Towards Black Holes in $4d$ Higher Spin Gauge Theory

V.E. Didenko

in collaboration with M.A. Vasiliev

Lebedev Institute, Moscow

Moscow, Lebedev Institute, May 22, 2009

Plan

- **Introduction.**

  Motivation. Einstein black holes.

- **Unfolded formulation for AdS_4 black hole.**

- **Higher spin (HS) equations in $d = 4$.**

- **Classical black holes from free HS theory.**

- **Solution of nonlinear equations.**

- **Conclusion**

Motivation

Gravity s=2	$\Longrightarrow$	Super-gravity	$\Longrightarrow$	Higher spin gauge theory (HS) $s = 0, 1/2, 1, \ldots, \infty$ **(Vasiliev)**

HS gauge theory:

Consistent theory of interacting massless fields $s = 0, 1/2, \ldots \infty$ in AdS space-time

Current status:

Formulated at the level of equations of motion for all spins in $d = 4$ Bosonic version is known in any d

String theory $\rightarrow$ **unbroken phase** $\rightarrow$ **HS theory** **(???)**

Lack of:

Action principle, quantization.

GR black holes $\rightarrow$ SUGRA black holes $\rightarrow$ HS black holes **???**

Obstacles:

1. HS does not have decoupled spin-2 sector $\rightarrow$ all higher spins involved in the equations of motion.

2. The interval $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ is not gauge invariant quantity in higher spin algebra.

3. Considerable technical difficulties – the equations are essentially non-local involving space-time derivatives of all orders.

Perturbative analysis available

HS theory $\rightarrow$ 0-th order vacua AdS $\rightarrow$ 1-st order free field Fronsdal equations $\rightarrow$ 2-nd order interactions $\rightarrow \ldots$

Classical black hole properties

Ex. $d = 4$ Kerr Solution

1. $g_{\mu\nu} = \eta_{\mu\nu}(x) + M h_{\mu\nu}(x)$ – no $O(M^2)$ terms $\Longrightarrow$ Einstein equations reduce to **free** $s = 2$ Pauli-Fierz eqs.

$$\Box h_{\mu\nu} - \partial_\mu \partial_\lambda h^\lambda{}_\nu - \partial_\nu \partial_\lambda h^\lambda{}_\mu = 0 \qquad (h_\mu{}^\mu = 0)$$

2. $h_{\mu\nu} = \frac{1}{U(x)} k_\mu(x) k_\nu(x)$ – factorized form. k^μ – Kerr-Schild vector

$$k_\mu k^\mu = 0\,, \qquad k^\mu D_\mu k_\nu = k^\mu \partial_\mu k_\nu = 0$$

3. BH provides Fronsdal fields $\phi_{\mu_1 \ldots \mu_s} = \frac{M}{U} k_{\mu_1} \ldots k_{\mu_s}$

$\mathrm{s} = 0 \Longrightarrow$ Klein-Gordon $\quad \Box \phi = 0$

$\mathrm{s} = 1 \Longrightarrow$ Maxwell $\quad \Box \phi_\mu - \partial_\lambda \partial_\mu \phi^\lambda = 0$

$\mathrm{s} = 2 \Longrightarrow$ Pauli-Fierz $\quad \Box \phi_{\mu\nu} - 2\partial_\lambda \partial_{(\mu} \phi_{\nu)}{}^\lambda = 0$

$\mathrm{s} = \mathrm{s} \Longrightarrow$ Fronsdal $\quad \Box \phi_{\mu_1 \ldots \phi_{\mu_s}} - s \partial_\lambda \partial_{(\mu_1} \phi^\lambda{}_{\mu_2 \ldots \mu_s)} = 0$

4. Kerr-Schild presentation is also valid in AdS

$$g_{\mu\nu} = \eta^{AdS}_{\mu\nu}(x) + \frac{M}{U} k_\mu k_\nu \,, \qquad k^\mu k_\mu = 0 \,, \qquad k^\mu \mathcal{D}_\mu k_\nu = k^\mu D_\mu k_\nu = 0$$

