Catalysis of Black Holes Production at the LHC

Irina Aref'eva

Steklov Mathematical Institute, Moscow

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Motivation:

- **Small cross-section of BH production at the LHC**
- **Question: what we can do to $\uparrow$ cross-section**
- **Modify**
    - **TeV gravity**
    - **Conditions for collisions**

$$G_{\mu\nu} = 8\pi G_N \left(T_{\mu\nu} + T_{\mu\nu}^{(catalyst)} \right)$$

Modify:

- **Conditions for collisions at the LHC**

$$G_{\mu\nu} = 8\pi G_N (T_{\mu\nu} + \quad T_{\mu\nu}^{(\text{catalyst})})$$

$$T_{\mu\nu}^{(\text{catalyst})} \approx \Lambda g_{\mu\nu}$$

$$\Lambda > 0$$

$$\Lambda < 0$$

- **Colliding objects**

$$T_{\mu\nu}^{(p)} \Rightarrow T_{\mu\nu}^{(p,e,s\ldots)}$$

Outlook

(***of the introductory part*** *of the talk*):

- **TeV gravity**
- **Quantum Gravity**
- **Black holes production**

  (in flat background)

Outlook
(*of recent results in non flat background*):

- **Black holes production**

  Trapped Surface formation in dS

- **Trapped Surface formation in AdS**

  (few remarks in the context of AdS/CFT)

BLACK HOLE PRODUCTION

- **BH forms if the impact parameter *b* is comparable to the Schwarzschild radius r_s of a BH of mass *E*.**

- **The Thorn's hoop conjecture gives a rough estimate for classical geometrical cross-section**

$$\sigma(1+1 \to \mathrm{BH}) \sim \pi\, r_S^2$$

$$r_S^2 \sim \frac{M_{BH}}{M_D^2}$$

BLACK HOLE PRODUCTION

- To deal with BH creation in particles collisions we have to deal with trans-Planckian scales.
- Trans-Planckian collisions in standard QG have inaccessible energy scale and cannot be realized in usual conditions.
- N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, I. Antoniadis, 1998
- TeV Gravity to produce BH at Labs (1999)

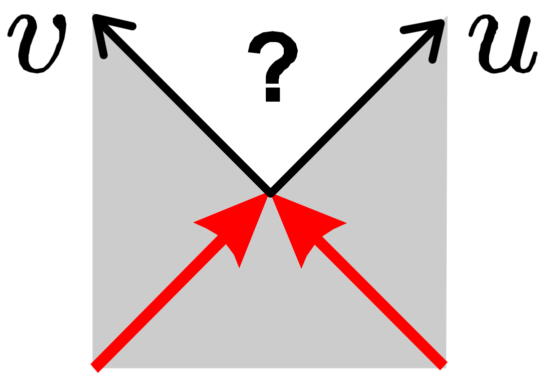
Banks, Fischler, hep-th/9906038
I.A., hep-th/9910269,
Giuduce, Rattazzi, Wells, hep-ph/0112161
Giddings, hep-ph/0106219
Dimopolos, Landsberg, hep-ph/0106295,

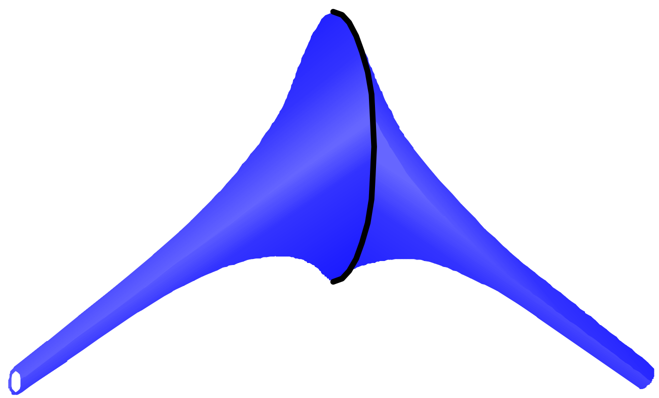
……………………………

D-dimensional Aichelburg-Sexl Shock Waves

$$ds^2 = -du\,dv + dx^{i^2} + \varphi(x^i)\delta(u)\,du^2, \quad \varphi(x^i) = \frac{c}{\rho^{D-4}}$$

Shock waves, **Penrose, D'Eath, Eardley, Giddings, …**





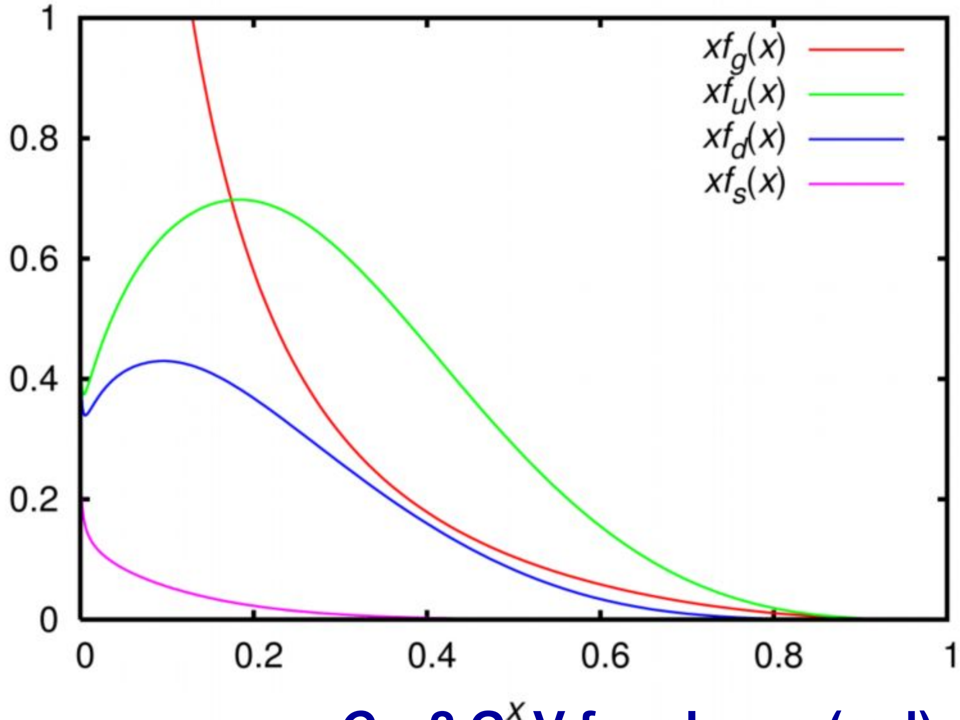
Classical geometric cross-section $\sigma \propto \pi r_s^2$

