On massive gravity and bigravity in three dimensions

Yu. M. Zinoviev

Institute for High Energy Physics, Protvino, Russia

Based on arXiv:1205.6892

Outlook

1. Massless case
   - Gravity
   - Bigravity
2. Massive case
   - Frame-like gauge invariant formalism
   - Gravitational interactions
   - Self-interaction
   - Beyond linear approximation
3. Conclusion

Outlook

1. Massless case
   - Gravity
   - Bigravity
2. Massive case
   - Frame-like gauge invariant formalism
   - Gravitational interactions
   - Self-interaction
   - Beyond linear approximation
3. Conclusion

Outlook

1. Massless case
   - Gravity
   - Bigravity
2. Massive case
   - Frame-like gauge invariant formalism
   - Gravitational interactions
   - Self-interaction
   - Beyond linear approximation
3. Conclusion

Massless gravity in a frame-like formalism

- Dual variables $\omega_\mu{}^{ab} \to \omega_\mu{}^a = \varepsilon^{abc}\omega_\mu{}^{bc}$.
- Free Lagrangian and gauge transformations ($\sigma = \pm 1$):

$$\sigma \mathcal{L}_0 = \frac{1}{2}\left\{ {}^{\mu\nu}_{ab} \right\} \omega_\mu{}^a \omega_\mu{}^b - \varepsilon^{\mu\nu\alpha}\omega_\mu{}^a D_\nu h_\alpha{}^a - \frac{\Lambda}{2}\left\{ {}^{\mu\nu}_{ab} \right\} h_\mu{}^a h_\nu{}^b$$

$$\delta_0 h_\mu{}^a = D_\mu \hat{\xi}^a + \varepsilon_\mu{}^{ab}\hat{\eta}^b, \qquad \delta_0 \omega_\mu{}^a = D_\mu \hat{\eta}^a - \Lambda \varepsilon_\mu{}^{ab}\hat{\xi}^b$$

where $\left\{ {}^{\mu\nu}_{ab} \right\} = e^\mu{}_a e^\nu{}_b - e^\mu{}_b e^\nu{}_a$ and $[D_\mu, D_\nu]\xi^a = -\Lambda e_{[\mu}{}^a \xi_{\nu]}$.

- Cubic vertex

$$\begin{aligned} \mathcal{L}_1 &= \kappa_0 \left\{ {}^{\mu\nu\alpha}_{abc} \right\} [h_\mu{}^a \omega_\nu{}^b \omega_\alpha{}^c - \frac{\Lambda}{3} h_\mu{}^a h_\nu{}^b h_\alpha{}^c] \\ \delta_1 h_\mu{}^a &= -2\sigma\kappa_0 \varepsilon^{abc}[h_\mu{}^b \hat{\eta}^c + \omega_\mu{}^b \hat{\xi}^c] \\ \delta_1 \omega_\mu{}^a &= -2\sigma\kappa_0 \varepsilon^{abc}[\omega_\mu{}^b \hat{\eta}^c - \Lambda h_\mu{}^b \hat{\xi}^c] \end{aligned}$$

- No quartic vertices $\Longrightarrow$ we obtain complete theory.

Massless gravity in a frame-like formalism

- Dual variables $\omega_\mu{}^{ab} \to \omega_\mu{}^a = \varepsilon^{abc}\omega_\mu{}^{bc}$.
- Free Lagrangian and gauge transformations ($\sigma = \pm 1$):

$$\sigma\mathcal{L}_0 = \frac{1}{2}\left\{ {}^{\mu\nu}_{ab} \right\} \omega_\mu{}^a \omega_\mu{}^b - \varepsilon^{\mu\nu\alpha}\omega_\mu{}^a D_\nu h_\alpha{}^a - \frac{\Lambda}{2}\left\{ {}^{\mu\nu}_{ab} \right\} h_\mu{}^a h_\nu{}^b$$

$$\delta_0 h_\mu{}^a = D_\mu \hat{\xi}^a + \varepsilon_\mu{}^{ab}\hat{\eta}^b, \qquad \delta_0 \omega_\mu{}^a = D_\mu \hat{\eta}^a - \Lambda \varepsilon_\mu{}^{ab}\hat{\xi}^b$$

where $\left\{ {}^{\mu\nu}_{ab} \right\} = e^\mu{}_a e^\nu{}_b - e^\mu{}_b e^\nu{}_a$ and $[D_\mu, D_\nu]\xi^a = -\Lambda e_{[\mu}{}^a \xi_{\nu]}$.