Just as well,

$$\phi_{\mu_1 \dots \mu_s} = \frac{M}{U} k_{\mu_1} \dots k_{\mu_s}$$

satisfies free massless spin-s equations (**Metsaev**) in AdS_4

$$\Box \phi_{\mu_1 \dots \mu_s} - s D_\lambda D_{(\mu_1} \phi^\lambda{}_{\mu_2 \dots \mu_s)} = -2(s-1)(s+1)\lambda^2 \phi_{\mu_1 \dots \mu_s}$$

$\phi_{\mu_1 \dots \mu_s}(x)$ − **Black hole massless fields**

Program for HS black holes

| vacuum AdS_4 | $\rightarrow$ | black hole massless fields $\phi_{\mu_1 \dots \mu_s}(x)$ | $\rightarrow$ | HS corrections | $\rightarrow$ | $\dots$ |

Unfolded formulation

- First order coordinate independent differential equations (differential forms formalism)

- Assumes additional fields (generally infinitely many) that parameterize all on-shell derivatives of physical fields

Example: free massless scalar in Minkowski space-time $\Box\phi(x) = 0$

Unfolding $\rightarrow \varphi(x), \quad \varphi_\mu = \partial_\mu \varphi, \quad \varphi_{\mu\nu} = \partial_\mu \varphi_\nu, \ldots \quad \varphi_{\mu_1 \ldots \mu_n} = \partial_{\mu_1} \varphi_{\mu_2 \ldots \mu_n},$

$\ldots$

Set of fields: $\phi, \quad \phi_\mu, \ldots \quad \varphi_{\mu_1 \ldots \mu_n}, \ldots$

Consistency condition: $\varphi_{\mu_1 \ldots \mu_n}$ − symmetric

Equations of motion: ${\varphi^\mu}_{\mu\mu_3 \ldots \mu_n} = 0$

To perform perturbative analysis of black holes in HS theory one has to have explicit expressions for all AdS_4 derivatives of AdS_4-Kerr black hole fields (vierbein, curvature). Therefore, one needs AdS_4 covariant description of classical black hole to proceed.

Strategy

1. **Find unfolded, AdS_4 covariant, coordinate free description of a black hole** **(arxiv: 0801.2213, 0901.2172 [hep-th])**

2. **Choose AdS_4 black hole with its generalization for the rest of spins as the first order solution in HS perturbative expansion**

3. **Calculate HS nonlinear corrections**

Unfolded formulation of AdS_4 black hole

Let $K_{AB}(x) = K_{BA}(x)\,,\quad A = (\alpha, \dot\alpha) = 1\ldots 4$ be an AdS_4 global symmetry parameter

$$D_0 K_{AB} = 0\,, \qquad D_0^2 = 0$$

Lorentz components $\longrightarrow$ $K_{AB} = K_{BA} = \begin{pmatrix} \lambda^{-1}\kappa_{\alpha\beta} & V_{\alpha\dot\beta} \\ V_{\beta\dot\alpha} & \lambda^{-1}\bar\kappa_{\dot\alpha\dot\beta} \end{pmatrix}$

$4d$ **Kerr black hole:** $\quad g_{mn} = \eta^{AdS}_{\mu\nu} + \frac{2M}{U} k_\mu k_\nu$

$$\frac{2}{U} = \frac{\lambda^2}{\sqrt{-\kappa^2}} + \frac{\lambda^2}{\sqrt{-\bar\kappa^2}}\,, \qquad k_{\alpha\dot\alpha} = \frac{1}{(V^- V^+)} V^-_{\alpha\dot\alpha}\,, \quad \text{where} \quad V^{\pm}_{\alpha\dot\alpha} = \pi^{\pm}_{\alpha}{}^{\gamma} \bar\pi^{\pm}_{\dot\alpha}{}^{\dot\gamma} V_{\gamma\dot\gamma}$$

and

$$\pi^{\pm}_{\alpha\beta} = \frac{1}{2}\left(\epsilon_{\alpha\beta} \pm \frac{\kappa_{\alpha\beta}}{\sqrt{-\kappa^2}}\right)\,, \qquad \kappa^2 = \frac{1}{2}\kappa_{\alpha\beta}\kappa^{\alpha\beta}$$