$$b \le r_S \propto E^{1/D-3}$$

$$D = 7 \quad \sigma \propto 100\,pb$$

Parton Distribution Functions

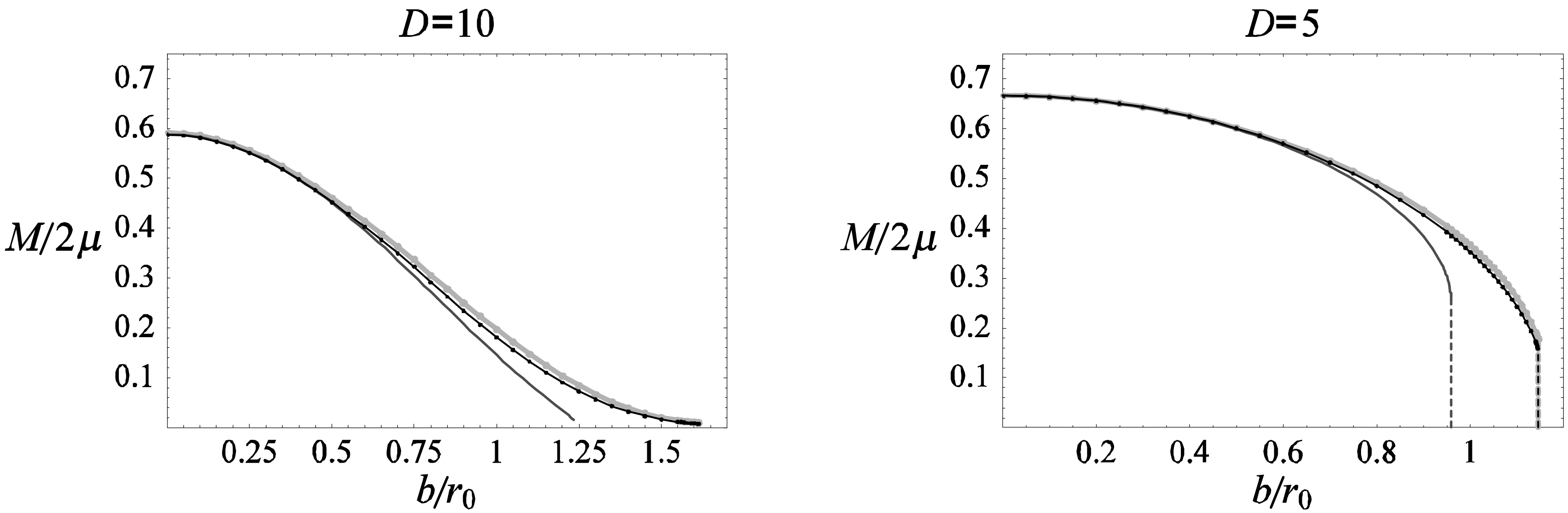
$$\sigma_{pp\to X} \propto \sum_{ij} \int dx_1 \int dx_2 f_i(x_1) f_i(x_2) \sigma_{ij\to X}(x_1, x_2)$$



Q = 2 GeV for gluons (red), up (green), down (blue), and strange (violet) quarks

From:

Inelasticity



The ratio of the mass of the BH to the initial energy of the collision as a function of the impact parameter divided by r_{sh} (the Schwarzschild radius)

Eardley, Yoshino, Randall

Catalyze of BH production due to an anisotropy

Modify:

Conditions for collisions at the LHC

$$G_{\mu\nu} = 8\pi G_N (T_{\mu\nu} + T_{\mu\nu}^{(\mathrm{catalyst})})$$

$$\Lambda > 0$$

I.A., Bagrov, E.Guseva, 0905.1087

$$T_{\mu\nu}^{(\mathrm{catalyst})} \approx \Lambda g_{\mu\nu}$$

Nastase; Gubser, Pufu, Yarom, 08,09; Alvarez-Gaume, Gomes,.. 08; Shuryak 08, …….

$$\Lambda < 0$$

Colliding objects - shock waves with e,s,…

In flat space: Yoshino, Mann 06; Yoshino, Zelnikov V.Frolov 07.

With lambda: work in progress, I.A., L.Joukowskaya

Framework (in words)

- Shock-waves as an approximation for ultrarelativistic particles.

- We analyze possible influence of cosmological constant on the process of a trapped surface formation in collisions of two ultrarelativistic particles

Metric of the space-time with shock wave

- M4 space-time with a shock wave

$$ds^2 = -dudv + dx^{i\,2} + \varphi(x^i)\delta(u)\,du^2, \quad \varphi(x^i) = \frac{c}{\rho^{D-4}}$$

