Strings, Branes and Gauge Theories

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Introduction

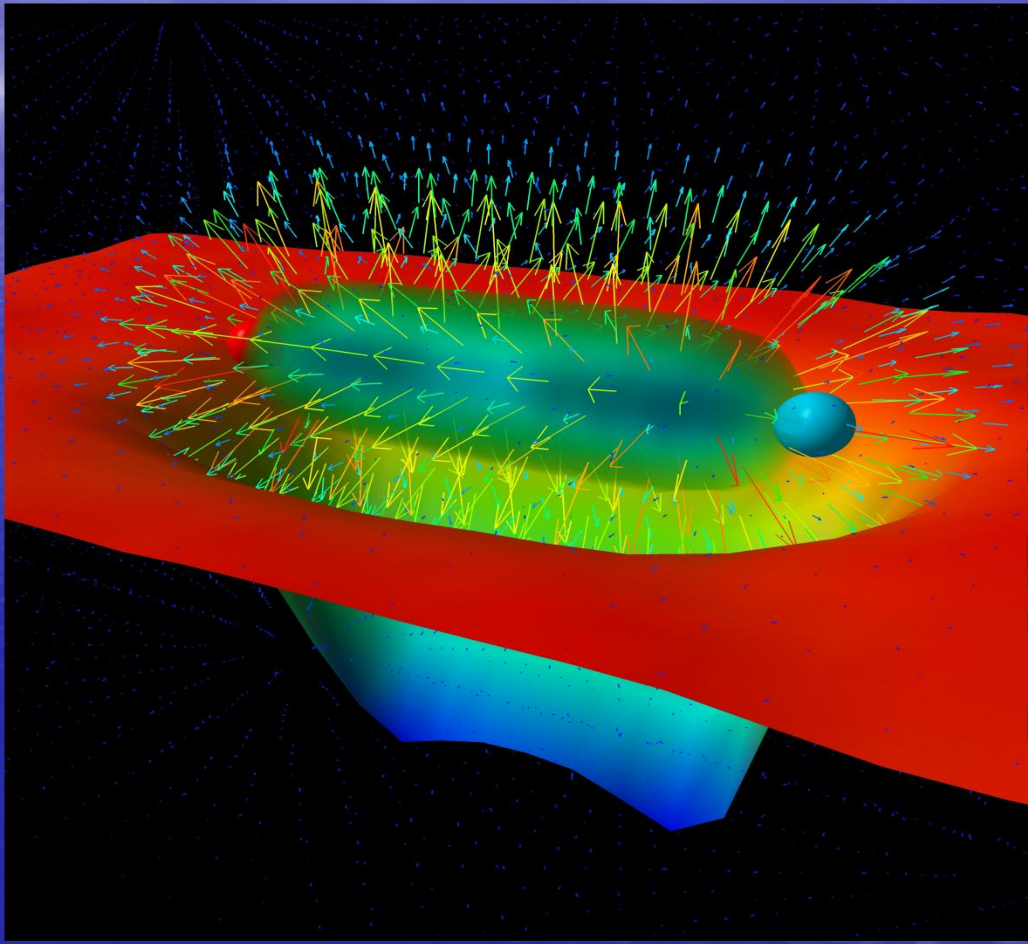
- String theory was invented to describe strong interactions, but in 1973-74 two important developments changed its course.
- Due to discovery of the Asymptotic Freedom, QCD emerged as the field theory for the strong interactions.

  Gross & Wilczek; Politzer

- Yoneya, and Scherk and Schwarz showed that closed string theory describes quantum gravity.
- Some 25 years later these two seemingly different faces of string theory, the gauge theoretic and the gravitational, have merged in the context of the AdS/CFT correspondence and its extensions. In this talk I review both the basic concepts and some of the recent progress in this field.

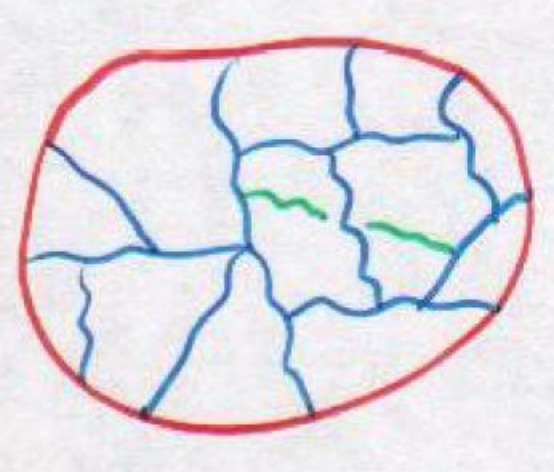
QCD and String Theory

- At short distances, much smaller than 1 fermi, the quark-antiquark potential is approximately Coulombic, due to the Asymptotic Freedom.
- At large distances the potential should be linear (Wilson) due to formation of confining flux tubes.

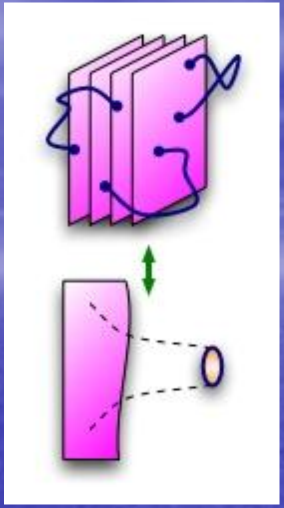


Large N Gauge Theories

- Connection of gauge theory with string theory is strengthened in `t Hooft's generalization from 3 colors (SU(3) gauge group) to N colors (SU(N) gauge group).
- Make N large, while keeping the `t Hooft coupling $\lambda = g^2_{\rm YM} N$ fixed.
- The probability of snapping a flux tube by quark-antiquark creation (meson decay) is 1/N. The string coupling is 1/N.
- Yet, the planar diagrams needed in the large N limit seem very difficult to sum explicitly.