Einstein equations satisfied for any $K_{AB}(x)$

Type of a black hole (M – real)

1. ${K_A}^C {K_C}^B \neq C {\delta_A}^B$ – **stationary (Kerr)**

2. ${K_A}^C {K_C}^B = C {\delta_A}^B$ – **static (Schwarzschild)** $\Longrightarrow$

$$r^2 - V^2 = 1\,, \qquad \bar{\kappa}_{\dot\alpha}{}^{\dot\gamma} V_{\beta\dot\gamma} + V^{\gamma}{}_{\dot\alpha}\kappa_{\gamma\beta} = 0\,, \qquad -r^2 = \kappa^2 = \bar{\kappa}^2$$

Weyl tensor $\longrightarrow$ $C_{\alpha(4)} = \frac{M}{r^5}\kappa_{\alpha\alpha}\kappa_{\alpha\alpha}\,, \qquad \bar{C}_{\dot\alpha(4)} = \frac{M}{r^5}\bar{\kappa}_{\dot\alpha\dot\alpha}\bar{\kappa}_{\dot\alpha\dot\alpha}$

HS equations

- **Star-product operation**

  Let $Y_A = (y_\alpha, \bar{y}_{\dot\alpha})$ and $Z_A = (z_\alpha, \bar{z}_{\dot\alpha})$ be commuting variables.

$$(f \star g)(Y,Z) = \int f(Y+s, Z+s) g(Y+t, Z-t) e^{s_A t^A} \longrightarrow$$

  associative algebra with

$$[Z_A, Z_B]_\star = -[Y_A, Y_B]_\star = 2\epsilon_{AB}\,, \qquad [Y_A, Z_B]_\star = 0$$

  Klein operators $\longrightarrow$

  $v = \exp\left(z_\alpha y^\alpha\right)$, $\quad \bar{v} = \exp\left(\bar{z}_{\dot\alpha} \bar{y}^{\dot\alpha}\right)$ with the following properties

$$v \star v = \bar{v} \star \bar{v} = 1\,, \qquad v \star f(y,z) = f(-y,-z) \star v\,, \qquad \bar{v} \star f(\bar{y},\bar{z}) = f(-\bar{y},-\bar{z}) \star \bar{v}$$

  Singular Klein operators $\longrightarrow$ $v = \delta(y) \star \delta(z)\,, \quad \bar{v} = \delta(\bar{y}) \star \delta(\bar{z})$

$$\delta(y) \star \delta(y) = 1\,, \quad \delta(y) \star f(y,z) = f(-y,z) \star \delta(y)$$

$$\hat{f}(y,z) = f(y,z) \star \delta(y) = \int d^2u f(u,z) e^{-u_\alpha y^\alpha} \longleftarrow$$ **Fourier transformation**

- **Free equations ($Z = 0$)**

HS linearized field strengths (twisted-adjoint module)

$$C(y, \bar{y}|x) = \sum_{n,m=0}^{\infty} \frac{1}{n!m!} C_{\alpha(n),\dot{\alpha}(m)} y^{\alpha} \dots y^{\alpha} \bar{y}^{\dot{\alpha}} \dots \bar{y}^{\dot{\alpha}} ,$$

$$\tilde{\mathcal{D}}_0 C \equiv dC - w_0 \star C + C \star \tilde{w}_0 = 0 ,$$

where $\tilde{f}(y, \bar{y}) = f(-y, \bar{y}) \leftarrow$ **twist operator**

$w_0(y, \bar{y}|x)$ is an AdS_4 connection

$$w_0 = -\frac{1}{8}(\omega_{\alpha\alpha} y^{\alpha} y^{\alpha} + \bar{\omega}_{\dot{\alpha}\dot{\alpha}} \bar{y}^{\dot{\alpha}} \bar{y}^{\dot{\alpha}} - 2h_{\alpha\dot{\alpha}} y^{\alpha} \bar{y}^{\dot{\alpha}}) , \qquad dw_0 - w_0 \star \wedge w_0 = 0$$

