Time Irreversibility Problem and Functional Formulation of Classical and Quantum Theory

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V.L. Ginzburg
Physics - Uspekhi , 42 (4) 353 (1999)

“What problems of physics and astrophysics seem now to be especially important and interesting”

V.L. Ginzburg:

“Three ‘great' problems:

First, I mean the increase of entropy,
time irreversibility and the `time arrow'.

Second is the problem of interpretation and comprehension of quantum mechanics.

And third is the question of
the relationship between physics and biology and, specifically,
the problem of reductionism.”

I.E.Tamm, M.A. Markov, E.L.Feinberg,…

PLAN

Time Irreversibility Problem

Non-Newtonian (functional) Classical Mechanics

‘t Hooft deterministic cellular automaton approach

Functional (random) quantum theory

Functional Probabilistic General Relativity

- **I. V. , “Randomness in classical mechanics and quantum mechanics”, *Found. Phys.*, 41:3 (2011), 516.**

- **M. Ohya, I. Volovich, Mathematical foundations of quantum information and computation and its applications to nano- and bio-systems, Springer, 2011.**

Newton Equation

$$m \frac{d^2}{dt^2} x = F(x),$$

$$x = x(t),$$

$$p = m \frac{dx}{dt},$$

$$(M = R^{2n}, \varphi_t)$$

Phase space (q,p), Hamilton dynamical flow

Why Newton`s mechanics can not be true?

- **Newton`s equations of motions use real numbers while one can observe only rationals. (s.i.)**

- **Classical uncertainty relations**

- **Time irreversibility problem**

- **Singularities in general relativity**

- **Try to solve these problems by developing a new, non-Newtonian mechanics.**

Classical Uncertainty Relations

$$\Delta q > 0, \quad \Delta p > 0$$

$$\Delta t > 0$$

Newton`s Classical Mechanics

Motion of a point body is described by the trajectory in the phase space.

Solutions of the equations of Newton or Hamilton.

Idealization: Arbitrary real numbers—non observable.

- **Newton`s mechanics deals with non-observable (non-physical) quantities.**

Real Numbers

- **A real number is an infinite series, which is unphysical:**

$$t = \sum_n a_n \frac{1}{10^n}, \quad a_n = 0,1,\ldots,9.$$

$$m \frac{d^2}{dt^2} x(t) = F$$

Rational numbers. P-Adic numbers

V.S. Vladimirov, I.V.V., E.I. Zelenov,

B. Dragovich, A.Yu. Khrennikov, S.V. Kozyrev, V.Avetisov, A.Bikulov,

A.P.Zubarev,…Witten, Freund, Frampton,…

- **Journal: “p-Adic Numbers,…”**

Time Irreversibility Problem

The time irreversibility problem is the problem of how to explain the irreversible behaviour of macroscopic systems from the time-symmetric microscopic laws: $t \to -t$

Newton, Schrodinger Eqs –- reversible

Navier-Stokes, Boltzmann, diffusion, Entropy increasing --- irreversible

$$\frac{\partial u}{\partial t} = \Delta u .$$

Time Irreversibility Problem

Boltzmann, Maxwell, Poincar´e, Bogolyubov, Kolmogorov, von Neumann, Landau, Prigogine, Feynman, Kozlov,…

Poincar´e, Landau, Prigogine, Ginzburg, Feynman: Problem is open.
We will never solve it (Poincare)
Quantum measurement? (Landau)

Lebowitz, Goldstein, Bricmont:
Problem was solved by Boltzmann

Boltzmann`s answers to:

Loschmidt: **statistical viewpoint**

Poincare—Zermelo: **extremely long Poincare recurrence time**

Coarse graining

- **Not convincing…**

Ergodic Theory

- **Boltzmann, Poincare, Hopf, Kolmogorov, Anosov, Arnold, Sinai, Kozlov,…:**

- **Ergodicity, mixing,… for various important deterministic mechanical and geometrical dynamical systems**

Bogolyubov method

1. Newton to Liouville Eq.

   Bogolyubov (BBGKI) hierarchy

2. Thermodynamic limit (infinite number of particles)

3. The condition of weakening of initial correlations between particles in the distant past

4. Functional conjecture

5. Expansion in powers of density

   Divergences.

