

# *Interference of Bogoliubov quasiparticles in the antinodal region of superconducting cuprates*

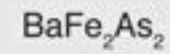
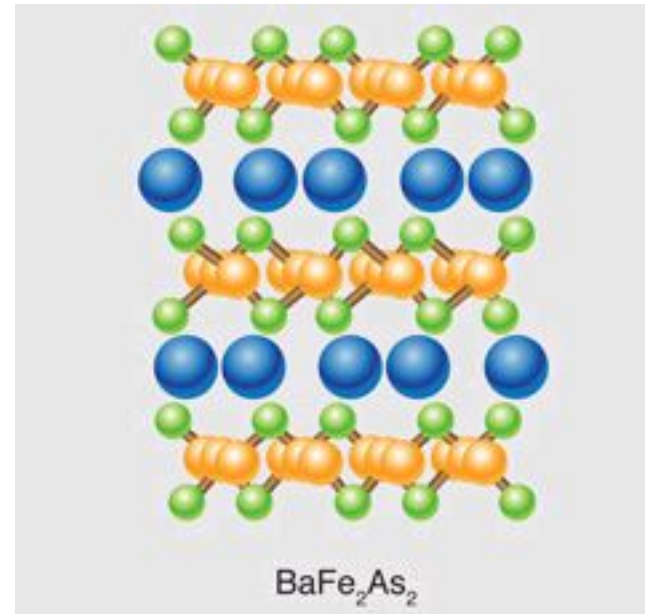
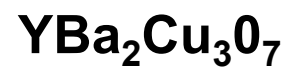
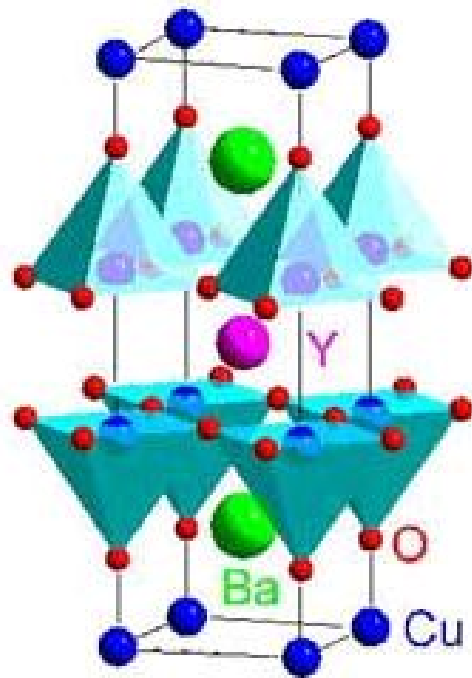
*Yu. V. Kopaeu*

*Solid state physics department*

*P.N. Lebedev Physical Institute of Russian Academy of Sciences*

- 1. Phase diagram of the cuprates*
- 2. Superconducting pairing with large total momentum  $K$ .  
Mirror nesting, kinematical constraint, oscillation of Coulomb potential.*
- 3. Coexisting of the Cooper and  $K$ -pairing channels.*
- 4. Topology of superconducting order parameter.*
- 5. FT-STM and AC-ARPES.*
- 6. Interference of Bogoliubov quasiparticles in the nodal region.*
- 7. Interference of Bogoliubov quasiparticles in the antinodal region.*
- 8. Stripe and checkerboard structures.*

## Crystal structure of cuprates and Fe-pnictides



# Phase diagram of high- $T_c$ cuprates

*Competing ordered states: proximity of the structure and energy of the ground state*

