

# Black hole as a supercollider

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## Plan of talk

Energy in the centre of mass frame and its growth

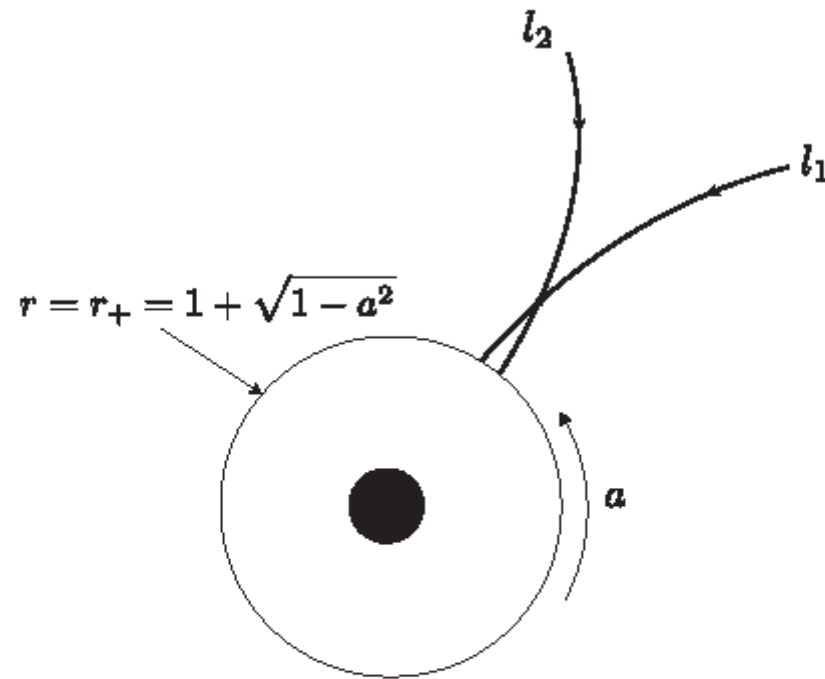
From particular metrics to generic rotating and charged black holes

Strong and weak versions of effect

Kinematic explanation

Geometric explanation

Collisions near inner horizon, role of bifurcation point



$$-2(1 + \sqrt{1 + a}) < l < 2(1 + \sqrt{1 - a})$$

FIG. 1. Schematic picture of two particles falling into a black hole with angular momentum  $a$  (per unit black hole mass) and colliding near the horizon. The allowed range of  $l$  for geodesics falling into the black hole is also given.

Banados, Silk, West

## Energy in centre of mass frame

$$(E_{\text{c.m.}}, 0, 0, 0) = mu_{(1)}^i + mu_{(2)}^i,$$

$$u^i u_i = 1$$

$$E_{\text{c.m.}} = m \sqrt{2} \sqrt{1 + u_{(1)}^i u_{(2)i}}$$

$$E_{cm}^2 = |P_m P^m|$$

Total momentum

$$P_m = p_{(1)m} + p_{(2)m}$$

## Schwarzschild metric (a=0)

Schwarzschild

$$E_{cm} = 2m\sqrt{5}$$

Baushev 2008

No effect

Kerr (rotation)

Reissner-Nordstrom (charge)

## Kerr metric

### Kerr

$$(E_{\text{c.m.}}^{\text{Kerr}})^2 = \frac{2m_0^2}{r(r^2 - 2r + a^2)} [2a^2(1 + r) - 2a(l_2 + l_1) - l_2 l_1(-2 + r) + 2(-1 + r)r^2 - \sqrt{2(a - l_2)^2 - l_2^2 r + 2r^2} \sqrt{2(a - l_1)^2 - l_1^2 r + 2r^2}].$$

On horizon both numerator and denominator=0

Extremal case:  $a=1$

$$E_{\text{c.m.}}^{\text{Kerr}}(r \rightarrow r_+) = \sqrt{2}m_0 \sqrt{\frac{l_2 - 2}{l_1 - 2} + \frac{l_1 - 2}{l_2 - 2}}$$

critical value  $l = 2$

BSW: extremal Kerr

Grib and Pavlov: also nonextremal

O. Z.: generic rotating, charged (even for radial motion)

BSW effect (2009): both particles move towards horizon, needs fine-tuning between energy and momentum

T. Piran and J. Shanam, PRD 1977

1 particle moves away from horizon, no fine-tuning required

## Extremal versus nonextremal

Problems with attaining extremality,  $a=0,998$  (Thorne)