D-Branes vs. Geometry



- Dirichlet branes (Polchinski) led string theory back to gauge theory in the mid-90's.
- A stack of N Dirichlet 3-branes realizes $\mathcal{N}$=4 supersymmetric SU(N) gauge theory in 4 dimensions. It also creates a curved background of 10-d theory of closed superstrings (artwork by E.Imeroni)

$$ds^2 = \left(1 + \frac{L^4}{r^4}\right)^{-1/2} \left(-(dx^0)^2 + (dx^i)^2\right) + \left(1 + \frac{L^4}{r^4}\right)^{1/2} \left(dr^2 + r^2 d\Omega_5^2\right)$$

which for small r approaches $AdS_5 \times \mathbf{S}^5$

who radius is related to the coupling by $L^4 = g_{\rm YM}^2 N \alpha'^2$

Super-Conformal Invariance

- In the $\mathcal{N}$=4 SYM theory there are 6 scalar fields (it is useful to combine them into 3 complex scalars: Z, W, V) and 4 gluinos interacting with the gluons. All the fields are in the adjoint representation of the SU(N) gauge group.
- The Asymptotic Freedom is canceled by the extra fields; the beta function is exactly zero for any complex coupling. The theory is invariant under scale transformations $x^{\mu} \to a\, x^{\mu}$. It is also invariant under space-time inversions. The full super-conformal group is SU(2,2|4).

The AdS/CFT Duality

Maldacena; Gubser, IK, Polyakov; Witten

- Relates conformal gauge theory in 4 dimensions to string theory on 5-d Anti-de Sitter space times a 5-d compact space. For the $\mathcal{N}$=4 SYM theory this compact space is a 5-d sphere.
- The geometrical symmetry of the AdS_5 space realizes the conformal symmetry of the gauge theory.
- The AdS_d space is a hyperboloid

$$(X^0)^2 + (X^d)^2 - \sum_{i=1}^{d-1} (X^i)^2 = L^2 \, .$$

- Its metric is

$$ds^2 = \frac{L^2}{z^2} \left(dz^2 - (dx^0)^2 + \sum_{i=1}^{d-2} (dx^i)^2 \right)$$

- When a gauge theory is strongly coupled, the radius of curvature of the dual AdS_5 and of the 5-d compact space becomes large: $\frac{L^2}{\alpha'} \sim \sqrt{g^2_{\rm YM} N}$

- String theory in such a weakly curved background can be studied in the effective (super)-gravity approximation, which allows for a host of explicit calculations. Corrections to it proceed in powers of $\frac{\alpha'}{L^2} \sim \lambda^{-1/2}$

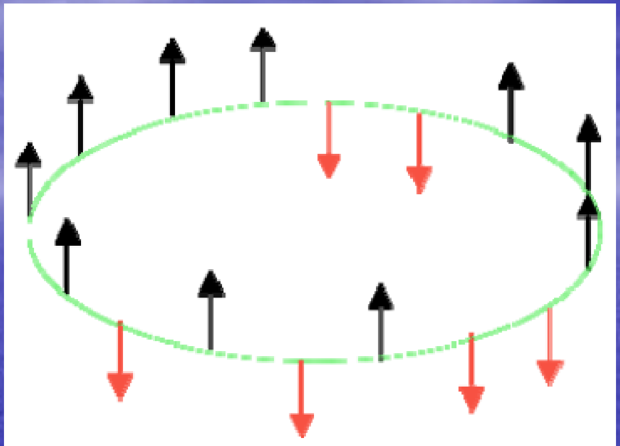
- Feynman graphs instead develop a weak coupling expansion in powers of λ. At weak coupling the dual string theory becomes difficult.

- For an introduction to gauge/gravity duality, see the article by I.K. and J. Maldacena in the January 2009 issue of Physics Today.
- Gauge invariant operators in the CFT_4 are in one-to-one correspondence with fields (or extended objects) in AdS_5
- Operator dimensions are an important set of quantitites $$\langle \mathcal{O}_{\Delta_1}(x_1)\mathcal{O}_{\Delta_2}(x_2)\rangle = \frac{\delta_{\Delta_1,\Delta_2}}{|x_1 - x_2|^{2\Delta_1}}$$
- The operator dimension is related to mass of the corresponding field in AdS space:

$$\Delta_\pm = 2 \pm \sqrt{4 + m^2 L^2}$$

Exact Integrability

- Perturbative calculations of operator dimensions are mapped to integrable spin chains, suggesting exact integrability of the $\mathcal{N}$=4 SYM theory. Minahan, Zarembo; Beisert, Staudacher



- For operators Tr (ZZZWZW…ZW) the Heisenberg spin chain emerges at 1 loop. Higher loops correct the Hamiltonian but seem to preserve its integrability (infinite number of conserved charges). Such spin chains are solvable via Bethe Ansatz.

$$H_2 = \frac{\lambda}{8\pi^2} \sum_{j=1}^{L} (1 - P_{j,j+1})$$

- This meshes nicely with earlier findings of integrability in certain subsectors of QCD. Lipatov; Faddeev, Korchemsky; Braun, Derkachov, Manashov; Belitsky

- The integrability also holds on the string side of the duality, i.e. for large values of the `t Hooft coupling. Bena, Polchinski, Roiban; …
- This suggests that it is present for all values of the coupling.
- This plausible assumption has led to some precision tests of the AdS/CFT correspondence and will hopefully lead to more (see other talks at this conference).

