# A Comparative Study of Laplacians in Riemannian and Antisymplectic Geometry

Klaus Bering

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Poisson Vs. Anti-Poiss Laplacian The Odd Scalar  $\nu_{
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# A Comparative Study of Laplacians in Riemannian and Antisymplectic Geometry



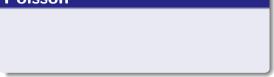
2 Riemannian Geometry



Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$ The  $\Delta$  Operator

## Darboux Coordinates

### Poisson





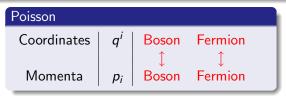
Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator



 $\begin{array}{l} \mbox{Poisson Vs. Anti-Poisson} \\ \mbox{Laplacian} \\ \mbox{The Odd Scalar } \nu_{\rho} \\ \mbox{The } \Delta \mbox{ Operator} \end{array}$ 

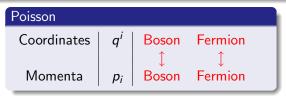
Poisson			
Coordinates	q <sup>i</sup>	Boson ↑	Fermion ↑
Momenta	p <sub>i</sub>	Boson	Fermion

Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$ The  $\Delta$  Operator



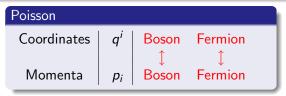


Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$ The  $\Delta$  Operator





Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator



Anti-Poisson					
Fields	$\phi^{\alpha}$	Boson	Fermion		
		$\uparrow$	\$		
Antifields	$\phi^*_{\alpha}$	Fermion	Boson		

 $\begin{array}{l} \mbox{Poisson Vs. Anti-Poisson} \\ \mbox{Laplacian} \\ \mbox{The Odd Scalar } \nu_{\rho} \\ \mbox{The } \Delta \mbox{ Operator} \end{array}$ 

# Darboux Coordinates

Poisson				Poisson Bracket $\{,\}_{PB}$
Coordinates	q <sup>i</sup>	Boson	Fermion	
Momenta	p <sub>i</sub>	↓ Boson	↓ Fermion	
Anti-Poisson				
Fields	$\phi^{\alpha}$	Boson	Fermion	
Antifields	$\phi^*_{lpha}$	↓ Fermion	↓ Boson	
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Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

Poisson		Poisson Bracket $\{,\}_{PB}$
Coordinates	$\uparrow \qquad \uparrow \qquad \uparrow \qquad \uparrow \qquad \uparrow$	
Momenta	p <sub>i</sub> Boson Fermi	on $\{q^{i},q^{j}\}_{PB} = 0$ $\{q^{i},p_{j}\}_{PB} = \delta^{i}_{j}$ $\{p_{i},p_{j}\}_{PB} = 0$
		$\{p_i, p_j\}_{PB} = 0$
Anti-Poisson		
Fields	$\phi^{lpha} \mid \begin{array}{c c} \operatorname{Boson} & \operatorname{Fermi} \\ \uparrow & \uparrow \end{array}$	on
Antifields	$\phi_{\alpha}^{*}$ Fermion Bosc	n
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Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

Poisson				Poisson Bracket { , } <sub>PB</sub>
Coordinate		1	Fermion ↓	$\{q^i,q^j\}_{PB} = 0$
Momenta		; Boson	Fermion	$ \begin{array}{rcl} \{q^{i},q^{j}\}_{PB} &=& 0\\ \{q^{i},p_{j}\}_{PB} &=& \delta^{i}_{j}\\ \{p_{i},p_{j}\}_{PB} &=& 0 \end{array} \end{array} $
Anti-Poissor	า			Antibracket $(, )_{AB}$
Anti-Poissor Fields	ו $\phi^{lpha}$	Boson ↓	Fermion ↓	Antibracket (, ) <sub>AB</sub>
		Boson ↓ Fermion	Fermion ↓ Boson	Antibracket (, ) <sub>AB</sub>
Fields	$\phi^{lpha}$	\$	$\uparrow$	Antibracket (, ) <sub>AB</sub>

Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

Poisson	Poisson Bracket $\{ , \}_{PB}$
Coordinates $q^i$ BosonFermion $\uparrow$ $\uparrow$ $\uparrow$ Momenta $p_i$ BosonFermion	$ \begin{array}{llllllllllllllllllllllllllllllllllll$
Anti-Poisson	Antibracket ( , ) <sub>AB</sub>
Anti-PoissonFields $\phi^{\alpha}$ BosonFermionAntifields $\phi^{*}_{\alpha}$ FermionBoson	Antibracket $(, )_{AB}$ $(\phi^{\alpha}, \phi^{\beta})_{AB} = 0$ $(\phi^{\alpha}, \phi^{*}_{\beta})_{AB} = \delta^{\alpha}_{\beta}$

Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

Poisson				Poisson Bracket { , } <sub>PB</sub>
Coordinate	s q <sup>i</sup>	1	Fermion ↓	$\{q^i,q^j\}_{PB} = 0$
Momenta	p <sub>i</sub>	Boson	Fermion	$ \begin{array}{rcl} \{q^{i},q^{j}\}_{PB} &=& 0\\ \{q^{i},p_{j}\}_{PB} &=& \delta^{i}_{j}\\ \{p_{i},p_{j}\}_{PB} &=& 0 \end{array} $
				"Comma is a Boson"
Anti-Poissor	I			Antibracket ( , ) <sub>AB</sub>
Fields	$\phi^{lpha}$	Boson ↓	Fermion ↓	
	$\phi^{lpha}$	¢	Fermion ↓ Boson	Antibracket $(, )_{AB}$ $(\phi^{\alpha}, \phi^{\beta})_{AB} = 0$ $(\phi^{\alpha}, \phi^{*}_{\beta})_{AB} = \delta^{\alpha}_{\beta}$ $(\phi^{*}_{\alpha}, \phi^{*}_{\beta})_{AB} = 0$

Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

Poisson				Poisson Bracket { , } <sub>PB</sub>
Coordinate	s q	i Boson ↑	Fermion ↑	$\left(a^{i}a^{j}\right) = 0$
Momenta	p	Boson	Fermion	$\begin{array}{rcl} \{q^{i},q^{j}\}_{PB} &=& 0\\ \{q^{i},p_{j}\}_{PB} &=& \delta^{i}_{j}\\ \{p_{i},p_{j}\}_{PB} &=& 0 \end{array}$
				"Comma is a Boson"
Anti-Poisson				Antibracket ( , ) <sub>AB</sub>
Fields Antifields	$\phi^{lpha} \ \phi^{st}_{lpha}$	¢	Fermion ↓ Boson	$egin{array}{rcl} (\phi^lpha,\phi^eta)_{AB}&=&0\ (\phi^lpha,\phi^sta)_{AB}&=&\delta^lpha_eta\ (\phi^sta,\phi^sta)_{AB}&=&0 \end{array}$
				$(\phi^*_{\alpha}, \phi^*_{\beta})_{AB} = 0$ "Comma is a Fermion"

Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

#### General Coordinates

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Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

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### General Coordinates

### **Poisson Bracket**

$$\{z^{A}, z^{B}\}_{PB} = \omega^{AB}$$
  
$$\{F, G\}_{PB} = (F\frac{\overleftarrow{\partial r}}{\partial z^{A}})\omega^{AB}(\frac{\overrightarrow{\partial \ell}}{\partial z^{B}}G)$$

 $\begin{array}{l} \mbox{Poisson Vs. Anti-Poisson} \\ \mbox{Laplacian} \\ \mbox{The Odd Scalar } \nu_{\rho} \\ \mbox{The } \Delta \mbox{ Operator} \end{array}$ 

### General Coordinates

#### Poisson Bracket

$$\{z^{A}, z^{B}\}_{PB} = \omega^{AB}$$
  
 
$$\{F, G\}_{PB} = (F\frac{\overrightarrow{\partial r}}{\partial z^{A}})\omega^{AB}(\frac{\overrightarrow{\partial \ell}}{\partial z^{B}}G)$$

### Antibracket

$$(z^{A}, z^{B})_{AB} = E^{AB}$$
  
(F, G)<sub>AB</sub> =  $(F \frac{\overleftarrow{\partial^{r}}}{\partial z^{A}}) E^{AB} (\frac{\overrightarrow{\partial^{\ell}}}{\partial z^{B}} G)$ 

Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

### General Coordinates

#### Poisson Bracket

$$\{z^{A}, z^{B}\}_{PB} = \omega^{AB}$$
  
 
$$\{F, G\}_{PB} = (F\frac{\overleftarrow{\partial r}}{\partial z^{A}})\omega^{AB}(\frac{\overrightarrow{\partial \ell}}{\partial z^{B}}G)$$

**Grassmann-parity** 
$$\varepsilon(\omega^{AB}) = \varepsilon_A + \varepsilon_B$$
 **Even**

#### Antibracket

$$(z^{A}, z^{B})_{AB} = E^{AB}$$
  
(F, G)\_{AB} = (F  $\frac{\partial^{r}}{\partial z^{A}}$ )  $E^{AB}(\frac{\partial^{\ell}}{\partial z^{B}}G)$ 

Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

## General Coordinates

#### Poisson Bracket

$$\{z^{A}, z^{B}\}_{PB} = \omega^{AB}$$
  
$$\{F, G\}_{PB} = (F\frac{\overleftarrow{\partial r}}{\partial z^{A}})\omega^{AB}(\frac{\overrightarrow{\partial \ell}}{\partial z^{B}}G)$$

Grassmann-parity 
$$\varepsilon(\omega^{AB}) = \varepsilon_A + \varepsilon_B$$
 Even

#### Antibracket

$$(z^{A}, z^{B})_{AB} = E^{AB}$$
  
(F, G)\_{AB} = (F  $\frac{\overleftarrow{\partial r}}{\partial z^{A}}$ )  $E^{AB}(\frac{\overrightarrow{\partial \ell}}{\partial z^{B}}G)$ 

**Grassmann-parity**  $\varepsilon(E^{AB}) = \varepsilon_A + \varepsilon_B + 1$  **Odd** 

Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

#### General Coordinates

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 $\begin{array}{l} \mbox{Poisson Vs. Anti-Poisson} \\ \mbox{Laplacian} \\ \mbox{The Odd Scalar } \nu_{\rho} \\ \mbox{The } \Delta \mbox{ Operator} \end{array}$ 