Jacobson et al, Berti et al: difficulties in realization

Grib and Pavlov: nonextremal Kerr

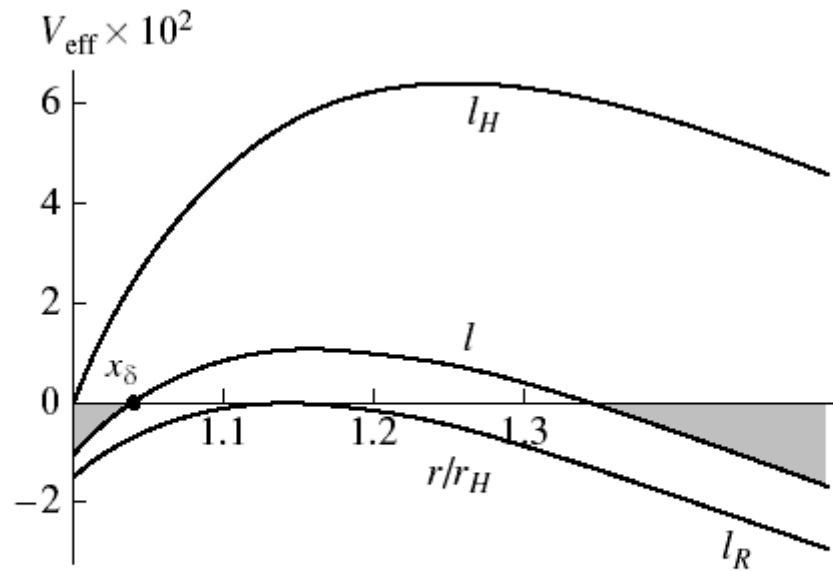
Extremal case: collision near horizon    proper time diverges

$10^3$  larger than that of the LHC one must wait only  $\approx 10^8$  s.

$$E_{\text{c.m.}} \approx \frac{m}{\sqrt{\delta}} \sqrt{\frac{2(l_H - l_2)}{1 - \sqrt{1 - A^2}}}$$

$$l_1 = l_H - \delta,$$





The effective potential for  $A = 0.95$  and  $l_R \approx 2.45$ ,  $l = 2.5$ ,  $l_H \approx 2.76$ . Allowed zones for  $l = 2.5$  are shown by the gray color.

### Multiple scattering (Grib and Pavlov)

$$\begin{aligned}
 -2 \left( 1 + \sqrt{1 + A} \right) &= l_L \leq l \leq l_R \\
 &= 2 \left( 1 + \sqrt{1 - A} \right).
 \end{aligned}$$

$$A = 1 - \epsilon$$

$$\begin{aligned}
 l_H - l_R &= 2 \frac{\sqrt{1 - A}}{A} \left( \sqrt{1 - A} + \sqrt{1 + A} - A \right) \\
 &\approx 2(\sqrt{2} - 1)\sqrt{\epsilon}, \quad \epsilon \rightarrow 0.
 \end{aligned}$$

# Acceleration of particles as universal property of rotating black holes

O. Z., PRD 2010

Role of horizon

Universality of black hole physics

Unified approach to nonextremal versus extremal black holes

$$ds^2 = -N^2 dt^2 + g_{\phi\phi}(d\phi - \omega dt)^2 + dl^2 + g_{zz} dz^2.$$

equatorial plane  $\theta = \frac{\pi}{2}$  ( $z = 0$ ) is a symmetry one.

$$u_0 \equiv -E$$

$$u_\phi \equiv L$$

conserved quantities

Integrals of geodesic equations

$$g_{mn} u^m u^n = -1$$

$$\dot{t} = u^0 = \frac{E - \omega L}{N^2}$$

$$\dot{\phi} = \frac{L}{g_{\phi\phi}} + \frac{(-\omega^2 L + E\omega)}{N^2}$$

$$j^2 = \frac{(E - \omega L)^2}{N^2} - \delta - \frac{L^2}{g_{\phi\phi}}$$

$\delta = 1$  for timelike

$$E_{CM} = \sqrt{2}m_0 \sqrt{1 - g_{\mu\nu} U_{(1)}^\mu U_{(2)}^\nu}$$

$$\frac{E_{cm}^2}{2m^2} = c + 1 - Y, \quad c = \frac{X}{N^2}$$

$$X = X_1 X_2 - Z_1 Z_2$$

$$X_i \equiv E_i - \omega L_i,$$

$$Z_i = \sqrt{(E_i - \omega L_i)^2 - N^2 b_i}, \quad b_i = 1 + \frac{L_i^2}{g_{\phi\phi}},$$

$$Y = \frac{L_1 L_2}{g_{\phi\phi}}.$$

## Different limiting transitions

- 1) Let, for generic  $L_i$ , one approaches the horizon, so  $N \rightarrow 0$ .

$$\left(\frac{E_{cm}^2}{2m^2}\right)_H = 1 + \frac{b_{1(H)}(L_{2(H)} - L_2)}{2(L_{1H} - L_1)} + \frac{b_{2(H)}(L_{(1)H} - L_1)}{2(L_{2(H)} - L_2)} - \frac{L_1 L_2}{(g_{\phi\phi})_H}, \quad L_{i(H)} \equiv \frac{E_i}{\omega_H}$$

$$L_1 = L_{1(H)}(1 - \varepsilon), \quad \varepsilon \ll 1, \quad L_2 \neq L_{2(H)}$$

$$\left(\frac{E_{cm}^2}{2m^2}\right)_H \approx \frac{b_{1(H)}(L_{2(H)} - L_2)}{2L_{1(H)}\varepsilon}.$$

$$\lim_{L_1 \rightarrow L_{1(H)}} \lim_{N \rightarrow 0} E_{cm} = \infty.$$

2) Let us take  $L_1 \rightarrow L_{1(H)}$  first and, then, consider the limit  $N \rightarrow 0$ .

