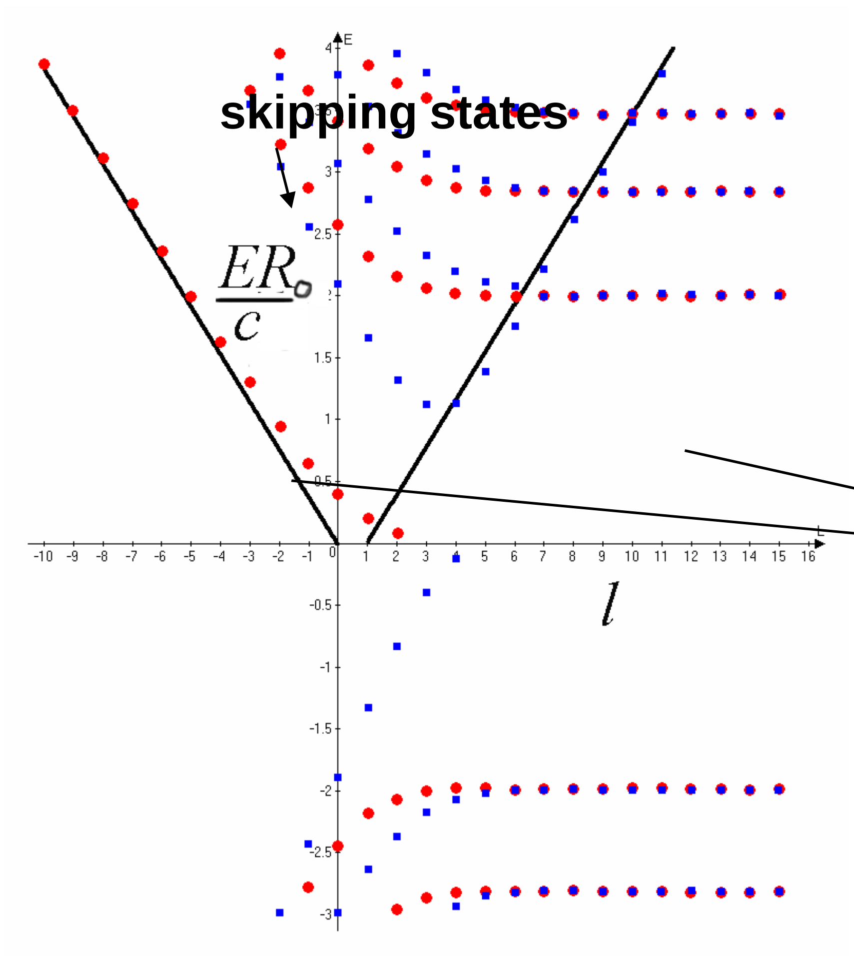
The quasiclassic quantization in the antidot R_0 :

$$k_\parallel = 2\pi\left(j - \tau/2\right)/2\pi R$$

Φ/Φ_0 – the number of magnetic flux quanta through the antidot.

$\Phi = \pi B R_0^2$

Energy of bulk and edge electrons in the antidot in magnetic field



skipping states

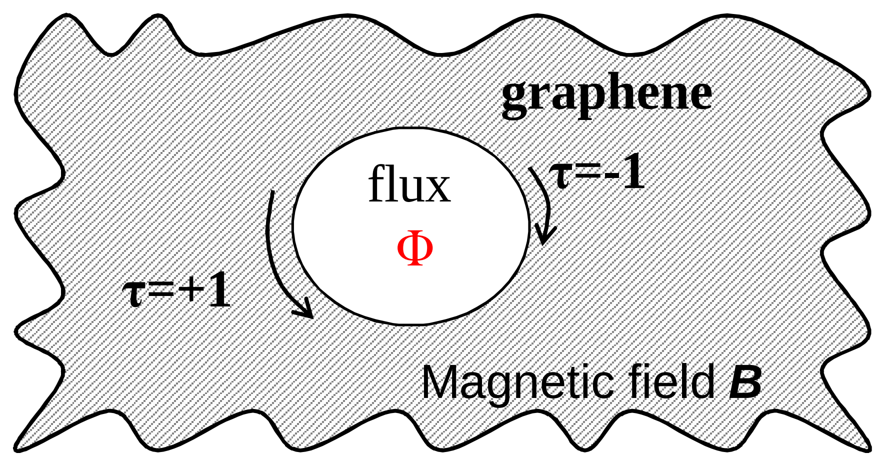
Left valley: red $\tau = -1$

Right valley: blue $\tau = 1$

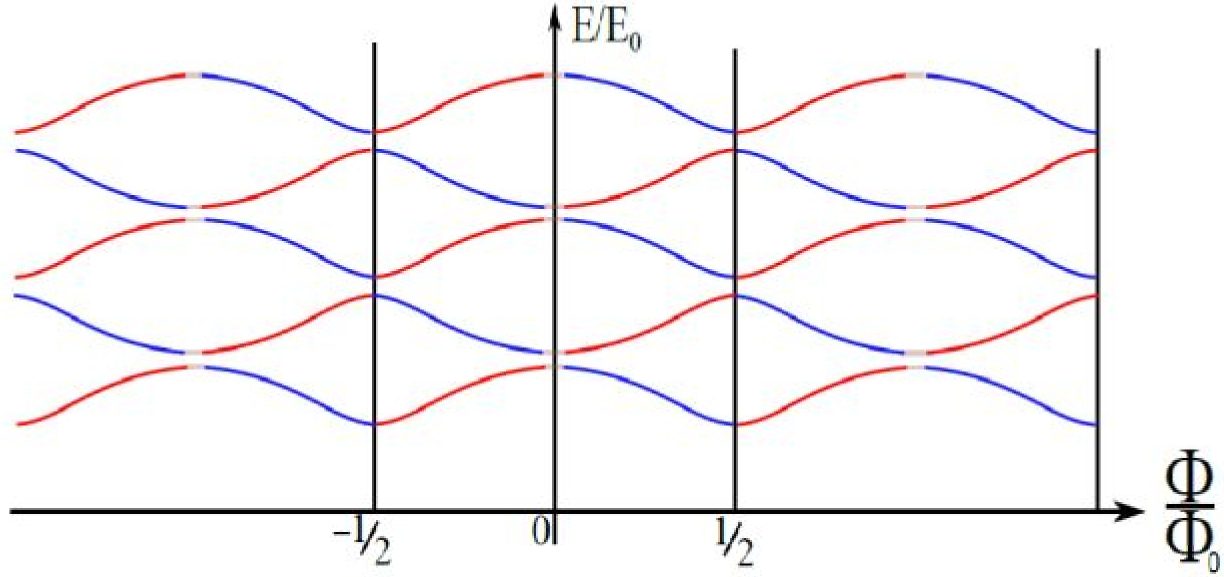
parameters : $a = 0.2$, $\Phi / \Phi_0 = -2$

Asymptotes of edge states in AD:

$$ER = \tau 2a(j + \Phi / \Phi_0 - \tau / 2)$$



The quantization of the Tamm-Dirac states in AD: magnetic “band structure”