- Cubic vertex

$$\begin{aligned} \mathcal{L}_1 &= \kappa_0 \left\{ {}^{\mu\nu\alpha}_{abc} \right\} [h_\mu{}^a \omega_\nu{}^b \omega_\alpha{}^c - \frac{\Lambda}{3} h_\mu{}^a h_\nu{}^b h_\alpha{}^c] \\ \delta_1 h_\mu{}^a &= -2\sigma\kappa_0 \varepsilon^{abc}[h_\mu{}^b \hat{\eta}^c + \omega_\mu{}^b \hat{\xi}^c] \\ \delta_1 \omega_\mu{}^a &= -2\sigma\kappa_0 \varepsilon^{abc}[\omega_\mu{}^b \hat{\eta}^c - \Lambda h_\mu{}^b \hat{\xi}^c] \end{aligned}$$

- No quartic vertices $\Longrightarrow$ we obtain complete theory.

Massless gravity in a frame-like formalism

- Dual variables $\omega_\mu{}^{ab} \to \omega_\mu{}^a = \varepsilon^{abc}\omega_\mu{}^{bc}$.
- Free Lagrangian and gauge transformations ($\sigma = \pm 1$):

$$\sigma\mathcal{L}_0 = \frac{1}{2}\left\{ {}^{\mu\nu}_{ab} \right\} \omega_\mu{}^a\omega_\mu{}^b - \varepsilon^{\mu\nu\alpha}\omega_\mu{}^a D_\nu h_\alpha{}^a - \frac{\Lambda}{2}\left\{ {}^{\mu\nu}_{ab} \right\} h_\mu{}^a h_\nu{}^b$$

$$\delta_0 h_\mu{}^a = D_\mu\hat{\xi}^a + \varepsilon_\mu{}^{ab}\hat{\eta}^b, \qquad \delta_0\omega_\mu{}^a = D_\mu\hat{\eta}^a - \Lambda\varepsilon_\mu{}^{ab}\hat{\xi}^b$$

where $\left\{ {}^{\mu\nu}_{ab} \right\} = e^\mu{}_a e^\nu{}_b - e^\mu{}_b e^\nu{}_a$ and $[D_\mu, D_\nu]\xi^a = -\Lambda e_{[\mu}{}^a\xi_{\nu]}$.

- Cubic vertex

$$\begin{aligned}
\mathcal{L}_1 &= \kappa_0\left\{ {}^{\mu\nu\alpha}_{abc} \right\}[h_\mu{}^a\omega_\nu{}^b\omega_\alpha{}^c - \frac{\Lambda}{3}h_\mu{}^a h_\nu{}^b h_\alpha{}^c] \\
\delta_1 h_\mu{}^a &= -2\sigma\kappa_0\varepsilon^{abc}[h_\mu{}^b\hat{\eta}^c + \omega_\mu{}^b\hat{\xi}^c] \\
\delta_1\omega_\mu{}^a &= -2\sigma\kappa_0\varepsilon^{abc}[\omega_\mu{}^b\hat{\eta}^c - \Lambda h_\mu{}^b\hat{\xi}^c]
\end{aligned}$$

- No quartic vertices $\Longrightarrow$ we obtain complete theory.

Massless gravity in a frame-like formalism

- Dual variables $\omega_\mu{}^{ab} \to \omega_\mu{}^a = \varepsilon^{abc}\omega_\mu{}^{bc}$.
- Free Lagrangian and gauge transformations ($\sigma = \pm 1$):

$$\sigma\mathcal{L}_0 = \frac{1}{2}\left\{ {}^{\mu\nu}_{ab} \right\} \omega_\mu{}^a \omega_\mu{}^b - \varepsilon^{\mu\nu\alpha}\omega_\mu{}^a D_\nu h_\alpha{}^a - \frac{\Lambda}{2}\left\{ {}^{\mu\nu}_{ab} \right\} h_\mu{}^a h_\nu{}^b$$

$$\delta_0 h_\mu{}^a = D_\mu \hat{\xi}^a + \varepsilon_\mu{}^{ab}\hat{\eta}^b, \qquad \delta_0 \omega_\mu{}^a = D_\mu \hat{\eta}^a - \Lambda \varepsilon_\mu{}^{ab}\hat{\xi}^b$$

where $\left\{ {}^{\mu\nu}_{ab} \right\} = e^\mu{}_a e^\nu{}_b - e^\mu{}_b e^\nu{}_a$ and $[D_\mu, D_\nu]\xi^a = -\Lambda e_{[\mu}{}^a \xi_{\nu]}$.