2a) Nonextremal case

It follows from the finiteness of the geometrical scalars

$$R \quad R_{mn} \quad R^{mn}$$

$$\omega = \omega_H + BN^2 + \dots$$

One cannot reach horizon. Of no interest

2b)

$$\omega = \omega_H - B_1N + B_2N^2 + \dots$$

Kerr:  $B_1 = M^{-1}$

$$\frac{E_{cm}^2}{2m^2} \approx \frac{(E_2 - \omega_H L_2)}{N} \left[ B_1 \frac{E_1}{\omega_H} - \sqrt{\left( \frac{E_1^2}{\omega_H^2} B_1^2 - b_1 \right)} \right].$$

$$\lim_{N \rightarrow 0} \lim_{L_1 \rightarrow L_1(H)} E_{cm} = \infty.$$

$$L = L_{(H)}$$

Proper time to approach horizon:

$$\tau \sim \int \frac{dl N}{Z} \sim l \rightarrow \infty$$

$$\Delta\phi \approx \frac{EB_1}{\sqrt{\left( \frac{E^2}{\omega_H^2} B_1^2 - b_1 \right)}} \int \frac{dl}{N}$$

$$N \approx N_0 \exp(-Al)$$

$$\Delta\phi \rightarrow \infty.$$



limits  $\varepsilon \rightarrow 0$  and  $N \rightarrow 0$ .

3) Small but nonzero

$\varepsilon$  and  $N$

$$Z^2 \geq 0$$

$$0 \leq N \leq \frac{E\varepsilon}{\sqrt{b_H}}.$$

4)

$$L_1 = L_{1(H)}, L_2 = L_{2(H)}.$$

Either horizon (nonextremal) is unreachable or  $E$  is finite (extremal).

## Agreement with results for Kerr metric

$$g_{00} = -\left(1 - \frac{2M}{r}\right), g_{0\phi} = -\frac{2Ma}{r}, g_{\phi\phi} = r^2 + a^2 + \frac{2Ma^2}{r}, \omega = -\frac{g_{0\phi}}{g_{\phi\phi}}.$$

$$N^2 = \frac{(r - r_H)(r - r_C)}{r^2 + a^2 + \frac{2M}{r}a^2},$$

where  $r$  is the Boyer-Lindquist coordinate,  $r_H = M + \sqrt{M^2 - a^2}$ ,  $r_C = M - \sqrt{M^2 - a^2}$

horizon value  $\omega_H = \frac{a}{2Mr_H}$

$$\left(\frac{E_{cm}}{2m}\right)_H = \sqrt{1 + \frac{M(l_1 - l_2)^2}{2r_C(l_1 - l_H)(l_2 - l_H)}}$$

$$0 \leq r - r_H \leq \frac{a^2 \varepsilon^2}{r_H \sqrt{1 - \frac{a^2}{M^2}}}$$

$$l_{(H)} = \frac{2r_H}{a}$$

$$l = \frac{L}{M}$$

$$\frac{E_{cm}^2}{2m^2} \approx \frac{(2 - l_2)}{2N} (2 - \sqrt{2}).$$

$$l_H = 2$$

$$\Delta\phi \approx \frac{M\sqrt{2}}{r-M} \rightarrow \infty$$

# Acceleration of particles by nonrotating charged black holes

O. Z. JETP Letters 2010

Role of rotation

$$L_1 = \frac{E_1}{\omega_H}$$

If  $\omega_H \rightarrow 0$ ,  $L_1 \rightarrow \infty$ ,

Angular momentum versus charge

Reissner-Nordström

Pure radial motion

not only  $\omega_H = 0$  but also  $L_1 = L_2 = 0$  for both colliding particles

particles charged, nongeodesic motion

$$ds^2 = -dt^2 f + \frac{dr^2}{f} + r^2 d\omega^2$$

$$f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$$

$$mu^0 = m\dot{t} = \frac{1}{f} \left( E - \frac{qQ}{r} \right),$$

$$m^2 \dot{r}^2 = \left( E - \frac{qQ}{r} \right)^2 - m^2 f.$$

$$\frac{E_{cm}^2}{2m^2} = 1 + \frac{X_1 X_2 - Z_1 Z_2}{fm^2}$$

$$X_i = E_i - \frac{q_i Q}{r}, \quad Z_i = \sqrt{X_i^2 - m^2 f}.$$

1) Let  $f \rightarrow 0$ . Then, we obtain from the (5), (6) that

$$\frac{E_{cm(H)}^2}{2m^2} = 1 + \frac{1}{2} \left[ \frac{q_{2(H)} - q_2}{q_{1(H)} - q_1} + \frac{q_{1(H)} - q_{11}}{q_{2(H)} - q_{22}} \right]$$

$$r_{i(H)} \equiv \frac{E r_H}{Q}$$

$$r_H = M + \sqrt{M^2 - Q^2} \geq Q$$

$$q_{(H)} > E$$

If particle comes from infinity,  $E \geq m$

$$q_{(H)} > m$$

“overcharged”