*Spin antiferromagnet (AF)*

*Weak pseudogap (wPG)*

*Orbital antiferromagnet (DDW – charge current density wave)*

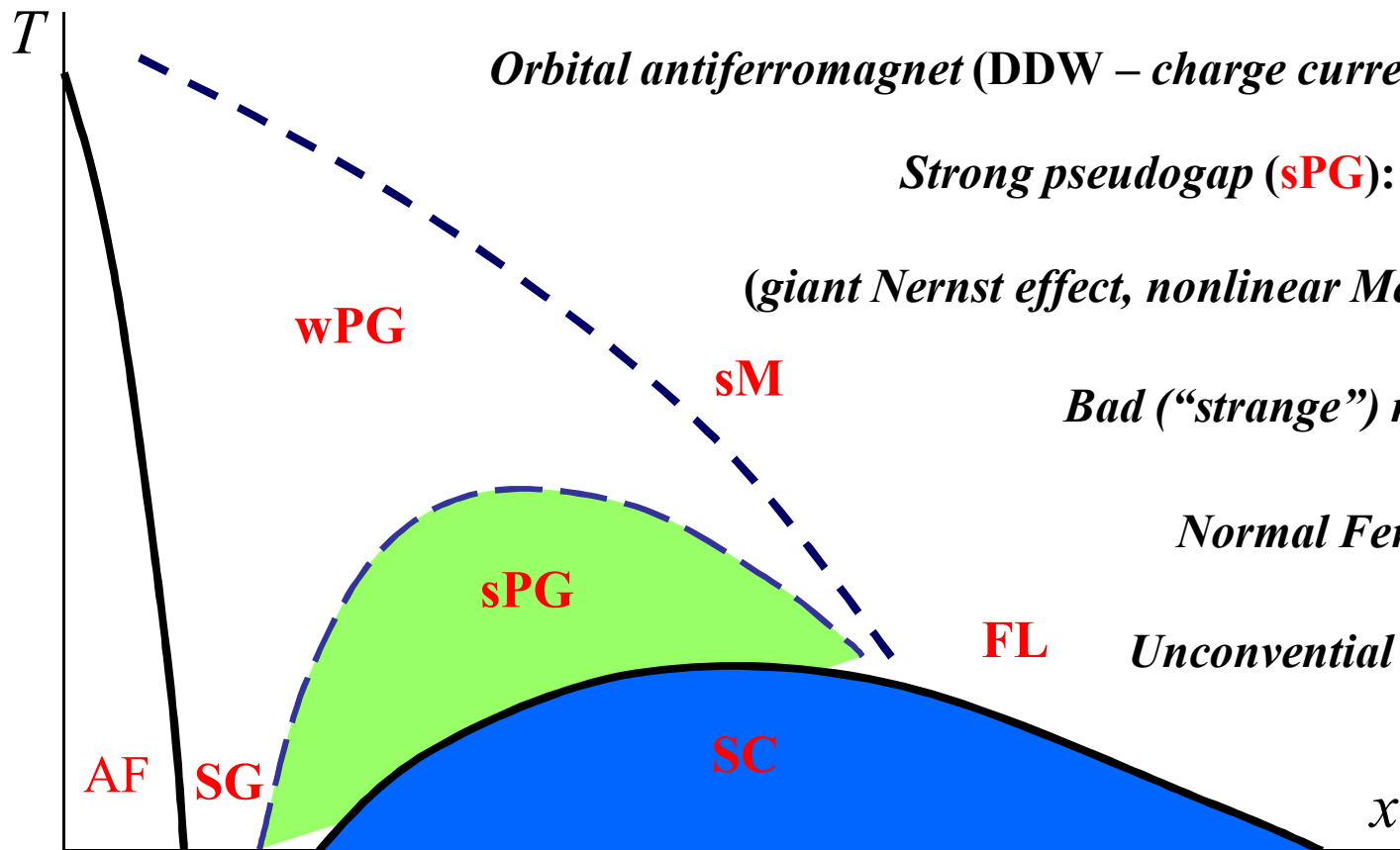
*Strong pseudogap (sPG):*

*(giant Nernst effect, nonlinear Meissner effect)*

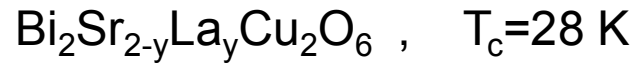
*Bad (“strange”) metal (sM)*

*Normal Fermi liquid (FL)*

*Unconventional superconductor (SC)*



# Anomalous diamagnetism and giant Nernst effect of the strong pseudogap state

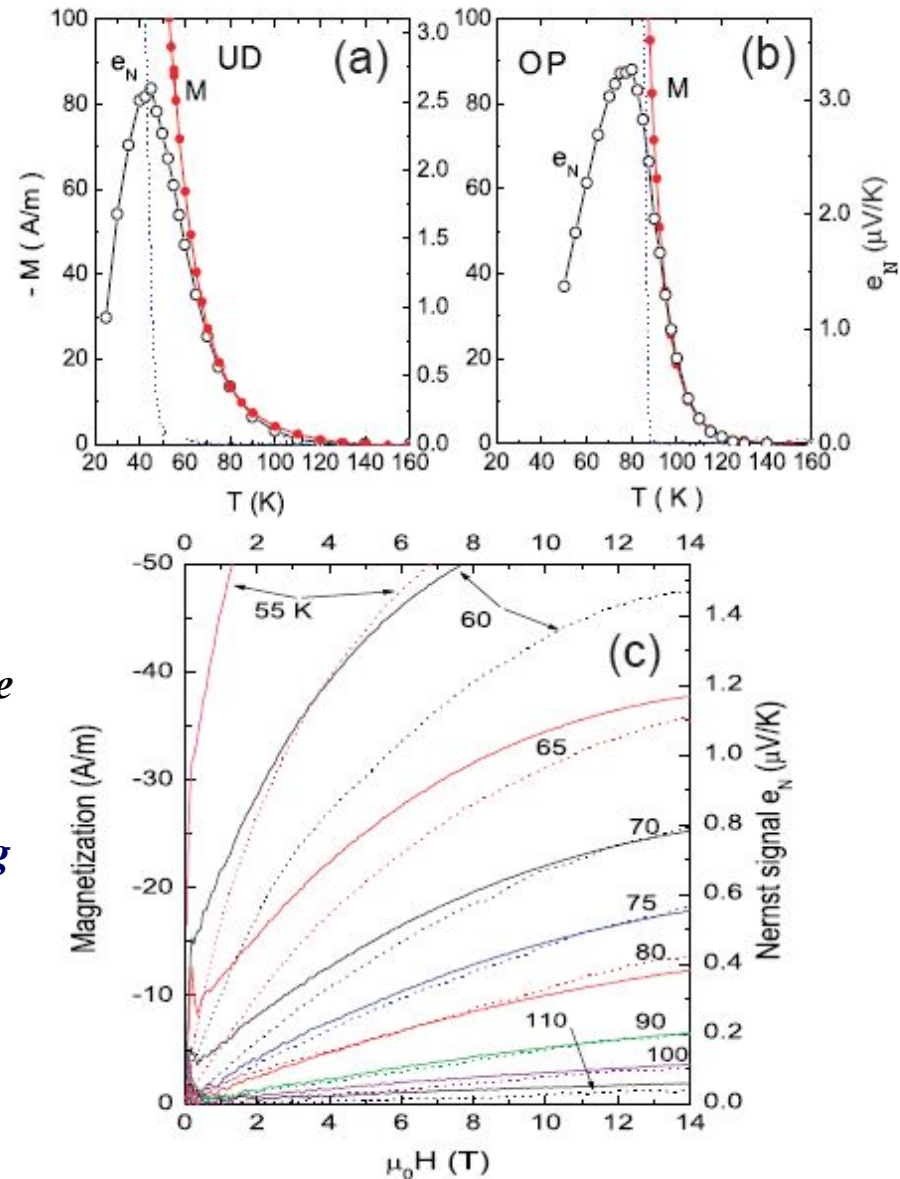


*Nernst response is observable up to  $\approx 100\text{ K}$*

*Anomalous diamagnetism, observable above the coherence loss temperature, disappears below weak pseudogap crossover temperature*

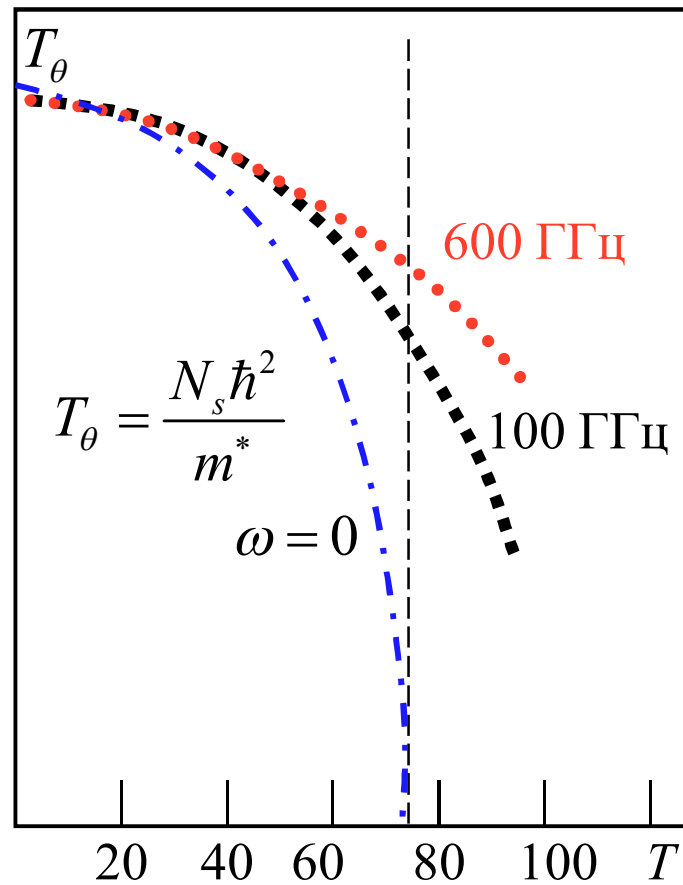
*Strong competition between superconducting and strong pseudogap states*

Z.A. Xu et al., Nature **406**, 486 (2000)