A.S.Trushechkin.

Stochastic Limit

and Irreversibility

Accardi, I.V., Yu.G.Lu, Kozyrev,

Pechen,…

Noncommutative probability

Quantum Field Theory, Quantum Optics

Starobinsky, Woodard, B.L.Hu,…

Functional Formulation of Classical Mechanics

Usual approaches to the irreversibility problem:

Start from Newton Eq. Gas of particles

Derive Boltzmann Eq.

This talk: Irreversibility for one particle

Modification of the Newton approach to Classical mechanics: Functional formulation

We attempt the following solution of the irreversibility problem:
a formulation of microscopic dynamics which is irreversible in time: Non-Newtonian Functional Approach.

Functional formulation of classical mechanics

- **Here the physical meaning is attributed not to an individual trajectory but only to a bunch of trajectories or to the distribution function on the phase space. The fundamental equation of the microscopic dynamics in the proposed "functional" approach is not the Newton equation but the Liouville or Fokker-Planck-Kolmogorov (Langevin, Smoluchowski) equation for the distribution function of the single particle.**

States and Observables in Functional Classical Mechanics

$$(q, p) \quad \in \quad \mathbb{R}^2 \qquad \text{(phase space)}.$$

$$\rho = \rho(q, p, t) \quad \text{state of a classical particle}$$

$$\rho \geq 0, \quad \int_{\mathbb{R}^2} \rho(q, p, t) dq dp = 1, \; t \in \mathbb{R}.$$

States and Observables in Functional Classical Mechanics

$$\overline{f}(t) = \int f(q,p)\rho(q,p,t)dqdp\,.$$

$f(q,p)$ is a function

Not a generalized function

Fundamental Equation in Functional Classical Mechanics

$$\frac{\partial \rho}{\partial t} = -\frac{p}{m}\frac{\partial \rho}{\partial q} + \frac{\partial V(q)}{\partial q}\frac{\partial \rho}{\partial p}.$$

Looks like the Liouville equation which is used in statistical physics
to describe a gas of particles but here we use it to describe a <u>single particle</u>.(moon,…)

Instead of Newton equation. <u>No trajectories!</u>

Cauchy Problem for Free Particle

$$\rho\big|_{t=0} = \rho_0(q,p)\,.$$

$$\rho_0(q,p) = \frac{1}{\pi ab} e^{-\frac{(q-q_0)^2}{a^2}} e^{-\frac{(p-p_0)^2}{b^2}}\,.$$

Average Value and Dispersion

$$\overline{q} = \int q\rho_0(q,p)dqdp = q_0\,, \quad \overline{p} = \int p\rho_0(q,p)dqdp = p_0\,,$$

$$\Delta q^2 = \overline{(q-\overline{q})^2} = \frac{1}{2}a^2, \quad \Delta p^2 = \overline{(p-\overline{p})^2} = \frac{1}{2}b^2\,.$$

Free Motion

$$\frac{\partial \rho}{\partial t} = -\frac{p}{m}\frac{\partial \rho}{\partial q}$$

$$\rho(q,p,t) = \rho_0(q - \frac{p}{m}t, p)\,.$$

Delocalization

$$\rho_c(q,t) = \int \rho(q,p,t)dp = \frac{1}{\sqrt{\pi}\sqrt{a^2 + \frac{b^2t^2}{m^2}}} \exp\{-\frac{(q - q_0 - \frac{p_0}{m}t)^2}{(a^2 + \frac{b^2t^2}{m^2})}\}$$