If, say,  $q_1 = q_{1(H)}(1 - \delta)$  with  $\delta \ll 1$  and  $q_2 \neq q_{2(H)}$ , the energy  $E_{cm(H)} \sim \frac{1}{\sqrt{\delta}}$  can be made as large as one likes. Thus, we have that

$$\lim_{q_1 \rightarrow q_{1(H)}} \lim_{r \rightarrow r_H} E_{cm} = \infty.$$

2) Let now  $q_1 = q_{1(H)}$  from the very beginning,  $q_2 \neq q_{2(H)}$ .

Nonextremal horizon unreachable

Extremal horizon

$$\frac{E_{cm}^2}{2m^2} = 1 + \frac{X_{2(H)}}{m^2(1 - \frac{r_H}{r})} [E_1 - \sqrt{(E_1^2 - m^2)}] + O((1 - \frac{r_H}{r})).$$

$$\lim_{r \rightarrow r_H} \lim_{q_1 \rightarrow q_{1(H)}} E_{cm} = \infty.$$

3)

$$q_1 = q_{1(H)}, q_2 = q_{2(H)}$$

Effect is absent

## LIMITING TRANSITIONS FOR TIME AND CONDITIONS OF COLLISION

- 1) E(center of mass) tends to infinity
- 2) Collision does occur

Particle 1 starts at  $t=0$ ,  $r=r(\text{initial})$

Particle 2 starts at the later moment  $t = t_0 > 0$  from the same point

Collision on horizon

$$t_0 = t_1 - t_2 > 0, t_1 = \int_{r_f}^{r_i} \frac{dr X_1}{f \sqrt{X_1^2 - m^2 f}}, t_2 = \int_{r_f}^{r_i} \frac{dr X_2}{f \sqrt{X_2^2 - m^2 f}}$$



## Kinematic explanation

$$\begin{aligned} E_{\text{c.m.}}^2 &= -(p_1^\mu + p_2^\mu)(p_{1\mu} + p_{2\mu}) \\ &= m_1^2 + m_2^2 - 2m_1m_2u_1^\mu u_{2\mu}. \end{aligned}$$

$$\gamma = -u_1^\mu u_{2\mu} = \frac{1}{\sqrt{1-w^2}}$$

BSW effect occurs if  $w \rightarrow 1$   $w$  is relative velocity

$$w^2 = 1 - \frac{(1-v_1^2)(1-v_2^2)}{[1-v_1v_2(\vec{n}_1\vec{n}_2)]^2}.$$

$$\vec{v}_1 = v_1\vec{n}_1$$

The most interesting case:

$$v_1 < 1,$$

$$v_2 \rightarrow 1$$

Collision of rapid particle with target

$$ds^2 = -N^2 dt^2 + g_{\phi\phi}(d\phi - \omega dt)^2 + dl^2 + g_{zz} dz^2.$$

Attached to observer

$$\begin{aligned} h_{(0)\mu} &= -N(1, 0, 0, 0), \\ h_{(1)\mu} &= (0, 1, 0, 0), \\ h_{(2)\mu} &= \sqrt{g_{zz}}(0, 0, 0, 1), \\ h_{(3)\mu} &= \sqrt{g_{\phi\phi}}(-\omega, 0, 0, 1). \end{aligned}$$

$$\begin{aligned} -u_\mu h_{(0)}^\mu &= \frac{E - \omega L}{N}, \\ u_\mu h_{(3)}^\mu &= \frac{L}{\sqrt{g_{\phi\phi}}}. \end{aligned}$$

If

$$V_m = h_{m(0)}$$

then

$$V_m X^{(3)m} = 0$$

ZAMO

$$v^{(i)} = v_{(i)} = \frac{u^\mu h_{\mu(i)}}{-u^\mu h_{\mu(0)}}.$$

$$E - \omega L = \frac{N}{\sqrt{1 - v^2}},$$

Horizon limit

$$N \rightarrow 0$$

1) Usual particle,

$$E \neq w_+ L \quad v \rightarrow 1$$

2) Critical particle

$$E = w_+ L \quad v \rightarrow v_0 < 1$$

## Collision between massive and massless particles (electron and photon)

$$E - wL = NE_{kin}$$

$$n_0 - wL = nN$$

$$E_{kin} = \frac{m}{\sqrt{1 - V^2}}$$

$n$  Measured by local observer

$E_{kin}$  Measured by local observer

$E$  conserved

$n_0$  conserved

Usual:  $V \rightarrow 1$   $E_{kin} \rightarrow \infty$

Usual:  $n \rightarrow \infty$

Critical:  $V \rightarrow V_+ < 1$

Critical:  $n$  finite

$E_{kin}$  finite

Play of two factors: Doppler effect (DE) and gravitational blue shift (GB)