# *Manifestation of superconducting properties of the strong pseudogap*

*Frequency dependence of superfluid density  $N_s$*

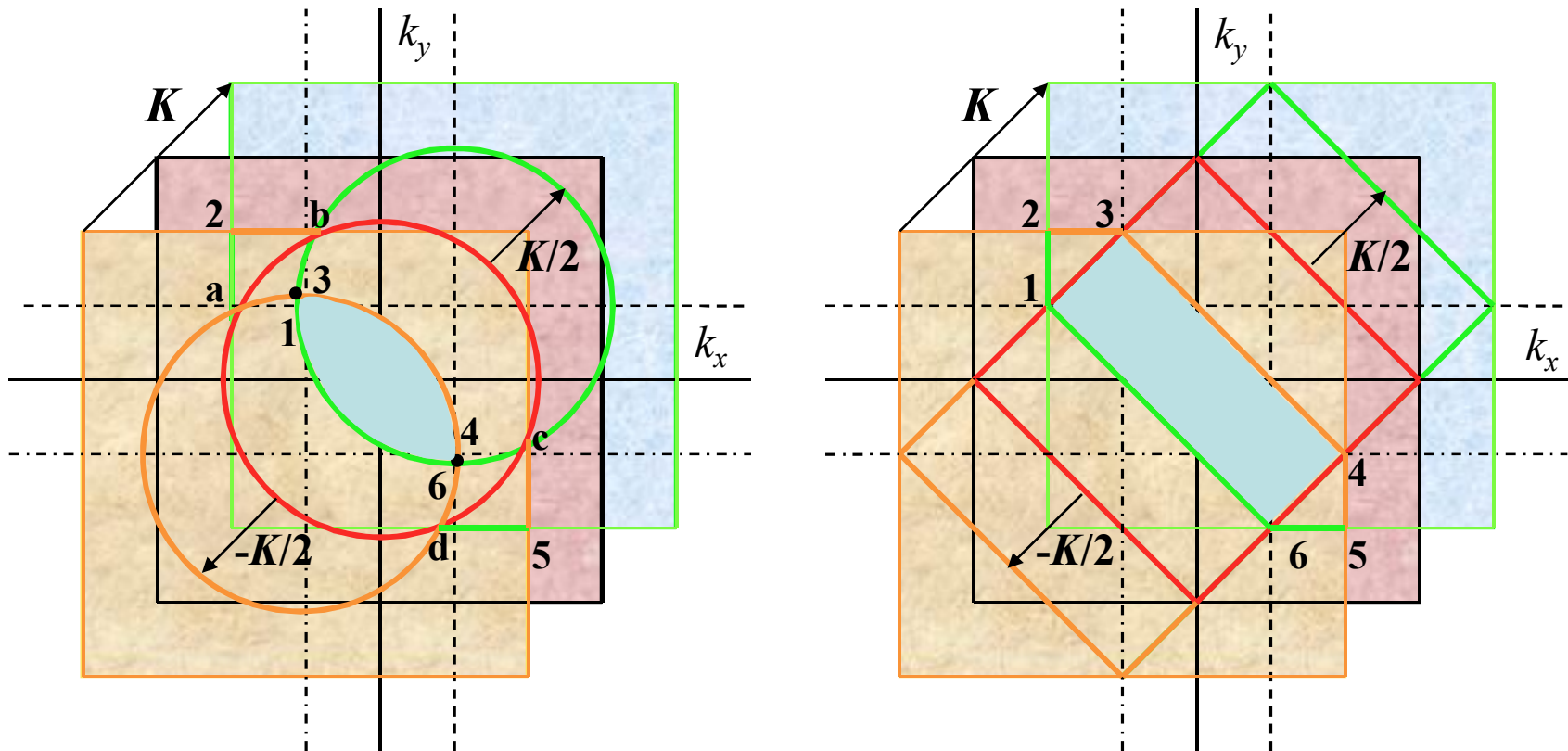


# Superconducting pairing with large momentum (K-pairing)

*Kinematical constraint. "Pair" Fermi contour*

$$\varepsilon(k_x, k_y) = \frac{\hbar^2}{2m^*} (k_x^2 + k_y^2)$$

$$\varepsilon(k_x, k_y) = -2t(\cos k_x + \cos k_y)$$

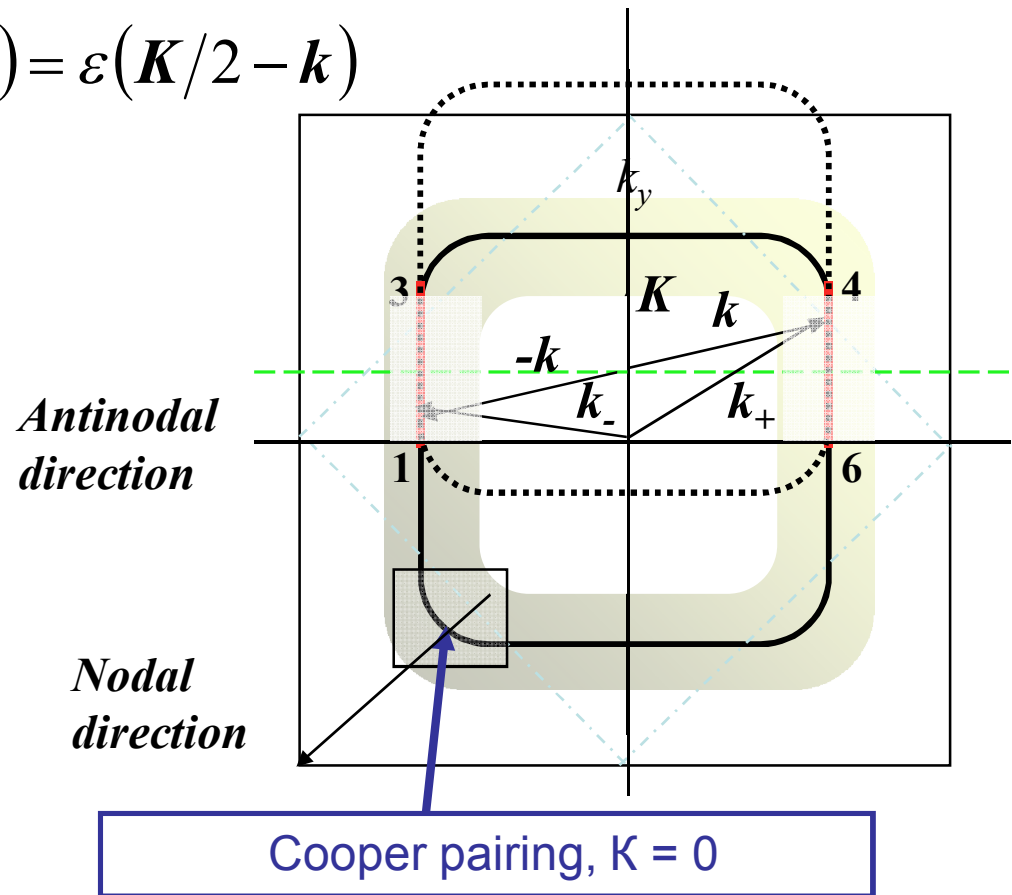
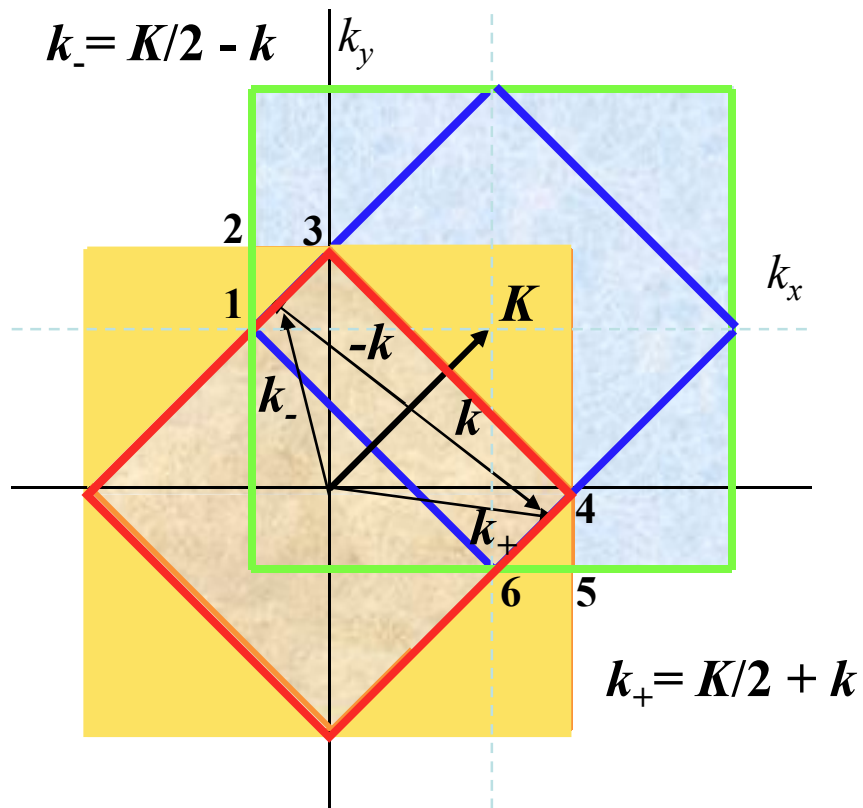


# K-pairing. Mirror nesting

*Kinematical constraint* + *Insulation induced constraint*

*logarithmic singularity in the SC K-pairing channel*

$$\varepsilon(\mathbf{K}/2 + \mathbf{k}) = \varepsilon(\mathbf{K}/2 - \mathbf{k})$$



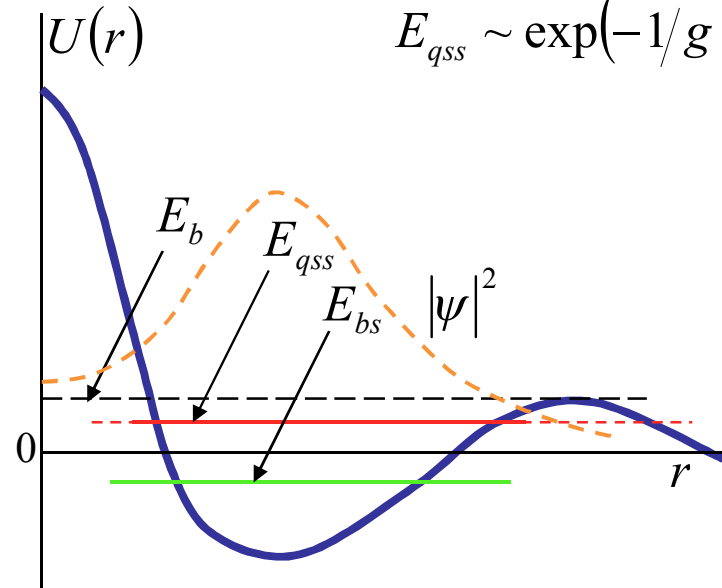
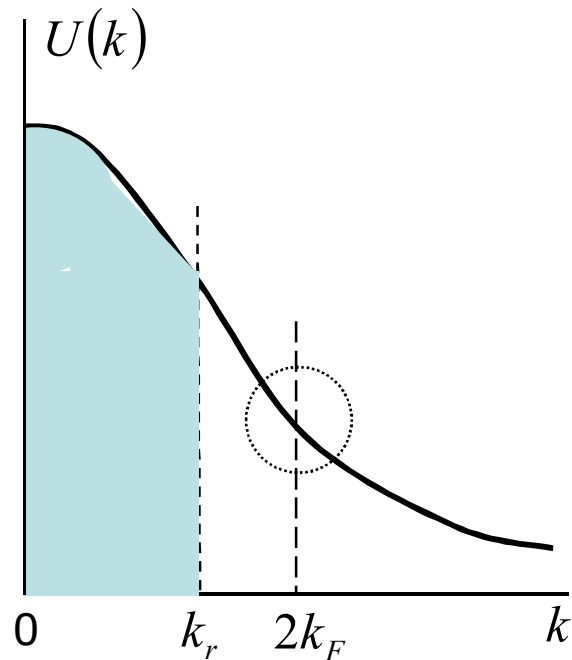
## Pairing Coulomb repulsion

**Bound state:**

$$E_{bs} \sim \exp(-1/gU_0)$$

**Quasi-stationary states:**

$$E \rightarrow E_{qss} - i\Gamma/2$$

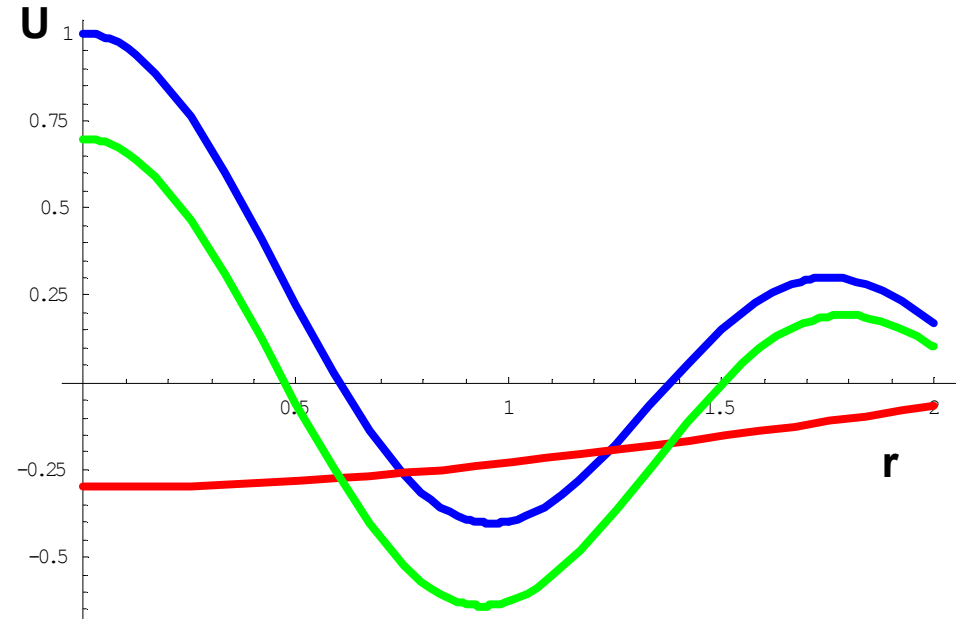
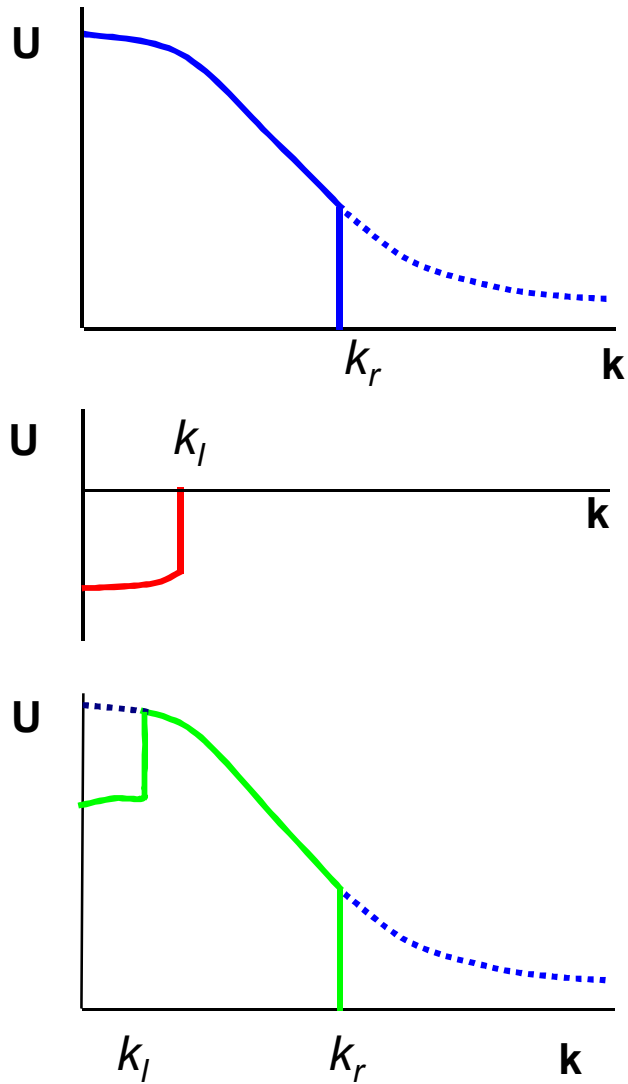