Global symmetry $\rightarrow$ $\quad \delta C = \epsilon_0 \star C - c \star \tilde{\epsilon}_0 , \qquad \mathcal{D}_0 \epsilon_0(y, \bar{y}|x) = 0,$

$\mathcal{D}_0 = d - w_0$

Note:

Any $\epsilon_0(y, \bar{y}|x)$ **generates solution of the twisted-adjoint module**

$$C(y, \bar{y}|x) = c_1 \epsilon_0(y, \bar{y}|x) \star \delta(y) + c_2 \epsilon_0(y, \bar{y}|x) \star \delta(\bar{y}) .$$

- **Nonlinear HS equations ($Z \neq 0$)**

$$w(y,\bar{y}|x) \to W(y,\bar{y},z,\bar{z}|x)\,, \qquad C(y,\bar{y}|x) \to B(y,\bar{y},z,\bar{z}|x)$$

Pure gauge compensator 1-form $\to$ $\quad S(Z,Y|x) = S_\alpha dz^\alpha + \bar{S}_{\dot\alpha} dz^{\dot\alpha}$

Introducing $A = d + W + S$, HS nonlinear equations take the form

$$A \star \wedge A = \mathcal{R}(B, v, \bar{v})$$

Component form (bosonic eqs.) $\to$

$$dW - W \star \wedge W = 0\,, \qquad dB - W \star B + B \star \tilde{W} = 0\,,$$

$$dS_\alpha - [W, S_\alpha]_\star = 0\,, \quad d\bar{S}_{\dot\alpha} - [W, \bar{S}_{\dot\alpha}]_\star = 0\,,$$

$$S_\alpha \star S^\alpha = 2(1 + B \star v)\,, \quad \bar{S}_{\dot\alpha} \star \bar{S}^{\dot\alpha} = 2(1 + B \star \bar{v})\,, \quad [S_\alpha, \bar{S}_{\dot\alpha}]_\star = 0\,,$$

$$B \star \tilde{S}_\alpha + S_\alpha \star B = 0\,, \quad B \star \tilde{\bar{S}}_{\dot\alpha} + \bar{S}_{\dot\alpha} \star B = 0\,,$$

Dynamical potentials and field strengths: $W(Y,Z|x)_{|Z=0}\,, B(Y,Z|x)_{|Z=0}$

Black holes from the free HS theory

$K_{AB}(x) \to AdS_4$ **global symmetry parameter** $(D_0 K_{AB} = 0) \Longrightarrow$

$$\mathcal{D}_0 f(K_{AB}Y^AY^B) = 0$$

generates solution of the twisted-adjoint module

$$C(y,\bar{y}|x) = M f(K_{AB}Y^AY^B) \star \delta(y) \qquad \leftarrow \qquad \text{complex solution}$$

$C(y,\bar{y}|x)$ **describes two** AdS_4**-Kerr-Taub-NUT black holes in** $s=2$ **sector and HS generalization for the rest of integer spins** (Didenko, Matveev, Vasiliev)

$$m \sim \mathbf{Re}M\,, \qquad n \sim \mathbf{Im}M$$

Schwarzschild case $\to M$ - real, $K_A{}^C K_C{}^B = \delta_A{}^B$

Choose $\to f = \exp\left(\frac{1}{2}K_{AB}Y^AY^B\right)$. It gives the real solution

$$C = \frac{M}{r}\exp\left(\frac{1}{2}\kappa^{-1}_{\alpha\alpha}y^\alpha y^\alpha + \frac{1}{2}\bar{\kappa}^{-1}_{\dot{\alpha}\dot{\alpha}}\bar{y}^{\dot{\alpha}}\bar{y}^{\dot{\alpha}} - \kappa^{-1}_{\alpha\gamma}v^\gamma{}_{\dot{\alpha}}y^\alpha\bar{y}^{\dot{\alpha}}\right) \to \mathbf{C}_{\alpha(2n)} = \frac{M}{r}(\kappa^{-1}_{\alpha\alpha})^{\mathbf{n}}\,,$$

$$\bar{\mathbf{C}}_{\dot{\alpha}(2n)} = \frac{M}{r}(\bar{\kappa}^{-1}_{\dot{\alpha}\dot{\alpha}})^{\mathbf{n}}\,, \qquad \phi_{\mu_1\ldots\mu_n} = \frac{M}{r}\mathbf{k}_{\mu_1}\ldots\mathbf{k}_{\mu_n} \qquad \to \textbf{BH Fronsdal fields}$$