$$\Delta q^2(t) = \frac{1}{2}(a^2 + \frac{b^2t^2}{m^2})$$

Newton`s Equation for Average

$$\overline{q}(t) = \int q\rho_c(q,t)dq = q_0 + \frac{p_0}{m}t\,, \quad \overline{p}(t) = \int p\rho_m(p,t)dp = p_0\,.$$

$$\frac{d^2}{dt^2}\overline{q}(t) = 0\,,$$

Comparison with Quantum Mechanics

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}$$

$$\rho_q(x,t) = |\psi(x,t)|^2 = \frac{1}{\sqrt{\pi}\sqrt{a^2 + \frac{\hbar^2 t^2}{a^2 m^2}}}\exp\{-\frac{(x - x_0 - \frac{p_0}{m}t)^2}{(a^2 + \frac{\hbar^2 t^2}{a^2 m^2})}\}$$

$$a^2 b^2 = \hbar^2$$

$$W(q,p,t) = \rho(q,p,t)$$

Liouville and Newton. Characteristics

$$\frac{\partial \rho}{\partial t} + \sum_{i=1}^{k} \frac{\partial}{\partial x^i}(\rho v^i) = 0$$

$$\dot{x} = v(x)$$

$$\rho|_{t=0} = \rho_0(x)$$

$$\rho(x, t) = \rho_0(\varphi_{-t}(x))$$

Corrections to Newton`s Equations

Non-Newtonian Mechanics

$$\rho_0(q,p) = \delta_\epsilon(q-q_0)\delta_\epsilon(p-p_0)$$

$$\delta_\epsilon(q) = \frac{1}{\sqrt{\pi}\epsilon} e^{-q^2/\epsilon^2} \ ,$$

Proposition 1. Newton's Equations

$$\lim_{\epsilon \longrightarrow 0} \int f(q,p)\rho(q,p,t)dqdp = f(\varphi_t(q_0,p_0)) \ .$$

Corrections to Newton`s Equations

$$\frac{\partial \rho}{\partial t} = -p\frac{\partial \rho}{\partial q} + \lambda q^2 \frac{\partial \rho}{\partial p}$$

$$\dot{p}(t) + \lambda q(t)^2 = 0\,,\ \ \dot{q}(t) = p(t)\,.$$

Corrections to Newton`s Equations

Proposition 2.