$$E_{qss} \sim \exp(-1/g U_{qss})$$

**Asymmetry of tunnel current-voltage characteristic.  
Analogy with Gamov's theory of alpha-decay**

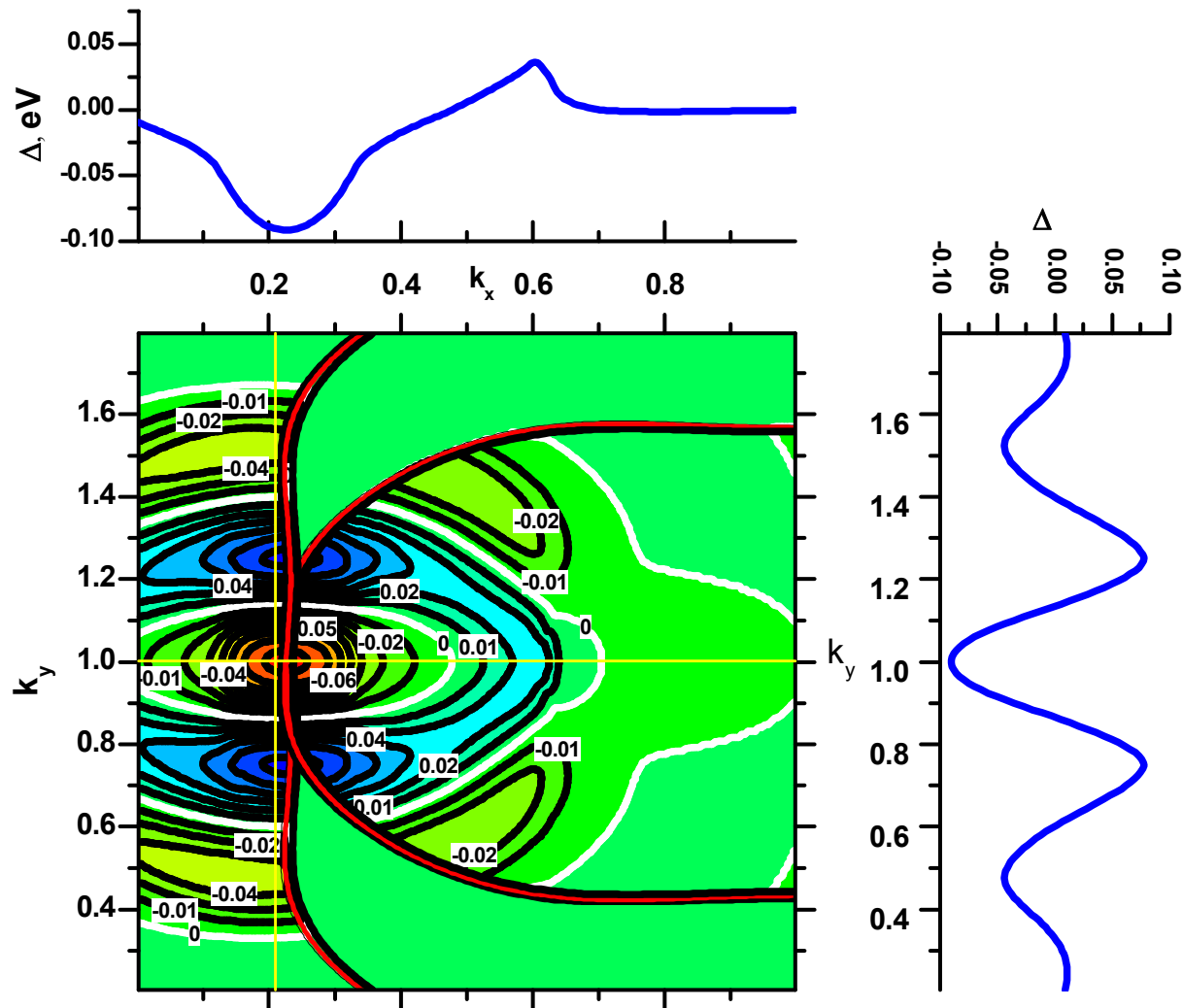


*Symmetrization of oscillating potential by electron-phonon interaction (impurity scattering  $k_l \sim 1/l$ )*

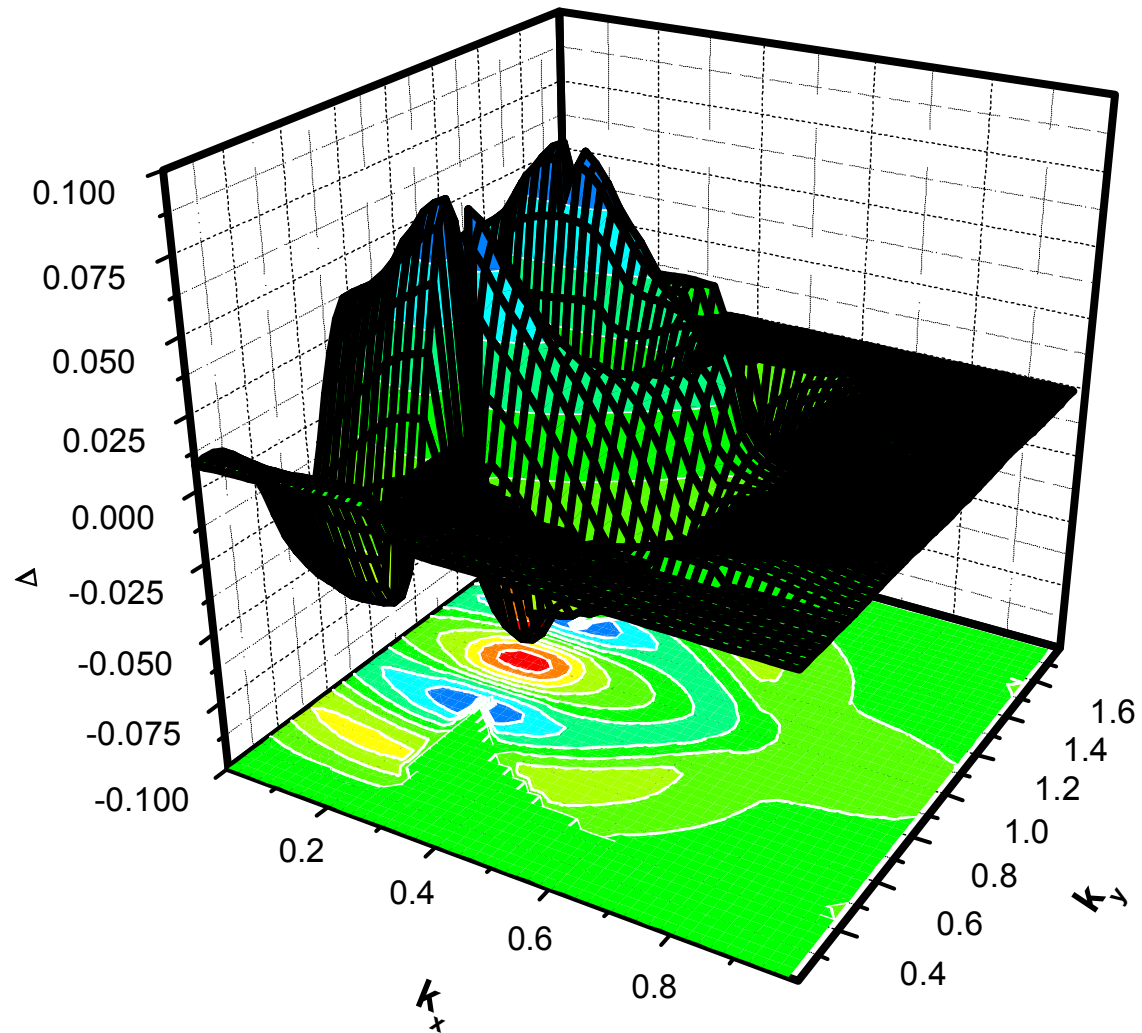


## *Topology of the superconducting order parameter*

$$\Delta_K(\mathbf{k}) = -\frac{1}{2} \sum_{\mathbf{k}'} \frac{U(\mathbf{k} - \mathbf{k}') \Delta_K(\mathbf{k}')}{\sqrt{\xi_K^2(\mathbf{k}') + |\Delta_K(\mathbf{k}')|^2}} \quad \oplus \text{ Essential dependence on the momentum of the relative motion of K-pair}$$



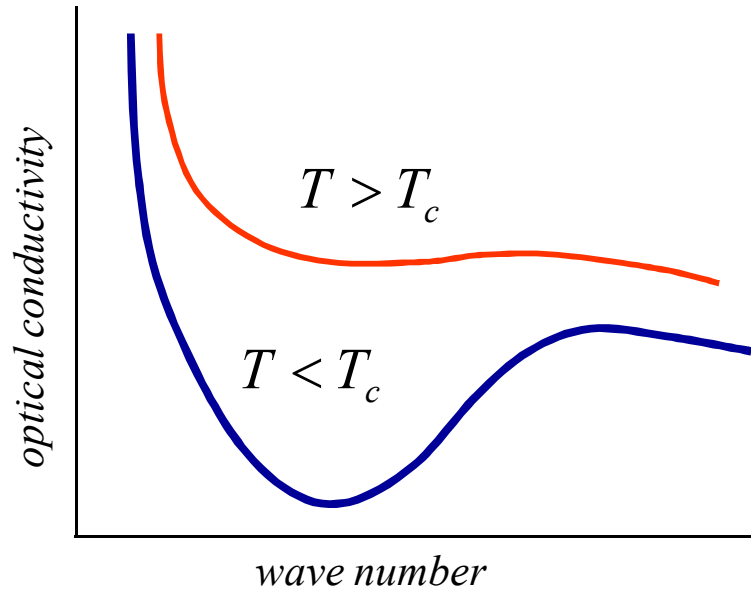
# *Topology of the superconducting order parameter*