- dS$_4$ space-time with a shock wave as submanifold ($-2uv + \vec{x}^2 = a^2$) of 5-dim. space-time with metric

$$ds^2 = -2dudv + d\vec{x}^2 + F(\vec{x})\delta(u)du^2$$

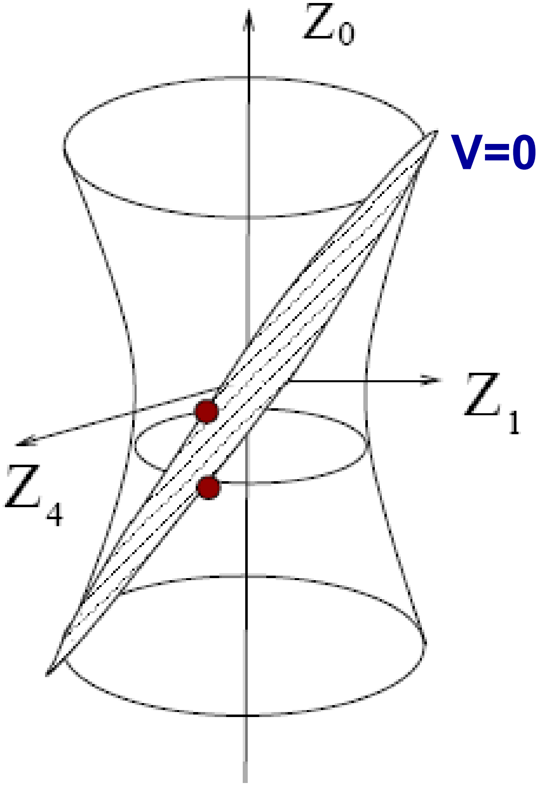
- The shape of the shock-wave is obtained by boosting of the Schwarzschild solution in dS:

$$F(x^i) = -2 + \frac{x^4}{a}\ln\left(\frac{a + x^4}{a - x^4}\right)$$

Hotta-Tanake, 92
Sfetsos, 95
Griffiths,Podolsky, 97
Horowitz, Itzhaki, 99
Emparan. 01

Shock wave in dS

$$a^2 = -Z_0^2 + \sum_{M=1}^{D} Z_M^2$$



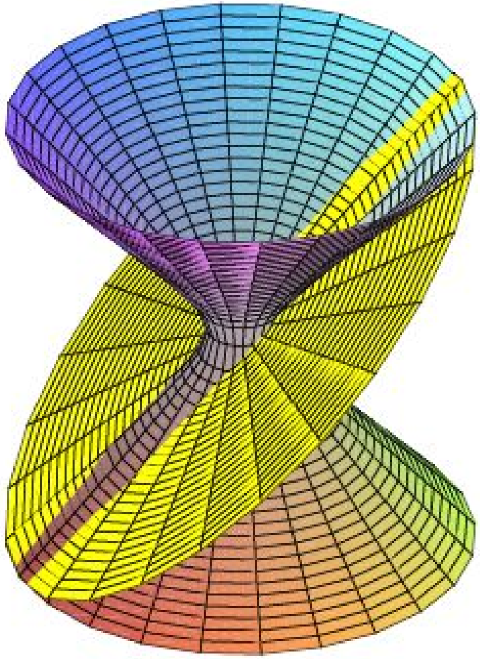


Figure 1: One shock wave in the de Sitter space presented as a hyperboloid is located on the intersection of the hyperboloid and the plane $Z_0 - Z_1 = 0.Z_2$ and Z_3 are suppressed At fixed "Z_0-time" this cross section consists of two points (small red ball in the left picture)