- Cubic vertex

$$\begin{aligned} \mathcal{L}_1 &= \kappa_0 \left\{ {}^{\mu\nu\alpha}_{abc} \right\} [h_\mu{}^a \omega_\nu{}^b \omega_\alpha{}^c - \frac{\Lambda}{3} h_\mu{}^a h_\nu{}^b h_\alpha{}^c] \\ \delta_1 h_\mu{}^a &= -2\sigma\kappa_0 \varepsilon^{abc}[h_\mu{}^b \hat{\eta}^c + \omega_\mu{}^b \hat{\xi}^c] \\ \delta_1 \omega_\mu{}^a &= -2\sigma\kappa_0 \varepsilon^{abc}[\omega_\mu{}^b \hat{\eta}^c - \Lambda h_\mu{}^b \hat{\xi}^c] \end{aligned}$$

- No quartic vertices $\Longrightarrow$ we obtain complete theory.

Massless bigravity — linear approximation

- General $d \geq 3$ case see e.g. Boulanger e.a. 2000.
- Four independent cubix vertices:

- Example of cross-interaction

$$\mathcal{L}_1 = \kappa_2 \left\{ {}^{\mu\nu\alpha}_{abc} \right\} [h_\mu{}^a \Omega_\nu{}^b \Omega_\alpha{}^c + 2 f_\mu{}^a \omega_\nu{}^b \Omega_\alpha{}^c - \frac{\Lambda}{2} h_\mu{}^a f_\nu{}^b f_\alpha{}^c]$$

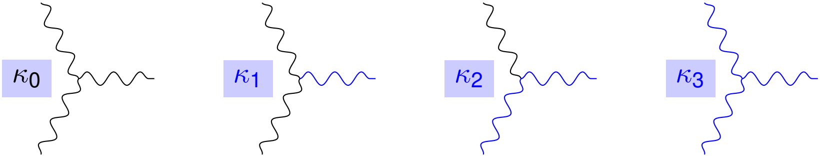
Corrections to gauge transformations, e.g.:

$$\delta_1 \omega_\mu{}^a = -2\sigma\kappa_2 \varepsilon^{abc} \Omega_\mu{}^b \eta^c, \qquad \delta_1 \Omega_\mu{}^a = -2\kappa_2 \varepsilon^{abc} \omega_\mu{}^b \eta^c$$

and similarly for $h_\mu{}^a$ and $f_\mu{}^a$.

Massless bigravity — linear approximation

- General $d \geq 3$ case see e.g. Boulanger e.a. 2000.
- Four independent cubix vertices:



- Example of cross-interaction

$$\mathcal{L}_1 = \kappa_2 \left\{ {}^{\mu\nu\alpha}_{abc} \right\} [h_\mu{}^a \Omega_\nu{}^b \Omega_\alpha{}^c + 2 f_\mu{}^a \omega_\nu{}^b \Omega_\alpha{}^c - \frac{\Lambda}{2} h_\mu{}^a f_\nu{}^b f_\alpha{}^c]$$

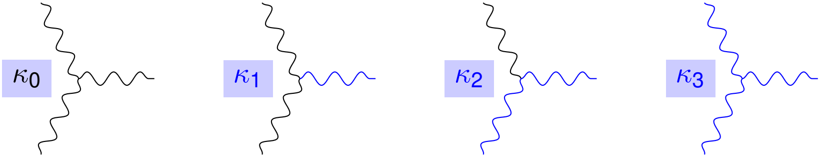
Corrections to gauge transformations, e.g.:

$$\delta_1 \omega_\mu{}^a = -2\sigma\kappa_2 \varepsilon^{abc} \Omega_\mu{}^b \eta^c, \qquad \delta_1 \Omega_\mu{}^a = -2\kappa_2 \varepsilon^{abc} \omega_\mu{}^b \eta^c$$

and similarly for $h_\mu{}^a$ and $f_\mu{}^a$.

Massless bigravity — linear approximation

- General $d \geq 3$ case see e.g. Boulanger e.a. 2000.
- Four independent cubix vertices:



- Example of cross-interaction

$$\mathcal{L}_1 = \kappa_2 \left\{ {}^{\mu\nu\alpha}_{abc} \right\} [h_\mu{}^a \Omega_\nu{}^b \Omega_\alpha{}^c + 2 f_\mu{}^a \omega_\nu{}^b \Omega_\alpha{}^c - \frac{\Lambda}{2} h_\mu{}^a f_\nu{}^b f_\alpha{}^c]$$

Corrections to gauge transformations, e.g.:

$$\delta_1 \omega_\mu{}^a = -2\sigma\kappa_2 \varepsilon^{abc} \Omega_\mu{}^b \eta^c, \qquad \delta_1 \Omega_\mu{}^a = -2\kappa_2 \varepsilon^{abc} \omega_\mu{}^b \eta^c$$

and similarly for $h_\mu{}^a$ and $f_\mu{}^a$.