## General Coordinates

### **Poisson Case**

$$\omega^{BA} = -(-1)^{\varepsilon_A \varepsilon_B} \omega^{AB}$$

### Antisymmetric

Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

# General Coordinates

Poisson Case

$$\omega^{BA} = -(-1)^{\varepsilon_A \varepsilon_B} \omega^{AB}$$

• Antisymmetric

Inverse 2-form  

$$\omega = \frac{1}{2} dz^A \omega_{AB} \wedge dz^B$$

$$\omega_{BA} = (-1)^{(\varepsilon_A+1)(\varepsilon_B+1)} \omega_{AB}$$

### • Skewsymmetric

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Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

# General Coordinates

Poisson Case

$$\omega^{BA} = -(-1)^{\varepsilon_A \varepsilon_B} \omega^{AB}$$

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$$\omega_{BA} = (-1)^{(\varepsilon_A+1)(\varepsilon_B+1)} \omega_{AB}$$

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#### • Skewsymmetric

#### **Anti-Poisson Case**

$$E^{BA} = -(-1)^{(\varepsilon_A+1)(\varepsilon_B+1)}E^{AB}$$

### Antiskewsymmetric

Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

# General Coordinates

#### Poisson Case

$$\omega^{BA} = -(-1)^{\varepsilon_A \varepsilon_B} \omega^{AB}$$

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$$\omega_{BA} = (-1)^{(\varepsilon_A+1)(\varepsilon_B+1)} \omega_{AB}$$

#### Skewsymmetric

#### Anti-Poisson Case

$$E^{BA} = -(-1)^{(\varepsilon_A+1)(\varepsilon_B+1)}E^{AB}$$

#### • Antiskewsymmetric

 Morally Symmetric like the Riemannian Case

Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

# General Coordinates

Poisson Case

$$\omega^{BA} = -(-1)^{\varepsilon_A \varepsilon_B} \omega^{AB}$$

• Antisymmetric

$$\omega_{BA} = (-1)^{(\varepsilon_A + 1)(\varepsilon_B + 1)} \omega_{AB}$$

Inverse 2-form  $\omega = \frac{1}{2} dz^A \omega_{AB} \wedge dz^B$ 

#### • Skewsymmetric

Anti-Poisson Case

$$E^{BA} = -(-1)^{(\varepsilon_A+1)(\varepsilon_B+1)}E^{AB}$$

- Antiskewsymmetric
- Morally Symmetric like the Riemannian Case

**Inverse 2-form**  $E = \frac{1}{2} dz^A E_{AB} \wedge dz^B$ 

$$E_{BA} = -(-1)^{\varepsilon_A \varepsilon_B} E_{AB}$$

Antisymmetric

Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

# General Coordinates

Poisson Case

$$\omega^{BA} = -(-1)^{\varepsilon_A \varepsilon_B} \omega^{AB}$$

Antisymmetric

Inverse 2-form 
$$\omega = \frac{1}{2} dz^A \omega_{AB} \wedge dz^B$$
  
 $\omega_{BA} = (-1)^{(\varepsilon_A + 1)(\varepsilon_B + 1)} \omega_{AB}$ 

#### • Skewsymmetric

Anti-Poisson Case

$$E^{BA} = -(-1)^{(\varepsilon_A+1)(\varepsilon_B+1)}E^{AB}$$

- Antiskewsymmetric
- Morally Symmetric like the Riemannian Case

Inverse 2-form  $E = \frac{1}{2} dz^A E_{AB} \wedge dz^B$ 

$$E_{BA} = -(-1)^{\varepsilon_A \varepsilon_B} E_{AB}$$

- Antisymmetric
- Morally Skewsymmetric like the Symplectic Case

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### Jacobi Identity and Closeness Relation

Jacobi Identity for Poisson Bracket

$$\sum_{\text{cycl. } f,g,h} (-1)^{\varepsilon_f \varepsilon_h} \{f, \{g,h\}\} = 0$$

Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$ The  $\Delta$  Operator

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### Jacobi Identity and Closeness Relation

Jacobi Identity for Poisson Bracket

$$\sum_{\text{rcl. } f,g,h} (-1)^{\varepsilon_f \varepsilon_h} \{f, \{g,h\}\} = 0$$

# Jacobi Identity for Antibracket

Cy

$$\sum_{\text{cycl. } f,g,h} (-1)^{(\varepsilon_f+1)(\varepsilon_h+1)}(f,(g,h)) = 0$$

 $\begin{array}{l} \mbox{Poisson Vs. Anti-Poisson} \\ \mbox{Laplacian} \\ \mbox{The Odd Scalar } \nu_{\rho} \\ \mbox{The } \Delta \mbox{ Operator} \end{array}$ 

### Jacobi Identity and Closeness Relation

Jacobi Identity for Poisson Bracket

$$\sum_{\text{ycl. } f,g,h} (-1)^{\varepsilon_f \varepsilon_h} \{f, \{g,h\}\} = 0$$

# Symplectic Case Closed 2-form: $d\omega = 0$

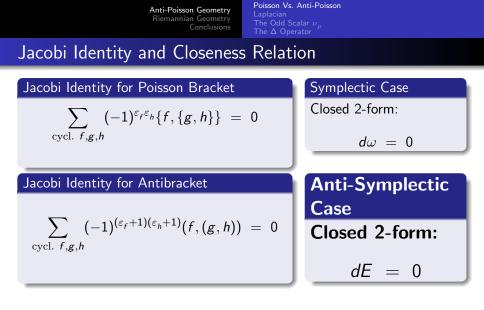
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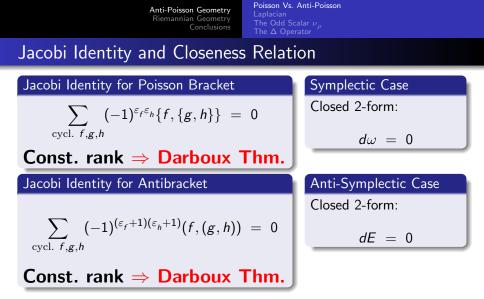
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#### Jacobi Identity for Antibracket

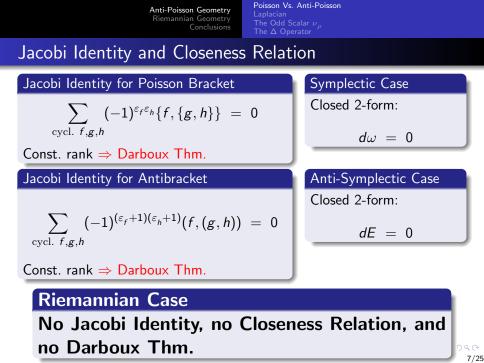
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$$\sum_{\text{cycl. } f,g,h} (-1)^{(\varepsilon_f+1)(\varepsilon_h+1)}(f,(g,h)) = 0$$





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Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

### Laplacian

### Even Laplacian in Riemannian Case

$$\Delta_{
ho} = rac{(-1)^{arepsilon_{A}}}{
ho} rac{\overrightarrow{\partial^{\ell}}}{\partial z^{A}} 
ho g^{AB} rac{\overrightarrow{\partial^{\ell}}}{\partial z^{B}}$$

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Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

## Laplacian

#### Even Laplacian in Riemannian Case

$$\Delta_{\rho} = \frac{(-1)^{\varepsilon_{A}}}{\rho} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{A}} \rho g^{AB} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{B}}$$

### **Odd Laplacian in Anti-Poisson Case**

$$2\Delta_{\rho} = \frac{(-1)^{\varepsilon_{A}}}{\rho} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{A}} \rho E^{AB} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{B}}$$

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Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

### Laplacian

#### Even Laplacian in Riemannian Case

$$\Delta_{\rho} = \frac{(-1)^{\varepsilon_{A}}}{\rho} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{A}} \rho g^{AB} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{B}}$$

# • Canonical density $\rho_g := \sqrt{g} := \sqrt{\operatorname{sdet}(g_{AB})}$

#### Odd Laplacian in Anti-Poisson Case

$$2\Delta_{\rho} = \frac{(-1)^{\varepsilon_{A}}}{\rho} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{A}} \rho E^{AB} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{B}}$$

Poisson Vs. Anti-Poissor Laplacian The Odd Scalar  $\nu_{\rho}$ The  $\Delta$  Operator

# Laplacian

Even Laplacian in Riemannian Case

$$\Delta_{\rho} = \frac{(-1)^{\varepsilon_{A}}}{\rho} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{A}} \rho g^{AB} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{B}}$$
  
o Canonical density  $\rho_{g} := \sqrt{g} := \sqrt{\operatorname{sdet}(g_{AB})}$ 

Odd Laplacian in Anti-Poisson Case

$$2\Delta_{\rho} = \frac{(-1)^{\varepsilon_{A}}}{\rho} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{A}} \rho E^{AB} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{B}}$$

• No canonical density!

Poisson Vs. Anti-Poisso Laplacian The Odd Scalar  $\nu_{\rho}$ The  $\Delta$  Operator

# Laplacian

Even Laplacian in Riemannian Case

$$\Delta_{\rho} = \frac{(-1)^{\varepsilon_{A}}}{\rho} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{A}} \rho g^{AB} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{B}}$$

• Canonical density  $ho_g := \sqrt{g} := \sqrt{\operatorname{sdet}(g_{AB})}$ 

•  $\Delta_{\rho}^2$  is a 4th-order operator.

Odd Laplacian in Anti-Poisson Case

$$2\Delta_{\rho} = \frac{(-1)^{\varepsilon_{A}}}{\rho} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{A}} \rho E^{AB} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{B}}$$

No canonical density!

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Poisson Vs. Anti-Poisso Laplacian The Odd Scalar  $\nu_{\rho}$ The  $\Delta$  Operator

# Laplacian

#### Even Laplacian in Riemannian Case

$$\Delta_{\rho} = \frac{(-1)^{\varepsilon_{A}}}{\rho} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{A}} \rho g^{AB} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{B}}$$

• Canonical density  $ho_g := \sqrt{g} := \sqrt{\operatorname{sdet}(g_{AB})}$ 

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Odd Laplacian in Anti-Poisson Case

$$2\Delta_{\rho} = \frac{(-1)^{\varepsilon_{A}}}{\rho} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{A}} \rho E^{AB} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{B}}$$

• No canonical density!