Two Shock waves in dS

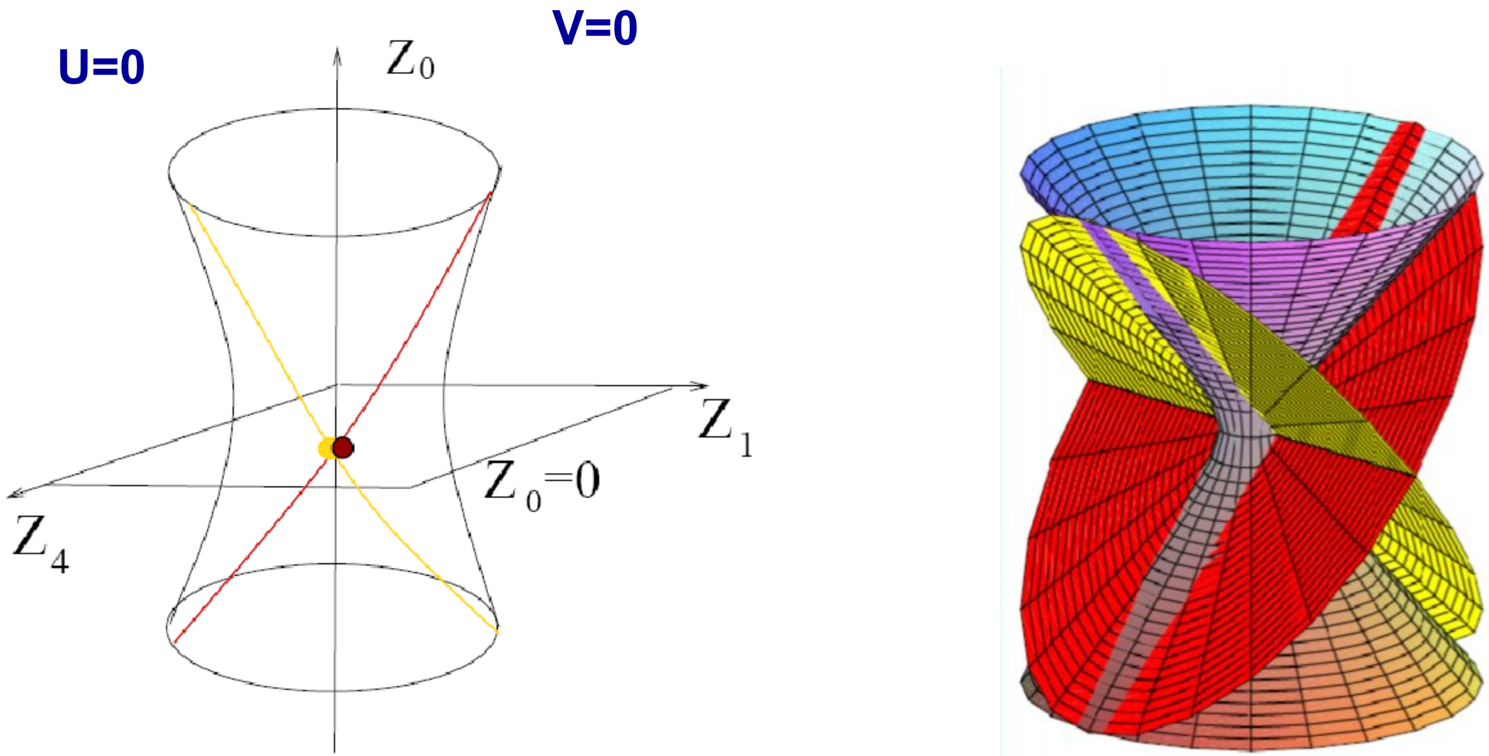


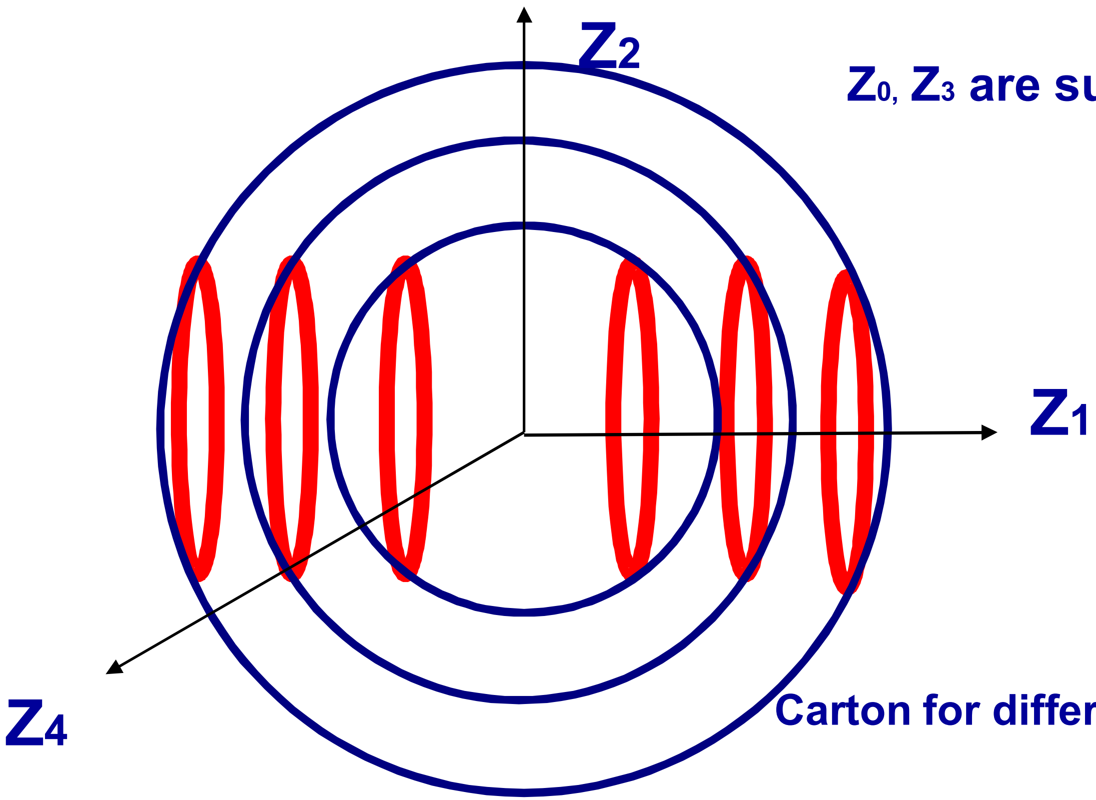
Figure 2: Two shock waves in the de Sitter space. A collision of two shock waves takes place at $Z_0 = 0$ and corresponds to a collision of red and yellow balls.

Shock wave in dS (nonexpanding shock waves).