Massless bigravity — quadratic approximation

- General massless bigravity has one relation:

$$\kappa_1{}^2 + \kappa_2{}^2 - \sigma\kappa_0\kappa_2 - \kappa_1\kappa_3 = 0$$

  so we have two independent coupling constants and a kind of "mixing angle".

- But in $d = 3$ cubic vertex for two massless spin 2 and one massive one does not exists. (For general $d \geq 4$ case see e.g. R. Metsaev arXiv:1205.3131).
- Thus we have to put $\kappa_1 = 0$ that leaves us with

Massless bigravity — quadratic approximation

- General massless bigravity has one relation:

$$\kappa_1{}^2 + \kappa_2{}^2 - \sigma\kappa_0\kappa_2 - \kappa_1\kappa_3 = 0$$

  so we have two independent coupling constants and a kind of "mixing angle".

- But in $d = 3$ cubic vertex for two massless spin 2 and one massive one does not exists. (For general $d \geq 4$ case see e.g. R. Metsaev arXiv:1205.3131).

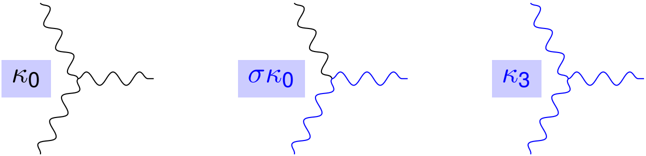
- Thus we have to put $\kappa_1 = 0$ that leaves us with

Massless bigravity — quadratic approximation

- General massless bigravity has one relation:

$$\kappa_1{}^2 + \kappa_2{}^2 - \sigma\kappa_0\kappa_2 - \kappa_1\kappa_3 = 0$$

  so we have two independent coupling constants and a kind of ”mixing angle”.
- But in $d = 3$ cubic vertex for two massless spin 2 and one massive one does not exists. (For general $d \geq 4$ case see e.g. R. Metsaev arXiv:1205.3131).
- Thus we have to put $\kappa_1 = 0$ that leaves us with



Frame-like gauge invariant formalism

- Set of fields: $(\Omega_\mu{}^a, f_\mu{}^a)$, (B^a, A_μ) and (π^a, φ).
- Lagrangian:

$$\begin{aligned}\mathcal{L}_0 &= \frac{1}{2}\left\{ {}^{\mu\nu}_{ab} \right\} \Omega_\mu{}^a \Omega_\nu{}^b - \varepsilon^{\mu\nu\alpha} \Omega_\mu{}^a D_\nu f_\alpha{}^a + \frac{1}{2} B_a{}^2 - \varepsilon^{\mu\nu\alpha} B_\mu D_\nu A_\alpha - \\ &- \frac{1}{2}\pi_a{}^2 + \pi^\mu D_\mu \varphi + m\varepsilon^{\mu\nu\alpha}[-2\Omega_{\mu\nu} A_\alpha + B_\mu f_{\nu\alpha}] + 2M\pi^\mu A_\mu + \\ &+ \frac{M^2}{2}\left\{ {}^{\mu\nu}_{ab} \right\} f_\mu{}^a f_\nu{}^b + 2mMe^\mu{}_a f_\mu{}^a \varphi + 3m^2\varphi^2\end{aligned}$$

  where $M^2 = 2m^2 - \Lambda$.
- Gauge transformations:

$$\begin{aligned}\delta_0 \Omega_\mu{}^a &= D_\mu \eta^a + M^2 \varepsilon_\mu{}^{ab} \xi^b \\ \delta_0 f_\mu{}^a &= D_\mu \xi^a + \varepsilon_\mu{}^{ab} \eta^b + 2me_\mu{}^a \xi \\ \delta_0 B^a &= -2m\eta^a, \qquad \delta_0 A_\mu = D_\mu \xi + m\xi_\mu \\ \delta_0 \pi^a &= 2mM\xi^a, \qquad \delta_0 \varphi = -2M\xi\end{aligned}$$

Frame-like gauge invariant formalism

- Set of fields: $(\Omega_\mu{}^a, f_\mu{}^a)$, (B^a, A_μ) and (π^a, φ).
- Lagrangian:

$$\begin{aligned}\mathcal{L}_0 &= \frac{1}{2}\left\{ {}^{\mu\nu}_{ab} \right\} \Omega_\mu{}^a \Omega_\nu{}^b - \varepsilon^{\mu\nu\alpha}\Omega_\mu{}^a D_\nu f_\alpha{}^a + \frac{1}{2}B_a{}^2 - \varepsilon^{\mu\nu\alpha} B_\mu D_\nu A_\alpha - \\ &- \frac{1}{2}\pi_a{}^2 + \pi^\mu D_\mu \varphi + m\varepsilon^{\mu\nu\alpha}[-2\Omega_{\mu\nu}A_\alpha + B_\mu f_{\nu\alpha}] + 2M\pi^\mu A_\mu + \\ &+ \frac{M^2}{2}\left\{ {}^{\mu\nu}_{ab} \right\} f_\mu{}^a f_\nu{}^b + 2mMe^\mu{}_a f_\mu{}^a \varphi + 3m^2\varphi^2\end{aligned}$$

where $M^2 = 2m^2 - \Lambda$.