• 
$$\Delta^2_
ho$$
 is a 1st-order operator.

Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

# Laplacian

### Even Laplacian in Riemannian Case

$$\Delta_{\rho} = \frac{(-1)^{\varepsilon_{A}}}{\rho} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{A}} \rho g^{AB} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{B}}$$

• Canonical density  $ho_g := \sqrt{g} := \sqrt{\operatorname{sdet}(g_{AB})}$ 

•  $\Delta_{\rho}^2$  is a 4th-order operator.

Odd Laplacian in Anti-Poisson Case

$$2\Delta_{\rho} = \frac{(-1)^{\varepsilon_{A}}}{\rho} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{A}} \rho E^{AB} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{B}}$$

- No canonical density!
- $\Delta_{\rho}^2$  is a 1st-order operator.
- When is  $\Delta_{\rho}^2 = 0$  nilpotent?

Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

# Laplacian

Even Laplacian in Riemannian Case

$$\Delta_{\rho} = \frac{(-1)^{\varepsilon_{A}}}{\rho} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{A}} \rho g^{AB} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{B}}$$

• Canonical density  $ho_g := \sqrt{g} := \sqrt{\operatorname{sdet}(g_{AB})}$ 

•  $\Delta_{\rho}^2$  is a 4th-order operator.

Odd Laplacian in Anti-Poisson Case

$$\mathbf{2}\Delta_{\rho} = \frac{(-1)^{\varepsilon_{A}}}{\rho} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{A}} \rho E^{AB} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{B}}$$

- No canonical density!
- $\Delta_{\rho}^2$  is a 1st-order operator.
- When is  $\Delta_{\rho}^2 = 0$  nilpotent?

Anti-Poisson Geometry Riemannian Geometry Conclusions Poisson Vs. Anti-Poisso Laplacian The Odd Scalar  $\nu_{\rho}$ The  $\Delta$  Operator

	Even Geometry	Odd Geometry
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	$g=dz^Ag_{AB}ee dz^B$	$g = dz^A g_{AB} \lor dz^B$
Riemannian	$\varepsilon(g_{AB}) = \varepsilon_A + \varepsilon_B$	$\varepsilon(g_{AB}) = \varepsilon_A + \varepsilon_B + 1$
Covariant	$g_{BA} = -(-1)^{(\varepsilon_A + 1)(\varepsilon_B + 1)}g_{AB}$	$g_{BA} = (-1)^{arepsilon_A arepsilon_B} g_{AB}$
Metric	Antiskewsymmetric	Symmetric
	No Closeness Relation	No Closeness Relation
Inverse	$\varepsilon(g^{AB}) = \varepsilon_A + \varepsilon_B$	$\varepsilon(g^{AB}) = \varepsilon_A + \varepsilon_B + 1$
Riemannian	$g^{BA} = (-1)^{\varepsilon_A \varepsilon_B} g^{\widetilde{A}B}$	$g^{BA} = (-1)^{(\varepsilon_A + 1)(\varepsilon_B + 1)} g^{AB}$
Contravariant	Symmetric	Skewsymmetric
Metric	Even Laplacian	No Laplacian
	$\omega = \frac{1}{2} dz^A \omega_{AB} \wedge dz^B$	$E = \frac{1}{2} dz^A E_{AB} \wedge dz^B$
Symplectic	$\varepsilon(\omega_{AB}) = \varepsilon_A + \varepsilon_B$	$\varepsilon(E_{AB}) = \varepsilon_A + \varepsilon_B + 1$
Covariant	$\omega_{BA} = (-1)^{(\varepsilon_A + 1)(\varepsilon_B + 1)} \omega_{AB}$	$E_{BA} = -(-1)^{\varepsilon_A \varepsilon_B} E_{AB}$
Two–Form	Skewsymmetric	Antisymmetric
	Closeness Relation	Closeness Relation
Inverse	$\varepsilon(\omega^{AB}) = \varepsilon_A + \varepsilon_B$	$\varepsilon(E^{AB}) = \varepsilon_A + \varepsilon_B + 1$
Symplectic	$\omega^{BA} = -(-1)^{\varepsilon_A \varepsilon_B} \omega^{AB}$	$E^{BA} = -(-1)^{(\varepsilon_A+1)(\varepsilon_B+1)}E^{AB}$
Contravariant	Antisymmetric	Symmetric
Tensor	No Laplacian	Odd Laplacian

Anti-Poisson Geometry Riemannian Geometry Conclusions Poisson Vs. Anti-Poisso Laplacian The Odd Scalar  $\nu_{\rho}$ The  $\Delta$  Operator

	Even Geometry	Odd Geometry
Riemannian Covariant Metric	$\begin{array}{c} g = dz^{A}g_{AB} \lor dz^{B} \\ \varepsilon(g_{AB}) = \varepsilon_{A} + \varepsilon_{B} \\ g_{BA} = -(-1)^{(\varepsilon_{A}+1)(\varepsilon_{B}+1)}g_{AB} \\ \text{Antiskewsymmetric} \\ \text{No Closeness Relation} \end{array}$	$g = dz^{A}g_{AB} \lor dz^{B}$ $\varepsilon(g_{AB}) = \varepsilon_{A} + \varepsilon_{B} + 1$ $g_{BA} = (-1)^{\varepsilon_{A}\varepsilon_{B}}g_{AB}$ Symmetric No Closeness Relation
Inverse Riemannian Contravariant Metric	$\begin{split} \varepsilon(g^{AB}) &= \varepsilon_A + \varepsilon_B \\ g^{BA} &= (-1)^{\varepsilon_A \varepsilon_B} g^{AB} \\ \text{Symmetric} \\ \text{Even Laplacian} \end{split}$	$ \begin{array}{c} \varepsilon(g^{AB}) = \varepsilon_A + \varepsilon_B + 1 \\ g^{BA} = (-1)^{(\varepsilon_A + 1)(\varepsilon_B + 1)}g^{AB} \\ \text{Skewsymmetric} \\ \text{No Laplacian} \end{array} $
Symplectic Covariant Two–Form	$\begin{split} \omega &= \frac{1}{2} dz^A \omega_{AB} \wedge dz^B \\ \varepsilon(\omega_{AB}) &= \varepsilon_A + \varepsilon_B \\ \omega_{BA} &= (-1)^{(\varepsilon_A + 1)(\varepsilon_B + 1)} \omega_{AB} \\ \text{Skewsymmetric} \\ \text{Closeness Relation} \end{split}$	$ \begin{array}{c} E = \frac{1}{2} dz^A E_{AB} \wedge dz^B \\ \varepsilon(E_{AB}) = \varepsilon_A + \varepsilon_B + 1 \\ E_{BA} = -(-1)^{\varepsilon_A \varepsilon_B} E_{AB} \\ Antisymmetric \\ Closeness Relation \end{array} $
Inverse Symplectic Contravariant Tensor	$\begin{array}{c} \varepsilon(\omega^{AB}) = \varepsilon_A + \varepsilon_B \\ \omega^{BA} = -(-1)^{\varepsilon_A \varepsilon_B} \omega^{AB} \\ \text{Antisymmetric} \\ \text{No Laplacian} \end{array}$	$ \begin{split} \varepsilon(E^{AB}) &= \varepsilon_A + \varepsilon_B + 1 \\ E^{BA} &= -(-1)^{(\varepsilon_A + 1)(\varepsilon_B + 1)} E^{AB} \\ \text{Symmetric} \\ \text{Odd Laplacian} \end{split} $

 Anti-Poisson Geometry
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 The Odd Scalar  $\nu_{\rho}$ 

	Even Geometry	Odd Geometry
Riemannian Covariant Metric	$\begin{array}{c} g = dz^{A}g_{AB} \lor dz^{B} \\ \varepsilon(g_{AB}) = \varepsilon_{A} + \varepsilon_{B} \\ g_{BA} = -(-1)^{(\varepsilon_{A}+1)(\varepsilon_{B}+1)}g_{AB} \\ \text{Antiskewsymmetric} \\ \text{No Closeness Relation} \end{array}$	$\begin{array}{c} g = dz^{A}g_{AB} \lor dz^{B} \\ \varepsilon(g_{AB}) = \varepsilon_{A} + \varepsilon_{B} + 1 \\ g_{BA} = (-1)^{\varepsilon_{A}\varepsilon_{B}}g_{AB} \\ \text{Symmetric} \\ \text{No Closeness Relation} \end{array}$
Inverse Riemannian Contravariant Metric	$\begin{split} \varepsilon(g^{AB}) &= \varepsilon_A + \varepsilon_B \\ g^{BA} &= (-1)^{\varepsilon_A \varepsilon_B} g^{AB} \\ \text{Symmetric} \\ \text{Even Laplacian} \end{split}$	$ \begin{array}{c} \varepsilon(g^{AB}) = \varepsilon_A + \varepsilon_B + 1 \\ g^{BA} = (-1)^{(\varepsilon_A + 1)(\varepsilon_B + 1)}g^{AB} \\ \text{Skewsymmetric} \\ \text{No Laplacian} \end{array} $
Symplectic Covariant Two–Form	$\begin{split} \omega &= \frac{1}{2} dz^A \omega_{AB} \wedge dz^B \\ \varepsilon(\omega_{AB}) &= \varepsilon_A + \varepsilon_B \\ \omega_{BA} &= (-1)^{(\varepsilon_A + 1)(\varepsilon_B + 1)} \omega_{AB} \\ \text{Skewsymmetric} \\ \text{Closeness Relation} \end{split}$	$ \begin{array}{c} E = \frac{1}{2} dz^A E_{AB} \wedge dz^B \\ \varepsilon(E_{AB}) = \varepsilon_A + \varepsilon_B + 1 \\ E_{BA} = -(-1)^{\varepsilon_A \varepsilon_B} E_{AB} \\ Antisymmetric \\ Closeness Relation \end{array} $
Inverse Symplectic Contravariant Tensor	$\begin{array}{c} \varepsilon(\omega^{AB}) = \varepsilon_A + \varepsilon_B \\ \omega^{BA} = -(-1)^{\varepsilon_A \varepsilon_B} \omega^{AB} \\ \text{Antisymmetric} \\ \text{No Laplacian} \end{array}$	$ \begin{array}{c} \varepsilon(E^{AB}) = \varepsilon_A + \varepsilon_B + 1 \\ E^{BA} = -(-1)^{(\varepsilon_A + 1)(\varepsilon_B + 1)} E^{AB} \\ \text{Symmetric} \\ \text{Odd Laplacian} \end{array} $