Solving nonlinear HS equations

Main idea: The function $F_k = \exp\left(\frac{1}{2}K_{AB}Y^AY^B\right)$ generates invariant subspace in the star-product algebra and provides suitable ansatz for solving nonlinear HS equations

Properties of F_k

1. $F_k \star F_k = F_k \quad \leftarrow$ **projector**
2. $\mathcal{D}_0 F_k = 0 \quad \leftarrow$ **by definition**
3. **Generates subalgebra of the form** $F_k\phi(a|x)$, where $a_A = Z_A + K_A{}^B Y_B$

$$(F_k\phi_1(a|x)) \star (F_k\phi_2(a|z)) = F_k(\phi_1(a|x) * \phi_2(a|x))$$

$*$ - is Fock induced associative star-product operation on the space of a_A - oscillators

4. $\mathcal{D}_0(F_k\phi(a|x)) = F_k(\hat{d} - \frac{1}{2}dK^{AB}\frac{\partial^2}{\partial a^A \partial a^B})\phi(a|x)\,, \qquad \hat{d}a_A = 0$

* - properties

1. **associativity** $\rightarrow$ $(\phi_1 * \phi_2) * \phi_3 = \phi_1 * (\phi_2 * \phi_3)$

$$[a_A, a_B]_* = 2\epsilon_{AB}$$

2. **Admits Klein operators of the form**

$$K = \frac{1}{r}\exp\left(\frac{1}{2}\kappa^{-1}_{\alpha\alpha}a^{\alpha}a^{\alpha}\right), \qquad \bar{K} = \frac{1}{r}\exp\left(\frac{1}{2}\bar{\kappa}^{-1}_{\dot{\alpha}\dot{\alpha}}\bar{a}^{\dot{\alpha}}\bar{a}^{\dot{\alpha}}\right)$$

$$K * K = \bar{K} * \bar{K} = 1\,, \quad \{K, a_{\alpha}\}_* = \{\bar{K}, \bar{a}_{\dot{\alpha}}\}_* = 0$$

3. **Differential** $\rightarrow$ $\mathcal{Q} = \hat{d} - \frac{1}{2}dK^{AB}\frac{\partial^2}{\partial a^A \partial a^B}$

$$\mathcal{Q}(f(a|x) * g(a|x)) = \mathcal{Q}f(a|x) * g(a|x) + f(a|x) * \mathcal{Q}g(a|x)\,,$$

$$\mathcal{Q}^2 = 0\,, \qquad \mathcal{Q}a_A = 0\,, \qquad \mathcal{Q}K = 0$$

The Ansatz

$$B = MF_k \star \delta(y)\,,$$

$$S_\alpha = z_\alpha + F_k \sigma_\alpha(a|x)\,, \qquad \bar{S}_{\dot\alpha} = \bar{z}_{\dot\alpha} + F_k \bar{\sigma}_{\dot\alpha}(\bar{a}|x)\,,$$

$$W = w_0(y,\bar{y}|x) + F_k(\omega(a|x) + \bar{\omega}(\bar{a}|x))\,,$$ w_0 **is the** AdS_4 **connection**

HS equations reduce to "$3d$ massive equations" :

$$[s_\alpha, s_\beta]_* = 2\epsilon_{\alpha\beta}(1 + M \cdot K)\,,$$

$$\mathcal{Q}s_\alpha - [\omega, s_\alpha]_* = 0\,,$$

$$\mathcal{Q}\omega - \omega * \wedge\omega = 0\,,$$

where $s_\alpha \equiv a_\alpha + \sigma_\alpha(a|x)$ - the so called deformed oscillators (Wigner).