Thermal gauge theory

- Thermal CFT is described by a black hole in AdS_5

$$ds^2_{BH} = \frac{L^2}{z^2}\left(\frac{dz^2}{1 - z^4/z_h^4} - (1 - z^4/z_h^4)(dx^0)^2 + \sum_{i=1}^{3}(dx^i)^2\right)$$

- The CFT temperature is identified with the Hawking T of the horizon located at z_h
- Any event horizon contains Bekenstein-Hawking entropy $S_{BH} = \frac{2\pi A_h}{\kappa^2}$
- A brief calculation gives the entropy density $s = \frac{\pi^2}{2} N^2 T^3$

Gubser, IK, Peet

- This is interpreted as the strong coupling limit of

$$s = \frac{2\pi^2}{3} f(\lambda) N^2 T^3$$

- The weak `t Hooft coupling behavior of the interpolating function is determined by Feynman graph calculations in the $\mathcal{N}$=4 SYM theory

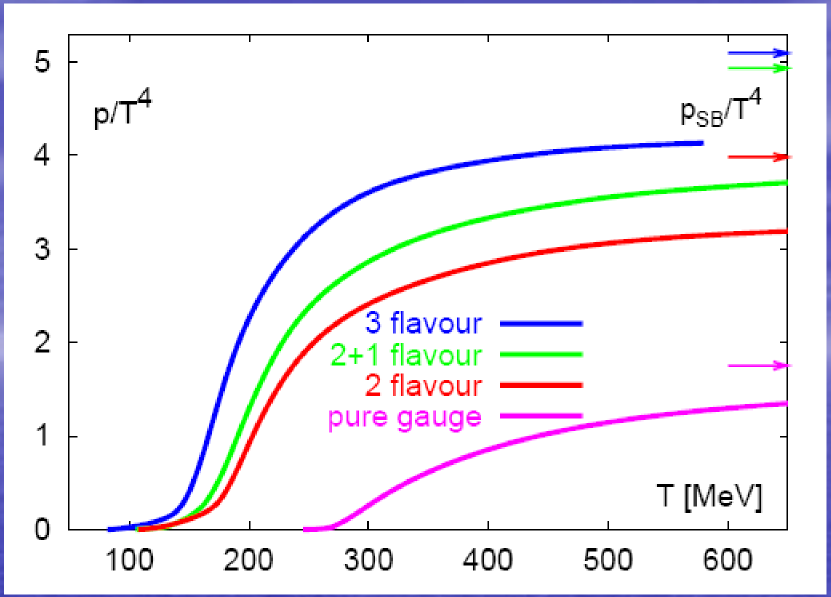
$$f(\lambda) = 1 - \frac{3}{2\pi^2}\lambda + \frac{3+\sqrt{2}}{\pi^3}\lambda^{3/2} + \ldots$$

- We deduce from AdS/CFT duality that

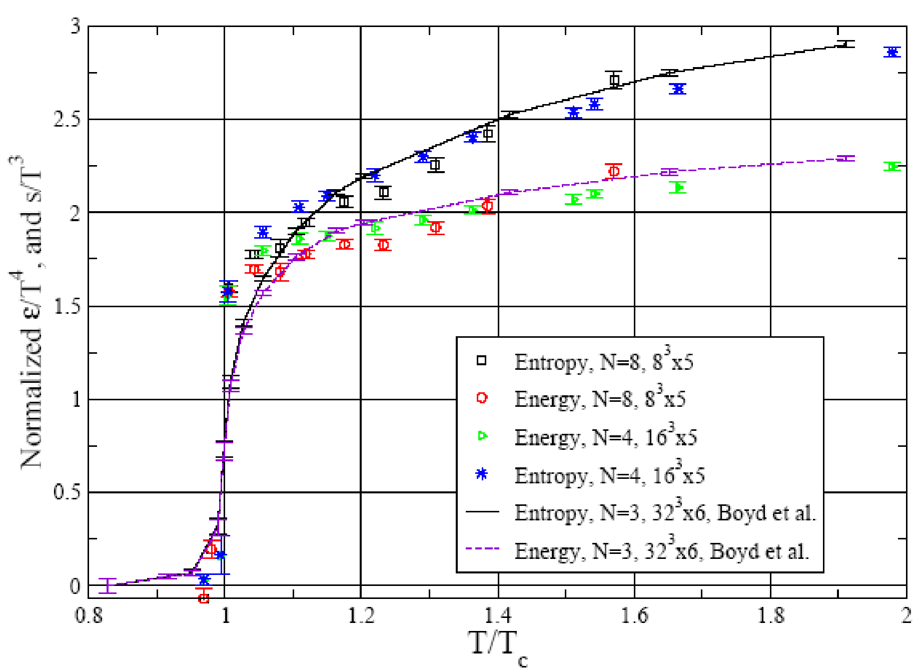
$$\lim_{\lambda\to\infty} f(\lambda) = \frac{3}{4}$$

- The entropy density is multiplied only by ¾ as the coupling changes from zero to infinity. Gubser, IK, Tseytlin

- A similar reduction of entropy by strong-coupling effects is observed in lattice non-supersymmetric gauge theories for N=3: the arrows show free field values.
  Karsch (hep-lat/0106019).