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### The $2 \times 2 = 4$ Classical Geometries and their Symmetries

	Even Geometry	Odd Geometry
	$g=dz^Ag_{AB}ee dz^B$	$g = dz^A g_{AB} \lor dz^B$
Riemannian	$\varepsilon(\mathbf{g}_{AB}) = \varepsilon_A + \varepsilon_B$	$\varepsilon(g_{AB}) = \varepsilon_A + \varepsilon_B + 1$
Covariant	$g_{BA} = -(-1)^{(\varepsilon_A+1)(\varepsilon_B+1)}g_{AB}$	$g_{BA} = (-1)^{\varepsilon_A \varepsilon_B} g_{AB}$
Metric	Antiskewsymmetric	Symmetric
	No Closeness Relation	No Closeness Relation
Inverse	$\varepsilon(g^{AB}) = \varepsilon_A + \varepsilon_B$	$\varepsilon(g^{AB}) = \varepsilon_A + \varepsilon_B + 1$
Riemannian	$g^{BA} = (-1)^{arepsilon_A arepsilon_B} g^{AB}$	$g^{BA} = (-1)^{(\varepsilon_A + 1)(\varepsilon_B + 1)} g^{AB}$
Contravariant	Symmetric	Skewsymmetric
Metric	Even Laplacian	No Laplacian
	$\omega = \frac{1}{2} dz^A \omega_{AB} \wedge dz^B$	$E = \frac{1}{2} dz^A E_{AB} \wedge dz^B$
Symplectic	$\varepsilon(\omega_{AB}) = \varepsilon_A + \varepsilon_B$	$\varepsilon(E_{AB}) = \varepsilon_A + \varepsilon_B + 1$
Covariant	$\omega_{BA} = (-1)^{(\varepsilon_A + 1)(\varepsilon_B + 1)} \omega_{AB}$	$E_{BA} = -(-1)^{\varepsilon_A \varepsilon_B} E_{AB}$
Two–Form	Skewsymmetric	Antisymmetric
	Closeness Relation	Closeness Relation
Inverse	$arepsilon(\omega^{AB}) = arepsilon_A + arepsilon_B$	$\varepsilon(E^{AB}) = \varepsilon_A + \varepsilon_B + 1$
Symplectic	$\omega^{BA} = -(-1)^{arepsilon_A arepsilon_B} \omega^{AB}$	$E^{BA} = -(-1)^{(\varepsilon_A+1)(\varepsilon_B+1)}E^{AB}$
Contravariant	Antisymmetric	Symmetric
Tensor	No Laplacian	Odd Laplacian

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### The $2 \times 2 = 4$ Classical Geometries and their Symmetries

$\frac{dz^B}{dz^B} + 1$ $\mathcal{g}_{AB}$
<sub>B</sub> + 1
g <sub>AB</sub>
ation
<sub>B</sub> + 1
<sup>B</sup> <sup>+1)</sup> g <sup>AB</sup>
ic
n
dz <sup>B</sup>
<sub>B</sub> + 1
в Е <sub>АВ</sub>
c
ion
$_{B} + 1$
$E_{B}^{(+1)}E^{AB}$
$E^{B+1)}E^{AB}$

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Anti-Poisson Geometry<br/>Riemannian Geometry<br/>ConclusionsPoisson Vs. Anti-Poiss<br/>Laplacian<br/>The Odd Scalar  $\nu_{\rho}$ 

	Even Geometry	Odd Geometry
Riemannian Covariant Metric	$g = dz^{A}g_{AB} \lor dz^{B}$ $\varepsilon(g_{AB}) = \varepsilon_{A} + \varepsilon_{B}$ $g_{BA} = -(-1)^{(\varepsilon_{A}+1)(\varepsilon_{B}+1)}g_{AB}$ Antiskewsymmetric No Closeness Relation	$g = dz^{A}g_{AB} \lor dz^{B}$ $\varepsilon(g_{AB}) = \varepsilon_{A} + \varepsilon_{B} + 1$ $g_{BA} = (-1)^{\varepsilon_{A}\varepsilon_{B}}g_{AB}$ Symmetric No Closeness Relation
Inverse Riemannian Contravariant Metric	$\begin{aligned} \varepsilon(g^{AB}) &= \varepsilon_A + \varepsilon_B \\ g^{BA} &= (-1)^{\varepsilon_A \varepsilon_B} g^{AB} \\ \text{Symmetric} \\ \text{Even Laplacian} \end{aligned}$	$\begin{array}{c} \varepsilon(g^{AB}) = \varepsilon_A + \varepsilon_B + 1\\ g^{BA} = (-1)^{(\varepsilon_A+1)(\varepsilon_B+1)}g^{AB}\\ \text{Skewsymmetric}\\ \text{No Laplacian} \end{array}$
Symplectic Covariant Two–Form	$ \begin{split} \omega &= \frac{1}{2} dz^A \omega_{AB} \wedge dz^B \\ \varepsilon(\omega_{AB}) &= \varepsilon_A + \varepsilon_B \\ \omega_{BA} &= (-1)^{(\varepsilon_A + 1)(\varepsilon_B + 1)} \omega_{AB} \\ \text{Skewsymmetric} \\ \text{Closeness Relation} \end{split} $	$ \begin{array}{l} E = \frac{1}{2} dz^A E_{AB} \wedge dz^B \\ \varepsilon(E_{AB}) = \varepsilon_A + \varepsilon_B + 1 \\ E_{BA} = -(-1)^{\varepsilon_A \varepsilon_B} E_{AB} \\ Antisymmetric \\ Closeness Relation \end{array} $
Inverse Symplectic Contravariant Tensor	$\begin{array}{c} \varepsilon(\omega^{AB}) = \varepsilon_A + \varepsilon_B \\ \omega^{BA} = -(-1)^{\varepsilon_A \varepsilon_B} \omega^{AB} \\ \text{Antisymmetric} \\ \text{No Laplacian} \end{array}$	$ \begin{aligned} \varepsilon(E^{AB}) &= \varepsilon_A + \varepsilon_B + 1 \\ E^{BA} &= -(-1)^{(\varepsilon_A + 1)(\varepsilon_B + 1)} E^{AB} \\ \text{Symmetric} \\ \text{Odd Laplacian} \end{aligned} $

 
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 The  $\Delta$  Operator

### The $2 \times 2 = 4$ Classical Geometries and their Symmetries

	Even Geometry	Odd Geometry
	$g = dz^A g_{AB} \vee dz^B$	$g = dz^A g_{AB} \vee dz^B$
Riemannian	$\varepsilon(\mathbf{g}_{AB}) = \varepsilon_A + \varepsilon_B$	$\varepsilon(g_{AB}) = \varepsilon_A^{AB} + \varepsilon_B^{A} + 1$
Covariant	$g_{BA} = -(-1)^{(\varepsilon_A+1)(\varepsilon_B+1)}g_{AB}$	$g_{BA}^{}=(-1)^{arepsilon_Aarepsilon_B}g_{AB}^{}$
Metric	Antiskewsymmetric	Symmetric
	No Closeness Relation	No Closeness Relation
Inverse	$\varepsilon(g^{AB}) = \varepsilon_A + \varepsilon_B$	$\varepsilon(g^{AB}) = \varepsilon_A + \varepsilon_B + 1$
Riemannian	$g^{BA}=(-1)^{arepsilon_Aarepsilon_B}g^{AB}$	$g^{BA} = (-1)^{(\varepsilon_A + 1)(\varepsilon_B + 1)} g^{AB}$
Contravariant	Symmetric	Skewsymmetric
Metric	Even Laplacian	No Laplacian
	$\omega = rac{1}{2} dz^A \omega_{AB} \wedge dz^B$	$E = \frac{1}{2} dz^A E_{AB} \wedge dz^B$
Symplectic	$\varepsilon(\omega_{AB}) = \varepsilon_A + \varepsilon_B$ $\omega_{BA} = (-1)^{(\varepsilon_A + 1)(\varepsilon_B + 1)} \omega_{AB}$	$\varepsilon(E_{AB}) = \varepsilon_A + \varepsilon_B + 1$
Covariant	$\omega_{BA} = (-1)^{(\varepsilon_A + 1)(\varepsilon_B + 1)} \omega_{AB}$	$E_{BA} = -(-1)^{\varepsilon_A \varepsilon_B} E_{AB}$
Two–Form	Skewsymmetric	Antisymmetric
	Closeness Relation	Closeness Relation
Inverse	$\varepsilon(\omega^{AB}) = \varepsilon_A + \varepsilon_B$	$\varepsilon(E^{AB}) = \varepsilon_A + \varepsilon_B + 1$
Symplectic	$\omega^{BA} = -(-1)^{\varepsilon_A \varepsilon_B} \bar{\omega}^{AB}$	$E^{BA} = -(-1)^{(\varepsilon_A+1)(\varepsilon_B+1)}E^{AB}$
Contravariant	Antisymmetric	Symmetric
Tensor	No Laplacian	Odd Laplacian