Note: $3d$ **HS equations around the vacuum** $B_0 = \nu = const$ **were considered by Prokushkin and Vasiliev and were shown to provide massive field dynamics with the mass scale** ν

Final solution

$$\sigma_\alpha = M\frac{a^+_\alpha}{r}\int_0^1 \mathrm{d}t\,\exp\,(\frac{t}{2}\kappa^{-1}_{\beta\beta}a^\beta a^\beta)\,, \qquad a^+_\alpha = \pi_\alpha^{+\beta}a_\beta \qquad \Rightarrow \qquad [\sigma_\alpha, \sigma_\beta]_* = \mathbf{0}$$

$$\omega = \Omega^{\alpha\alpha}(s_\alpha * s_\alpha - a_\alpha * a_\alpha) + f_0 + c.c. \qquad \rightarrow \qquad \boldsymbol{\Omega}^{\alpha\alpha}\sigma_\alpha * \sigma_\alpha = \mathbf{0}\,,$$

where

$\Omega_{\alpha\alpha} = \frac{1}{8}d\tau_\alpha{}^\gamma\tau_{\gamma\alpha}\,, \qquad \tau_{\alpha\alpha} = \frac{\kappa_{\alpha\alpha}}{r}\,. \qquad (\Omega_{\alpha\alpha}, \bar{\Omega}_{\dot\alpha\dot\alpha})$ - $sp(2) \oplus sp(2)$ flat connection

and

$$\mathbf{df_0} = \frac{M}{16}\mathbf{d}\tau^{\alpha\gamma}\wedge\mathbf{d}\tau_\gamma{}^\alpha\tau_{\alpha\alpha}$$

Straightforward star-product calculation leads to

$$S_\alpha = z_\alpha + MF_k \frac{a^+_\alpha}{r} \int_0^1 \mathrm{dt}\, \exp\,(\frac{t}{2}\kappa^{-1}_{\beta\beta} a^\beta a^\beta)\,,$$

$$\bar{S}_{\dot\alpha} = \bar{z}_{\dot\alpha} + MF_k \frac{\bar{a}^+_{\dot\alpha}}{r} \int_0^1 \mathrm{dt}\, \exp\,(\frac{t}{2}\bar\kappa^{-1}_{\dot\beta\dot\beta} \bar{a}^{\dot\beta} \bar{a}^{\dot\beta})\,,$$

$$B = \frac{M}{r} \exp\,(\frac{1}{2}\kappa^{-1}_{\alpha\beta} y^\alpha y^\beta + \frac{1}{2}\bar\kappa^{-1}_{\dot\alpha\dot\beta} \bar{y}^{\dot\alpha}\bar{y}^{\dot\beta} - \kappa^{-1}_{\alpha\gamma} v^\gamma{}_{\dot\alpha} y^\alpha \bar{y}^{\dot\alpha})\,,$$

$$W = W_0 + (\frac{M}{8r} F_k d\tau^{\alpha\beta} \pi^{+\alpha}_{\beta} a_\alpha a_\alpha \int_0^1 dt(1-t)\exp\,(\frac{t}{2}\kappa^{-1}_{\beta\beta} a^\beta a^\beta) + F_k f_0 + c.c.)\,,$$

The solution is of first order in M corresponding to a very specific gauge fixing in HS equations

Conclusion

- The new exact solution of $4d$ bosonic HS theory is presented. It is constructed to correspond to Schwarzschild black hole in spin two sector and to natural generalization for the rest of Fronsdal fields at free level and is called HS black hole. The HS corrections cancel each other (in a specific gauge) for this solution reducing HS nonlinear equations to free ones.

- The crucial element of the proposed construction is a Fock vacuum in the star–product algebra which is available for Schwarzschild black hole. It allows us to project $4d$ equations to $3d$ ones upon inducing the inner star–product operation. This reduction suggests an interesting duality between AdS_3 massive fields of a mass scale ν and $4d$ HS black hole of mass M

$$\nu = \lambda G M$$

- The construction admits natural generalization for the Kerr and supersymmetric cases as well as for higher dimensions (black rings??)