$$< q(t) >= q_{\text{Newton}}(t) - \frac{\lambda}{4}\epsilon^2 t^2$$

$$q_{\text{Newton}}(t) = q_0 + p_0 t - \frac{\lambda}{2} q_0^2 t^2$$

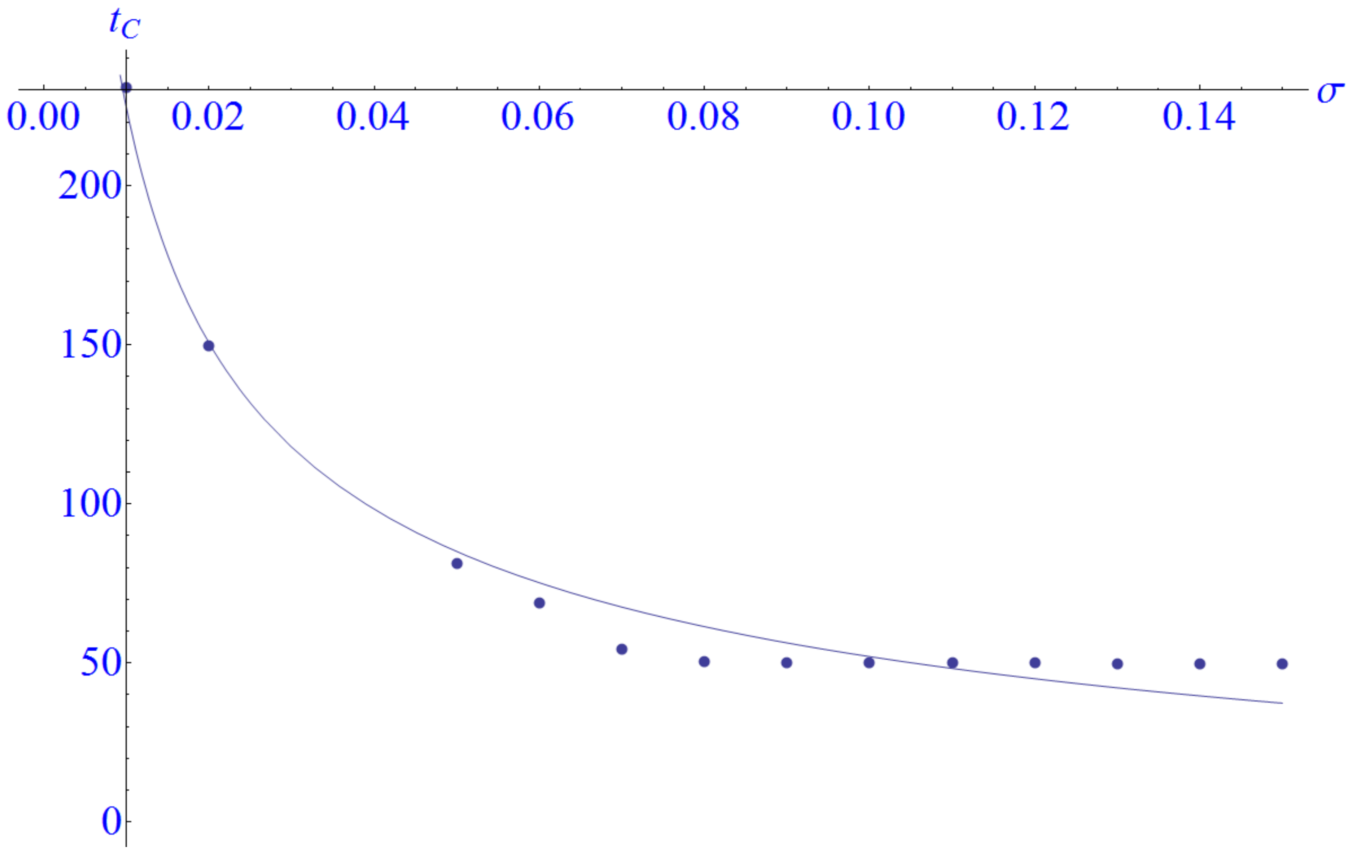
$-\frac{\lambda}{4}\epsilon^2 t^2$ is the correction to the Newton solution

Corrections

$$m\frac{d^2}{dt^2} < q(t) > = < F(q)(t) >$$

E. Piskovsky,

A. Mikhailov



$$|\langle q\rangle(t_C,\sigma) - q_{KB}(t_C,b)| = 0.1 q_{(KB)}(0,\sigma)$$

$$t_C = O(1/\sqrt{\sigma})$$

Stability of the Solar System?

- **Kepler, Newton, Laplace, Poincare,…,Kolmogorov, Arnold,…---stability?**

- **Solar System is unstable, chaotic (J.Laskar)**
- Laskar's equations had some 150,000 terms. Laskar's work showed that the Earth's orbit (as well as the orbits of all the inner planets) is chaotic and that an error as small as 15 metres in measuring the position of the Earth today would make it impossible to predict where the Earth would be in its orbit in just over 100 million years' time.

- **Newton`s approach: Empty space (vacuum) and point particles.**
- **Reductionism: For physics, biology economy, politics (freedom, liberty,…)**

- **This approach: No empty space. Probability distribution. Collective phenomena. Subjective.**

Fokker-Planck-Kolmogorov versus Newton

$$\frac{\partial f}{\partial t} - \{H, f\} = \sigma \frac{\partial^2 f}{\partial p^2} + \gamma \frac{\partial (pf)}{\partial p}$$

G. 't Hooft, Duality between a deterministic cellular automaton and a bosonic quantum field theory in 1+1 dimensions, arXiv:1205.4107.