*Case 1: Electron is critical, photon is usual*

Finite redshift - DE . GB infinite  
Effect exists

*Case 2: Electron is usual, photon is critical*

DE infinite, GB finite Effect exists

*Case 3: Both particles are critical*

DE and GB finite, no effect

*4. Case 4: Both particles are usual*

Infinite DE compensated by infinite GB  
Outcome is finite, no effect

Ultimate manifestation of kinematic nature of BSW effect:  
target at rest

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2$$

$$\varphi = \frac{Q}{r}$$

$$f = 1 - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3},$$

Radial motion

$$m^2 \dot{r}^2 = -V_{eff} = X^2 - m^2 f.$$

$$X = E - q\varphi.$$

d  
i  
a

Equilibrium

$$V_{eff}(r_0) = 0.$$

$$V'_{eff}(r_0) = 0$$

perpetual turning point

## Analogue of innermost stable orbit in Kerr metric

$$V''_{eff}(r_0) = 0.$$

We are interested in points near horizon

$$-\frac{1}{2}V'_{eff}(r_0) = m\sqrt{f(r_0)}\frac{qQ}{r_0^2} - \frac{m^2}{2}f'(r_0).$$

Nonextremal: surface gravity  $k = \frac{f'(r_+)}{2} \neq 0$

On the horizon  $V'_{eff}(r_0) \rightarrow -m^2\kappa$

No such points

## Near-extremal

$$\kappa \ll Dx.$$

$$x = r_0 - r_+$$

$$x^3 \approx H^3 \kappa^2,$$

$$H^3 = \frac{3r_+^3}{4(-\Lambda)(1 - 2\Lambda r_+^2)}$$

$$\Lambda < 0$$

## Collisions with unbound energies

$$\frac{E_{c.m.}^2}{2m^2} = 1 + \frac{X_1 X_2 - Z_1 Z_2}{fm^2}$$

where

$$Z_i = \sqrt{X_i^2 - m^2 f}, \quad i = 1, 2.$$

$$Z_1 = 0$$

$$\frac{E^2}{2m^2} = 1 + \frac{X_2}{m\sqrt{f}}$$

$$E_{c.m.} \approx \sqrt{2mX_2} A\kappa^{-\frac{1}{3}}$$



2 particles, nonlocal picture

Role of gravity: to diminish velocity,  
Acceleration due to deceleration!

## KINEMATIC CENSORSHIP

1) Collision on horizon of extremal Kerr: infinite proper time

2) Now: if we take the limit of zero surface gravity or take extremal horizon from the very beginning: no timelike trajectory on lightlike surface