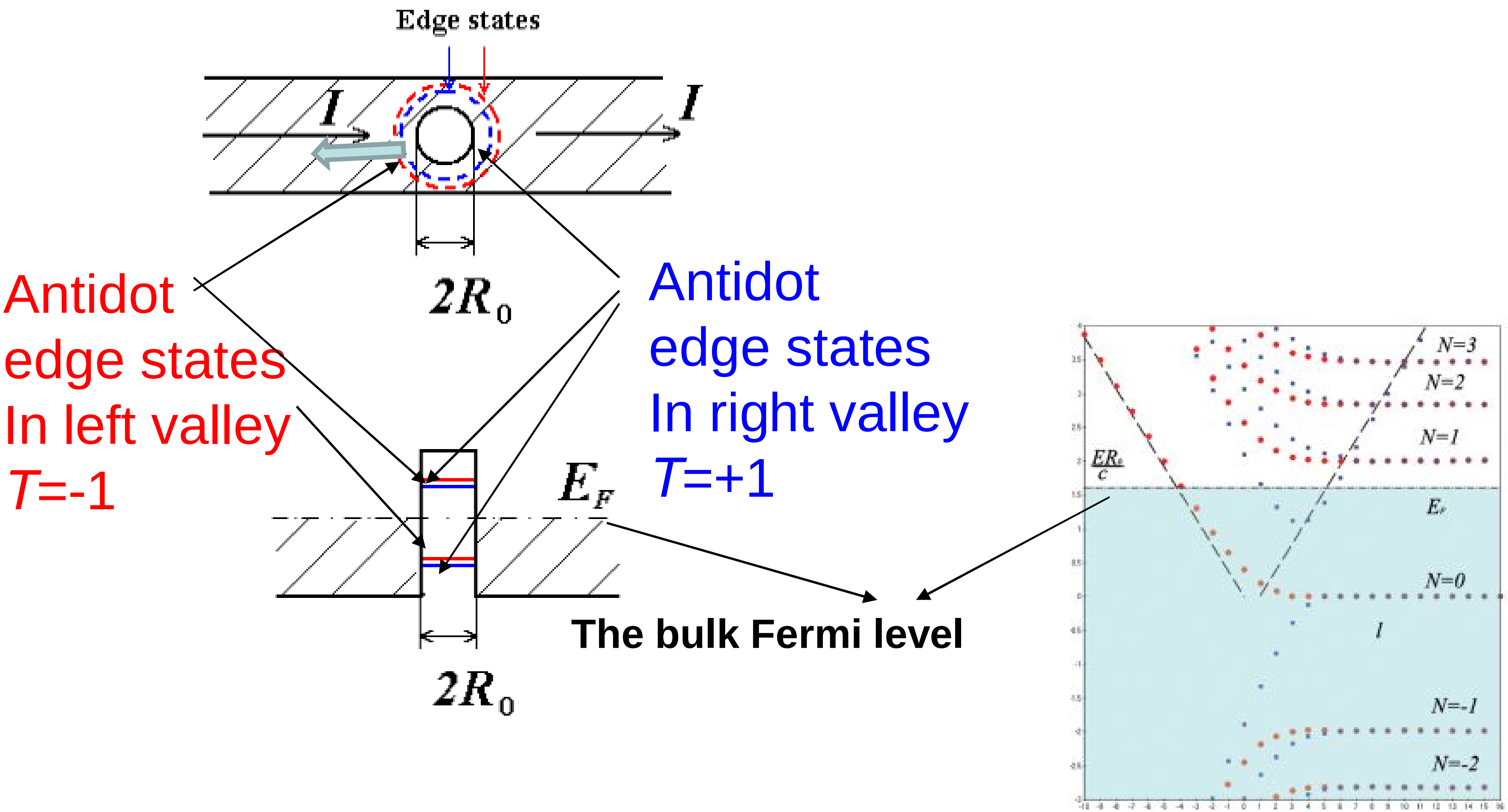
$$E_\tau R = \tau 2va(j + \Phi/\Phi_0 - \tau/2)$$

W/o intervalley scattering:

Gaps (anticrossings) as result of resonant intervalley scattering backscattering : a mechanism of conductance peaks at

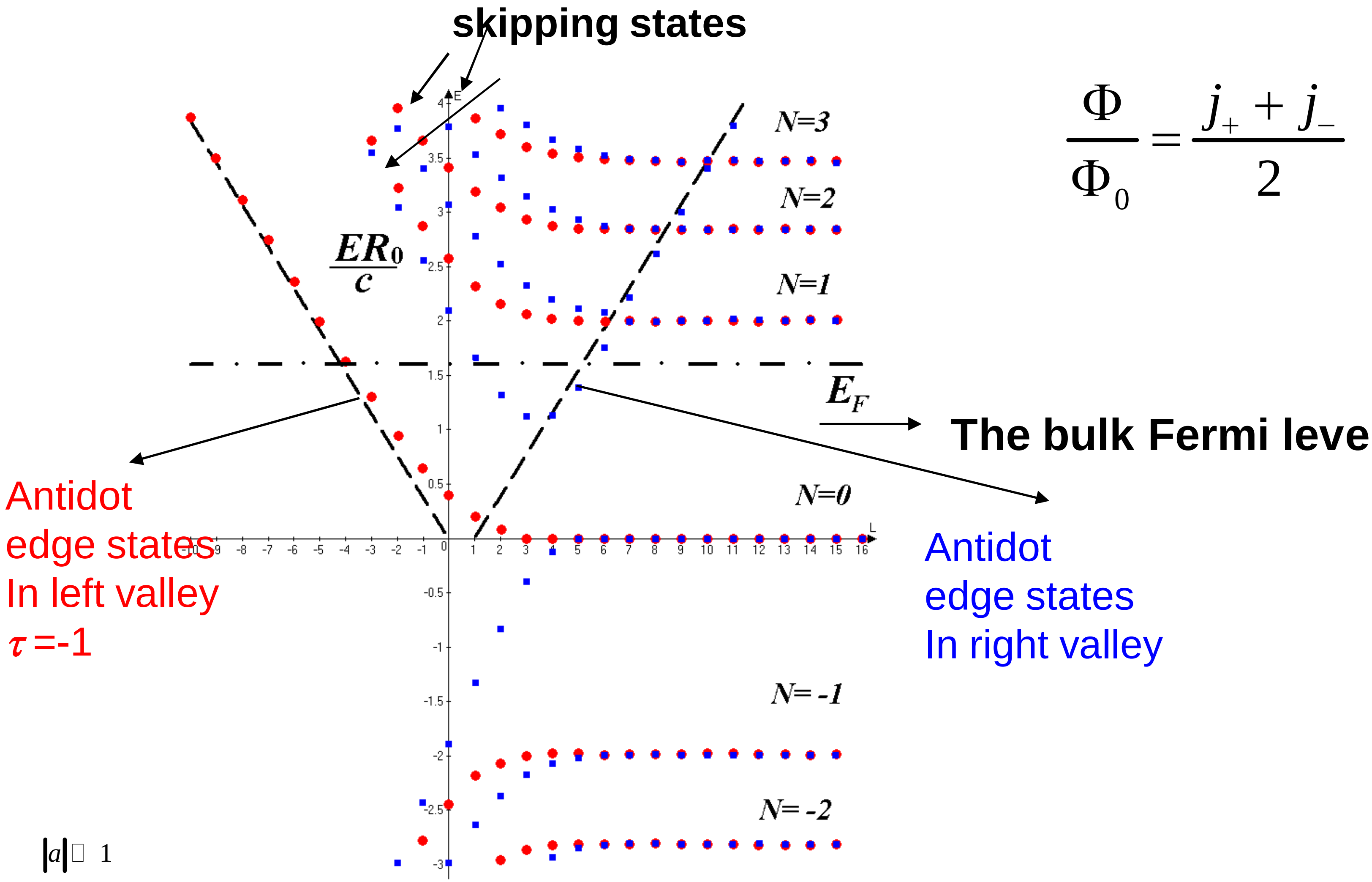
$$\frac{\Phi}{\Phi_0} = \frac{j_+ + j_-}{2}$$

Resonant blue–red back-scattering on antidot leds to conductance resonances and AB-type resistance oscillations



Resonant intervalley scattering does'nt depend on Fermi level and temperature

$$\frac{\Phi}{\Phi_0} = \frac{j_+ + j_-}{2}$$



$|a|$□ 1

CONCLUSION

Experiment:

•Resistance of nano-perforated graphene samples demonstrate the Aharonov-Bohm type magneto-oscillation with period corresponding to the flux quantum per hole area.