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	Even Geometry	Odd Geometry
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	$g = dz^A g_{AB}^A \lor dz^B$	$g = dz^A g_{AB}^{} \lor dz^B$
Riemannian	$\varepsilon(\mathbf{g}_{AB}) = \varepsilon_A + \varepsilon_B$	$\varepsilon(g_{AB}) = \varepsilon_A + \varepsilon_B + 1$
Covariant	$g_{BA} = -(-1)^{(\varepsilon_A+1)(\varepsilon_B+1)}g_{AB}$	$g_{BA} = (-1)^{arepsilon_A arepsilon_B} g_{AB}$
Metric	Antiskewsymmetric	Symmetric
	No Closeness Relation	No Closeness Relation
Inverse	$\varepsilon(g^{AB}) = \varepsilon_A + \varepsilon_B$	$\varepsilon(g^{AB}) = \varepsilon_A + \varepsilon_B + 1$
Riemannian	$g^{BA} = (-1)^{arepsilon_A arepsilon_B} g^{AB}$	$g^{BA} = (-1)^{(\varepsilon_A+1)(\varepsilon_B+1)}g^{AB}$
Contravariant	Symmetric	Skewsymmetric
Metric	Even Laplacian	No Laplacian
	$\omega = \frac{1}{2} dz^A \omega_{AB} \wedge dz^B$	$E = \frac{1}{2} dz^A E_{AB} \wedge dz^B$
Symplectic	$\varepsilon(\omega_{AB}) = \varepsilon_A + \varepsilon_B$ $\omega_{BA} = (-1)^{(\varepsilon_A + 1)(\varepsilon_B + 1)} \omega_{AB}$	$\varepsilon(E_{AB}) = \varepsilon_A + \varepsilon_B + 1$
Covariant	$\omega_{BA} = (-1)^{(\varepsilon_A + 1)(\varepsilon_B + 1)} \omega_{AB}$	$E_{BA} = -(-1)^{\varepsilon_A \varepsilon_B} E_{AB}$
Two–Form	Skewsymmetric	Antisymmetric
	Closeness Relation	Closeness Relation
Inverse	$\varepsilon(\omega^{AB}) = \varepsilon_A + \varepsilon_B$	$\varepsilon(E^{AB}) = \varepsilon_A + \varepsilon_B + 1$
Symplectic	$\omega^{BA} = -(-1)^{\varepsilon_A \varepsilon_B} \omega^{AB}$	$E^{BA} = -(-1)^{(\varepsilon_A^{-}+1)(\varepsilon_B^{-}+1)}E^{AB}$
Contravariant	Antisymmetric	Symmetric
Tensor	No Laplacian	Odd Laplacian

Poisson Vs. Anti-Poissor Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

# The Odd Scalar $u_{ ho}$

Odd Scalar in Antisymplectic Geometry

$$\nu_{\rho} := \nu_{\rho}^{(0)} + \frac{\nu^{(1)}}{8} - \frac{\nu^{(2)}}{24}$$

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(KB 2006)

Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

# The Odd Scalar $u_{ ho_1}$

Odd Scalar in Antisymplectic Geometry

$$\nu_{\rho} := \nu_{\rho}^{(0)} + \frac{\nu^{(1)}}{8} - \frac{\nu^{(2)}}{24}$$

### Terms built from *E* and $\rho$

$$u^{(\mathbf{0})}_{
ho} := rac{1}{\sqrt{
ho}} (\Delta_1 \sqrt{
ho})$$

(KB 2006)

Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{
ho}$  The  $\Delta$  Operator

# The Odd Scalar $u_{ ho}$

Odd Scalar in Antisymplectic Geometry

(KB 2006)

$$\nu_{\rho} := \nu_{\rho}^{(0)} + \frac{\nu^{(1)}}{8} - \frac{\nu^{(2)}}{24}$$

#### Terms built from E and $\rho$

$$\begin{split} \nu_{\rho}^{(0)} &:= \frac{1}{\sqrt{\rho}} (\Delta_1 \sqrt{\rho}) \\ \nu^{(1)} &:= (-1)^{\varepsilon_A} (\frac{\overrightarrow{\partial^{\ell}}}{\partial z^A} E^{AB} \frac{\overleftarrow{\partial^{r}}}{\partial z^B}) (-1)^{\varepsilon_B} \end{split}$$

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Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

# The Odd Scalar $\nu_{ ho}$

Odd Scalar in Antisymplectic Geometry

(KB 2006)

$$\nu_{\rho} := \nu_{\rho}^{(0)} + \frac{\nu^{(1)}}{8} - \frac{\nu^{(2)}}{24}$$

#### Terms built from E and $\rho$

$$\nu_{\rho}^{(0)} := \frac{1}{\sqrt{\rho}} (\Delta_{1}\sqrt{\rho})$$

$$\nu^{(1)} := (-1)^{\varepsilon_{A}} (\frac{\overrightarrow{\partial^{\ell}}}{\partial z^{A}} E^{AB} \frac{\overleftarrow{\partial^{r}}}{\partial z^{B}}) (-1)^{\varepsilon_{B}}$$

$$\nu^{(2)} := -(-1)^{\varepsilon_{B}} (\frac{\overrightarrow{\partial^{\ell}}}{\partial z^{A}} E_{BC}) (z^{C}, (z^{B}, z^{A}))$$

Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

# The Odd Scalar $u_{ ho}$

Odd Scalar in Antisymplectic Geometry

(KB 2006)

$$\nu_{\rho} := \nu_{\rho}^{(0)} + \frac{\nu^{(1)}}{8} - \frac{\nu^{(2)}}{24}$$

#### Terms built from E and $\rho$

$$\begin{aligned}
\nu_{\rho}^{(0)} &:= \frac{1}{\sqrt{\rho}} (\Delta_{1} \sqrt{\rho}) \\
\nu^{(1)} &:= (-1)^{\varepsilon_{A}} (\frac{\overrightarrow{\partial^{\ell}}}{\partial z^{A}} E^{AB} \frac{\overleftarrow{\partial^{r}}}{\partial z^{B}}) (-1)^{\varepsilon_{B}} \\
\nu^{(2)} &:= -(-1)^{\varepsilon_{B}} (\frac{\overrightarrow{\partial^{\ell}}}{\partial z^{A}} E_{BC}) (z^{C}, (z^{B}, z^{A})) \\
&= (-1)^{\varepsilon_{A} \varepsilon_{D}} (\frac{\overrightarrow{\partial^{\ell}}}{\partial z^{D}} E^{AB}) E_{BC} (E^{CD} \frac{\overleftarrow{\partial^{r}}}{\partial z^{A}})
\end{aligned}$$

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Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

### Classification of 2nd-order Differential Invariants

### Question

Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$ The  $\Delta$  Operator

### Classification of 2nd-order Differential Invariants

#### Question

What is the most general function  $\nu = \nu(z)$  such that

•  $\nu(z)$  is a scalar,

Poisson Vs. Anti-Poisson Laplacian **The Odd Scalar**  $\nu_{\rho}$ The  $\Delta$  Operator

### Classification of 2nd-order Differential Invariants

#### Question

- $\nu(z)$  is a scalar,
- $\nu(z)$  is a polynomial of the metric  $E_{AB}(z)$ , the density  $\rho(z)$ , their inverses, and *z*-derivatives thereof in the point *z*,

Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

### Classification of 2nd-order Differential Invariants

#### Question

- $\nu(z)$  is a scalar,
- $\nu(z)$  is a polynomial of the metric  $E_{AB}(z)$ , the density  $\rho(z)$ , their inverses, and z-derivatives thereof in the point z,
- $\nu$  is invariant under constant rescaling of the density  $\rho \rightarrow \lambda \rho$ , where  $\lambda$  is a *z*-independent parameter,

Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{
ho}$  The  $\Delta$  Operator

### Classification of 2nd-order Differential Invariants

#### Question

- $\nu(z)$  is a scalar,
- $\nu(z)$  is a polynomial of the metric  $E_{AB}(z)$ , the density  $\rho(z)$ , their inverses, and z-derivatives thereof in the point z,
- $\nu$  is invariant under constant rescaling of the density  $\rho \rightarrow \lambda \rho$ , where  $\lambda$  is a *z*-independent parameter,
- $\nu$  scales as  $\nu \rightarrow \lambda \nu$  under constant Weyl scaling  $E^{AB} \rightarrow \lambda E^{AB}$ , where  $\lambda$  is a *z*-independent parameter,

Poisson Vs. Anti-Poissor Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

### Classification of 2nd-order Differential Invariants

#### Question

- $\nu(z)$  is a scalar,
- $\nu(z)$  is a polynomial of the metric  $E_{AB}(z)$ , the density  $\rho(z)$ , their inverses, and z-derivatives thereof in the point z,
- $\nu$  is invariant under constant rescaling of the density  $\rho \rightarrow \lambda \rho$ , where  $\lambda$  is a *z*-independent parameter,
- $\nu$  scales as  $\nu \rightarrow \lambda \nu$  under constant Weyl scaling  $E^{AB} \rightarrow \lambda E^{AB}$ , where  $\lambda$  is a *z*-independent parameter,
- and each term in  $\nu$  contains precisely two *z*-derivatives?

Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{
ho}$  The  $\Delta$  Operator

### Classification of 2nd-order Differential Invariants

#### Question

What is the most general function  $\nu = \nu(z)$  such that

- $\nu(z)$  is a scalar,
- $\nu(z)$  is a polynomial of the metric  $E_{AB}(z)$ , the density  $\rho(z)$ , their inverses, and z-derivatives thereof in the point z,
- $\nu$  is invariant under constant rescaling of the density  $\rho \rightarrow \lambda \rho$ , where  $\lambda$  is a z-independent parameter,
- $\nu$  scales as  $\nu \rightarrow \lambda \nu$  under constant Weyl scaling  $E^{AB} \rightarrow \lambda E^{AB}$ , where  $\lambda$  is a *z*-independent parameter,
- and each term in  $\nu$  contains precisely two *z*-derivatives?

### Unique Answer (up to scaling)

#### (Batalin,KB 2008)

$$u = \alpha \nu_{
ho}$$

Poisson Vs. Anti-Poisso Laplacian The Odd Scalar  $\nu_{\rho}$ **The \Delta Operator** 

# The $\Delta$ Operator

### Question

# What is (the local form of) the most general differential operator $\Delta$

 Anti-Poisson Geometry Riemannian Geometry Conclusions The Δd Scalar The Δ Operator

### The $\Delta$ Operator

#### Question

What is (the local form of) the most general differential operator  $\boldsymbol{\Delta}$ 

• that takes scalar functions to scalar functions,

Anti-Poisson Geometry Riemannian Geometry Conclusions The Δd Scalar The Δ Operator

### The $\Delta$ Operator

#### Question

What is (the local form of) the most general differential operator  $\boldsymbol{\Delta}$ 

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- that takes scalar functions to scalar functions,
- that is Grassmann-odd,

Anti-Poisson Geometry Riemannian Geometry Conclusions The Δ Operator

### The $\Delta$ Operator

#### Question

What is (the local form of) the most general differential operator  $\boldsymbol{\Delta}$ 

- that takes scalar functions to scalar functions,
- that is Grassmann-odd,
- that is nilpotent,

Anti-Poisson Geometry Riemannian Geometry Conclusions The  $\Delta$  Operator

### The $\Delta$ Operator

#### Question

What is (the local form of) the most general differential operator  $\boldsymbol{\Delta}$ 

- that takes scalar functions to scalar functions,
- that is Grassmann-odd,
- that is nilpotent,
- that is of 2nd-order,

Anti-Poisson Geometry Riemannian Geometry Conclusions The Odd Scalar The Δ Operator

### The $\Delta$ Operator

#### Question

What is (the local form of) the most general differential operator  $\boldsymbol{\Delta}$ 

- that takes scalar functions to scalar functions,
- that is Grassmann-odd,
- that is nilpotent,
- that is of **2nd-order**,
- and the 2nd-order part is non-degenerate?