- Gauge transformations:

$$\begin{aligned}\delta_0 \Omega_\mu{}^a &= D_\mu \eta^a + M^2 \varepsilon_\mu{}^{ab}\xi^b \\ \delta_0 f_\mu{}^a &= D_\mu \xi^a + \varepsilon_\mu{}^{ab}\eta^b + 2me_\mu{}^a \xi \\ \delta_0 B^a &= -2m\eta^a, \qquad \delta_0 A_\mu = D_\mu \xi + m\xi_\mu \\ \delta_0 \pi^a &= 2mM\xi^a, \qquad \delta_0 \varphi = -2M\xi\end{aligned}$$

Frame-like gauge invariant formalism

- Set of fields: $(\Omega_\mu{}^a, f_\mu{}^a)$, (B^a, A_μ) and (π^a, φ).
- Lagrangian:

$$\begin{aligned} \mathcal{L}_0 &= \frac{1}{2}\left\{ {}^{\mu\nu}_{ab} \right\} \Omega_\mu{}^a \Omega_\nu{}^b - \varepsilon^{\mu\nu\alpha} \Omega_\mu{}^a D_\nu f_\alpha{}^a + \frac{1}{2} B_a{}^2 - \varepsilon^{\mu\nu\alpha} B_\mu D_\nu A_\alpha - \\ &- \frac{1}{2} \pi_a{}^2 + \pi^\mu D_\mu \varphi + m \varepsilon^{\mu\nu\alpha} [-2\Omega_{\mu\nu} A_\alpha + B_\mu f_{\nu\alpha}] + 2M\pi^\mu A_\mu + \\ &+ \frac{M^2}{2} \left\{ {}^{\mu\nu}_{ab} \right\} f_\mu{}^a f_\nu{}^b + 2mMe^\mu{}_a f_\mu{}^a \varphi + 3m^2 \varphi^2 \end{aligned}$$

  where $M^2 = 2m^2 - \Lambda$.
- Gauge transformations:

$$\begin{aligned} \delta_0 \Omega_\mu{}^a &= D_\mu \eta^a + M^2 \varepsilon_\mu{}^{ab} \xi^b \\ \delta_0 f_\mu{}^a &= D_\mu \xi^a + \varepsilon_\mu{}^{ab} \eta^b + 2m e_\mu{}^a \xi \\ \delta_0 B^a &= -2m\eta^a, \qquad \delta_0 A_\mu = D_\mu \xi + m\xi_\mu \\ \delta_0 \pi^a &= 2mM\xi^a, \qquad \delta_0 \varphi = -2M\xi \end{aligned}$$

Gravitational interactions

- Only standard interactions (up to terms that can be removed by field redefinitions):

$$e_\mu{}^a \Rightarrow h_\mu{}^a, \qquad D_\mu \Rightarrow \mathcal{D}_\mu, \qquad \mathcal{D}_\mu \xi^a = D_\mu \xi^a - 2\kappa_0 \varepsilon^{abc} \omega_\mu{}^b \xi^c$$

- Cubic vertex:

$$\mathcal{L}_1 = \kappa_2 \left\{ {}^{\mu\nu\alpha}_{abc} \right\} [h_\mu{}^a \Omega_\nu{}^b \Omega_\alpha{}^c + 2 f_\mu{}^a \omega_\nu{}^b \Omega_\alpha{}^c + \frac{M^2}{2} h_\mu{}^a f_\mu{}^b f_\alpha{}^c]$$