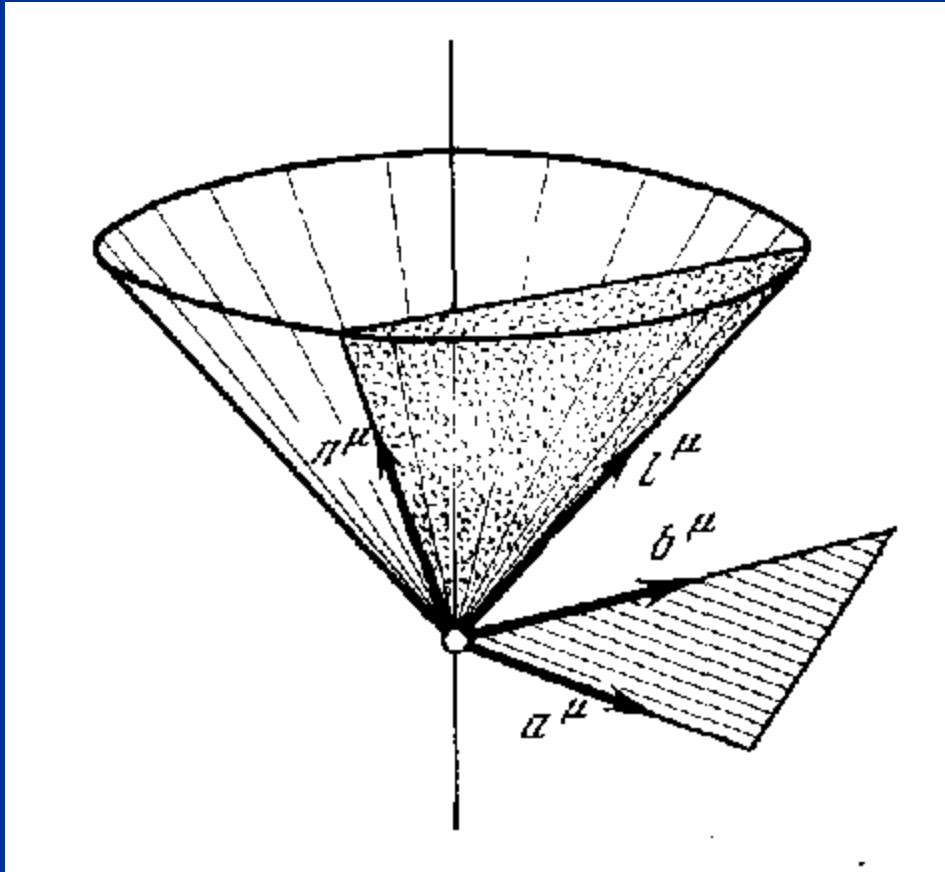
## Geometric explanation

$$g_{\alpha\beta} = -l_{\alpha}N_{\beta} - l_{\beta}N_{\alpha} + \sigma_{\alpha\beta}$$

lightlike vectors  $l^{\mu}, N^{\mu}$

spacelike vectors

$a^{\mu}, b^{\mu}$  orthogonal to them



## Four-velocity

$$u_i^\mu = \frac{l^\mu}{2\alpha_i} + \alpha_i N^\mu + s_i^\mu, \quad s_i^\mu = A_i a^\mu + B_i b^\mu$$

$$-(u_1 u_2) = \frac{1}{2} \left( \frac{\alpha_1}{\alpha_2} + \frac{\alpha_2}{\alpha_1} \right) - (s_1 s_2).$$

$$E_{c.m.}^2 = m_1^2 + m_2^2 + m_1 m_2 \left[ \frac{\alpha_1}{\alpha_2} + \frac{\alpha_2}{\alpha_1} - 2(s_1 s_2) \right].$$

$a=0$

Case 1  
always

For head-on collision  
means that particle  
cannot cross horizon

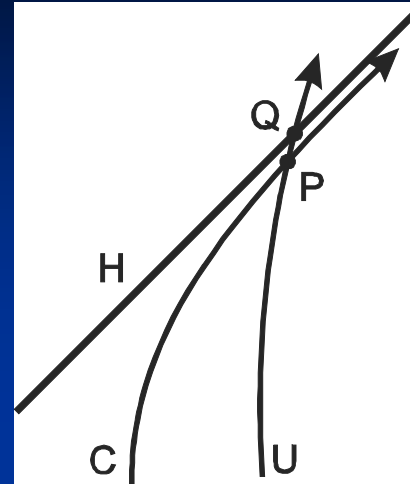
Case 2  
Special condition

## Case 2

$$\alpha_1 \rightarrow 0$$

Now, special condition

$$E_{c.m.}^2 \rightarrow \infty.$$



Kruskal-like coordinates

$$ds^2 = -CdXdY + \gamma_{ab}dx^a dx^b$$

$$Cu^X u^Y = 1$$

$$u^X \sim \alpha \rightarrow 0.$$

$$\tau \sim -\ln X \rightarrow \infty$$

## Reissner-Nordstrom

### Case 1

$$\alpha = \frac{X - Z}{m}$$

$$Z = \sqrt{X^2 - m^2 N^2}$$

$$X = E - \frac{qQ}{r}$$

On the horizon  $Z = X_H$

$$a \rightarrow 0$$

on horizon always

### Case 2

$$\alpha_H = \frac{2X_H}{m} > 0$$

On horizon

Special condition

## BSW effect versus Penrose process

What energy can be observed at infinity?

### 1) Radial motion in RN metric

$$ds^2 = -N^2 dt^2 + \frac{dr^2}{N^2} + r^2(d\theta^2 + d\phi^2 \sin^2 \theta).$$

$$u_{(i)}^\mu = \frac{l^\mu}{2\alpha_i} + \beta_i N^\mu,$$

$$\alpha_i = \frac{X_i - \varepsilon_i Z_i}{m_i}, \quad X_i \equiv E_i - q_i \varphi,$$

$$Z_i = \sqrt{X_i^2 - m_i^2 N^2},$$

$$m(\alpha_1 + \alpha_2) = \mu(\alpha_3 + \alpha_4),$$
$$m\left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2}\right) = \mu\left(\frac{1}{\alpha_3} + \frac{1}{\alpha_4}\right).$$