Anti-Poisson Geometry Riemannian Geometry Conclusions The  $\Delta$  Operator

### The $\Delta$ Operator

#### Question

What is (the local form of) the most general differential operator  $\Delta$ 

- that takes scalar functions to scalar functions,
- that is Grassmann-odd,
- that is nilpotent,
- that is of **2nd-order**,
- and the 2nd-order part is non-degenerate?

# Unique Answer (modulo an odd constant) (Batalin,KB 2007)

$$\Delta = \Delta_{
ho} + 
u_{
ho}$$

Anti-Poisson Geometry Riemannian Geometry Conclusions The Odd Scalar The Δ Operator

# The $\Delta$ Operator

#### Question

What is (the local form of) the most general differential operator  $\Delta$ 

- that takes scalar functions to scalar functions,
- that is Grassmann-odd,
- that is nilpotent,
- that is of **2nd-order**,
- and the 2nd-order part is non-degenerate?

# Unique Answer (modulo an odd constant) (Batalin,KB 2007)

$$\Delta = \Delta_{\rho} + \nu_{\rho}$$

The  $\Delta$  Operator = Odd Laplacian + Odd Scalar.

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Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

### Quantum Master Equation

# **Exponential form**

 $\Delta e^{rac{i}{\hbar}W} = 0$ 



Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

### Quantum Master Equation

 $\Delta e^{\frac{i}{\hbar}W} = 0$ 

#### Exponential form

### Additive form

$$\frac{1}{2}(W,W) = i\hbar\Delta_{\rho}W + \hbar^{2}\nu_{\rho}$$

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### Quantum Master Equation

 $\Delta e^{\frac{i}{\hbar}W} = 0$ 

Exponential form

Additive form

$$\frac{1}{2}(W,W) = i\hbar\Delta_{\rho}W + \hbar^{2}\nu_{\rho}$$

Odd scalar  $\nu_{\rho}$  enters at 2-loop.

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# Quantum Master Equation

Exponential form

 $\Delta e^{rac{i}{\hbar}W} = 0$ 

# Quantum Master Action

$$W = S + \sum_{n=1}^{\infty} \hbar^n M_n$$

#### Additive form

$$\frac{1}{2}(W,W) = i\hbar\Delta_{\rho}W + \hbar^{2}\nu_{\rho}$$

Odd scalar  $\nu_{\rho}$  enters at 2-loop.

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# Quantum Master Equation

Exponential form

 $\Delta e^{rac{i}{\hbar}W} = 0$ 

Quantum Master Action  

$$W = S + \sum_{n=1}^{\infty} \hbar^n M_n$$

#### Additive form

$$\frac{1}{2}(W,W) = i\hbar\Delta_{\rho}W + \hbar^{2}\nu_{\rho}$$

Odd scalar  $\nu_{\rho}$  enters at 2-loop.

### **Infinite Tower of Master Equations**

$$(S,S) = 0$$

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# Quantum Master Equation

Exponential form

 $\Delta e^{rac{i}{\hbar}W} = 0$ 

Quantum Master Action  

$$W = S + \sum_{n=1}^{\infty} \hbar^n M_n$$

#### Additive form

$$\frac{1}{2}(W,W) = i\hbar\Delta_{\rho}W + \hbar^{2}\nu_{\rho}$$

Odd scalar  $\nu_{\rho}$  enters at 2-loop.

Infinite Tower of Master Equations

 $(S,S) = 0 \leftarrow Classical Master Equation$ 

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Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

# Quantum Master Equation

Exponential form

 $\Delta e^{rac{i}{\hbar}W} = 0$ 

Quantum Master Action  

$$W = S + \sum_{n=1}^{\infty} \hbar^n M_n$$

#### Additive form

$$\frac{1}{2}(W,W) = i\hbar\Delta_{\rho}W + \hbar^{2}\nu_{\rho}$$

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$$(S,S) = 0 \leftarrow (M_1,S) = i(\Delta_{\rho}S)$$

Classical Master Equation

Poisson Vs. Anti-Poissor Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

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#### Infinite Tower of Master Equations

$$\begin{array}{rcl} (S,S) &=& 0 &\leftarrow & \text{Classical Master Equation} \\ (M_1,S) &=& i(\Delta_{\rho}S) \\ (M_2,S) &=& i(\Delta_{\rho}M_1) - \frac{1}{2}(M_1,M_1) \ + \ \nu_{\rho} \end{array}$$

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Poisson Vs. Anti-Poissor Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

# Quantum Master Equation

Exponential form

A

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$$(M_1,S) = i(\Delta_{\rho}S)$$
$$(M_2,S) = i(\Delta_{\rho}M_1) - \frac{1}{2}(M_1,M_1) + \nu_{\rho}$$
$$n \ge 3: (M_n,S) = i(\Delta_{\rho}M_{n-1}) - \frac{1}{2}\sum_{r=1}^{n-1}(M_r,M_{n-r})$$

Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

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# Khudaverdian's $\Delta_E$ Operator

The  $\triangle$  Operator

$$\Delta = \Delta_{\rho} + \nu_{\rho}$$

= Odd Laplacian + Odd Scalar = built from E and  $\rho$ .

Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

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# Khudaverdian's $\Delta_E$ Operator

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### **Curious Fact**

 $\sqrt{\rho}\Delta \frac{1}{\sqrt{\rho}}$  is independent of

Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

# Khudaverdian's $\Delta_E$ Operator

# **Def:** Khudaverdian's $\Delta_E$ **Operator**

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Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

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Def: Khudaverdian's  $\Delta_E$  Operator

• In Darboux Coordinates:

$$\Delta_{E} := \Delta_{1} = (-1)^{\varepsilon_{lpha}} rac{\overrightarrow{\partial^{\ell}}}{\partial \phi^{lpha}} rac{\overrightarrow{\partial^{\ell}}}{\partial \phi^{st}_{lpha}}$$

(Khudaverdian 1997)

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Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

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### (Khudaverdian 1997)

• In General Coordinates:

$$\Delta_E := \Delta_1 + rac{
u^{(1)}}{8} - rac{
u^{(2)}}{24}$$
(KB 2006)

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$$\frac{\sqrt{\rho}\Delta\frac{1}{\sqrt{\rho}}}{\text{is independent of }\rho!}$$

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Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

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### Proporties of $\Delta_E$

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Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

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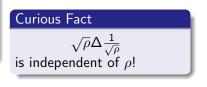
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(KB 200

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Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

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$$\Delta_E$$
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Poisson Vs. Anti-Poisson Laplacian The Odd Scalar  $\nu_{\rho}$  The  $\Delta$  Operator

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The Even Scalar  $u_{
m g}$ Particle in Curved Space

# A Comparative Study of Laplacians in Riemannian and Antisymplectic Geometry



2 Riemannian Geometry





The Even Scalar  $\nu_{\rho}$ Particle in Curved Space

# The Even Scalar $\nu_{\rho}$

Even Scalar in Riemannian Geometry with density  $\rho$ 

$$\nu_{\rho} := \nu_{\rho}^{(0)} + \frac{\nu^{(1)}}{4} - \frac{\nu^{(2)}}{8} - \frac{\nu^{(3)}}{16}$$

The Even Scalar  $\nu_{\rm g}$ Particle in Curved Space

# The Even Scalar $\nu_{\rho}$

Even Scalar in Riemannian Geometry with density ho

$$\nu_{\rho} := \nu_{\rho}^{(0)} + \frac{\nu^{(1)}}{4} - \frac{\nu^{(2)}}{8} - \frac{\nu^{(3)}}{16}$$

$$u^{(0)}_{
ho} := rac{1}{\sqrt{
ho}} (\Delta_1 \sqrt{
ho})$$

The Even Scalar  $\nu_{\rm p}$ Particle in Curved Space

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$$egin{array}{rcl} 
u_{
ho}^{(0)} & := & rac{1}{\sqrt{
ho}}(\Delta_1\sqrt{
ho}) \ 
u^{(1)} & := & (-1)^{arepsilon_A}(rac{\partial^\ell}{\partial z^A}g^{AB}rac{\partial^r}{\partial z^B})(-1)^{arepsilon_B} \end{array}$$

The Even Scalar  $\nu_{\rm p}$ Particle in Curved Space

# The Even Scalar $\nu_{\rho}$

Even Scalar in Riemannian Geometry with density ho

$$u_{\rho} := \nu_{\rho}^{(0)} + \frac{\nu^{(1)}}{4} - \frac{\nu^{(2)}}{8} - \frac{\nu^{(3)}}{16}$$

$$\nu_{\rho}^{(0)} := \frac{1}{\sqrt{\rho}} (\Delta_{1} \sqrt{\rho})$$

$$\nu^{(1)} := (-1)^{\varepsilon_{A}} (\frac{\overrightarrow{\partial^{\ell}}}{\partial z^{A}} g^{AB} \frac{\overleftarrow{\partial^{r}}}{\partial z^{B}}) (-1)^{\varepsilon_{B}}$$

$$\nu^{(2)} := -(-1)^{\varepsilon_{C}} (z^{C}, (z^{B}, z^{A})) (\frac{\overrightarrow{\partial^{\ell}}}{\partial z^{A}} g_{BC})$$