$$a^2 = -Z_0^2 + \sum_{M=1}^{D} Z_M^2$$

Z_0 fixed $\quad S^{D-1}$

Shock wave $Z_1+Z_0=0$ → Z_1 fixed $\quad S^{D-2}$

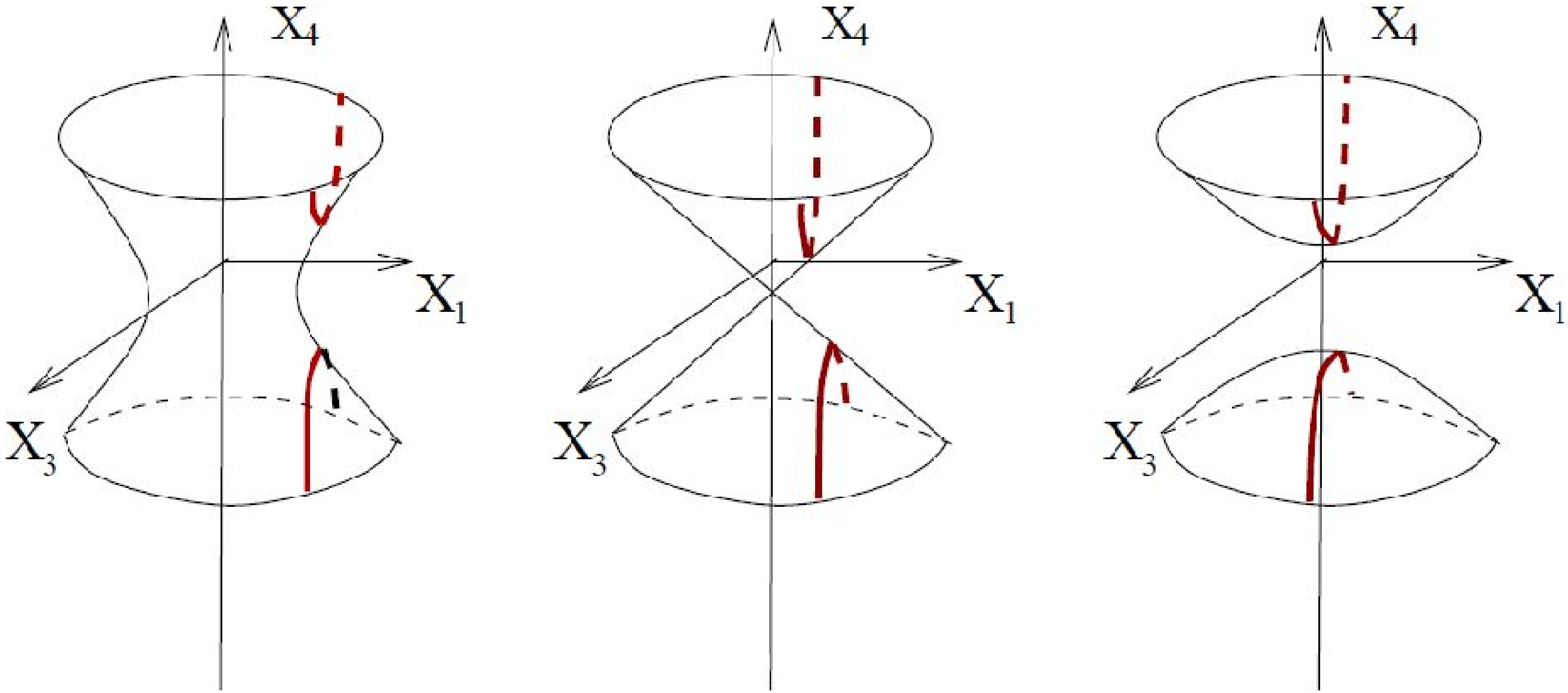


Z_0, Z_3 are suppressed

Carton for different Z_0 time

Shock wave in AdS

$$-a^2 = -2uv + \sum_{i=2}^{D-1}(X^i)^2 - (X^D)^2$$



Our goal

- The equation for trapped surface for two colliding shock waves in dS

$$ds^2 = -2dudv + d\vec{x}^2$$

$$+ F(\xi, \xi_1)\delta(u)du^2 + F(\xi, \xi_2)\delta(v)dv^2.$$

For this purpose: geodesics

n-field approach

- The null-geodesics could be derived from n-field-like Lagrangian:

$$\int d\tau\,[\frac{dX^M(\tau)}{d\tau}G_{MN}(X(\tau))\frac{dX^N(\tau)}{d\tau} - \lambda\left(X^M(\tau)g_{MN}X^N(\tau) - a^2\right)]$$

- Where:

$$G_{MN}[X] = g_{MN} + h_{MN}[X]$$

$$g_{MN} \;\; = \;\; -\delta^U_M\delta^V_N + \delta^N_M\delta^i_N, \quad h_{MN}[U,V,X] = \delta^U_M\delta^U_N F(X)\delta(U)$$

n-field approach

$$\mathcal{S} = \int d^D x \, (\partial n)^2, \quad n = (n_1, ... n_N)$$

$$n^2 = \frac{N}{\gamma^2}$$

$$\mathcal{S} = \int d^D x \left[(\partial n)^2 - \lambda \left(n^2 - \frac{N}{\gamma^2} \right) \right]$$

$$\int d^D x \left[\partial_\alpha X^M g_{MN} \partial^\alpha X^N - \lambda \left(X^M g_{MN} X^N - a^2 \right) \right]$$

Null-geodesics

- Based on this Lagrangian we can derive the equations of geodesics in following simple form:

$$\begin{aligned}
\ddot{u} &= -\frac{1}{2a^2}(-F + x^i F_{,i})\delta(u)\dot{u}^2 u \\
\ddot{v} - \frac{1}{F}\delta'(u)\dot{u}^2 - F_{,i}\delta(u)\dot{u}\dot{x}^i &= -\frac{1}{2a^2}(-F + x^i F_{,i})\delta(u)\dot{u}^2 v \\
\ddot{x}^i - \frac{1}{2}F_{,i}\delta(u)\dot{u}^2 &= -\frac{1}{2a^2}(-F + x^i F_{,i})\delta(u)\dot{u}^2 x^i
\end{aligned}$$

Null-geodesics

- Solution of these equations:

$$
\begin{aligned}
v &= v_0 + v_1 u + Q(x_0^j)\theta(u) + R(x_0^j)\theta(u)u \\
x^i &= x_0^i + x_1^i u + S_i \theta(u) u
\end{aligned}
$$

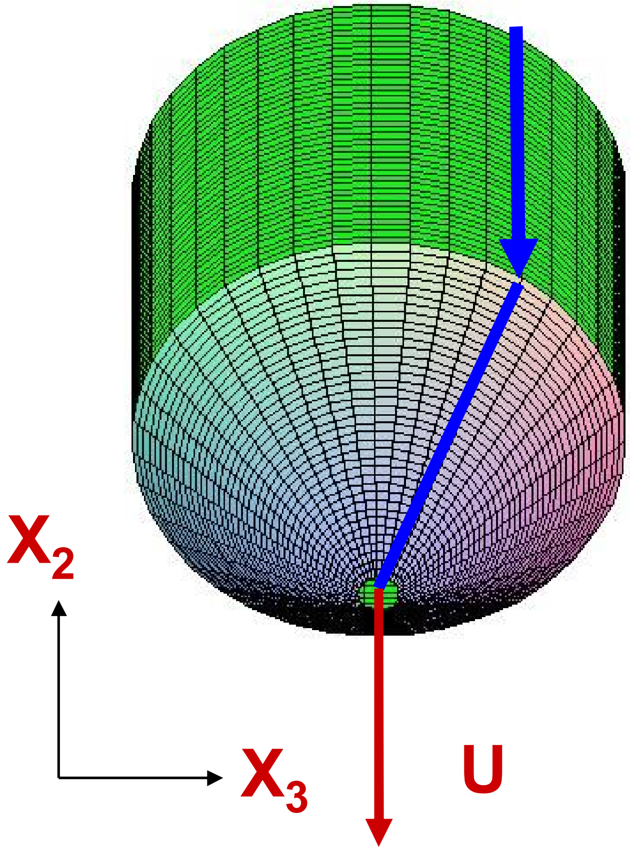
$$
\begin{aligned}
Q &= \frac{1}{2}F \\
R &= \frac{1}{2}F_{,i}x_1^i + \frac{1}{2a^2}(F - x_0^i F_{,i})v_0 + \frac{1}{8}F_{,i}^2 + \frac{1}{8a^2}(F^2 - (x_0^i F_{,i})^2) \\
S_i &= \frac{1}{2}F_{,i} + \frac{1}{2a^2}\left(F - x_0^j F_{,j}\right) x_0^i
\end{aligned}
$$