# Optical conductivity. Superfluid density

D.N. Basov, T. Timusk, 2005

*Effective number of carriers per planar Cu atom*



Material	$T_c$ (K)	$N_{tot}$	$N_{Drude}$	$N_s$	$N_s/N_{tot}$	$N_s/N_{Drude}$
$La_2CuO_{4.12}$	40	0.14	0.025	0.028	0.2	0.8
$Bi_2Sr_2CaCu_2O_8$	85	0.38	0.105	0.092	0.24	0.88
Y0.35	40	0.21	0.04	0.02	0.08	0.5
P0.5,Y0.2	35	0.23	0.05	0.017	0.07	0.34
$YBa_2Cu_3O_{7-\delta}$	92	0.44	0.093	0.082	0.19	0.89
Pr0.15	75	0.375	0.073	0.054	0.14	0.74
Pr0.35	40	0.25	0.045	0.02	0.08	0.44
$Tl_2Ba_2CaCu_2O_8$	110	0.54	0.13	0.115	0.21	0.88
Uncertainties	$\pm 2$	$\pm 0.03$	$\pm 0.01$	$\pm 0.001$	$\pm 0.001$	$\pm 0.04$

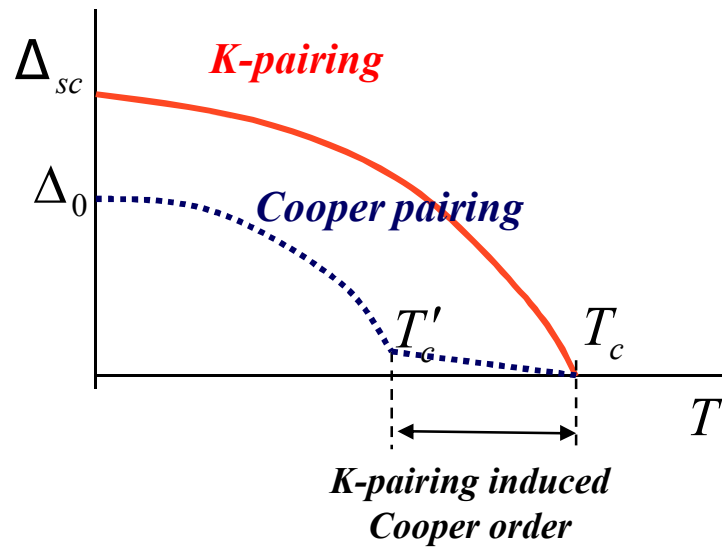
**Chemical potential shift:  $\delta\mu \sim |\Delta|$**   
*(instead of  $\delta\mu \sim \Delta^2/E_F$  in the BCS theory)*

G. Rietveld, N.Y. Chen, D. van der Marel,  
 PRL 69, 2578 (1992)

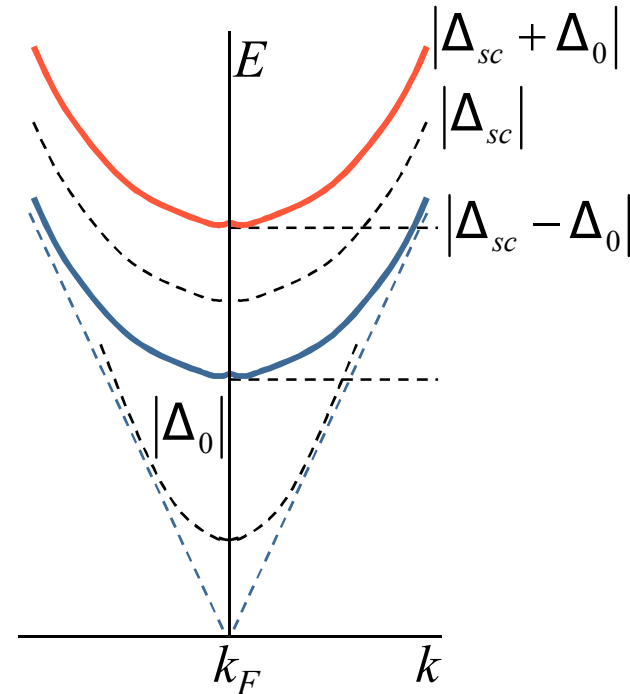
$\sigma_1(T > T_c) - \sigma_1(T < T_c) \neq 0, \quad \hbar\omega \sim 100\Delta$  (**“high energy problem”**, A. Leggett, 2006)

# Two-gap quasi-particle spectrum

$$|\Delta_{sc} - \Delta_0| \quad |\Delta_{sc} + \Delta_0| \quad \Delta_{pg}$$



*Redistribution of the spectral weight between the states with “large” and “small” superconducting gaps*



$$E_{\pm}(k) = \sqrt{\xi_K^2(k) + |\Delta_{sc}(k) \pm \Delta_0(k)|^2}$$

# Superfluid density of the biordered state

## Superfluid density

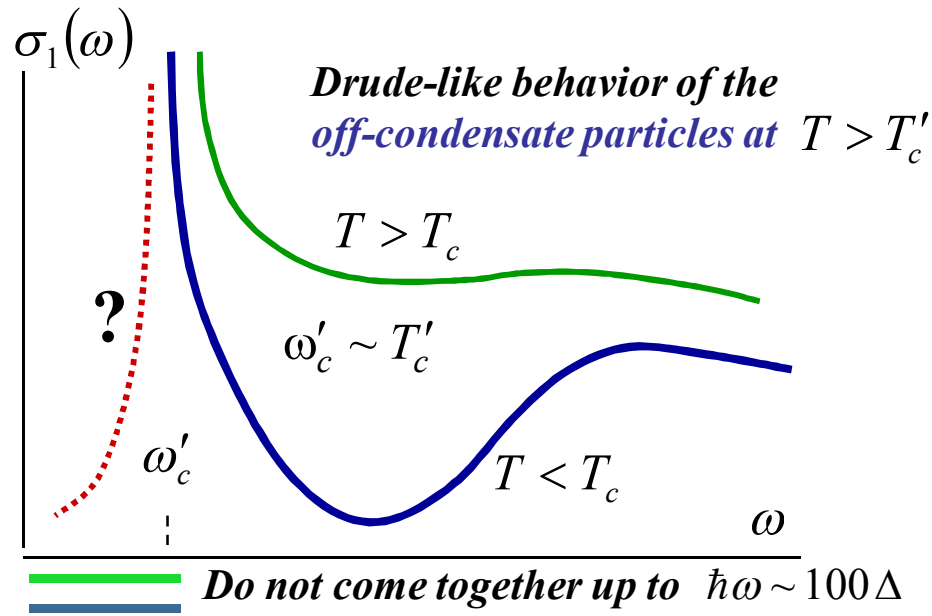
$$0 < T < T'_c : \rho_s \sim 1$$

$$T'_c < T < T_c : \rho_s \ll 1$$

## Optical conductivity

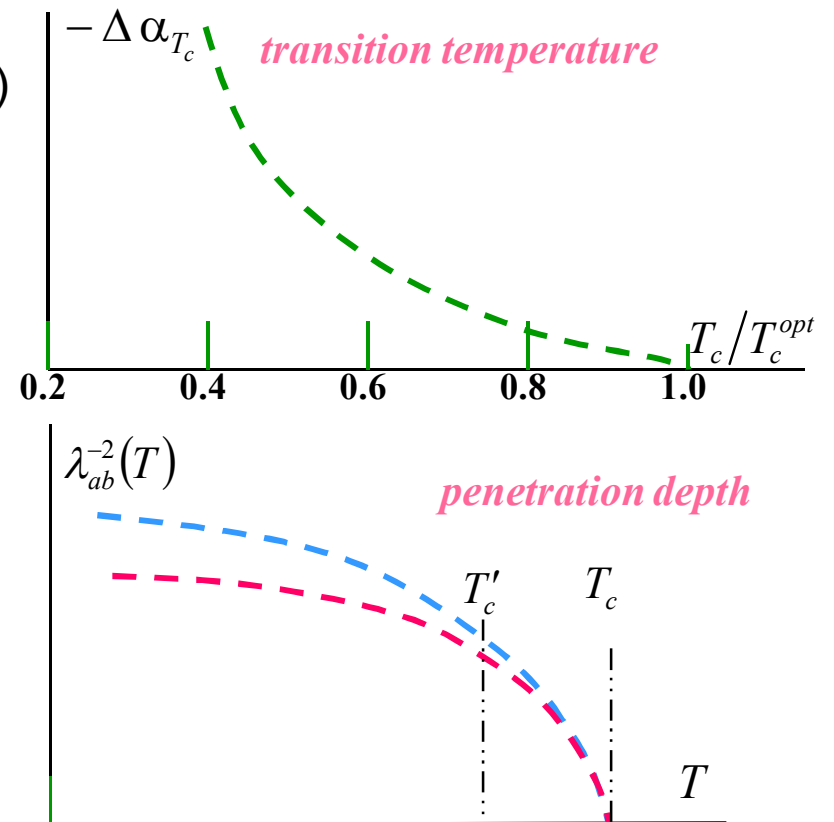
### Chemical potential shift

$$\delta\mu \sim |\Delta| \quad (\text{in the framework of BCS theory } \delta\mu \sim \Delta^2/E_F)$$