The Even Scalar  $\nu_{\rm g}$ Particle in Curved Space

# The Even Scalar $\nu_{\rho}$

Even Scalar in Riemannian Geometry with density ho

$$u_{\rho} := \nu_{\rho}^{(0)} + \frac{\nu^{(1)}}{4} - \frac{\nu^{(2)}}{8} - \frac{\nu^{(3)}}{16}$$

$$\begin{aligned}
\nu_{\rho}^{(0)} &:= \frac{1}{\sqrt{\rho}} (\Delta_{1} \sqrt{\rho}) \\
\nu^{(1)} &:= (-1)^{\varepsilon_{A}} (\frac{\overrightarrow{\partial^{\ell}}}{\partial z^{A}} g^{AB} \frac{\overleftarrow{\partial^{r}}}{\partial z^{B}}) (-1)^{\varepsilon_{B}} \\
\nu^{(2)} &:= -(-1)^{\varepsilon_{C}} (z^{C}, (z^{B}, z^{A})) (\frac{\overrightarrow{\partial^{\ell}}}{\partial z^{A}} g_{BC}) \\
&= -(-1)^{(\varepsilon_{A}+1)(\varepsilon_{D}+1)} (\frac{\overrightarrow{\partial^{\ell}}}{\partial z^{D}} g^{AB}) g_{BC} (g^{CD} \frac{\overleftarrow{\partial^{r}}}{\partial z^{A}})
\end{aligned}$$

The Even Scalar  $\nu_{\rm g}$ Particle in Curved Space

# The Even Scalar $\nu_{\rho}$

Even Scalar in Riemannian Geometry with density ho

$$u_{\rho} := \nu_{\rho}^{(0)} + \frac{\nu^{(1)}}{4} - \frac{\nu^{(2)}}{8} - \frac{\nu^{(3)}}{16}$$

#### Terms built from g and $\rho$

$$\begin{split} \nu_{\rho}^{(0)} &:= \frac{1}{\sqrt{\rho}} (\Delta_{1}\sqrt{\rho}) \\ \nu^{(1)} &:= (-1)^{\varepsilon_{A}} (\frac{\overrightarrow{\partial^{\ell}}}{\partial z^{A}} g^{AB} \frac{\overleftarrow{\partial^{r}}}{\partial z^{B}}) (-1)^{\varepsilon_{B}} \\ \nu^{(2)} &:= -(-1)^{\varepsilon_{C}} (z^{C}, (z^{B}, z^{A})) (\frac{\overrightarrow{\partial^{\ell}}}{\partial z^{A}} g_{BC}) \\ &= -(-1)^{(\varepsilon_{A}+1)(\varepsilon_{D}+1)} (\frac{\overrightarrow{\partial^{\ell}}}{\partial z^{D}} g^{AB}) g_{BC} (g^{CD} \frac{\overleftarrow{\partial^{r}}}{\partial z^{A}}) \\ \nu^{(3)} &:= (-1)^{\varepsilon_{A}} (g_{AB}, g^{BA}) \end{split}$$

The Even Scalar  $\nu_{\rm g}$ Particle in Curved Space

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Even Scalar in Riemannian Geometry with density ho

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The Even Scalar  $\nu_{\rho}$ Particle in Curved Space

# Interpretation of $\nu_{\rho}$ in terms of Scalar Curvature R

# Riemannian Case

 $\nu_{\rho_g} = -\frac{\pi}{4}$ 

The Even Scalar  $\nu_{\rho}$ Particle in Curved Space

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### Interpretation of $\nu_{\rho}$ in terms of Scalar Curvature R

Riemannian Case

$$\nu_{\rho_g} = -\frac{F}{Z}$$

### **Even Scalar Curvature**

$$R := (-1)^{\varepsilon_A} R_{AB} g^{BA}$$

for the Levi-Civita Connection  $\nabla$ , *i.e.*,  $\nabla$ is:

The Even Scalar  $\nu_{\rho}$ Particle in Curved Space

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# Interpretation of $u_{ ho}$ in terms of Scalar Curvature R

Riemannian Case

 $\nu_{\rho_g} = -\frac{R}{4}$ 

#### Even Scalar Curvature

$$R := (-1)^{\varepsilon_A} R_{AB} g^{BA}$$

for the Levi-Civita Connection  $\nabla$ , *i.e.*,  $\nabla$  is:

• metric,

The Even Scalar  $\nu_{\rho}$ Particle in Curved Space

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$$R := (-1)^{\varepsilon_A} R_{AB} g^{BA}$$

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- metric,
- and torsionfree.

The Even Scalar  $\nu_{\rho}$ Particle in Curved Space

# Interpretation of $u_{\rho}$ in terms of Scalar Curvature R

Riemannian Case

$$\nu_{\rho_g} = -\frac{R}{4}$$

Antisymplectic Case

$$2\nu_{\rho} = -\frac{\kappa}{4}$$

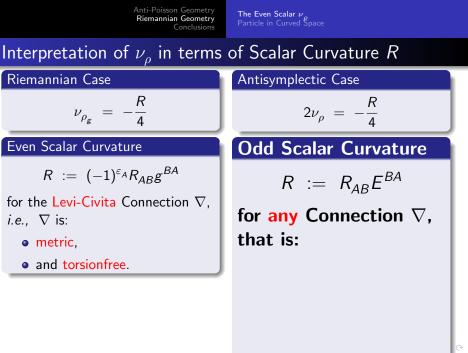
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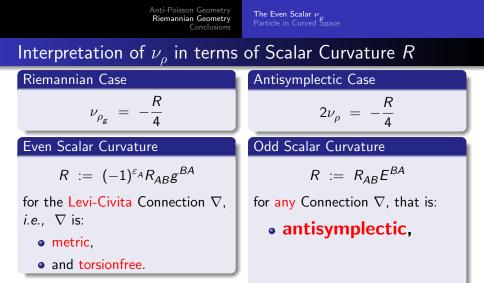
#### Even Scalar Curvature

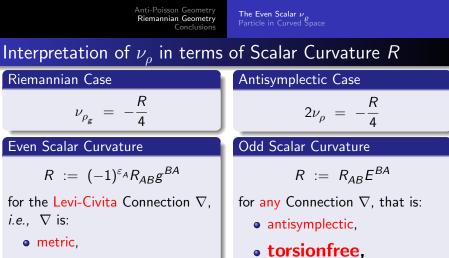
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- and torsionfree.







• and torsionfree.



# Interpretation of $u_{ ho}$ in terms of Scalar Curvature R

Riemannian Case

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for the Levi-Civita Connection  $\nabla$ , *i.e.*,  $\nabla$  is:

- metric,
- and torsionfree.

Antisymplectic Case

$$2\nu_{\rho} = -\frac{R}{4}$$

Odd Scalar Curvature

$$R := R_{AB}E^{BA}$$

for any Connection  $\nabla,$  that is:

- antisymplectic,
- torsionfree,
- and compatible with  $\rho$ .



# Interpretation of $u_{ ho}$ in terms of Scalar Curvature R

Riemannian Case

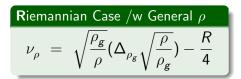
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# Interpretation of $u_{ ho}$ in terms of Scalar Curvature R

Riemannian Case

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Riemannian Case /w General 
$$\rho$$
  
 $\nu_{\rho} = \sqrt{\frac{\rho_{g}}{\rho}} (\Delta_{\rho_{g}} \sqrt{\frac{\rho}{\rho_{g}}}) - \frac{R}{4}$ 

Antisymplectic Case

$$\mathbf{2}\nu_{\rho} = -\frac{R}{4}$$

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The Even Scalar  $\nu_{\rho}$ Particle in Curved Space

### Classification of 2nd-order Differential Invariants

### Question

# What is the most general function $\nu = \nu(z)$ such that

The Even Scalar  $\nu_{e}$ Particle in Curved Space

### Classification of 2nd-order Differential Invariants

#### Question

What is the most general function  $\nu = \nu(z)$  such that

•  $\nu(z)$  is a scalar,

# Classification of 2nd-order Differential Invariants

### Question

- $\nu(z)$  is a scalar,
- $\nu(z)$  is a polynomial of the metric  $g_{AB}(z)$ , the density  $\rho(z)$ , their inverses, and *z*-derivatives thereof in the point *z*,

# Classification of 2nd-order Differential Invariants

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- $\nu$  is invariant under constant rescaling of the density  $\rho \rightarrow \lambda \rho$ , where  $\lambda$  is a *z*-independent parameter,

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# Classification of 2nd-order Differential Invariants

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# **Complete Solution**

$$\nu = \alpha \ \nu_{\rho} + \beta \ \nu_{\rho_{g}} + \gamma \ (\ln \frac{\rho}{\rho_{g}}, \ln \frac{\rho}{\rho_{g}})$$

The Even Scalar  $\nu_{\rm p}$ Particle in Curved Space

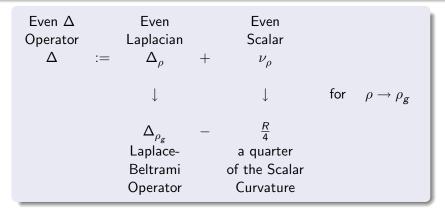


The Even Scalar  $\nu_{\rho}$ Particle in Curved Space

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The Even Scalar  $\nu_{
m e}$ Particle in Curved Space

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Even $\Delta$ Operator $\Delta$ :=	Even Laplacian $\Delta_ ho$	+	Even Scalar $ u_{ ho}$			
	$\downarrow$		$\downarrow$	for	$\rho \to \rho_{\rm g}$	
	$\Delta_{ ho_g}$ Laplace- Beltrami Operator	_	Ra quarterof the ScalarCurvature			
For comparison: Conformally Covariant Laplacian						
$\Delta_{ ho_g} - rac{(N-2)R}{(N-1)4}$						

The Even Scalar  $\nu_{
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Even Δ Operator Δ :=	Even Laplacian = $\Delta_{ ho}$	Even Scalar $+$ $ u_{ ho}$				
	$\downarrow$	$\downarrow$	for $\rho \rightarrow \rho_g$			
	$\Delta_{ ho_g}$ Laplace- Beltrami Operator	<ul> <li> <sup>R</sup>/<sub>4</sub> a quarter of the Scalar Curvature         </li> </ul>				
For comparison: Conformally Covariant Laplacian						
$\Delta_{ ho_g} - rac{(N)}{(N)}$	$(-2)R \ -1)4 \rightarrow$	$\Delta_{ ho_g} - rac{R}{4}$ for	$N \to \infty$ .			