## Outgoing particles

$$a = \frac{X - Z}{m} \quad Z = \sqrt{X^2 - m^2 N^2}$$

near horizon,  $a \rightarrow 0$  automatically

$$\alpha_4 \approx \frac{\mu_4}{m} \alpha_1$$

1) BSW effect,  $a_1 \rightarrow 0$

$$m \leq E < \frac{m^2}{E_1 + \sqrt{E_1^2 - m^2}}$$

- 2) Special situation: usual particles collide and produce light particles

$$E_4 > E$$

$$\sqrt{\frac{2N(\cosh \gamma_1 + \cosh \gamma_2)}{\alpha_1}} m < \mu \ll m.$$

$$N \ll 1$$

$$E_i = m \cosh g_i$$



## Rotating black holes

$$ds^2 = -N^2 dt^2 + g_{\phi\phi}(d\phi - \omega dt)^2 + dn^2 + g_{zz} dz^2$$

$$X = \frac{E - \omega mL}{m}$$

Critical particle:  $E = \omega_H mL$

Now,  
near horizon

$$Z = N \sqrt{B_1^2 L^2 - \left(1 + \frac{L^2}{g_{\phi\phi}^H}\right) + O(N^2)},$$

$$\omega = \omega_H - B_1 N + O(N^2)$$

$$E_4 \approx E_1 \frac{1 - \sqrt{1 - \frac{1}{B_1^2 g_{\phi\phi}^H}}}{1 + \sqrt{1 - \frac{1}{B_1^2 g_{\phi\phi}^H}}} < E_1.$$

Extremal Kerr metric

$$E_4 \approx E_1 \frac{2 - \sqrt{3}}{2 + \sqrt{3}} \approx 0.07 E_1.$$

## Complimentarity

BSW effect: large energy in CM frame near horizon, modest energy at infinity

Finite energy in CM frame near horizon but large energy at infinity

# Circular orbits and acceleration of particles by near-extremal dirty rotating black holes: general approach

Circular orbits in astrophysics. Kerr metric (Bardeen et al, 1972)

BSW effect: critical particle. Natural realization near horizon

ISCO innermost stable orbit

MBO marginally bound orbit

PhO photon orbit

$$ds^2 = -N^2 dt^2 + g_{\phi\phi}(d\phi - \omega dt)^2 + dl^2 + g_{zz} dz^2.$$

$$N^2 \dot{l}^2 \equiv -V_{eff} = (E - \omega L)^2 - bN^2, \quad b = \left(\gamma + \frac{L^2}{g_{\phi\phi}}\right).$$

$$dl = \frac{d\rho}{N}.$$

$$V_{eff}(\rho_0) = 0,$$
$$\frac{dV_{eff}}{d\rho}(\rho_0) = 0$$

For nonextremal BH

$$-\frac{1}{2} \frac{dV_{eff}}{d\rho} \rightarrow -b_+ \kappa \neq 0$$

No orbits near horizon

Near extremal, small surface gravity

ISCO

$$\frac{d^2 V_{eff}(\rho_0)}{d\rho^2} = 0.$$

$$x = \rho_0 - \rho_+$$

$$x : k^{2/3}$$

MBO

PhO

$E=m$

$x : k$

$x : k$

## PROPER DISTANCE.

$$\rho \approx \rho_+$$

Between horizon and MBO or PhO,

Proper distance is finite

Between horizon and ISCO,  $l : \ln \frac{1}{k}$

## Two scenarios of collisions

O –scenario. Collision on circular near-horizon orbit

H – scenario Collision on horizon with participation of particle that plunged to horizon from circular orbit

O - version

H - version

ISCO

$$E_{c.m.} : k^{-1/3}$$

$$E_{c.m.} : k^{-1/3}$$

MBO, PhO

$$E_{c.m.} : k^{-1/2}$$

Universality



## Collisions on inner horizon

Again, one of two particles should be critical. Then, the following cases are possible

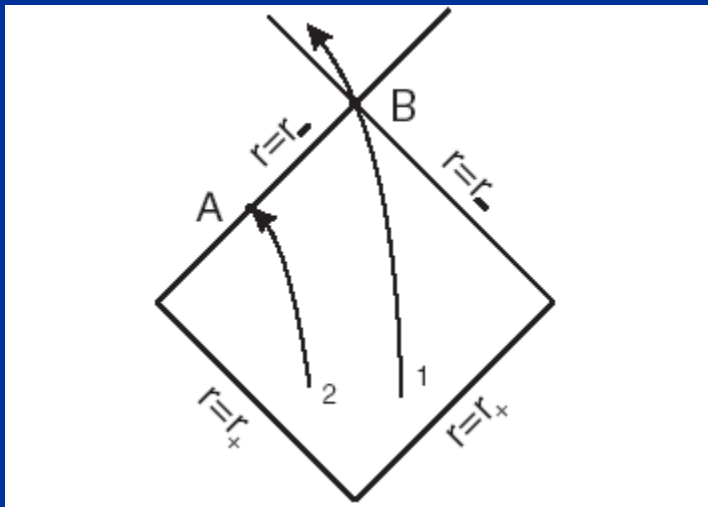


FIG. 1. Impossibility of the strong version of the BSW effect. The critical particle 1 passes through the bifurcation point whereas a usual one 2 hits the left horizon.