An exact correspondence

between the states of

a deterministic cellular automaton in 1+1

dimensions and those of a

bosonic quantum field theory.

Quantum field theories may be much closer

related to deterministic automata than what is usually thought possible.

1990,.1997,….2012.

G.'t Hooft

1 Classical Deterministic Cellular Automaton (CDCA)

We have to distinguish between the classical deterministic cellular automaton and the cellular automaton in Hilbert space. The state of the deterministic cellular automaton [1] is just a pair of integer valued functions $A^L(x)$ and $A^R(x)$ on the lattice $Z, x \in Z$. The dynamics is given by the shift $A^L(x) \to A^L(x+t)$ and $A^R(x) \to A^R(x-t)$.

We call this model the Classical Deterministic Cellular Automaton (CDCA).

2 Cellular Automaton in Hilbert Space (CAHS)

The following model of cellular automaton in the Hilbert space is suggested in [1]. The model is given by a pair $\{\mathcal{H}, U_t\}$ where $\mathcal{H}$ is a Hilbert space and U_t is a group of unitary operators in $\mathcal{H}$, $U_tU_\tau = U_{t+\tau}$ where t and τ are integers.

The Hilbert space $\mathcal{H}$ is defined as follows. Let $A^L(x)$ and $A^R(x)$ be a pair of an integer valued functions defined on the lattice of integers $Z\,, x \in Z$. We denote $A = (\{A^L(x)\}_{x\in Z}, \{A^R(x)\}_{x\in Z})$. Then the Hilbert space $\mathcal{H}$ is formed from the complex valued functions $\psi(A)$ with the scalar product

$$(\psi, \varphi) = \sum_A \bar{\psi}(A)\varphi(A)\,. \tag{1}$$

The action of the unitary operator U_t in $\mathcal{H}$ is defined by

$$U_t\psi(A) = \psi(A_t),\ A_t = (\{A^L(x+t)\}_{x\in Z}, \{A^R(x-t)\}_{x\in Z}) \tag{2}$$

3 Quantum Field Theory

In [1] it is discovered a remarkable realization of the massless two-dimensional lattice quantum field theory in the Hilbert space $\mathcal{H}$ of CAHS with the unitary group U_t being the quantum dynamics of the quantum field theory.

Instead of the canonical field and momentum operators one considers the left- and right movers $a^L(x), a^R(x)$ on the lattice, $x \in Z$, that obey the commutation rules

$$[a^L(x), a^L(y)] = \pm\frac{i}{2\pi}\delta_{y,x\pm1}, \ \ [a^R(x), a^R(y)] = \mp\frac{i}{2\pi}\delta_{y,x\pm1}. \tag{3}$$

One can construct a nontrivial realization of the commutation relations (3) in the Hilbert space $\mathcal{H}$ of the CAHS. Moreover the Heisenberg dynamical evolution of the operators $a^L(x), a^R(x)$ is provideed by the unitary operators U_t:

$$U_t a^L(x) U_t^* = a^L(x+t), \ U_t a^R(x) U_t^* = a^R(x-t). \tag{4}$$

There is a restriction in this construction that one considers only these elements of the Hilbert space of the automaton that are orthogonal to the edge states.

4 Discussion

Mathematics of the mapping between the formulation of the cellular automaton in the Hilbert space (CAHS) and quantum field theory looks rather clear and it is very interesting.

However in [1] is stressed that also a correspondence is established between the states of deterministic cellular automaton (i.e. CDCA?) and those of the two-dimensional lattice quantum field theory. This assertion is not clear.

The state of the deterministic cellular automaton is just a pair of functions $A^L(x)$ and $A^R(x)$ but not the function $\psi(A)$ from the Hilbert space $\mathcal{H}$ in CAHS. Moreover by using the Hilbert space formalism we invoke also the probabilistic interpretation which is hardly can be called deterministic one.