•The results are associated with the existence of the Tamm-type states of the Dirac electrons ("Tamm-Dirac states")

Theory: without magnetic field:

•The Tamm-Dirac edge states are predicted on any edge of graphene.

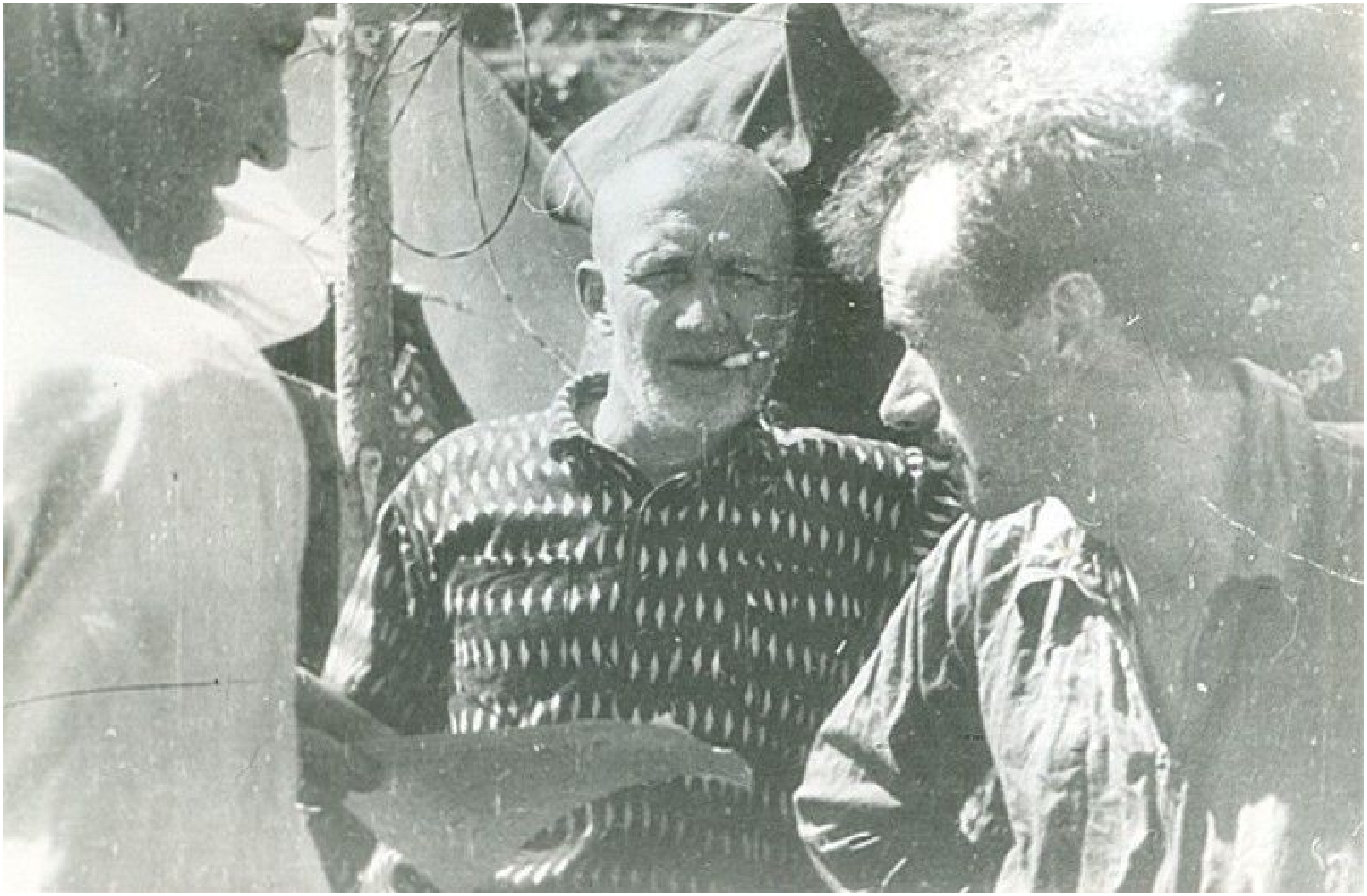
• These AD states from different valleys are occupied by electrons rotating clockwise or counter-clockwise around the antidot. The AD energies are quantized.

Theory: effect of magnetic field:

•Energy of edge states is controlled by flux through the AD.

•Intervalley resonant backscattering leads to gaps in magnetic dispersion of spectrum and the Aharonov-Bohm type effect in electron transport.

•The scenario of Tamm-Dirac edge states may explain the experimental data in the nano-thin graphite and graphene samples with nanohole.

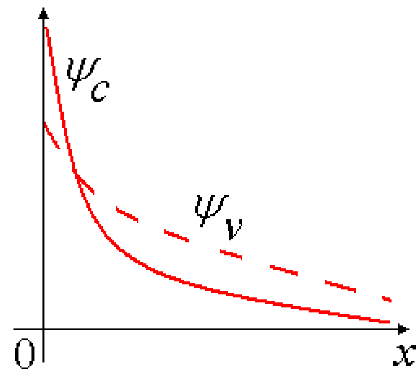


I.E. Tamm and P.A.M. Dirac,
Elbrus, 1936

Envelope Functions and Boundary Problem

$$\varphi_{micro} = \sum_n u_{n0}\left(\vec{r}\right)\psi_n\left(\vec{r}\right)$$

$$\begin{cases} \hat{H}\psi = E\psi \\ \hat{\Gamma}\psi\,|_S = 0 \end{cases}$$



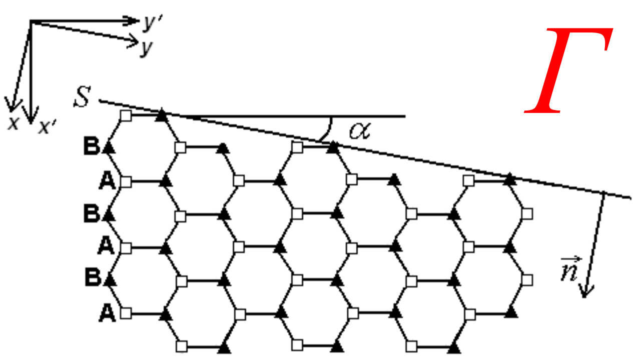
envelope functions column

$$\varphi_{micro} \rightarrow \begin{pmatrix} \psi_1\left(\vec{r}\right) \\ \vdots \\ \psi_n\left(\vec{r}\right) \\ \vdots \end{pmatrix}$$