- Corrections to gauge transfromations:

$$\begin{aligned}
\delta_1 \omega_\mu{}^a &= -2\sigma\kappa_2 \varepsilon^{abc} [\Omega_\mu{}^b \eta^c + M^2 f_\mu{}^b \xi^c] - 4m\sigma\kappa_2 \Omega_\mu{}^a \xi \\
\delta_1 h_\mu{}^a &= -2\sigma\kappa_2 \varepsilon^{abc} [f_\mu{}^b \eta^c + \Omega_\mu{}^b \xi^c] \\
\delta_1 \Omega_\mu{}^a &= -2\kappa_2 \varepsilon^{abc} [\Omega_\mu{}^b \hat{\eta}^c + \omega_\mu{}^b \eta^c + M^2 f_\mu{}^b \hat{\xi}^c + M^2 h_\mu{}^b \xi^c] \\
\delta_1 f_\mu{}^a &= -2\kappa_2 \varepsilon^{abc} [f_\mu{}^b \hat{\eta}^c + \Omega_\mu{}^b \hat{\xi}^c + h_\mu{}^b \eta^c + \omega_\mu{}^b \xi^c] + 4m\kappa_2 h_\mu{}^a \xi
\end{aligned}$$

Gravitational interactions

- Only standard interactions (up to terms that can be removed by field redefinitions):

$$e_\mu{}^a \Rightarrow h_\mu{}^a, \qquad D_\mu \Rightarrow \mathcal{D}_\mu, \qquad \mathcal{D}_\mu \xi^a = D_\mu \xi^a - 2\kappa_0 \varepsilon^{abc} \omega_\mu{}^b \xi^c$$

- Cubic vertex:

$$\mathcal{L}_1 = \kappa_2 \left\{ {}^{\mu\nu\alpha}_{abc} \right\} [h_\mu{}^a \Omega_\nu{}^b \Omega_\alpha{}^c + 2 f_\mu{}^a \omega_\nu{}^b \Omega_\alpha{}^c + \frac{M^2}{2} h_\mu{}^a f_\mu{}^b f_\alpha{}^c]$$

- Corrections to gauge transfromations:

$$\begin{aligned}
\delta_1 \omega_\mu{}^a &= -2\sigma\kappa_2 \varepsilon^{abc} [\Omega_\mu{}^b \eta^c + M^2 f_\mu{}^b \xi^c] - 4m\sigma\kappa_2 \Omega_\mu{}^a \xi \\
\delta_1 h_\mu{}^a &= -2\sigma\kappa_2 \varepsilon^{abc} [f_\mu{}^b \eta^c + \Omega_\mu{}^b \xi^c] \\
\delta_1 \Omega_\mu{}^a &= -2\kappa_2 \varepsilon^{abc} [\Omega_\mu{}^b \hat{\eta}^c + \omega_\mu{}^b \eta^c + M^2 f_\mu{}^b \hat{\xi}^c + M^2 h_\mu{}^b \xi^c] \\
\delta_1 f_\mu{}^a &= -2\kappa_2 \varepsilon^{abc} [f_\mu{}^b \hat{\eta}^c + \Omega_\mu{}^b \hat{\xi}^c + h_\mu{}^b \eta^c + \omega_\mu{}^b \xi^c] + 4m\kappa_2 h_\mu{}^a \xi
\end{aligned}$$

Gravitational interactions

- Only standard interactions (up to terms that can be removed by field redefinitions):

$$e_\mu{}^a \Rightarrow h_\mu{}^a, \qquad D_\mu \Rightarrow \mathcal{D}_\mu, \qquad \mathcal{D}_\mu \xi^a = D_\mu \xi^a - 2\kappa_0 \varepsilon^{abc} \omega_\mu{}^b \xi^c$$

- Cubic vertex:

$$\mathcal{L}_1 = \kappa_2 \left\{ {}^{\mu\nu\alpha}_{abc} \right\} [h_\mu{}^a \Omega_\nu{}^b \Omega_\alpha{}^c + 2 f_\mu{}^a \omega_\nu{}^b \Omega_\alpha{}^c + \frac{M^2}{2} h_\mu{}^a f_\mu{}^b f_\alpha{}^c]$$

- Corrections to gauge transfromations:

$$\begin{aligned}
\delta_1 \omega_\mu{}^a &= -2\sigma\kappa_2 \varepsilon^{abc} [\Omega_\mu{}^b \eta^c + M^2 f_\mu{}^b \xi^c] - 4m\sigma\kappa_2 \Omega_\mu{}^a \xi \\
\delta_1 h_\mu{}^a &= -2\sigma\kappa_2 \varepsilon^{abc} [f_\mu{}^b \eta^c + \Omega_\mu{}^b \xi^c] \\
\delta_1 \Omega_\mu{}^a &= -2\kappa_2 \varepsilon^{abc} [\Omega_\mu{}^b \hat{\eta}^c + \omega_\mu{}^b \eta^c + M^2 f_\mu{}^b \hat{\xi}^c + M^2 h_\mu{}^b \xi^c] \\
\delta_1 f_\mu{}^a &= -2\kappa_2 \varepsilon^{abc} [f_\mu{}^b \hat{\eta}^c + \Omega_\mu{}^b \hat{\xi}^c + h_\mu{}^b \eta^c + \omega_\mu{}^b \xi^c] + 4m\kappa_2 h_\mu{}^a \xi
\end{aligned}$$

