Graviton Corrections to Maxwell's Equations

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Classical Maxwell's Equations

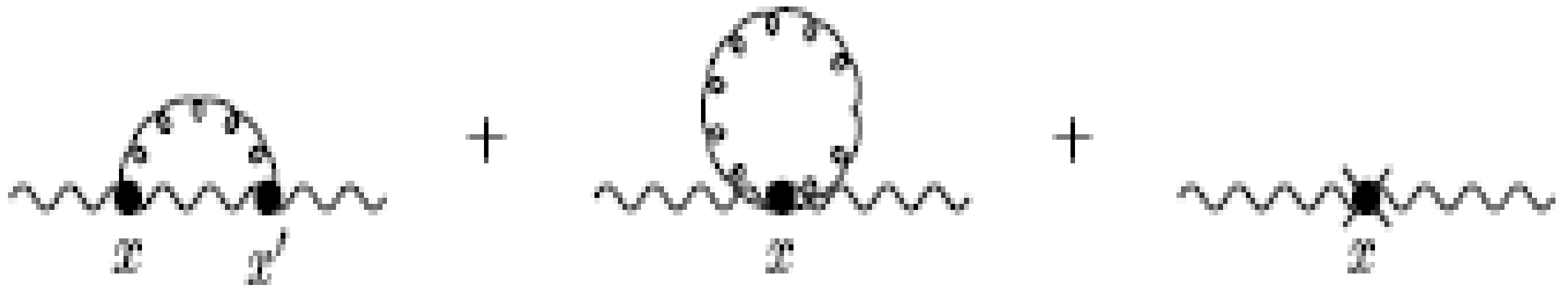
$\partial_\nu [\sqrt{-g}\, g^{\nu\rho} g^{\mu\sigma} F_{\rho\sigma}] = J^\mu$

1. Photons ($J^\mu = 0$)
   - $E^i(t,x) \sim \varepsilon^i e^{-ik(ct-x)}$
2. Static point charge ($J^\mu = q\delta^\mu{}_0 \delta^3(x)$)
   - $A_0(t,x) = q/[4\pi r]$
3. Instantaneous dipole
   - $J^0 = -\theta(t) p^i \partial_i \delta^3(x)$, $J^i = p^i \delta(t) \delta^3(x)$
   - $F_{ij}(t,x\} = -1/2\pi (\partial_i p_j - \partial_j p_i)\, \theta(t)\, \delta(r^2 - t^2)$

Quantum-Correcting Maxwell

- $\partial_\nu[\sqrt{-g}\, g^{\nu\rho} g^{\mu\sigma} F_{\rho\sigma}(x)]$
  $+ \int d^4x' [{}^\mu\Pi^\nu](x;x')\, A_\nu(x') = \mathcal{J}^\mu(x)$
  - 1PI 2-pt function: $i[{}^\mu\Pi^\nu](x;x')$
  - 1PI 1-pt function: $\mathcal{J}^\mu(x)$
- Specialization to flat space
  - $[{}^\mu\Pi^\nu](x;x') = (\eta^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu)\, \Pi(\Delta x^2)$
  - $\partial_\nu F^{\nu\mu}(x) + \int d^4x'\, \Pi(\Delta x^2)\, \partial'_\nu F^{\nu\mu}(x') = \mathcal{J}^\mu(x)$

No Integrals at One Loop!



$i[{}^{\mu}\Pi^{\nu}]_{3pt} = \partial_\kappa \partial'_\theta \{ i\kappa V^{\mu\rho\kappa\lambda\alpha\beta}\ i[{}_{\alpha\beta}\Delta_{\gamma\delta}]\ i\kappa V^{\nu\sigma\phi\theta\gamma\delta}\ \partial_\lambda \partial'_\phi\ i[{}_\rho\Delta_\sigma]\}$

$= -\kappa^2\Gamma^2(D/2-1)/16\pi^D\ (D-3)(D-2)^2 D$

$\bullet \{(D+1)\eta^{\mu\nu}/\Delta x^{2D} - 2D\Delta x^\mu \Delta x^\nu/\Delta x^{2D+2}\}$

$= \text{Same} \bullet 1/[2(D-1)]\ [\eta^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu]\ 1/\Delta x^{2D-2}$

Schwinger-Keldysh Formalism (converting in-out to in-in)