$\hat{H}$ - many-band effective-mass Hamiltonian

$$\mathrm{H}_W = c\vec{\sigma}\vec{p}$$

$\hat{\Gamma}$ - boundary operator.



$$\hat{\Gamma} = ?$$

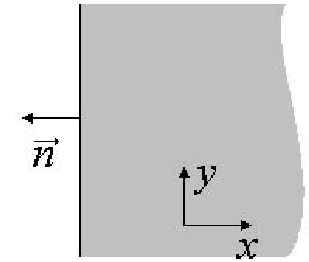
V.Volkov, T.Pinsker (1981)
B.Volkov, O.Pankratov (1985)
M.Berry, R. Mondragon (1987)
E. McCann, V. Fal'ko (2004)
A.Akhmerov, C. Beenakker (2008)
V.Volkov, I. Zagorodnev (2009)

Appendix: **The Tamm states in** the **Dirac band model**

$$\hat{H}_{4x4} = \begin{pmatrix} mc^{*2} & c^{*}\vec{\sigma}\vec{p} \\ c^{*}\vec{\sigma}\vec{p} & -mc^{*2} \end{pmatrix} \qquad \psi = \begin{pmatrix} \psi_c \\ \psi_v \end{pmatrix}$$

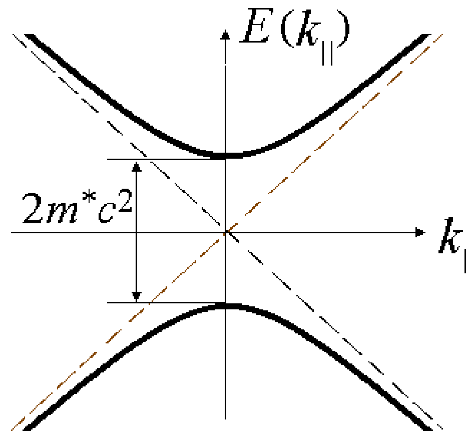
$\psi_c = \begin{pmatrix} \psi_{c1} \\ \psi_{c2} \end{pmatrix}$ c-spinor

$\psi_v = \begin{pmatrix} \psi_{v1} \\ \psi_{v2} \end{pmatrix}$ v- spinor

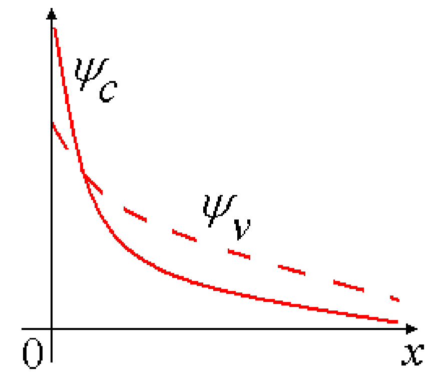


$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

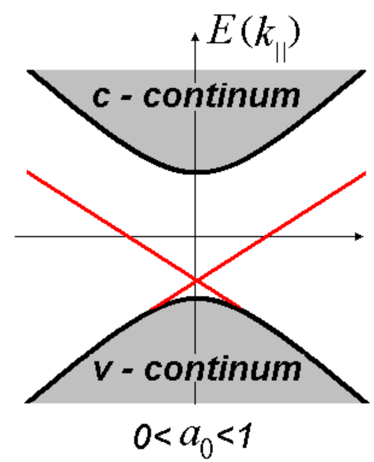


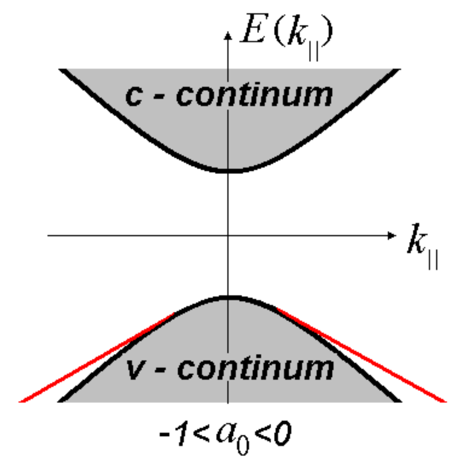
$$\hat{T} = i\begin{pmatrix} \sigma_2 & 0 \\ 0 & \sigma_2 \end{pmatrix}\hat{K}_0$$



$\hat{K}_0$ - complex conjugate

The Tamm spectra depend on a single boundary parameter a_0





$$\begin{cases} \hat{H}^{+} = \hat{H} \\ \hat{T}\hat{\Gamma}\hat{T}^{-1} = \hat{\Gamma} \end{cases} \Rightarrow \left(\psi_c + a_0\vec{\sigma}\vec{n}\psi_v\right)\Big|_S = 0$$

В. Волков, Т. Пинскер, ФТТ, 1981

V. Volkov, T. Pinsker, 1981

The Tamm-Dirac states on graphene semi-plane

$$H_{Dirac} = \begin{pmatrix} \vec{\sigma}\vec{p} & mc^2 \\ mc^2 & -\vec{\sigma}\vec{p} \end{pmatrix} \rightarrow \begin{pmatrix} H_W & 0 \\ 0 & -H_W \end{pmatrix} \qquad 2mc^2 \rightarrow 0$$

2x2 Weyl: $H_W = \vec{\sigma}\vec{p}$

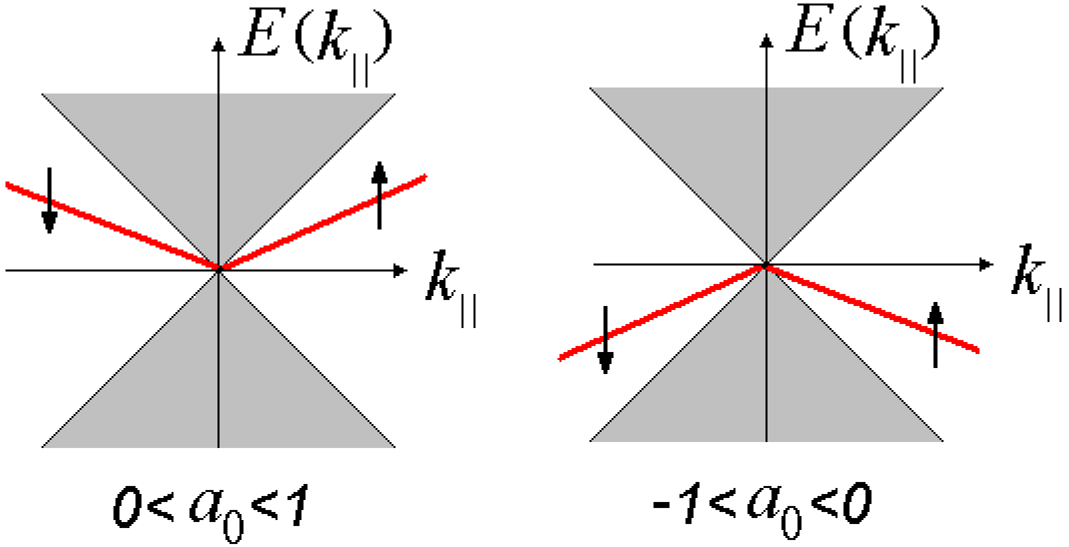
Boundary conditions:

$$\left(\psi_c + ie^{ia_0\vec{\sigma}\vec{n}}\psi_v\right)\Big|_S = 0$$

$$E = \frac{2a_0}{a_0^2+1}k_y,$$

$$\text{where } k_y\left(1-a_0^2\right) > 0$$

Tamm-Dirac spectra:



For the antidot R_0 in quasiclassics: $k_y = n/R_0$

Aharonov-Bohm Effect

Y. Aharonov and D. Bohm. PR **115**,485 (1959).

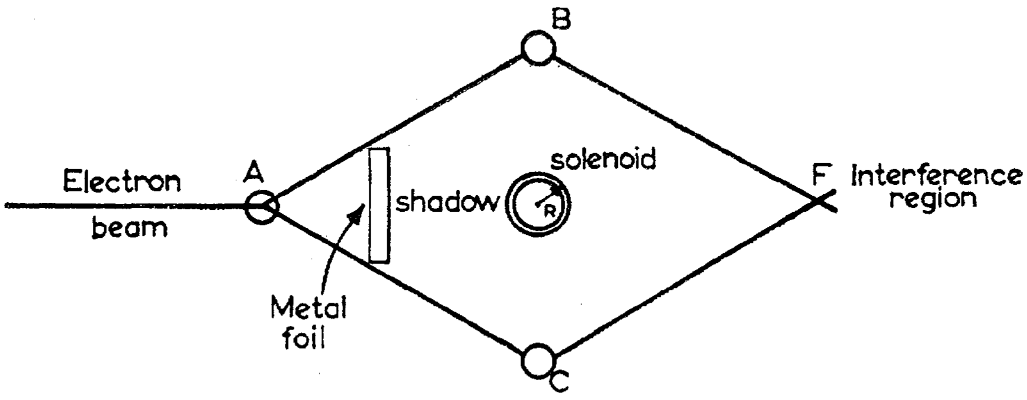


FIG. 2. Schematic experiment to demonstrate interference with time-independent vector potential.

$\Phi_0 = hc/e$

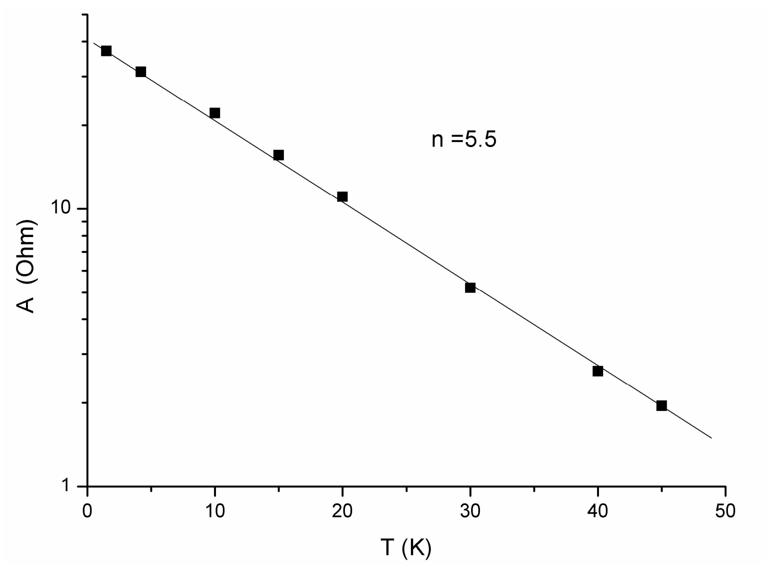
is the flux quantum

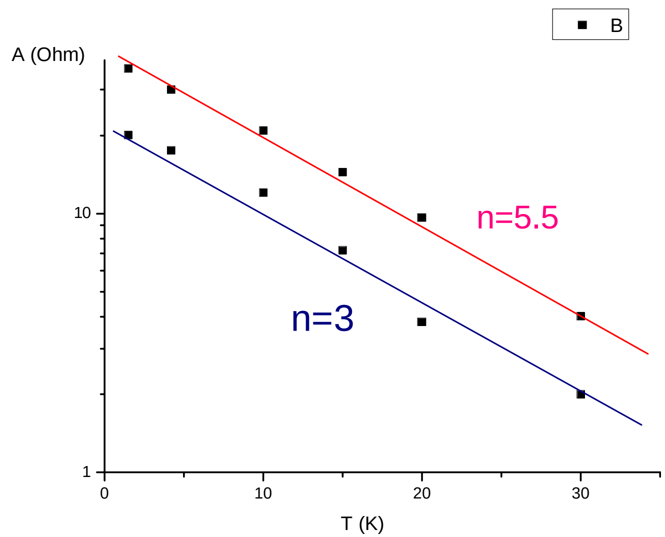
$$\psi_{1,2} = \psi^0_{1,2} \exp(iS_{1,2}/\hbar)$$

$$\Delta S/\hbar = -\frac{e}{c\hbar}\oint A dx = \int H ds = 2\pi\Phi(e/hc) = 2\pi \Phi/\Phi_0$$

the phase difference effect

The temperature dependence of the height of these peaks





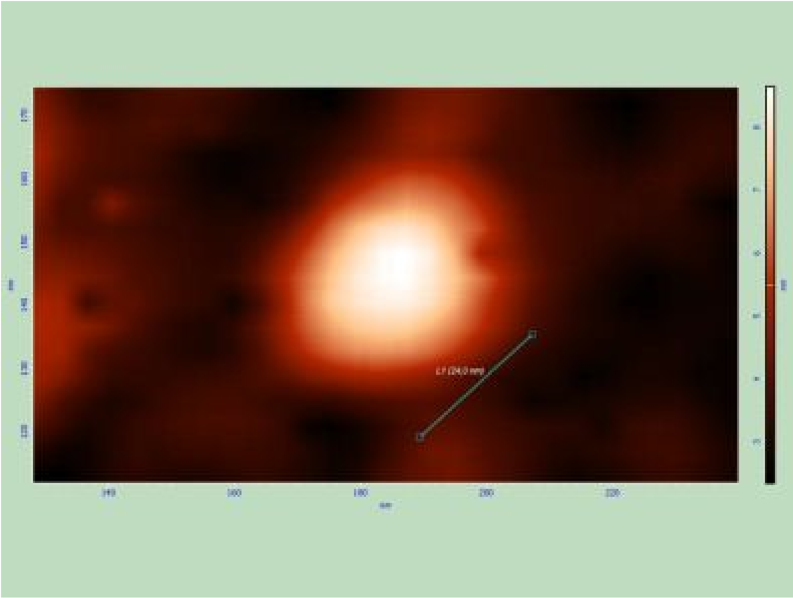
$A / A_0 = \exp(-T/T_0)$ with $T_0 = 20$ K