- N-dependence in the pure glue theory enters largely through the overall normalization.
  Bringoltz and Teper (hep-lat/0506034)



Shear Viscosity η of the Plasma

- In a comoving frame,

$$T_{ij} = \delta_{ij} p - \eta \left(\partial_i u_j + \partial_j u_i - \frac{2}{3} \delta_{ij} \partial_k u_k \right)$$

- Can be also determined through the Kubo formula

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d\mathbf{x}\, e^{i\omega t} \langle [T_{xy}(t, \mathbf{x}), T_{xy}(0,0)] \rangle$$

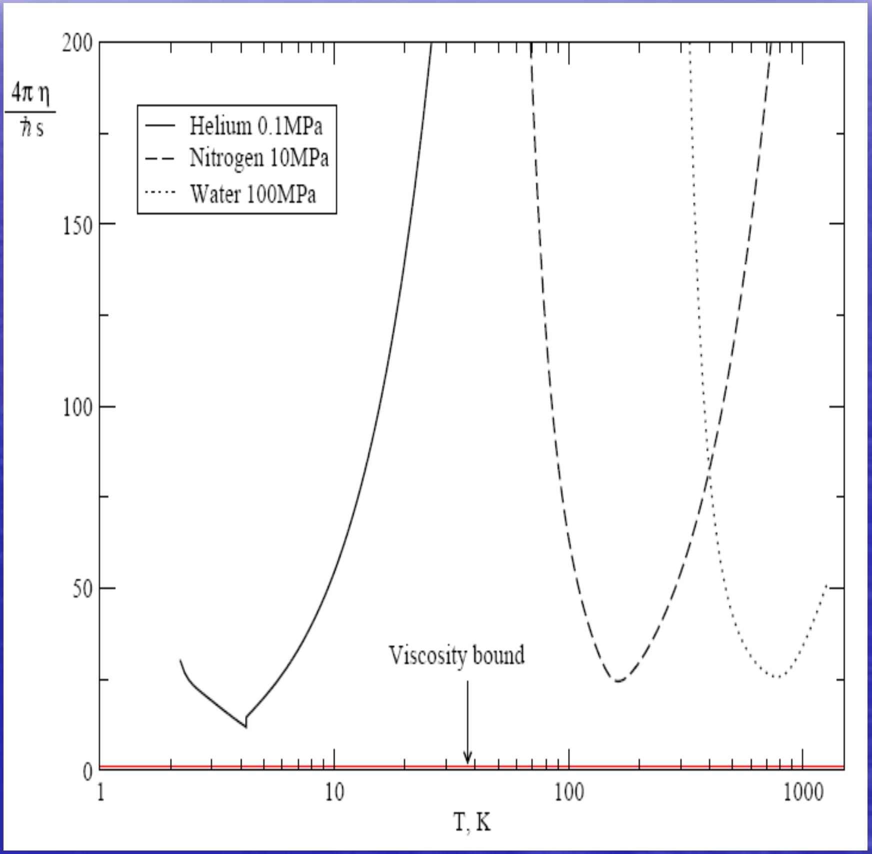
- For the $\mathcal{N}$=4 supersymmetric YM theory this 2-point function may be computed from graviton absorption by the 3-brane metric.
- At very strong coupling, Policastro, Son and Starinets found

$$\eta = \frac{\pi}{8} N^2 T^3$$

$$\frac{\eta}{s} = \frac{\hbar}{4\pi}$$

The RHIC Connection

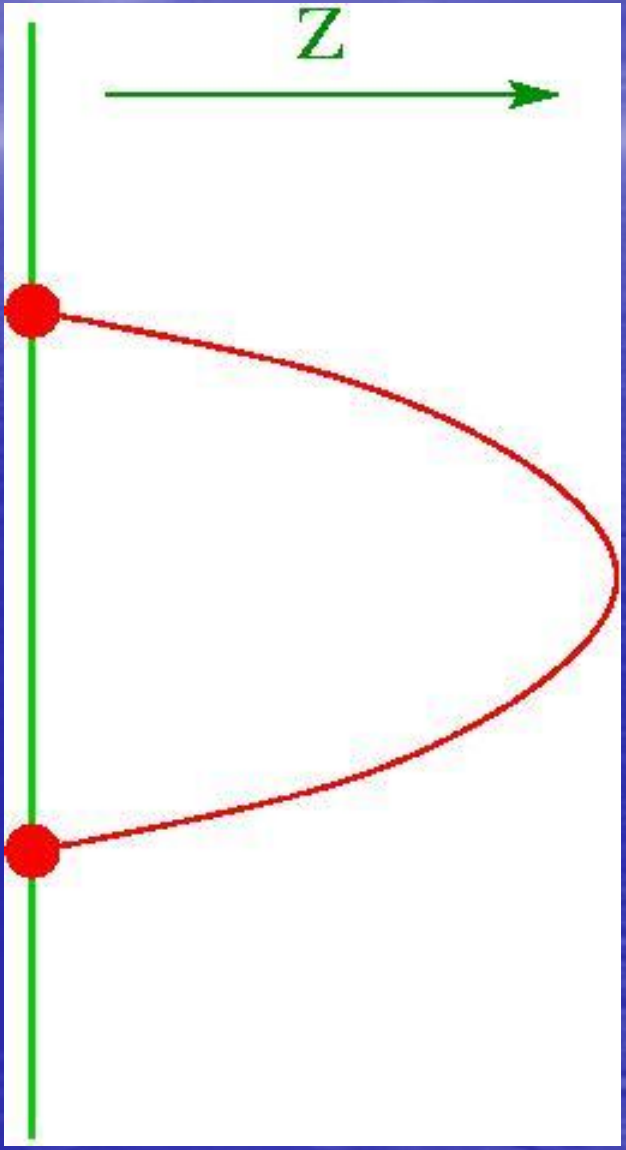
- For known fluids (e.g. helium, nitrogen, water) η/s is considerably higher.
- The quark-gluon plasma produced at RHIC is believed to be strongly coupled and to have low viscosity. Shuryak, Teaney, Gyulassy, McLerran, Hirano, …
- A new term has been coined, sQGP, to describe the state observed at RHIC. A lot of recent string theory research is devoted to developing intuition about this state.