- End of each line gets ± polarity
- Vertices all + (normal) or all – (-normal)
- Just replace Δx^2 in propagators
  - ++ ➔ $||x - x'||^2 - (|t - t'| - i\varepsilon)^2$
  - +- ➔ $||x - x'||^2 - (t - t' + i\varepsilon)^2$
  - -+ ➔ $||x - x'||^2 - (t - t' - i\varepsilon)^2$
  - -- ➔ $||x - x'||^2 - (|t - t'| + i\varepsilon)^2$
- In-out N-pt function gives 2^N in-in N-pt

SK Effective Field Equations

- $\Pi_{\text{in-out}} \to \Pi_{++} + \Pi_{+-}$
  - $t' > t \Rightarrow$ ++ and +- cancel
  - $\Delta r > \Delta t \Rightarrow$ ++ and +- cancel
  - $\Delta r < \Delta t \Rightarrow +- = -(++)^*$
- $\ln(\mu^2\Delta x^2)/\Delta x^2 = \frac{1}{8}\,\partial^2[\ln^2(\mu^2\Delta x^2) - 2\ln(\mu^2\Delta x^2)]$
  - $\ln(\mu^2\Delta x^2_{++}) - \ln(\mu^2\Delta x^2_{+-}) = i\pi\,\theta(\Delta t-\Delta r)$
  - $\ln^2(\mu^2\Delta x^2_{++}) - \ln^2(\mu^2\Delta x^2_{+-})$ $= i2\pi\,\theta(\Delta t-\Delta r)\ln[\mu^2(\Delta t^2-\Delta r^2)]$
- $\Pi(x;x') = G/24\pi^2\,\partial^6\{\theta(\Delta t-\Delta r)$ $[\ln[\mu^2(\Delta t^2-\Delta r^2)] - 1]\} + O(G^2)$

No Change for Photons

- $\partial_\nu F^{\nu\mu}(x) + \int d^4x' \, \Pi(x;x')\partial'_\nu F^{\nu\mu}(x') = \mathcal{J}^\mu(x)$
- $\partial_\nu F^{\nu\mu}(x) = 0$ works for $\mathcal{J}^\mu = 0$
- Should be true to all orders

UV Enhancement for Coulomb

- $\Phi(r) = q/4\pi r\,[1 + 2G/3\pi r^2 + O(G^2)]$
  - $\#G/r^2$ fixed by dimensionality
  - No dependence on finite part of counterterm
- Same sign as S-matrix techniques
  - Radkowski (1970)
  - Bjerrum-Bohr (2002)
- Opposite to RG flow from β-function
  - Robinson & Wilczek (2006)
  - Harst & Reuter (2011)
  - But other things matter

Light-Cone Pushed Out

- Point dipole p^i created at t=0
  - $J^0(t,x\} = -\theta(t)\, p^i \partial_i\, \delta^3(x)$
  - $J^i(t,x\} = p^i\, \delta(t)\delta^3(x)$
- Induced $F_{ij}(t,x) = -\theta(t)\, (\partial_i p_j - \partial_j p_i)$ X
  - 0 Loop: $\delta(r^2 - t^2)/2\pi$
  - 1 Loop: $-4G/3\pi\ \delta'(r^2 - t^2)/2\pi$
- Hence $\delta(r^2 - t^2 - 4G/3\pi) + O(G^2)$
  - Super-luminal but Poincare invariant

Smearing of the Light-Cone

- An old idea (Pauli 1956, Deser 1957)
  - $i\Delta[g](x;x') = 1/\{2\pi^2 \sigma[g](x;x')\} + O(\ln\sigma)$
  - $\int d^4x' \{i\Delta[g](x;x')\}^2$ doesn't exist
  - But $\_\Gamma[dg] \int d^4x' \{i\Delta[g](x;x')\}^2$ might
- Something quantum gravity should do
  - Light-cone is a quantum operator
  - S-luminal propagation thru fluctuations
- We might (eventually) build starships

Summary

- $\partial_\nu F^{\nu\mu}(x) + \int d^4x' \, \Pi(x;x') \partial'_\nu F^{\nu\mu}(x') = J^\mu(x)$
  - $\Pi = G/(24\pi^2)\, \partial^6 \{\theta(\Delta t - \Delta r)\, [\ln[\mu^2(\Delta t^2 - \Delta r^2)] - 1]\} + O(G^2)$
- Dynamical photons unchanged
- $\Phi(r) \rightarrow q/(4\pi r)\, [1 + 2G/(3\pi r^2) + O(G^2)]$
- Pulse $\rightarrow$ $r^2 = t^2 + 4G/3\pi + O(G^2)$
  - NB Poincare invariance inevitable in pert.
- Gauge dependence requires work
  - But S-matrix results gauge independent