The MOTIVATION

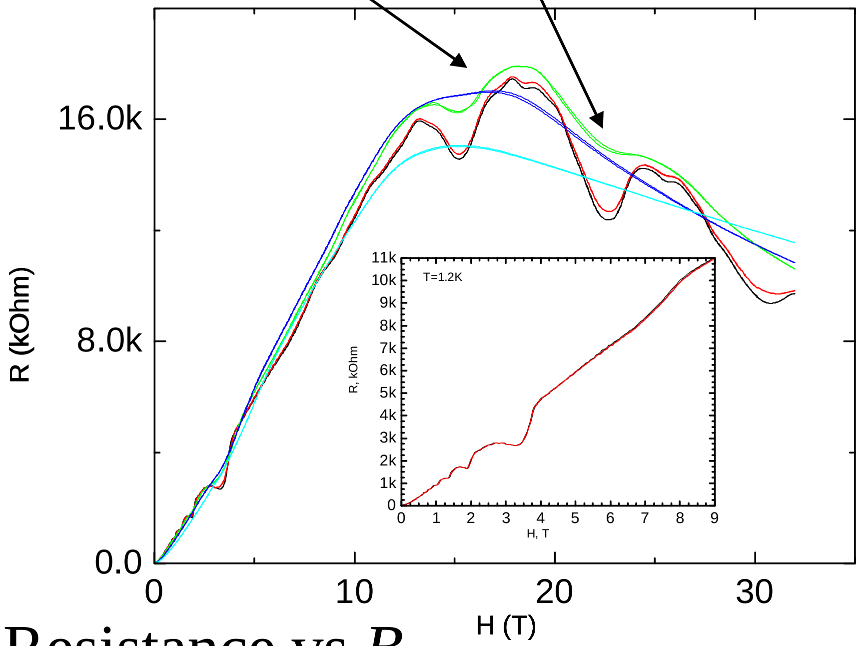
B-periodic resistance oscillations **in ultrathin graphite crystals** with columnar defects (**nanoholes**) were observed in magnetic field.

$\Delta\Phi \approx hc/e = \Phi_0$: ***the Aharonov-Bohm effect ?***

Yu.I. Latyshev, A.Yu. Latyshev, A.P. Orlov, A.A. Shchekin, V.A. Bykov, P. Monceau, K. van der Beck, M. Kontsikovskii, I. Monnet. JETP Letters, **90**, 526 (2009)



AFM image of a nanohole

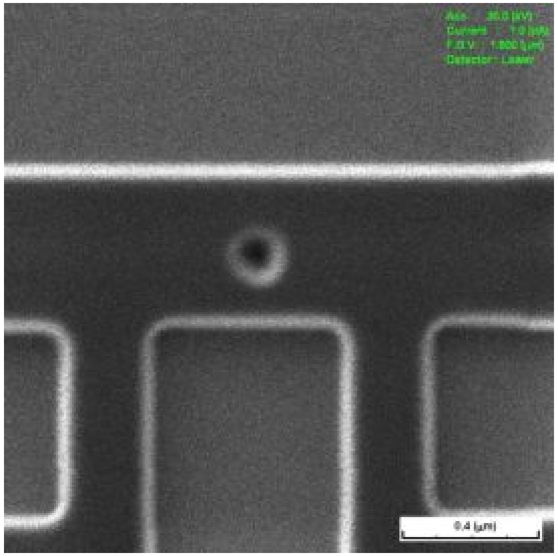


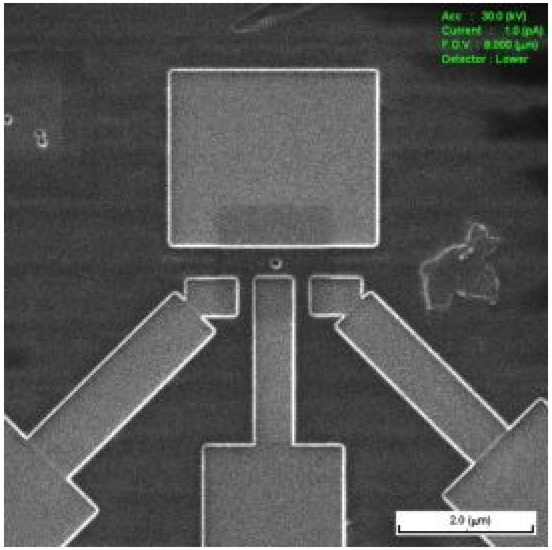
Resistance vs *B*,
T= 1.2 - 32 K, c= 10^9 def/см^2.

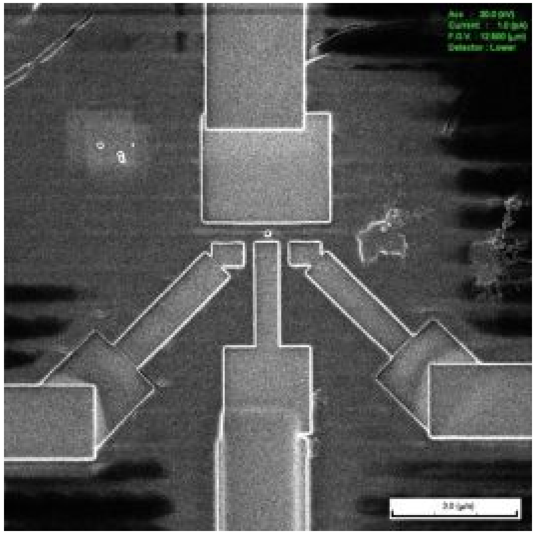
Mechanism of AB effect ??

The MOTIVATION: the AB-like effect in abnormal geometry.