The Even Scalar  $\nu_{\rho}$ Particle in Curved Space

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Riemannian version  $\Delta_g$  of Khudaverdian's  $\Delta_E$  Operator

### The $\triangle$ Operator

$$\Delta = \Delta_{\rho} + \nu_{\rho}$$

- = Even Laplacian + Even Scalar
- = built from g and  $\rho$ .

The Even Scalar  $\nu_{\rho}$ Particle in Curved Space

# Riemannian version $\Delta_g$ of Khudaverdian's $\Delta_E$ Operator

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# Curious Fact $\sqrt{\rho}\Delta \frac{1}{\sqrt{\rho}}$ is independent of $\rho!$

The Even Scalar  $\nu_{\rho}$ Particle in Curved Space

# Riemannian version $\Delta_g$ of Khudaverdian's $\Delta_E$ Operator

Definition of 
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 $\Delta_g := \Delta_1 + \frac{\nu^{(1)}}{4} - \frac{\nu^{(2)}}{8} - \frac{\nu^{(3)}}{16}$ 

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The Even Scalar  $\nu_{\rho}$ Particle in Curved Space

# Riemannian version $\Delta_g$ of Khudaverdian's $\Delta_E$ Operator

Definition of  $\Delta_g$ 

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The Even Scalar  $\nu_{\rm g}$ Particle in Curved Space

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### Proporties of $\Delta_g$

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Curious Fact  $\sqrt{\rho}\Delta \frac{1}{\sqrt{\rho}}$ is independent of  $\rho!$ 

The Even Scalar  $\nu_{\rm g}$ Particle in Curved Space

# Riemannian version $\Delta_g$ of Khudaverdian's $\Delta_E$ Operator

Definition of 
$$\Delta_g$$
  
$$\Delta_g := \Delta_1 + \frac{\nu^{(1)}}{4} - \frac{\nu^{(2)}}{8} - \frac{\nu^{(3)}}{16}$$

### Proporties of $\Delta_g$

- Δ<sub>g</sub> takes semidensities to semidensities.
- $\Delta_g$  is manifestly independent of  $\rho$ .
- NB!  $\Delta_g$  is not nilpotent.

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The Even Scalar  $\nu_{\rho}$ Particle in Curved Space

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# Particle in Curved Space

# **Classical Hamiltonian Action**

$$S_{cl} = \int dt \left( p_A \dot{z}^A - H_{cl} \right)$$
$$H_{cl} = \frac{1}{2} p_A p_B g^{BA}$$
$$\{ z^A, p_B \}_{PB} = \delta^A_B$$

The Even Scalar  $\nu_{\rho}$ Particle in Curved Space

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# Naive Quantum Hamiltonian

$$\hat{H}_{
ho} \;=\; rac{1}{2\sqrt{
ho(\hat{z})}}\; \hat{
ho}_{A} \; 
ho(\hat{z}) \; g^{AB}(\hat{z}) \; \hat{
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# Particle in Curved Space

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 $\{z^A$ 

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### Full Quantum Hamiltonian

$$\hat{H} = \hat{H}_
ho - rac{\hbar^2}{2} 
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The Even Scalar  $\nu_{\rho}$ Particle in Curved Space

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# Schrödinger Representation

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$$\frac{\hbar}{i} \frac{\overrightarrow{\partial^{\ell}}}{\partial z^{A}} = \sqrt{\rho(\hat{z})} \, \hat{p}_{A} \, \frac{(-1)^{\varepsilon_{A}}}{\sqrt{\rho(\hat{z})}}$$

### Naive Quantum Hamiltonian

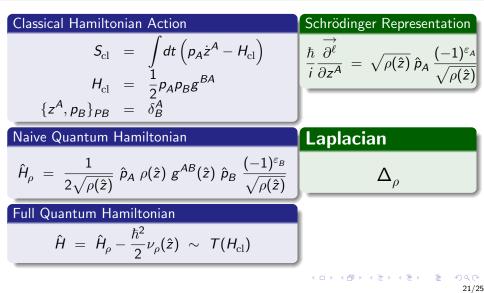
$$\hat{H}_{
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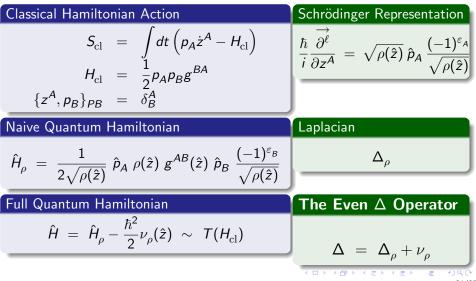
The Even Scalar  $\nu_{\rho}$ Particle in Curved Space

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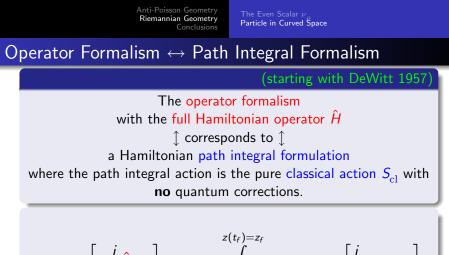


The Even Scalar  $\nu_{\rho}$ Particle in Curved Space

# Particle in Curved Space



Anti-Poisson Geometry Riemannian Geometry Particle in Curved Space Conclusions Operator Formalism ↔ Path Integral Formalism (starting with DeWitt 1957) The operator formalism with the full Hamiltonian operator  $\hat{H}$  $\uparrow$  corresponds to  $\uparrow$ a Hamiltonian path integral formulation where the path integral action is the pure classical action  $S_{cl}$  with no quantum corrections.



$$\langle z_f | \exp\left[-rac{i}{\hbar}\hat{H}\Delta t
ight] | z_i 
angle \ \sim \int\limits_{z(t_i)=z_i} [dz][dp] \ \exp\left[rac{i}{\hbar}S_{
m cl}[z,p]
ight]$$



# Operator Formalism \leftrightarrow Path Integral Formalism

(starting with DeWitt 1957)

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The operator formalism with the full Hamiltonian operator  $\hat{H}$  corresponds to a Hamiltonian path integral formulation where the path integral action is the pure classical action  $S_{\rm cl}$  with **no** quantum corrections.

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# Full Quantum Hamiltonian $\hat{H} = \hat{H}_{\rho} - \frac{\hbar^2}{2} \nu_{\rho}(\hat{z})$

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The Even Scalar  $\nu_{\rho}$ Particle in Curved Space

# Operator Formalism \leftrightarrow Path Integral Formalism

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Il Quantum Hamiltonian  
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$$S_{cl}[z, p] = \int dt \left( p_{A} \dot{z}^{A} - H_{cl} \right)$$$$

# A Comparative Study of Laplacians in Riemannian and Antisymplectic Geometry



2 Riemannian Geometry



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# Conclusions

# Odd $\triangle$ Operator in Antisymplectic Geometry

$$2\Delta = 2\Delta_{\rho} + 2\nu_{\rho} = 2\Delta_{\rho} - \frac{\kappa}{4}$$

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 $\exists \rightarrow$ 

# Conclusions

Odd  $\Delta$  Operator in Antisymplectic Geometry

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 $\exists \rightarrow$ 

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• Characterized by nilpotency

# Conclusions

Odd  $\Delta$  Operator in Antisymplectic Geometry

$$2\Delta = 2\Delta_{\rho} + 2\nu_{\rho} = 2\Delta_{\rho} - \frac{R}{4}$$

- Characterized by nilpotency
- and characterized by a  $\rho$  independence argument.

Anti-Poisson Geometry Riemannian Geometry Conclusions Conclusions Odd  $\Delta$  Operator in Even  $\triangle$  Operator in Antisymplectic Geometry **Riemannian Geometry**  $2\Delta = 2\Delta_{\rho} + 2\nu_{\rho} = 2\Delta_{\rho} - \frac{\kappa}{4}$ R  $\Delta = \Delta_{\rho} + \nu_{\rho} \rightarrow \Delta_{\rho_{g}}$ • Characterized by nilpotency • and characterized by a  $\rho$ independence argument.

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# Conclusions

### Even ∆ Operator in Riemannian Geometry

$$\Delta = \Delta_{\rho} + \nu_{\rho} \rightarrow \Delta_{\rho_g} - \frac{\kappa}{4}$$

• Characterized by a  $\rho$  independence argument.

Odd  $\Delta$  Operator in Antisymplectic Geometry

$$2\Delta = 2\Delta_{\rho} + 2\nu_{\rho} = 2\Delta_{\rho} - \frac{\kappa}{4}$$

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Anti-Poisson Geometry Riemannian Geometry Conclusions Conclusions Even  $\Delta$  Operator in Odd  $\Delta$  Operator in **Riemannian Geometry** Antisymplectic Geometry  $\Delta = \Delta_{\rho} + \nu_{\rho} \rightarrow \Delta_{\rho_{\sigma}} - \frac{R}{4}$  $2\Delta = 2\Delta_{\rho} + 2\nu_{\rho} = 2\Delta_{\rho} - \frac{R}{4}$  Characterized by nilpotency • Characterized by a  $\rho$ • and characterized by a  $\rho$ independence argument. independence argument. Particle in Curved Space  $\Delta$  is the full quantum Hamiltonian  $\hat{H} = \hat{H}_{\rho} - \frac{\hbar^2}{2} \nu_{\rho}(\hat{z})$ in the Schrödinger representation.

# Conclusions

### Even ∆ Operator in Riemannian Geometry

$$\Delta = \Delta_{\rho} + \nu_{\rho} \rightarrow \Delta_{\rho_g} - \frac{\kappa}{4}$$

• Characterized by a  $\rho$  independence argument.

Odd  $\Delta$  Operator in Antisymplectic Geometry

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### Particle in Curved Space

 $\Delta$  is the full quantum Hamiltonian

$$\hat{H} = \hat{H}_{\rho} - \frac{\hbar^2}{2} \nu_{\rho}(\hat{z})$$

in the Schrödinger representation.

Curvature term in Quantum Master Equation

$$(W, W) = 2i\hbar\Delta_{\rho}W - \hbar^2 \frac{R}{4}$$

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Important 2-loop effect.

# References

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