The quark anti-quark potential

- The z-direction of AdS is dual to the energy scale of the gauge theory: small z is the UV; large z is the IR.
- Because of the 5-th dimension z, the string picture applies even to theories that are conformal. The quark and anti-quark are placed at the boundary of Anti-de Sitter space (z=0), but the string connecting them bends into the interior (z>0). Due to the scaling symmetry of the AdS space, this gives Coulomb potential (Maldacena; Rey, Yee)

$$V(r) = -\frac{4\pi^2\sqrt{\lambda}}{\Gamma\left(\frac{1}{4}\right)^4 r}$$



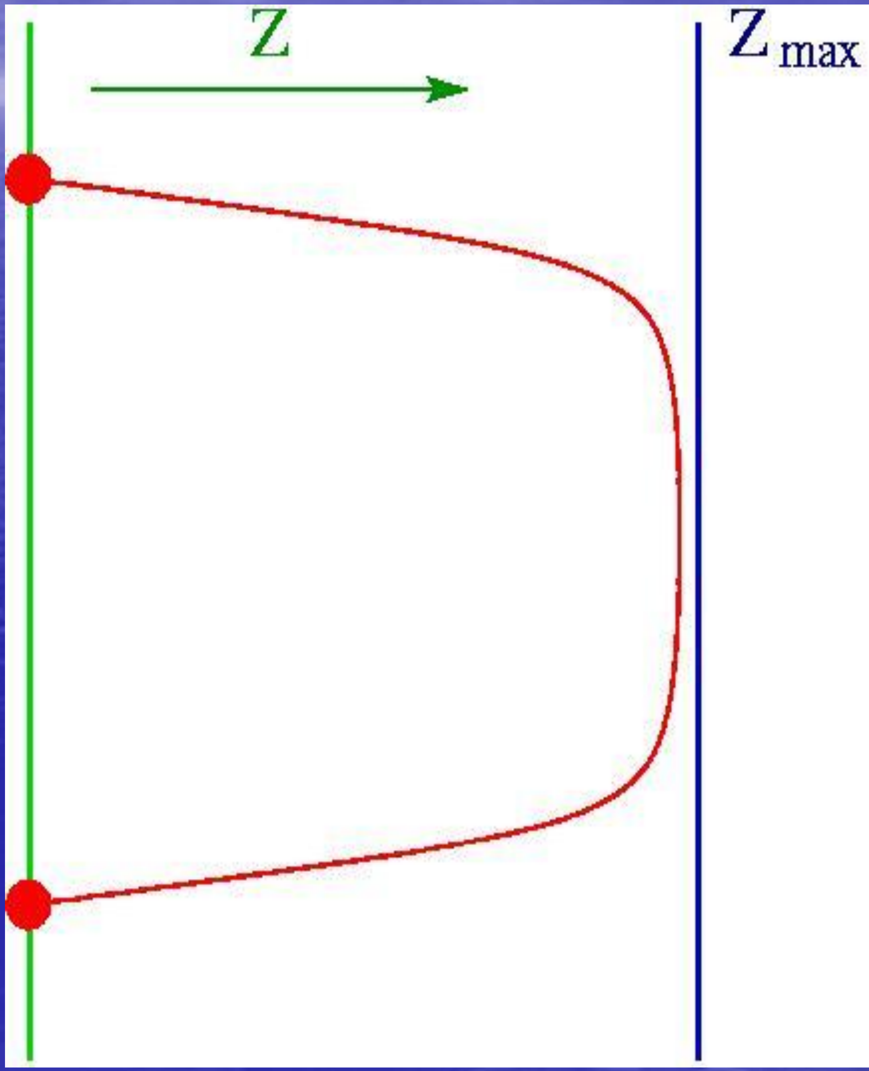
Towards Color Confinement

- It is possible to generalize the AdS/CFT correspondence in such a way that the quark-antiquark potential is linear at large distance but nearly Coulombic at small distance.
- The 5-d metric should have a warped form (Polyakov):

$$ds^2 = \frac{dz^2}{z^2} + a^2(z)\left(-(dx^0)^2 + (dx^i)^2\right)$$

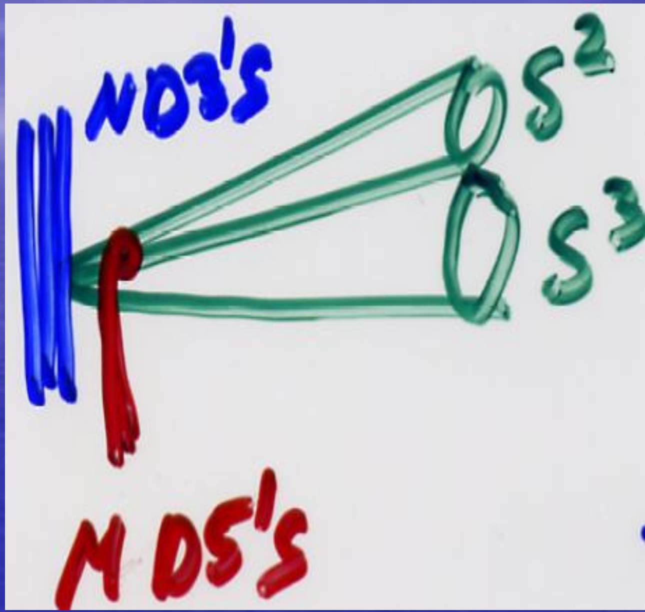
- The space ends at a maximum value of z where the warp factor is finite. Then the confining string tension is

$$\frac{a^2(z_{\rm max})}{2\pi\alpha'}$$



Confinement and χSB

- To break conformal invariance, change the gauge theory: add to the N D3-branes M D5-branes wrapped over the S^2 at the tip of the conifold.