FIB fabrication of a ultrathin single-hole graphite device





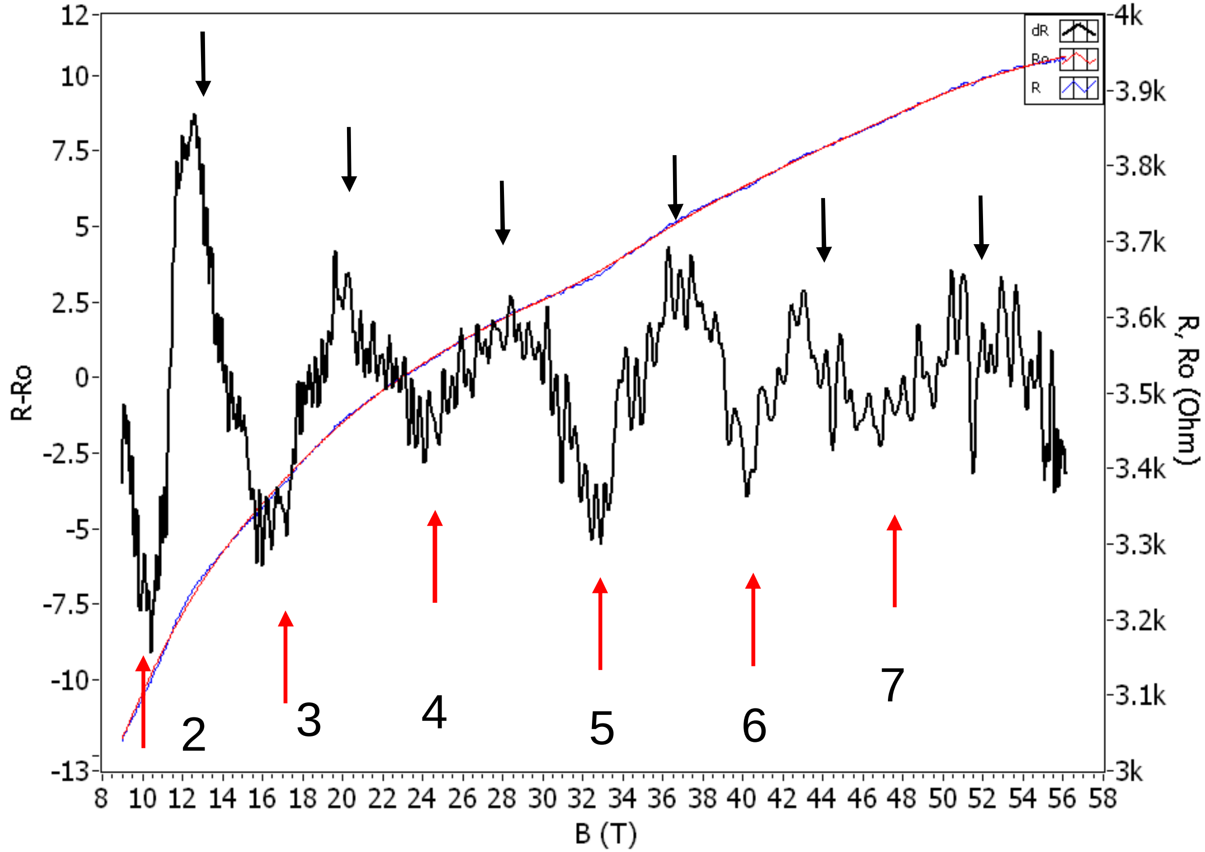


SEM picture of the single-hole structure in thin graphite flake

Thin graphite flake containes 30-50 graphene layers

Hole diameter is 40 nm.

AB oscillations on bi-graphene with columnar defects at pulsed fields



Graphite single crystal with columnar defects thinned to the **thickness of ≤ 1 nm** (**bi-graphene**) obtained by soft beam-plasma discharge.

Period of oscillations ΔB = 7.6 ±0.2 T

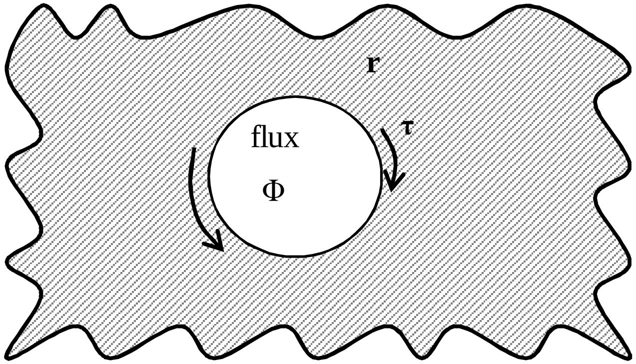
The effect does not depend on sample thickness.

That points to the surface contribution: Yu.I. Latyshev et al. 2011

Estimation of an effective ring width

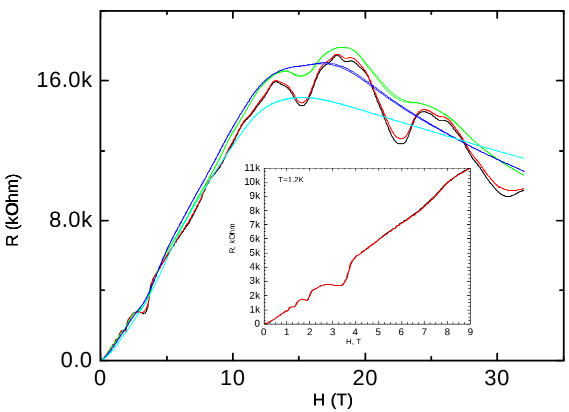
Sample No	ΔH, T	D_{geom} ,nm	D_{eff} ,nm	$(D_{eff} - D_{geom})/2$, nm
#1	9.0	20±2	24	2
#2	7.5	24±2	27	1.5
#3	3.2	37±3	41	2

D_{eff} has been calculated using $\Delta H \, \pi D^2 / 4 = \Phi_0$

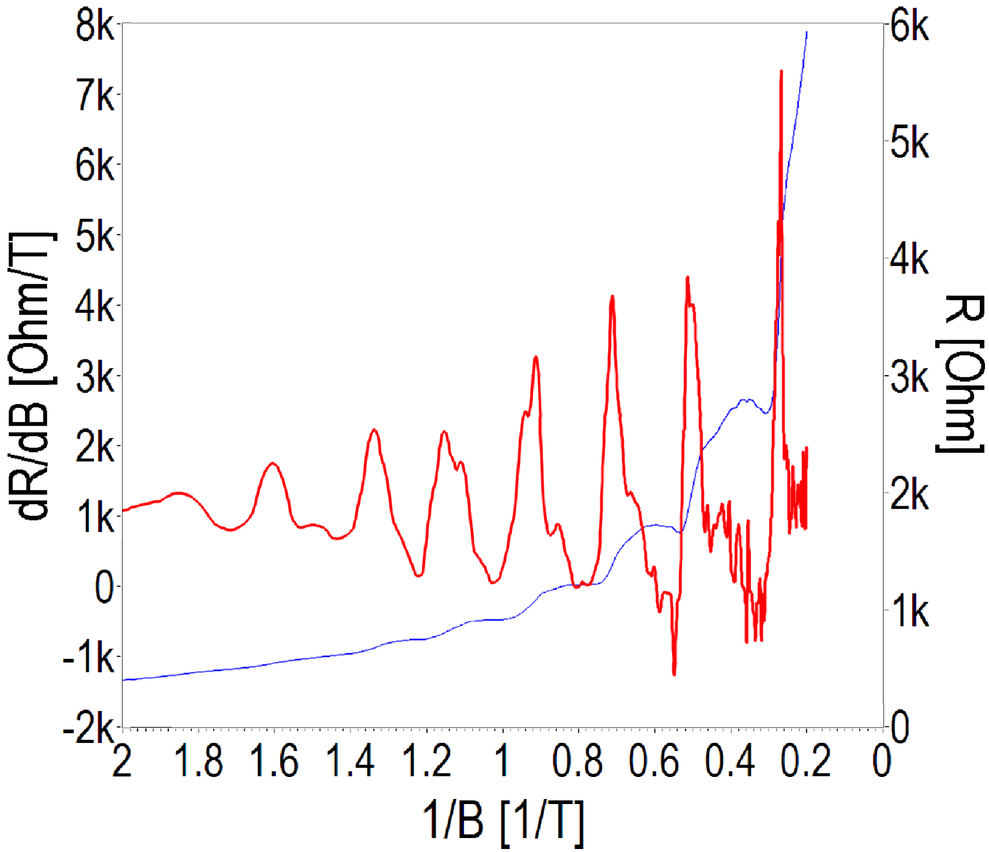


Experiment in low magnetic fields:

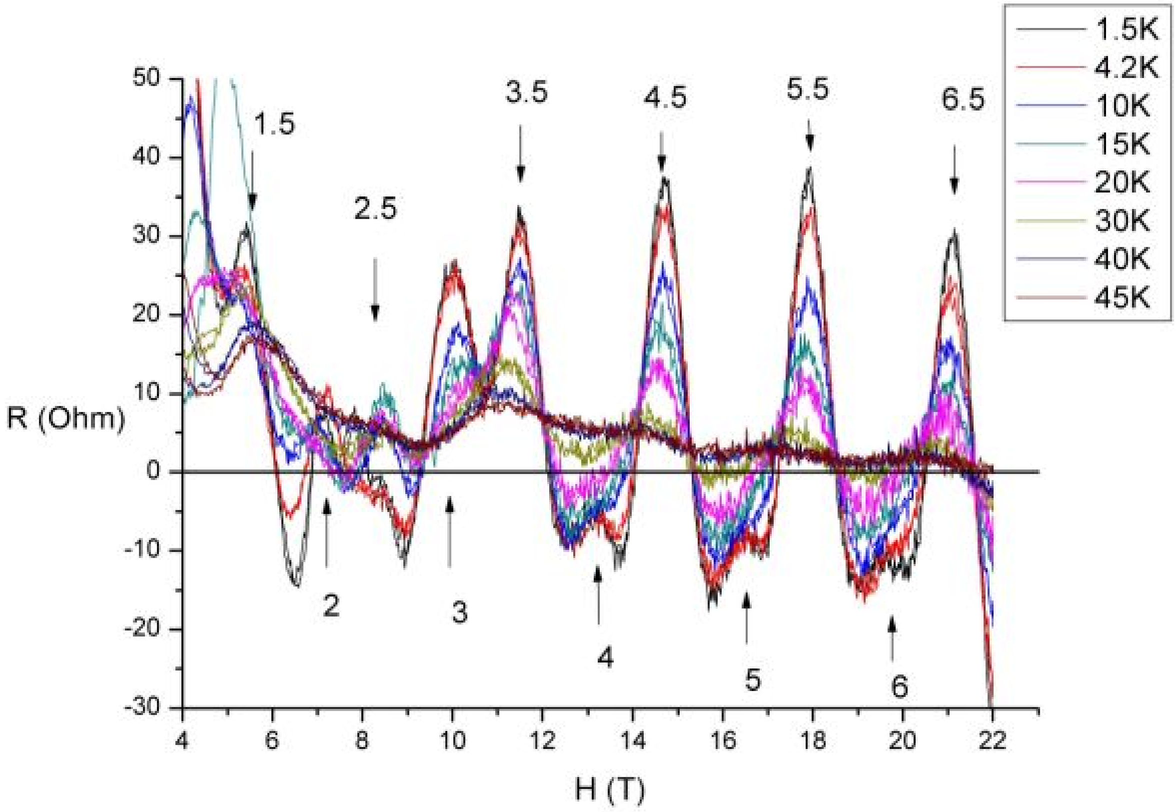
the Shubnikov-de Haas oscillations of resistance as a function of the inversed field for graphite sample #1



ShDG AB oscillations

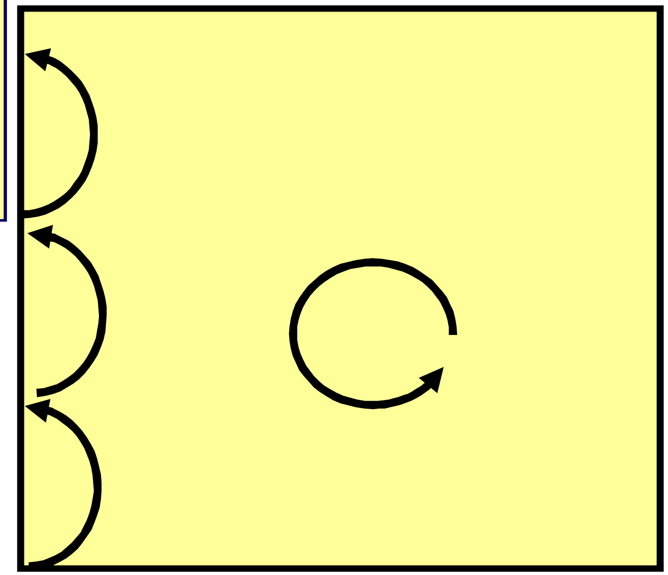


The H-periodic part of resistance of thin graphite sample #2 at various temperatures



The downward arrows: principal series $\Phi = (2n + 1)\,\Phi_0/2$,
the upward arrows: an additional series $\Phi = n\,\Phi_0$

Магнитные поверхностные (краевые) уровни и изгиб уровней Ландау

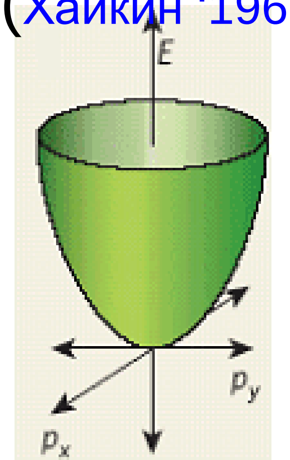


Скачущие орбиты электрона в магнитном поле вблизи края

$\Rightarrow$

магнитные поверхностные (краевые) уровни

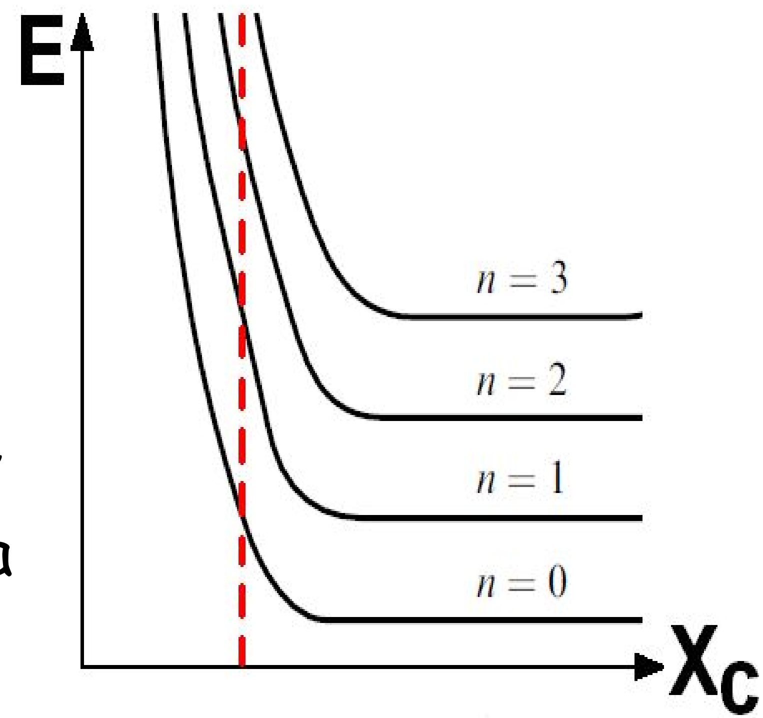
(Хайкин ‘1966, Гальперин’1982)



Граничное условие Ψ=0

$$x_c = \lambda^2 p_y = \frac{c}{eB} p_y$$

- Центр осциллятора



ВЫВОДЫ

1.Исследовано сопротивление образцов графена и тонкого графита разного типа с наноотверстиями в сильных магнитных полях.

2. **В магнитном квантовом пределе обнаружены B-периодические осцилляции сопротивления с периодом, отвечающим прохождению кванта магнитного потока через отверстие (эффект типа АБ)**.

3. Для основной серии осцилляций пики сопротивления соответствовали полуцелому количеству квантов потока.

4. Амплитуда осцилляций слабо зависела от температуры.

5. **Эффект АБ в образцах нестандартной (некольцевой) геометрии связывается с существованием краевых состояний Тамма-Дирака, сильно прижатых к краю отверстия.** Дираковские электроны в этих состояниях вращаются вокруг отверстия по часовой (в одной долине) или против часовой (в другой долине) стрелки.

6. Магнитный поток управляет фазой этих состояний, как в стандартном АВ эффекте, и скоростью вращения дираковских электронов.

7. **При полуцелом количестве квантов потока происходит междолинный резонанс в рассеянии назад, что объясняет пики в сопротивлении и их устойчивость к температуре**. Из сравнения с опытом извлечены параметры теории.