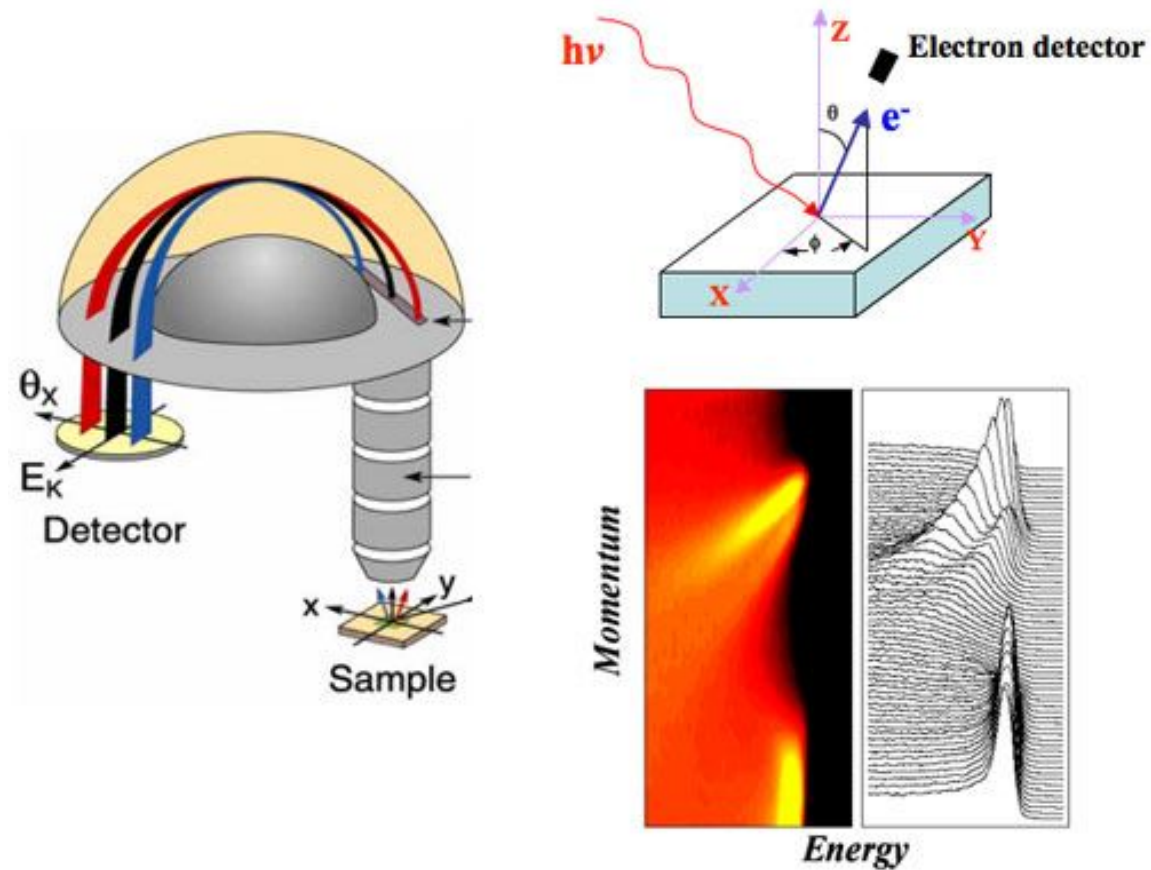
## Isotope effect

- **Phonon suppression of the scattering with small momentum transfers**
- **A contribution of the Cooper channel**



# Study of the electron structure in the momentum space

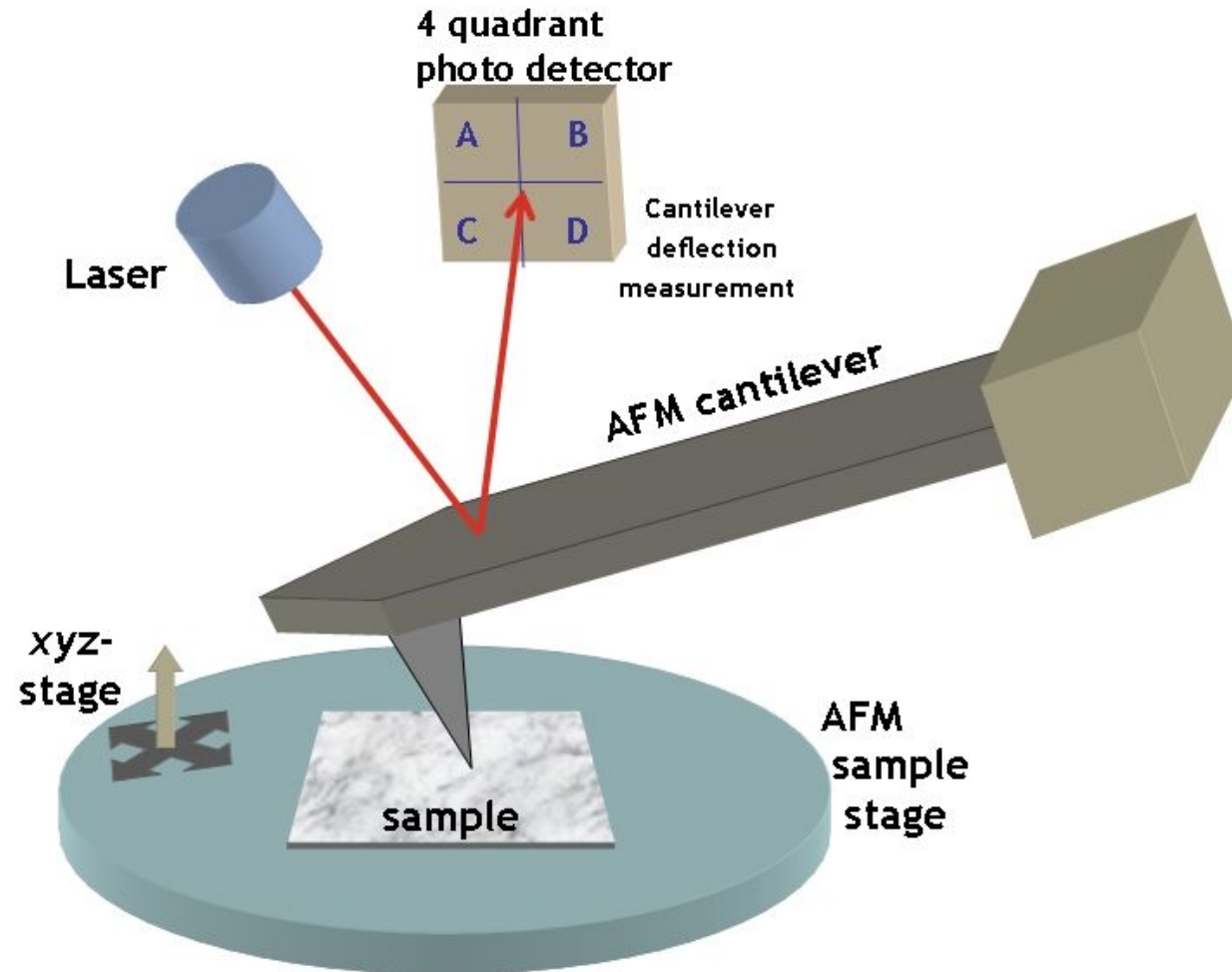
## Angle resolved photoemission spectroscopy (ARPES)



*Geometry of an ARPES experiment.*

*The emission direction of the photoelectron is specified by the polar ( $\theta$ ) and azimuth ( $\varphi$ ) angles*

## *Study of the electron structure in the real space*



*STM: invented by Gerd Karl Binnig and Heinrich Rohrer in earlier 1980 (Nobel prize, 1986).*



# *A comparison of FT-STM and AC-ARPES*

## *Fourier transform STM (FT-STM)*

$$\frac{dI(V, r)}{dV} = G(V, r) \quad G(r, \omega) = I_0 |M_{f,i}^r|^2 A(r, \omega)$$

$\omega = eV$ ,  $A(r, \omega)$  – local density of states (LDOS)

$$A(k, \omega) = \sum_{r_l \in L \times L} e^{ikr_l} A(r_l, \omega) \quad \text{- FT-STM}$$

## *Autocorrelated ARPES (AC-ARPES)*

$$I(\hat{e}, k, \omega) = I_0 |M_{f,i}^{\hat{e}}|^2 A(k, \omega) \quad C(q, \omega) = \int A(k+q, \omega) A(k, \omega) d^2k$$

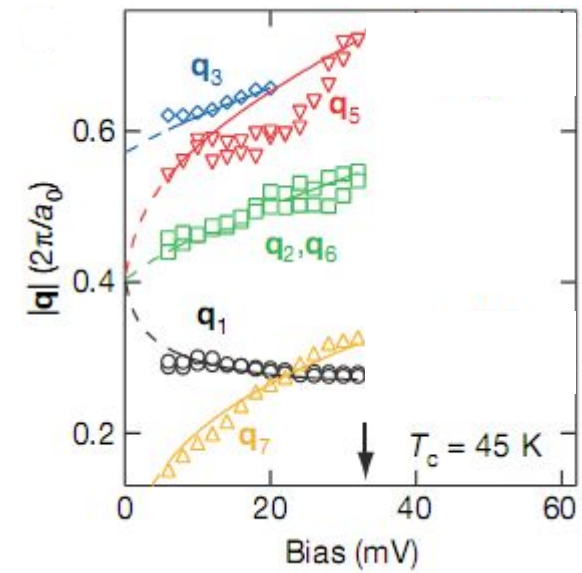
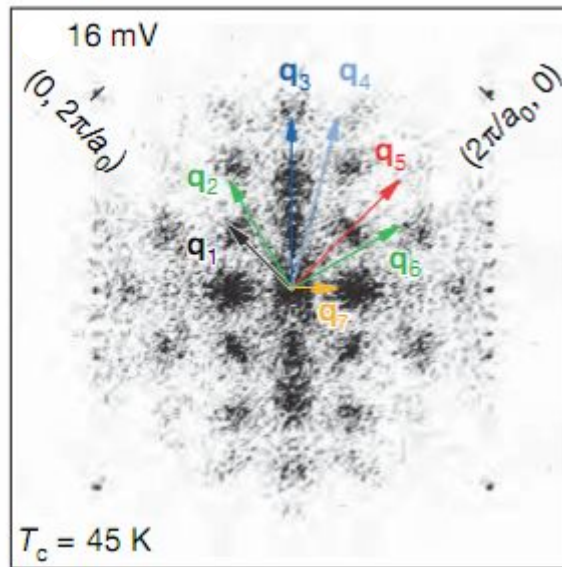
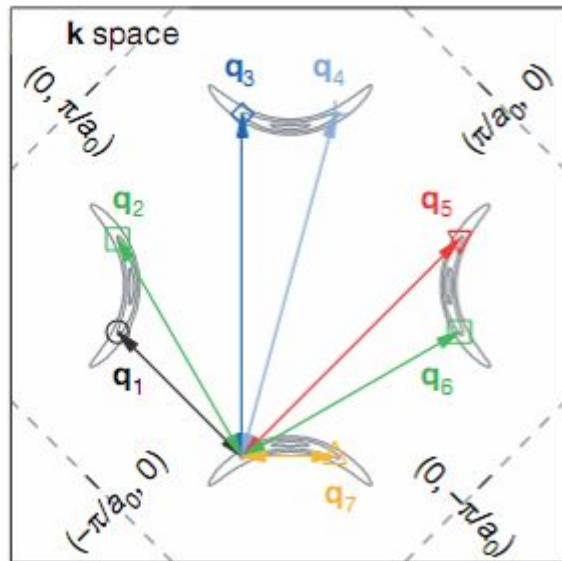
$C(q, \omega)$  – joined density of states (JDOS)

$$C(q, \omega) = -\frac{\delta\varepsilon(q)}{\pi} \text{Im} \Lambda(q, \omega), \quad \Lambda(q, \omega) = \int d^2r e^{iqr} G(r, \omega) G(-r, \omega)$$