- The 10-d geometry dual to the gauge theory on these branes is the warped deformed conifold (IK, Strassler)

$$ds^2_{10} = h^{-1/2}(t)\big(-(dx^0)^2 + (dx^i)^2\big) + h^{1/2}(t)ds^2_6$$

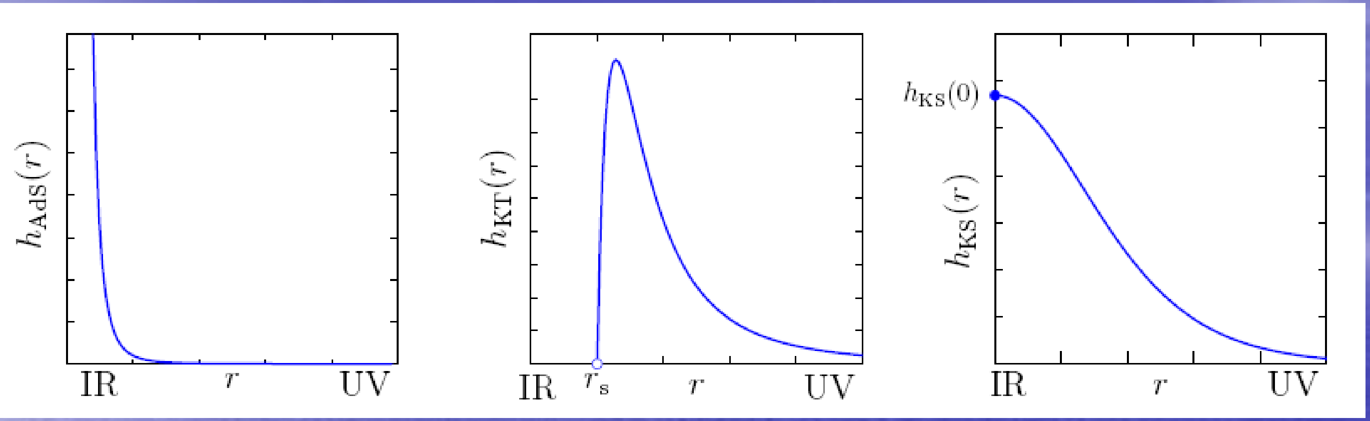
- ds^2_6 is the metric of the deformed conifold, a simple Calabi-Yau space defined by the following constraint on 4 complex variables:

$$\sum_{i=1}^{4} z_i^2 = \varepsilon^2$$

- The warp factor is finite at the `tip of the cigar' t=0, as required for the confinement: $h(t)= 2^{-8/3} \gamma I(t)$

$$I(t) = \int_t^\infty dx \frac{x \coth x - 1}{\sinh^2 x} (\sinh 2x - 2x)^{1/3} , \qquad \gamma = 2^{10/3} (g_s M \alpha')^2 \varepsilon^{-8/3}$$

- The string tension, is proportional to $h(t)^{-1/2}$ and is minimized at t=0. It blows up at large t (near the boundary) where space is `near-AdS.'
- Dimensional transmutation in the IR. The dynamically generated confinement scale is $\sim \varepsilon^{2/3}$
- The pattern of R-symmetry breaking is the same as in the SU(M) SYM theory: $Z_{2M} \to Z_2$



- Comparison of warp factors in the AdS, warped conifold, and warped deformed conifold cases. The warped conifold solution with ϵ =0 has an unacceptable naked singularity where h=0.
- This is how string theory tells us that the chiral symmetry breaking and dynamical scale generation must take place through turning on the deformation ϵ. The finiteness of the warp factor at r=0 translates into confinement.

- The graph of quark anti-quark potential is qualitatively similar to that found in numerical simulations of QCD. The upper graph, from the recent Senior Thesis of V. Cvicek shows the string theory result for the warped deformed conifold.
- The lower graph shows lattice QCD results by G. Bali et al with $r_0 \sim 0.5$ fm.

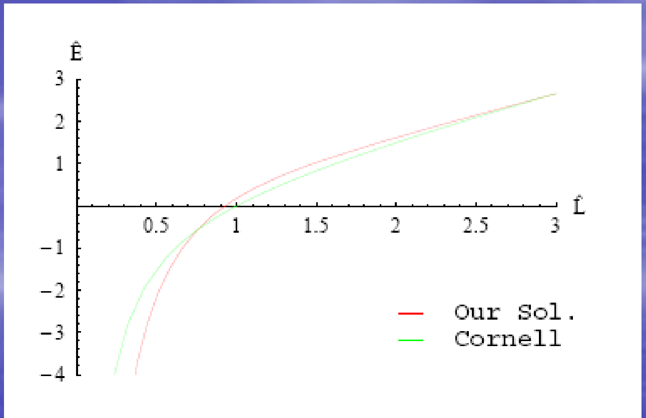
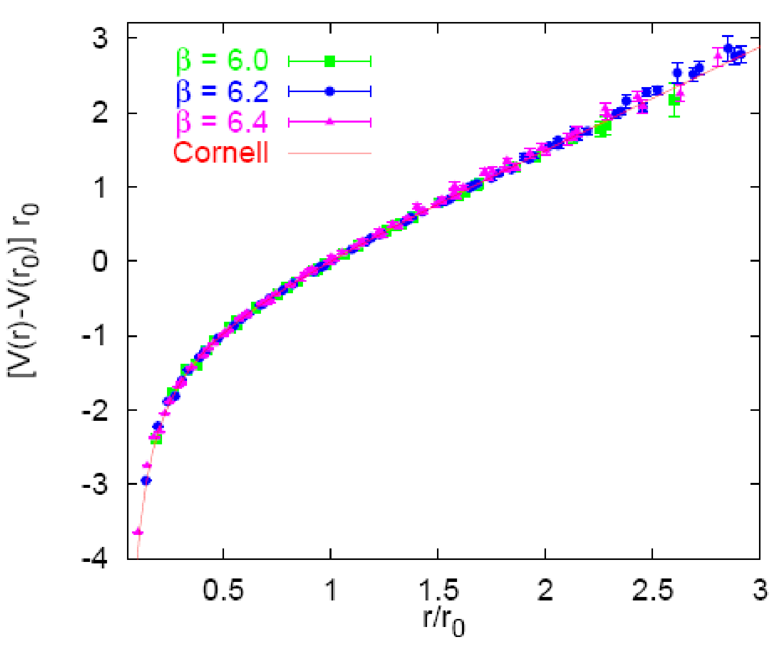