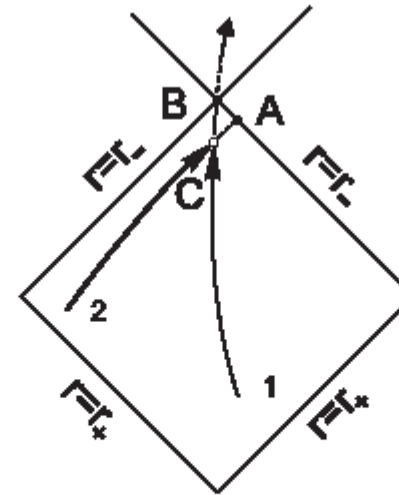


FIG. 2. The weak version of the BSW effect. Near-horizon collision between critical particle 1 and usual one 2.

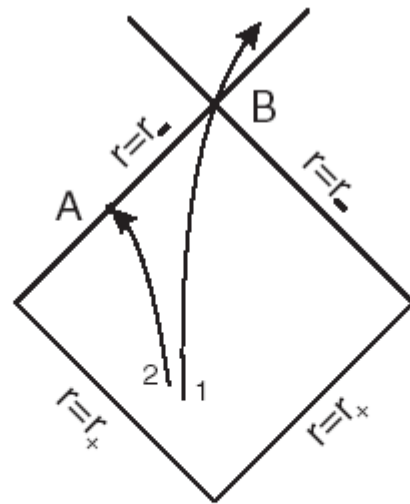


FIG. 3. Impossibility of the strong version of the PS effect. The critical particle 1 passes through the bifurcation point, whereas a usual one 2 hits the left horizon.

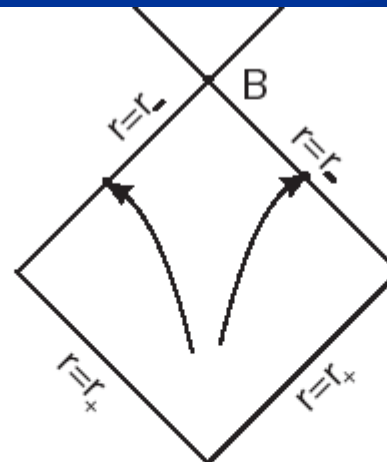


FIG. 4. Impossibility of the strong version of the PS effect. Two usual particles hit different branches of the horizon.

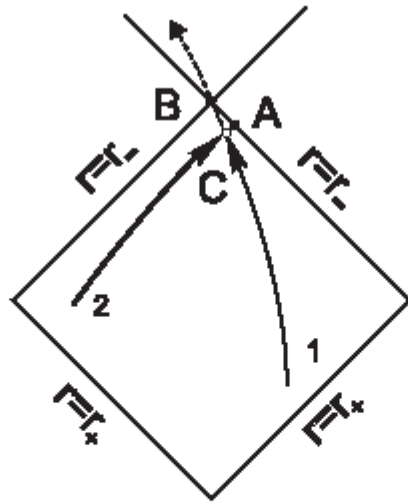


FIG. 5. The weak version of the PS effect. Near-horizon collision between critical particle 1 and usual one 2.

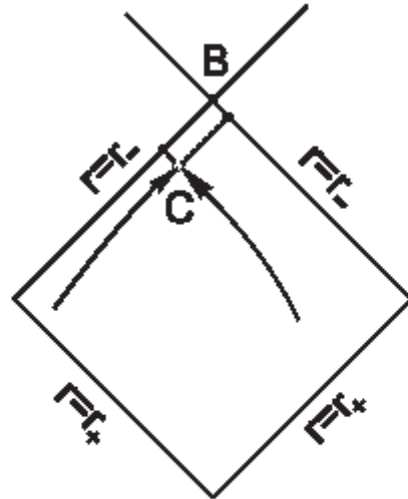


FIG. 6. The weak version of the PS effect. Collision between two usual particles near the left horizon.

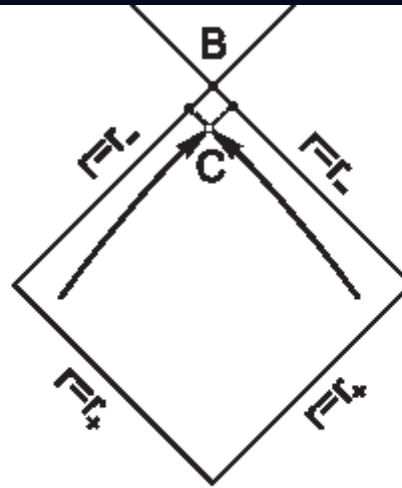


FIG. 7. The weak version of the PS effect. Collision between two usual particles near the bifurcation point.

Kinematic censorship

## Summary and conclusions

Main ingredients of the effect:

1) event horizon,

Universality of effect

2) critical particle

Rotating and charged BH – universal property

Extremal BH. Kinematic censorship: infinite proper time

Nonextremal BH: multiple scattering, narrow near-horizon strip

Kinematic explanation: in terms of relative velocity

Geometric explanation: local light cone

Potential applications: Probe of Planck scale, properties of galaxy nuclei, etc.

Thank you!