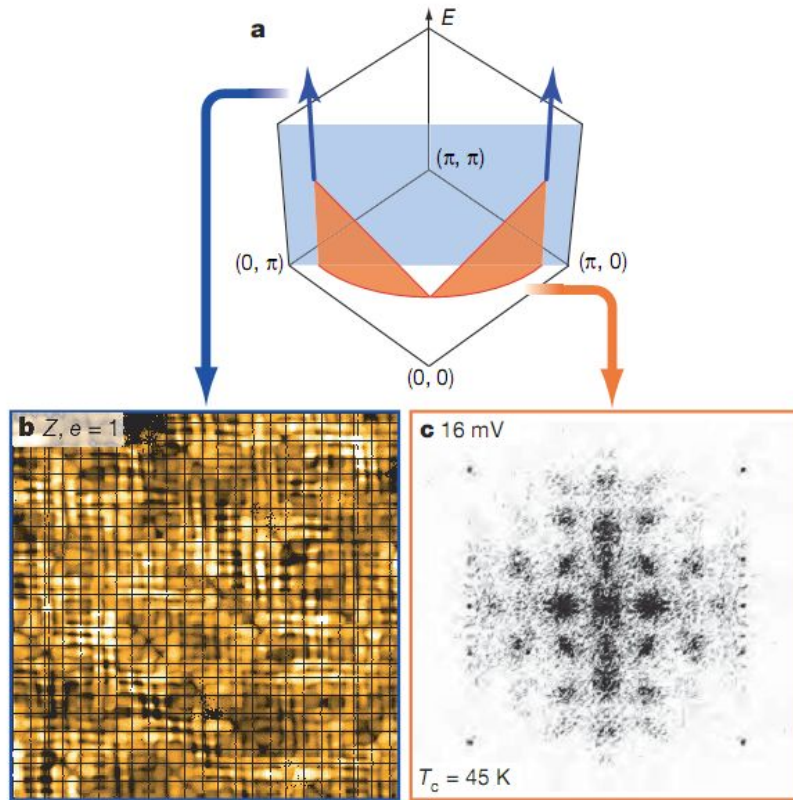
$|\delta\varepsilon(q)|^2$  – structure factor of the scattering center

$|\text{Im} \Lambda(q, \omega)|^2$  – describes the quasi-particle interference

# Bogoliubov Quasi-particle (BQP) interference

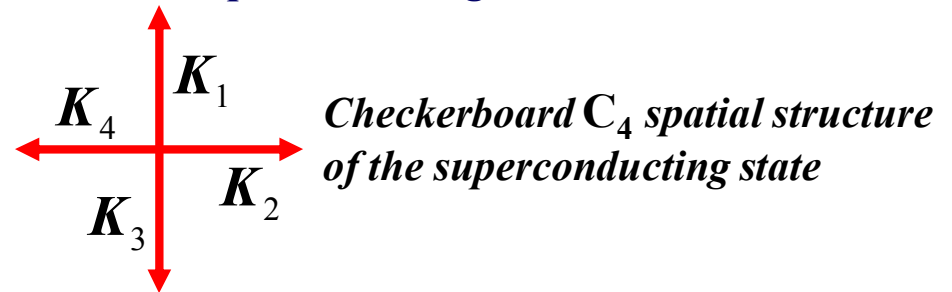


**Imaging of the electron structure of high- $T_c$   $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$  in real space and momentum space:** Y. Kohsaka et al., *Nature* **454**, 1072 (2008)

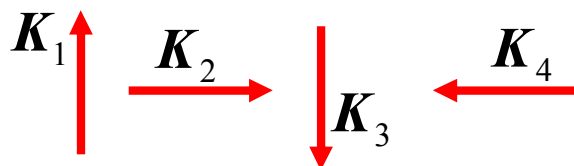


- pseudogap and superconducting states attributed with different regions of the momentum space
- reduction of the rotation symmetry during transition from the superconducting state into the pseudogap one:  $C_4 \rightarrow C_2$

**Coherent superconducting state**

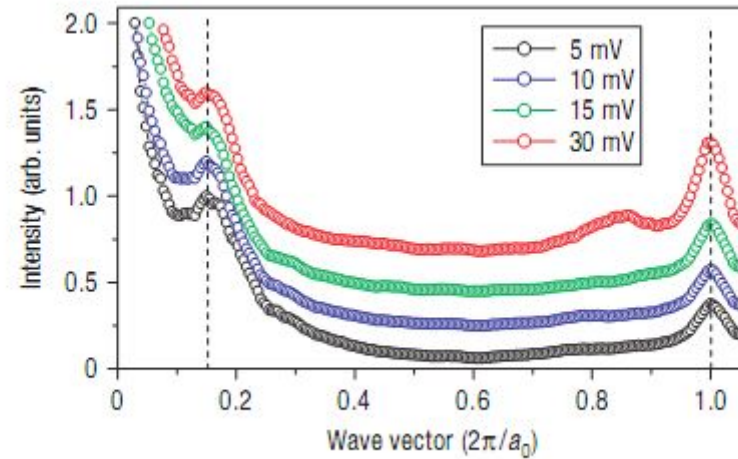
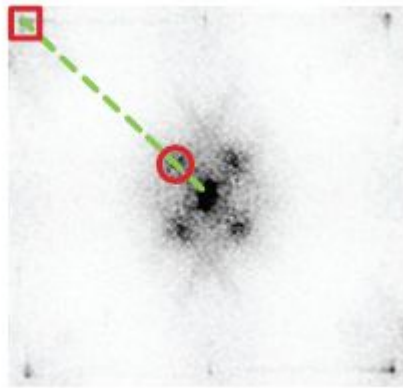


**Non-coherent states of the strong pseudogap**



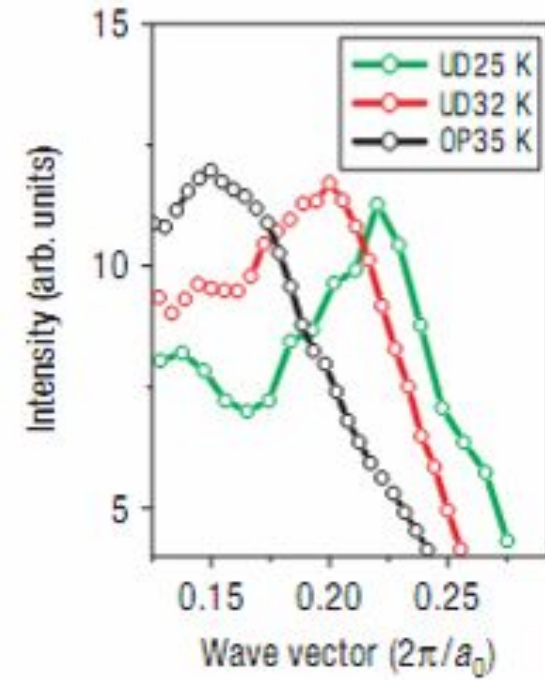
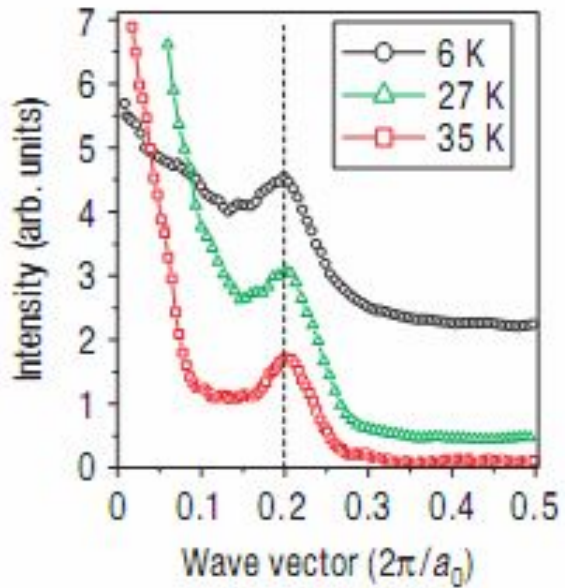
**Stripe  $C_2$  electron structure of the strong pseudogap in the real space**

## *Anatomy of the checkerboard in optimally doped Bi-2201*



W.D. Wise et al. , Nature Physics, V4, p.696, 2008. Charge-density-wave origin of cuprate Checkerboard visualized by scanning tunnelling microscopy.