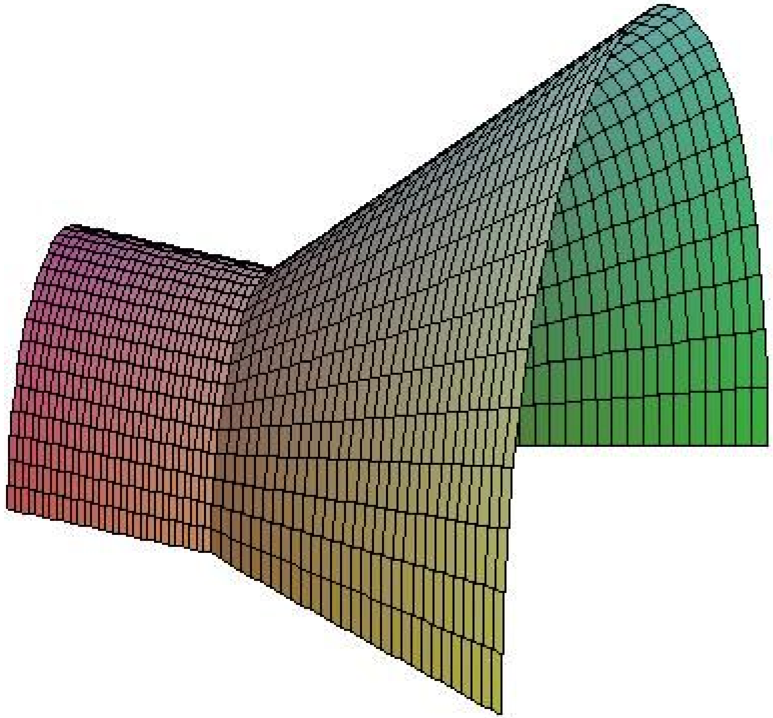
Null-geodesics

for simplicity $X_{i1} = 0$, this gives $V_0 = V_1 = 0$

$$
\begin{aligned}
V(U) &= V_f(X_{i0})\theta(U) + V_d(X_{i0})\theta(U)U \\
X^i(U) &= X_{i0} + X_{id}(X_{i0})\theta(U)U
\end{aligned}
$$

Behavior of geodesics dS/AdS





For each value of initial parameter X_{40}
we have different focal length

Independent coordinates

- Metric tensor to independent smooth coordinates. We can do it in two steps.
- Projection:

$$w = \frac{2au}{(x_4+a)},$$
$$\sigma = \frac{2av}{(x_4+a)},$$
$$\zeta = \frac{\sqrt{2}a(x_2+ix_3)}{x_4+a}$$

- Regularization:

$$\begin{aligned} w &= W, \\ \sigma &= \Sigma + H\,\theta(W) \\ &+ W\,\theta(W)\,H_\Upsilon H_{\bar{\Upsilon}}, \\ \zeta &= \Upsilon + W\,\theta(W)\,H_{\bar{\Upsilon}} \end{aligned}$$

Two shock waves

In independent coordinates

$$ds^2 = \frac{-2dw\,d\sigma + 2d\zeta\,d\bar{\zeta} + 2H_1(\zeta,\bar{\zeta})\,\delta(w)\,dw^2 + 2H_2(\zeta,\bar{\zeta})\,\delta(\sigma)\,d\sigma^2}{[1 - \frac{1}{2a^2}(w\sigma - \zeta\bar{\zeta})]^2}$$

$$H_i(\zeta,\bar{\zeta}) = H(\zeta,\bar{\zeta},\zeta_i,\bar{\zeta}_i)$$

$$H(\zeta,\bar{\zeta},0,0) = H(\zeta\bar{\zeta}) = \frac{1}{2}(1 + \frac{1}{2a^2}\zeta\bar{\zeta})F(\frac{1-\zeta\bar{\zeta}/2a^2}{1+\zeta\bar{\zeta}/2a^2})$$

Two shock waves

In smooth independent coordinates

$$ds^2 =$$

$$\frac{-2dW\, d\Sigma + 2|d\Upsilon + (\Sigma\theta(\Sigma) + W\theta(W))(H_{\Upsilon\bar{\Upsilon}}d\Upsilon + H_{\bar{\Upsilon}\bar{\Upsilon}}d\bar{\Upsilon})|^2}{\left[1 - \frac{1}{2a^2}(W\Sigma - \Upsilon\bar{\Upsilon} + (\Sigma\theta(\Sigma) + W\theta(W))G)\right]^2}$$

Trapped surface

- A trapped surface is a two dimensional spacelike surface whose two null normals have zero convergence (Neighbouring light rays, normal to the surface, must move towards one another)

- **Th. (Hawking-Penrose)** A spacetime ($M; g$) with a complete future null infinity which contains a closed trapped surface must contain a future event horizon, the interior of which contains the trapped surface

Trapped surface

The TS has two parts which lie in the regions

$$\Sigma < 0 \quad \text{and} \quad W < 0$$

They are defined in terms of two functions

$$\mathcal{S}_1 : \begin{cases} W = 0 \\ \Sigma = -\Psi_1(\Upsilon, \bar{\Upsilon}) \end{cases}, \qquad \mathcal{S}_2 : \begin{cases} \Sigma = 0 \\ W = -\Psi_2(\Upsilon, \bar{\Upsilon}) \end{cases}$$

Boundary condition:

$$\Psi_1(\Upsilon, \bar{\Upsilon})\Big|_{\mathcal{C}} = 0, \qquad \Psi_2(\Upsilon, \bar{\Upsilon})\Big|_{\mathcal{C}} = 0$$

The equation for trapped surface

$$(\triangle_{\mathbb{S}^2} + \frac{2}{a^2})\phi_{1,2}(\Upsilon, \bar{\Upsilon}) = 0,$$

$$\triangle_{\mathbb{S}^2} = 2(1 + \frac{\Upsilon\bar{\Upsilon}}{2a^2})^2 \partial_{\Upsilon\bar{\Upsilon}}$$

$$\phi_{1,2} = \frac{2\Psi_{1,2} - H_{1,2}}{1 + \frac{\Upsilon\bar{\Upsilon}}{2a^2}}$$