Self-interaction

- Non-standard (though similar in structure) interactions
- Cubic vertex:

$$\mathcal{L}_1 = \kappa_3 \left\{ {}^{\mu\nu\alpha}_{abc} \right\} [f_\mu{}^a \Omega_\nu{}^b \Omega_\alpha{}^c + \frac{M^2 + m^2}{3} f_\mu{}^a f_\nu{}^b f_\alpha{}^c]$$

- Note that in full Lagrangian there are terms proportional to m^2/M^2 so that partially massless limit $M \to 0$ is impossible.
- Corrections to gauge transformations:

$$\begin{aligned} \delta_1 \Omega_\mu{}^a &= -2\kappa_3 \varepsilon^{abc} [\Omega_\mu{}^b \eta^c + (M^2 + m^2) f_\mu{}^b \xi^c] - 2m\kappa_3 \Omega_\mu{}^a \xi \\ \delta_1 f_\mu{}^a &= -2\kappa_3 \varepsilon^{abc} [f_\mu{}^b \eta^c + \Omega_\mu{}^b \xi^c] + 2m\kappa_3 f_\mu{}^a \xi \end{aligned}$$

Self-interaction

- Non-standard (though similar in structure) interactions
- Cubic vertex:

$$\mathcal{L}_1 = \kappa_3 \left\{ {}^{\mu\nu\alpha}_{abc} \right\} [f_\mu{}^a \Omega_\nu{}^b \Omega_\alpha{}^c + \frac{M^2 + m^2}{3} f_\mu{}^a f_\nu{}^b f_\alpha{}^c]$$

- Note that in full Lagrangian there are terms proportional to m^2/M^2 so that partially massless limit $M \to 0$ is impossible.
- Corrections to gauge transformations:

$$\begin{aligned} \delta_1 \Omega_\mu{}^a &= -2\kappa_3 \varepsilon^{abc} [\Omega_\mu{}^b \eta^c + (M^2 + m^2) f_\mu{}^b \xi^c] - 2m\kappa_3 \Omega_\mu{}^a \xi \\ \delta_1 f_\mu{}^a &= -2\kappa_3 \varepsilon^{abc} [f_\mu{}^b \eta^c + \Omega_\mu{}^b \xi^c] + 2m\kappa_3 f_\mu{}^a \xi \end{aligned}$$

Self-interaction

- Non-standard (though similar in structure) interactions
- Cubic vertex:

$$\mathcal{L}_1 = \kappa_3 \left\{ {}^{\mu\nu\alpha}_{abc} \right\} [f_\mu{}^a \Omega_\nu{}^b \Omega_\alpha{}^c + \frac{M^2 + m^2}{3} f_\mu{}^a f_\nu{}^b f_\alpha{}^c]$$

- Note that in full Lagrangian there are terms proportional to m^2/M^2 so that partially massless limit $M \to 0$ is impossible.
- Corrections to gauge transformations:

$$\begin{aligned} \delta_1 \Omega_\mu{}^a &= -2\kappa_3 \varepsilon^{abc} [\Omega_\mu{}^b \eta^c + (M^2 + m^2) f_\mu{}^b \xi^c] - 2m\kappa_3 \Omega_\mu{}^a \xi \\ \delta_1 f_\mu{}^a &= -2\kappa_3 \varepsilon^{abc} [f_\mu{}^b \eta^c + \Omega_\mu{}^b \xi^c] + 2m\kappa_3 f_\mu{}^a \xi \end{aligned}$$

Self-interaction

- Non-standard (though similar in structure) interactions
- Cubic vertex:

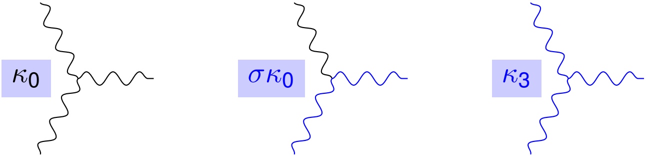
$$\mathcal{L}_1 = \kappa_3 \left\{ {}^{\mu\nu\alpha}_{abc} \right\} [f_\mu{}^a \Omega_\nu{}^b \Omega_\alpha{}^c + \frac{M^2 + m^2}{3} f_\mu{}^a f_\nu{}^b f_\alpha{}^c]$$