Figure 11: Comparison to the Cornell model



- All of this provides us with an exact solution of a class of 4-d large N confining supersymmetric gauge theories.
- This should be a good playground for studying strongly coupled gauge theory: a `hyperbolic cow' approximation to $\mathcal{N}=1$ supersymmetric gluodynamics.
- Some results on glueball spectra are already available, and further calculations are ongoing. Krasnitz; Caceres, Hernandez; Dymarsky, Melnikov; Berg, Haack, Muck; Benna, Dymarsky, IK, Soloviev
- Possible applications of these models to new physics include RS warped extra dimension models, KKLT moduli stabilization in flux compactifications, as well as warped throat D-brane cosmology (KKLMMT).

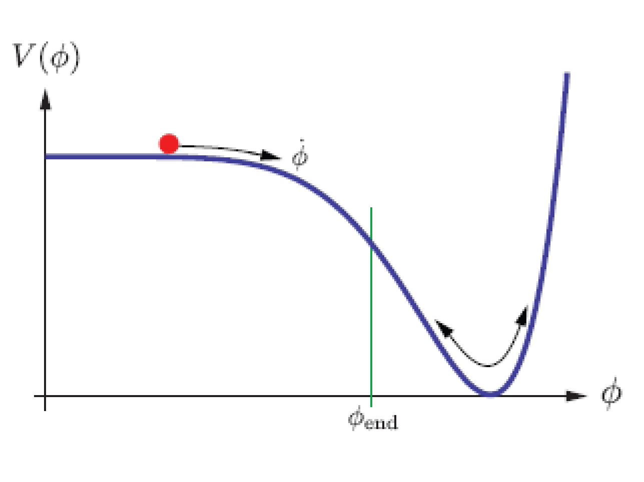
Cosmic Inflation

- A brief period of exponential expansion of the Universe by at least a factor of e^{60}.

$$ds^2 = -dt^2 + e^{2Ht}d\mathbf{x}^2$$

- This probably lasted only 10^{-32} seconds after the Big Bang.
- A plausible explanation of why the Universe is observed to be big and flat

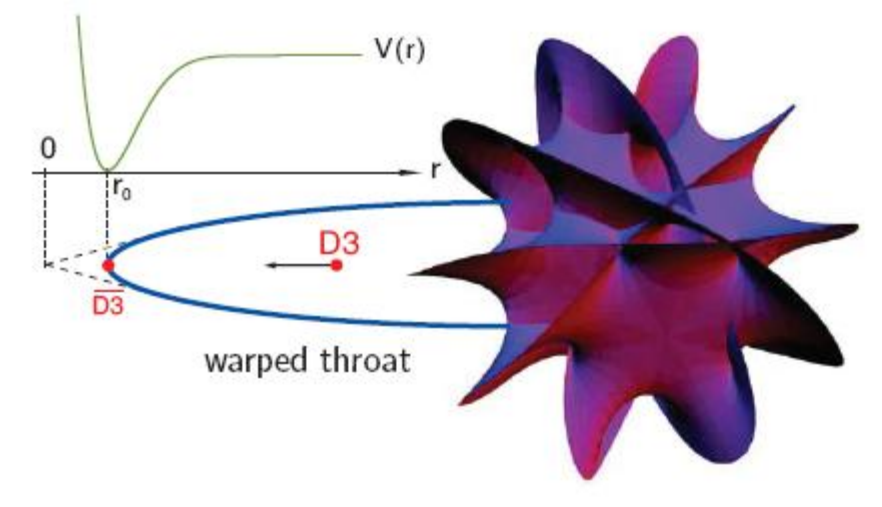
Guth; Linde; Albrecht, Steinhardt; Starobinsky; Mukhanov, Chibisov; … (early ideas by Gliner, …)



$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$$

D-brane Inflation

- Finding models with very flat potentials has proven to be difficult. Recent string theory constructions use moving D-branes. Dvali, Tye, …
- In the KKLT/KKLMMT model, the warped deformed conifold is embedded into a string compactification. An anti-D3-brane is added at the bottom to break SUSY and generate a potential. A D3-brane rolls in the throat. Its radial coordinate plays the role of an inflaton. Kachru, Kallosh, Linde, Maldacena, McAllister, Trivedi



Connection with Cosmic Strings

- A fundamental string at the bottom of the warped deformed conifold is dual to a confining string. A D-string is dual to a certain solitonic string due to spntaneously broken baryonic U(1) symmetry in the throat. There is also a rich spectrum of (p,q) strings.
- Upon embedding of the warped throat into a flux compactification, these objects can be used to model cosmic strings. Copeland, Myers, Polchinski; ...
- Detectable via gravity wave bursts? Damour, Vilenkin
- This throat is not the \`standard model throat' but another, \`inflationary throat,' dual to a hidden sector gauge theory with confining scale $\sim 10^{14}$ GeV.