Boundary conditions:

$$\Psi_1(\Upsilon, \bar{\Upsilon})\Big|_{\mathcal{C}} = 0, \qquad \Psi_2(\Upsilon, \bar{\Upsilon})\Big|_{\mathcal{C}} = 0$$

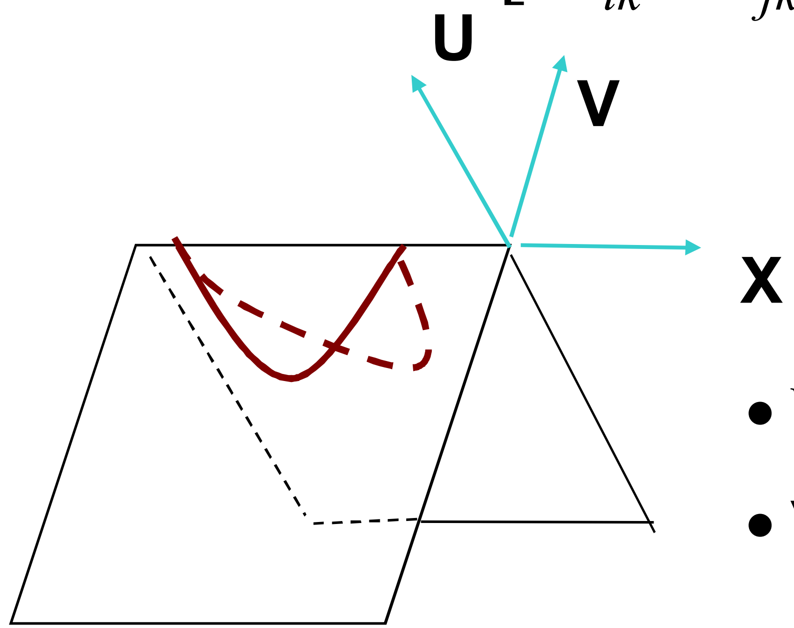
$$\partial_\Upsilon \Psi_1 \partial_{\bar{\Upsilon}} \Psi_2 |_{\mathcal{C}} = 1$$

For the AdS case analogous equation and its' solution

in: Gubser, Pufu, Yarom 0805.1551

Trapped surface in two Aichelburg-Sexl shock waves

$$ds^2 = -dUdV + [H_{ik}^{(1)} H_{jk}^{(1)} + H_{ik}^{(2)} H_{jk}^{(2)} - \delta_{ij}] dX^i dX^j,$$



- $\Psi_{1,2} > 0,\ X \in D,\ \ \Psi_{1,2} = 0,\quad X \in \partial D$
- $\nabla^2 \Psi_{1,2} = \delta^{(D-2)}(X - X_{(1,2)}),\quad X \in D,$

  the outer null normals have zero convergence
- $\nabla\Psi_1 \cdot \nabla\Psi_2 = 4 \quad X \in \partial D$

  no $\delta-$ function in convergence

Eardley, Giddings; Kang, Nastase,….

$$\left(1+\frac{\rho^2}{2a^2}\right)^2\left(\frac{\partial^2\phi}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial\phi}{\partial\rho}\right)+\frac{4\phi}{a^2}=0$$

$$\phi=\frac{A(\rho^2-2a^2)+B\left((\rho^2-2a^2)\ln\rho+4a^2\right)}{\rho^2+2a^2}$$

$$\Psi=\sqrt{2}p\left(1+\frac{\rho^2}{2a^2}\right)\left(-2+\frac{2a^2-\rho^2}{2a^2+\rho^2}\ln(\frac{2a^2}{\rho^2})\right)+\frac{1}{2a^2}A(\rho^2-2a^2)$$

$\Psi|_{\mathcal{C}}=0,$ $\mathcal{C}$ is a circle

★ $\partial_\Upsilon\Psi\partial_{\bar{\Upsilon}}\Psi|_{\mathcal{C}}=1.$

★ **gives**

$\rho=\rho_0=const.$

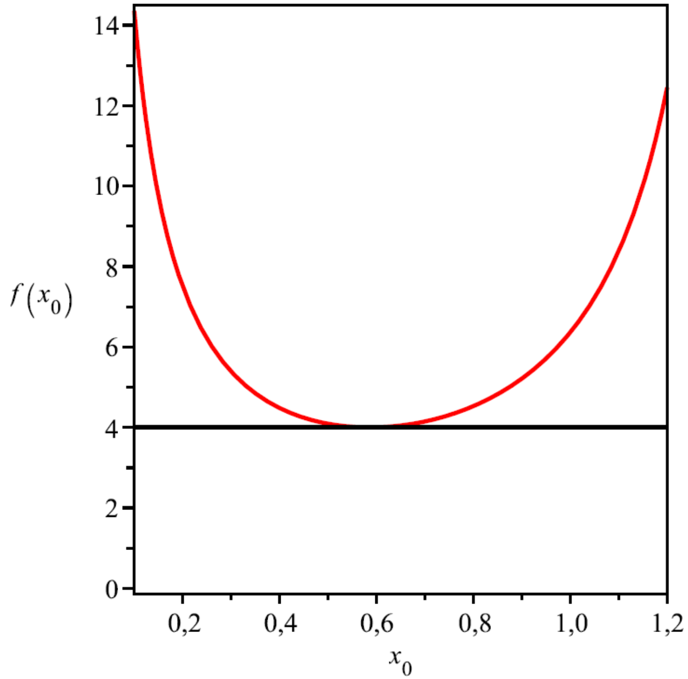
Solution ★(trapped surface equation)