- Note that in full Lagrangian there are terms proportional to m^2/M^2 so that partially massless limit $M \to 0$ is impossible.
- Corrections to gauge transformations:

$$\begin{aligned} \delta_1 \Omega_\mu{}^a &= -2\kappa_3 \varepsilon^{abc} [\Omega_\mu{}^b \eta^c + (M^2 + m^2) f_\mu{}^b \xi^c] - 2m\kappa_3 \Omega_\mu{}^a \xi \\ \delta_1 f_\mu{}^a &= -2\kappa_3 \varepsilon^{abc} [f_\mu{}^b \eta^c + \Omega_\mu{}^b \xi^c] + 2m\kappa_3 f_\mu{}^a \xi \end{aligned}$$

Beyond linear approximation

- In the massless case we have:



- Additional gauge symmetry:

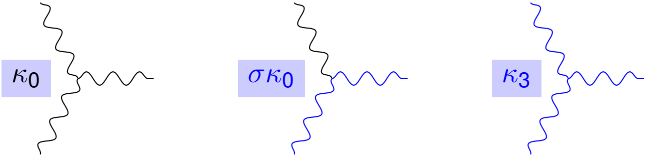
$$\begin{aligned} \delta_1 \omega_\mu{}^a &= -4m\kappa_0 \Omega_\mu{}^a \xi, \qquad \delta_1 \Omega_\mu{}^a = -2m\kappa_3 \Omega_\mu{}^a \xi \\ \delta_1 f_\mu{}^a &= 4\sigma m \kappa_0 h_\mu{}^a \xi + 2m\kappa_3 f_\mu{}^a \xi \end{aligned}$$

- Result:

$$4\sigma\kappa_0{}^2 + \kappa_3{}^2 = 0 \quad \Longrightarrow \quad \sigma = -1$$

Beyond linear approximation

- In the massless case we have:



- Additional gauge symmetry:

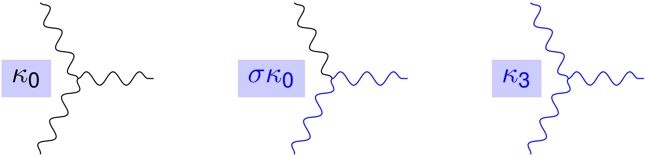
$$\delta_1 \omega_\mu{}^a = -4m\kappa_0 \Omega_\mu{}^a \xi, \qquad \delta_1 \Omega_\mu{}^a = -2m\kappa_3 \Omega_\mu{}^a \xi$$
$$\delta_1 f_\mu{}^a = 4\sigma m\kappa_0 h_\mu{}^a \xi + 2m\kappa_3 f_\mu{}^a \xi$$

- Result:

$$4\sigma\kappa_0{}^2 + \kappa_3{}^2 = 0 \quad \Longrightarrow \quad \sigma = -1$$

Beyond linear approximation

- In the massless case we have:



- Additional gauge symmetry:

$$\delta_1 \omega_\mu{}^a = -4m\kappa_0 \Omega_\mu{}^a \xi, \qquad \delta_1 \Omega_\mu{}^a = -2m\kappa_3 \Omega_\mu{}^a \xi$$
$$\delta_1 f_\mu{}^a = 4\sigma m\kappa_0 h_\mu{}^a \xi + 2m\kappa_3 f_\mu{}^a \xi$$

- Result:

$$4\sigma\kappa_0{}^2 + \kappa_3{}^2 = 0 \quad \Longrightarrow \quad \sigma = -1$$

Conclusion

- Constructive approach based on frame-like gauge invariant formalism does allows us systematicaly investigate consistent ghost-free theories.
- Such approach can be staightforwardly extended to massive higher spins, in this
  - No quartic vertices for $s \geq 2$
  - No extra fields $\Rightarrow$ no higher derivatives
- Similarly, it can be used for investigation of massive spin 2 (and higher) models in $d \geq 4$ dimensions.

Conclusion

- Constructive approach based on frame-like gauge invariant formalism does allows us systematicaly investigate consistent ghost-free theories.
- Such approach can be staightforwardly extended to massive higher spins, in this
  - No quartic vertices for $s \geq 2$
  - No extra fields $\Rightarrow$ no higher derivatives
- Similarly, it can be used for investigation of massive spin 2 (and higher) models in $d \geq 4$ dimensions.

Conclusion

- Constructive approach based on frame-like gauge invariant formalism does allows us systematicaly investigate consistent ghost-free theories.
- Such approach can be staightforwardly extended to massive higher spins, in this
  - No quartic vertices for $s \geq 2$
  - No extra fields $\Rightarrow$ no higher derivatives
- Similarly, it can be used for investigation of massive spin 2 (and higher) models in $d \geq 4$ dimensions.