In fact the description of the cellural automaton in the Hilbert space $\mathcal{H}$ might be called quasi-quantization of the classical deterministic cellural automaton. It leads to such typical quantum phenomena as interference of states.

Boltzmann and Bogolyubov Equations

- **A method for obtaining kinetic equations from the Newton equations of mechanics was proposed by Bogoliubov. This method has the following basic stages:**
- **Liouville equation for the distribution function of particles in a finite volume, derive a chain of equations for the distribution functions,**
- **pass to the infinite-volume, infinite number of particles limit,**
- **postulate that the initial correlations between the particles were weaker in the remote past,**
- **introduce the hypothesis that all many-particle distribution functions depend on time only via the one-particle distribution function, and use the formal expansion in power series in the density.**

- **Non-Newtonian Functional Mechanics:**
- **Finite volume. Two particles.**

- **A.Trushechkin.**

Liouville equation for two particles

$$\rho = \rho(x_1, x_2, t)$$

<u>Finite volume. </u> If $\rho(x_1, x_2, t)$ If

satisfies the Liouville equation then

$$f_1(x_1, t)$$

obeys to the following equation

$$\left(\frac{\partial}{\partial t} + \frac{p_1}{m}\frac{\partial}{\partial q_1}\right) f_1(x_1, t) =$$

$$= \frac{1}{V}\int_{\Omega_V} \frac{\partial \Phi(|q_1 - q_2|)}{\partial q_1} \frac{\partial}{\partial p_1} [f_1(\varphi^{(1)}_{t_0 - t}(x_1, x_2), t_0) f_1(\varphi^{(2)}_{t_0 - t}(x_1, x_2), t_0)] dq_2\, dp_2.$$

<u>Bogolyubov type equation for two particles in finite volume</u>

- **No classical determinism**
- **Classical randomness**

- **World is probabilistic (classical and quantum)**

Compare: Bohr, Heisenberg, von Neumann, Einstein,…

Single particle (moon,…)

$$\rho = \rho(q, p, t)$$

$$\frac{\partial \rho}{\partial t} = -\frac{p}{m}\frac{\partial \rho}{\partial q} + \frac{\partial V}{\partial q}\frac{\partial \rho}{\partial p}$$

$$\rho \,|_{t=0} = \frac{1}{\pi ab}\exp\{-\frac{(q-q_0)^2}{a^2} - \frac{(p-p_0)^2}{b^2}\}$$

- **Newton`s approach: Empty space (vacuum) and point particles.**
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Functional General Relativity

- **Fixed background** (M, g)

Geodesics in functional mechanics $\rho(x, p)$

Probability distributions of spacetimes $\rho(M, g)$

- **No fixed classical background spacetime.**
- **No Penrose—Hawking singularity theorems**
- **Stochastic geometry?**

Example

$$\dot{x} = x^2, \; x(t) = \frac{x_0}{1 - x_0 t} \quad \text{singular}$$

$$\rho(x,t) = C \exp\{-(\frac{x}{1-xt} - q_0)^2 / \varepsilon^2\},$$

nonsingular

Fixed classical spacetime?

- **A fixed classical background spacetime does not exists (Kaluza—Klein, Strings, Branes).**

There is a set of classical universes and a probability distribution $\rho(M, g_{\mu\nu})$ **which satisfies the Liouville equation (not Wheeler—De Witt).**

Stochastic inflation?

Conclusions

Functional formulation (non-Newtonian) of classical mechanics: distribution function instead of individual trajectories.

Fundamental equation: Liouville or FPK for a single particle. Irreversibility.

Newton equation—approximate for average values. Corrections to Newton`s trajectories.

Stochastic geometry, general relativity.

New quantum theory, kinetic theory, …

Information Loss in Black Holes

- **Hawking paradox**.

- **Particular case of the Irreversibility problem.**

- **Bogolyubov method of derivation of kinetic equations -- to quantum gravity.**
- Th.M. Nieuwenhuizen, I.V. (2005)