$$f(x_0) = \frac{a}{p},$$

where

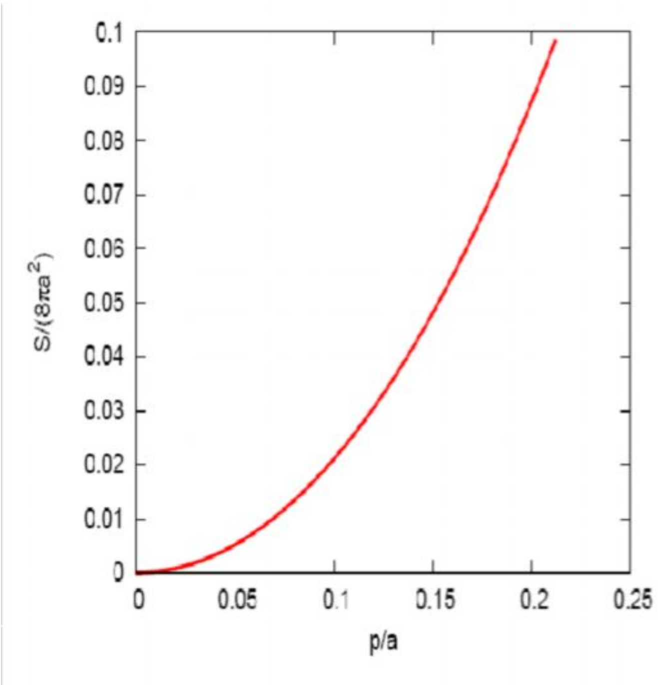
$$f(x_0) \equiv \frac{1}{\sqrt{2}x_0} \frac{(2 + x_0^2)^2}{2 - x_0^2}$$

$$x_0 = \rho_0/a$$

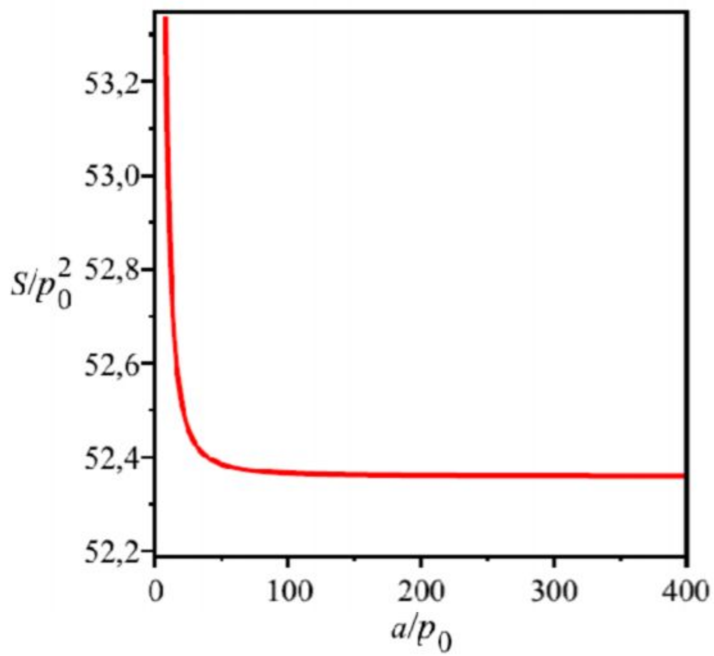


$$f'(x)|_{x=x_{min}} = 0, \quad x_{min} = 2 - \sqrt{2}$$

Solution to trapped surface equation



The area of the trapped surface in the units a^2 as a function of p/a



The area of the trapped surface as a function of the cosmological constant for fixed $p_0 = 0.01$

$$S_{M^4} \approx (G_4 E)^2$$

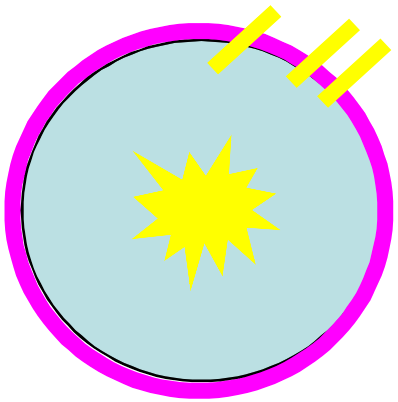
$$\Lambda > 0$$

$$S \approx 16\pi p^2 + \frac{2\pi}{9} \frac{p^2(3a^4 + 85a^2p^2 + 8p^4)}{a^2(a^2 - p^2)} + \dots$$

$$p \approx G_4 E$$

$$S_{M^5} \approx (G_5 E)^{3/2} \quad S_{AdS_5} \approx G_5 (\frac{a^3}{G_5})^{1/3} (Ea)^{2/3}$$

$$\Lambda < 0 \qquad = a\,(G_5 E a)^{2/3} = a^{5/3}\,(G_5 E)^{2/3}$$



Nastase; Shuryak, Sin, Zahed;
Kajantie, Louko,
Tahkokkalio; Grumiller, Romatshcke;
Gubser, Pufu, Yarom.

McLerran-Venugopalan model in AdS

$$\lim_{v\to 1} \gamma f\left(\gamma^2 (Y_0 + vY_1)^2\right) = \delta(Y_0 + Y_1) \int f(x^2) dx$$

$$\lim_{v\to 1} \left[\frac{\gamma}{\sqrt{\gamma^2 (Y_0 + vY_1)^2 + Y^2}} \right)$$

In $\mathcal{D}'(R^2)$

$$\frac{1}{\sqrt{w^2 + \epsilon^2 z^2}} = \delta(w) \ln \frac{4}{C\epsilon^2} + \frac{1}{|w|} + \delta(w) \ln \frac{C}{z^2} + \mathcal{O}(\epsilon^2)$$

.

$$\lim_{v\to 1} \left(\frac{\gamma}{\sqrt{\gamma^2 (Y_0 + vY_1)^2 + Y^2}} - \frac{\gamma}{\sqrt{\gamma^2 (Y_0 + vY_1)^2 + 1}} \right)$$

$$= \delta(Y_0 + Y_1) \ln Y^2$$

Conclusion

- **TeV Gravity opens new channels – BHs, ets.**
- **The important question on possible experimental signatures of spacetime nontrivial objects deserves further explorations**