Perturbing the Throat

- Gluing the throat into a Calabi-Yau manifold modifes the inflaton potential.
  Baumann, Dymarsky, Kachru, IK, McAllister
- There are large r perturbations of the throat geometry, which in the gauge theory correspond to adding irrelevant operators to the gauge theory.

$$\Delta \mathcal{K} = c \int d^4\theta \; M_{\rm UV}^{-\Delta} \; X^\dagger X \, \mathcal{O}_\Delta \qquad \Rightarrow \qquad \Delta V = c \; M_{\rm UV}^{-\Delta} |F_X|^2 \, \mathcal{O}_\Delta$$

- The structure of the full inflaton potential is

$$V(\phi) = V_{D3/\overline{D3}}(\phi) + H^2\phi^2 + \Delta V(\phi)$$

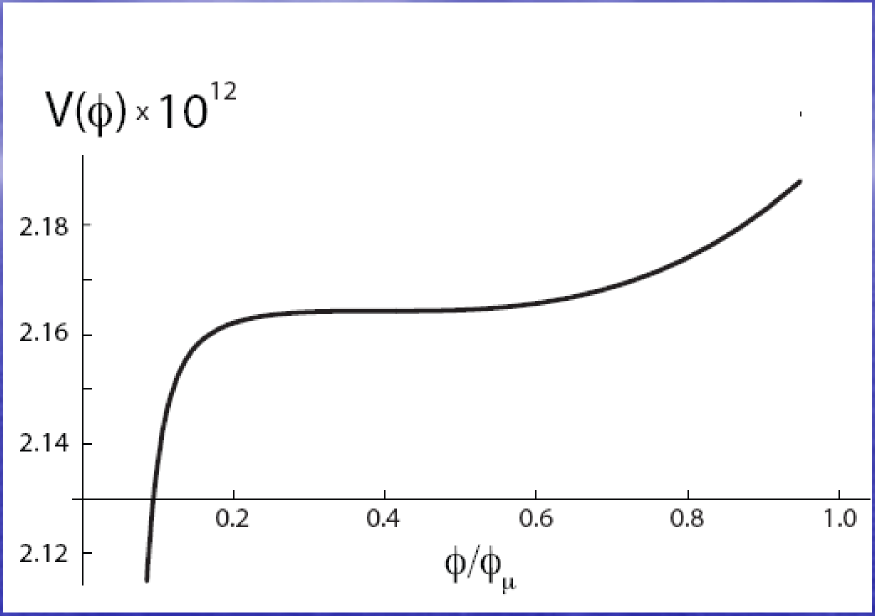
- The power of the perturbation to the potential is determined by the dimension of the dual operator

$$\Delta V = -c\, M_{\mathrm{UV}}^{-\Delta} |F_X|^2 \phi^{\Delta}$$

- In the warped conifold throat, the lowest dimension is 3/2. Balancing the two terms can lead to an inflection point.

$$\phi \equiv r\sqrt{\tfrac{3}{2}T_3}$$

- The effective potential for the inflaton generically has a local maximum and minium. It can be fine-tuned to have an inflection point.
- Motion near the inflection point can produce enough e-folds of inflation. Baumann, Dymarsky, IK, McAllister, Steinhardt



- Models of Inflection Point Inflation were also considered in string theory by Itzhaki and Kovetz; Linde and Westphal,

  and in MSSM inflation by Allahverdi, Enqvist, Garcia-Bellido and Mazumdar; …

M2 Branes and Chern-Simons

- AdS/CFT also leads to `solving' certain strongly interacting theories in 2+1 dimensions. These theories arise on coincident membranes in 11-dimensional M-theory. Due to recent progress we understand that they are certain supersymmetric versions of Chern-Simons gauge theory with group U(N) x U(N)

$$\frac{1}{2f}\epsilon^{\mu\nu\lambda}(A_\mu\partial_\nu A_\lambda + \frac{2i}{3}A_\mu A_\nu A_\lambda) - \frac{1}{2f}\epsilon^{\mu\nu\lambda}(\hat{A}_\mu\partial_\nu \hat{A}_\lambda + \frac{2i}{3}\hat{A}_\mu \hat{A}_\nu \hat{A}_\lambda)\Big]$$

Bagger, Lambert; Gustavsson; Van Raamsdonk; Aharony, Bergman, Jafferis, Maldacena

- When N is large, the dual description of the gauge theory is provided by the weakly curved $AdS_4 \times S^7$ background in 11-dimensional M-theory which is essentially described by Einstein gravity coupled to other fields.
- This leads to many non-trivial predictions, e.g. the number of degrees of freedom scales as $N^{3/2}$ I.K., A. Tseytlin
- Such models may provide approximations for strongly interacting many-body systems on a plane.

Conclusions

- Throughout its history, string theory has been intertwined with the theory of strong interactions.
- The AdS/CFT correspondence makes this connection precise. It makes many dynamical statements about strongly coupled conformal gauge theories.
- Extensions of AdS/CFT provide a new geometrical understanding of confinement, chiral symmetry breaking and other strong coupling phenomena.
- Physical applications of the new methods range from heavy ion collisions to cosmological inflation.
- Superconformal Chern-Simons gauge theories in 2+1 dimensions are dual to 4-d AdS backgrounds in M-theory.