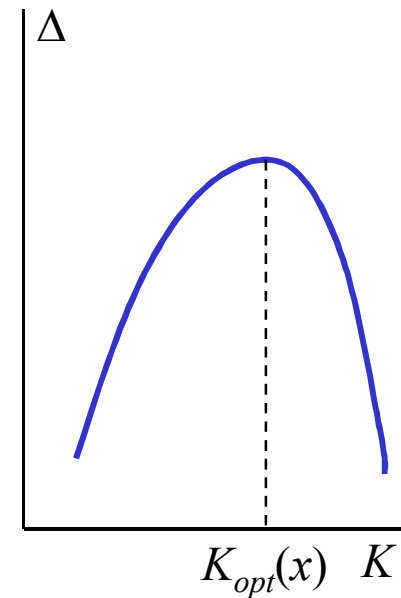
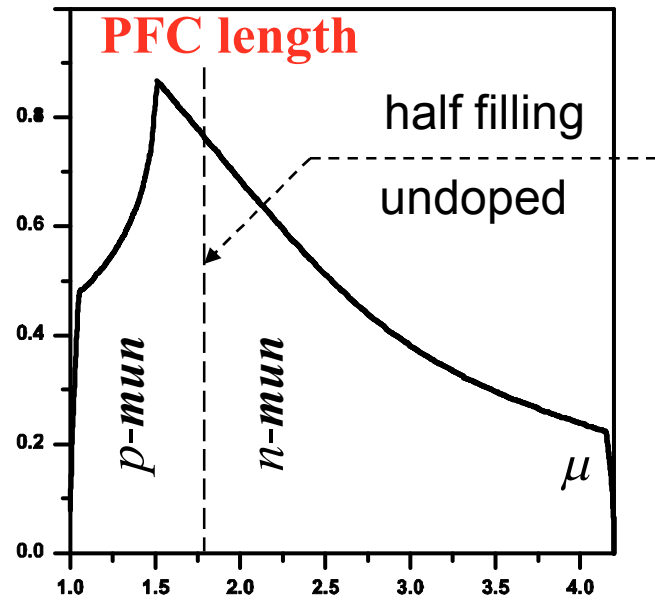
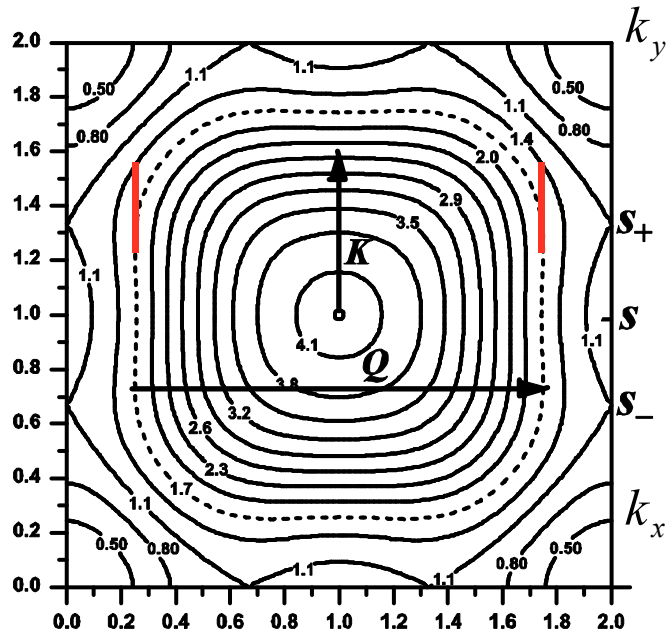
## *Doping dependence of the checkerboard*



## *Temperature independence of the checkerboard*

## *Distorted mirror nesting*

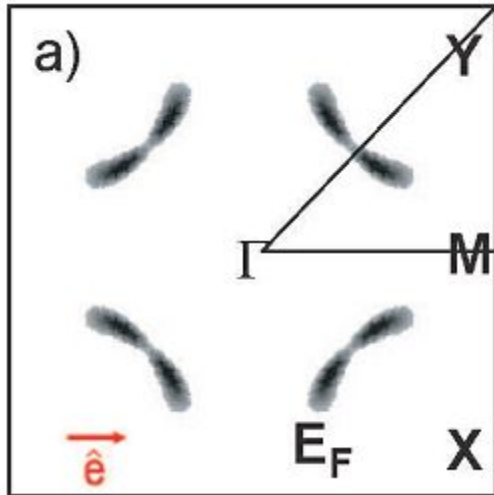
$$\varepsilon(k_x, k_y) = -2t(\cos k_x + \cos k_y) - 4t' \cos k_x \cos k_y - 2t''(\cos 2k_x + \cos 2k_y)$$



$$\Delta = \sqrt{\Delta_0(\Delta_0 - \delta)}$$

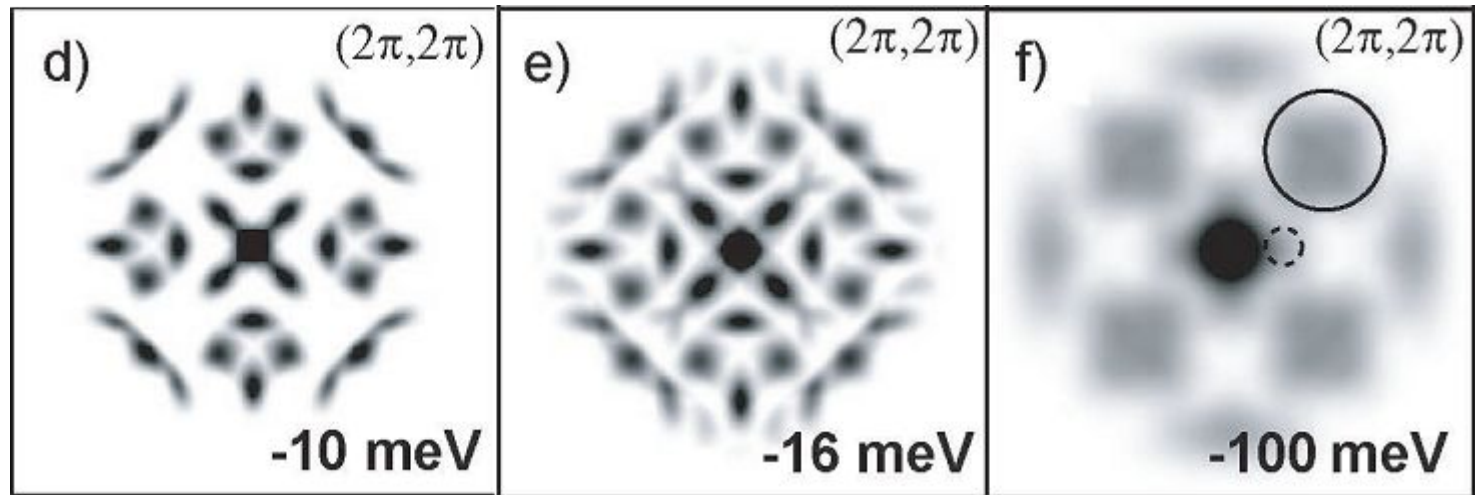
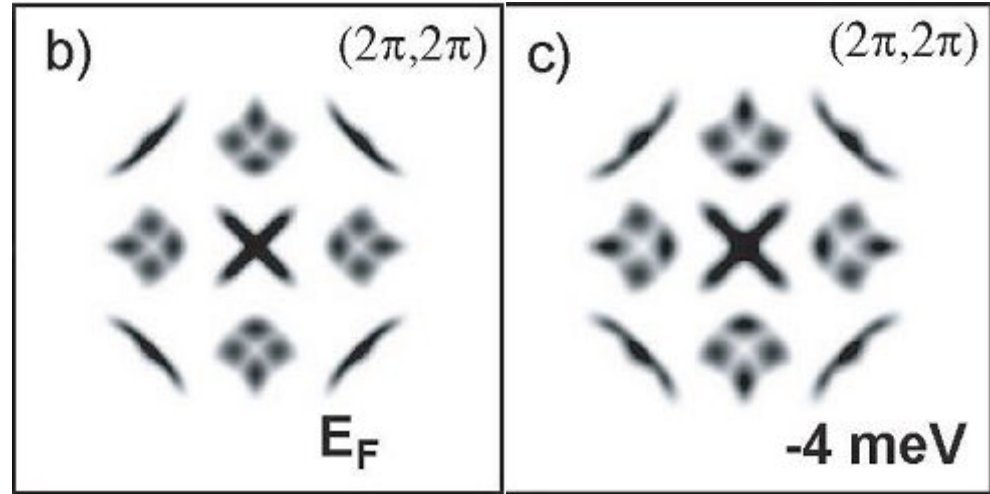
$\delta$  - mean-squared deviation of an isoline from perfect mirror nesting

# *A Comparison between Real and Momentum Space Photoemission Spectroscopies* (Phys. Rev. Lett. 96, 067005 (2006) )



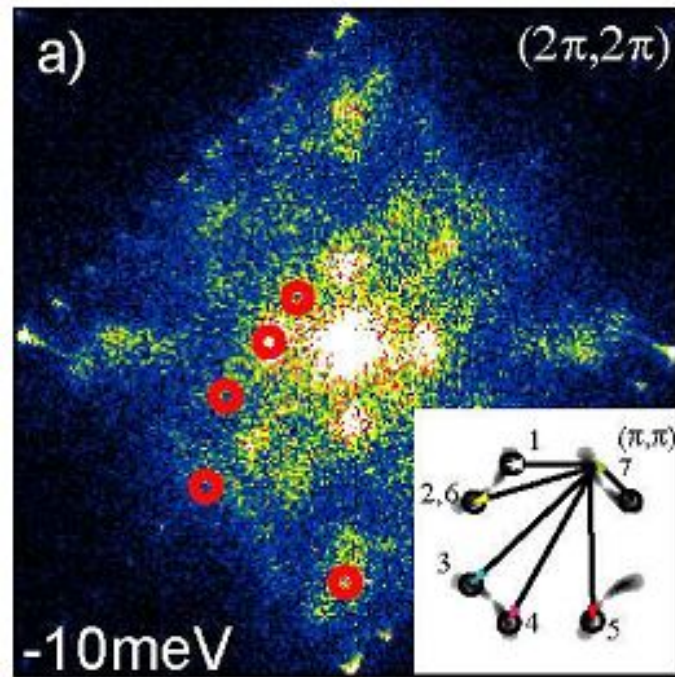
$$I(\hat{e}, \vec{k}, \omega) = I_0 |M_{f,i}^{\hat{e}}|^2 f(\omega) A(\vec{k}, \omega)$$

**a) ARPES; b) – f) AC- ARPES**

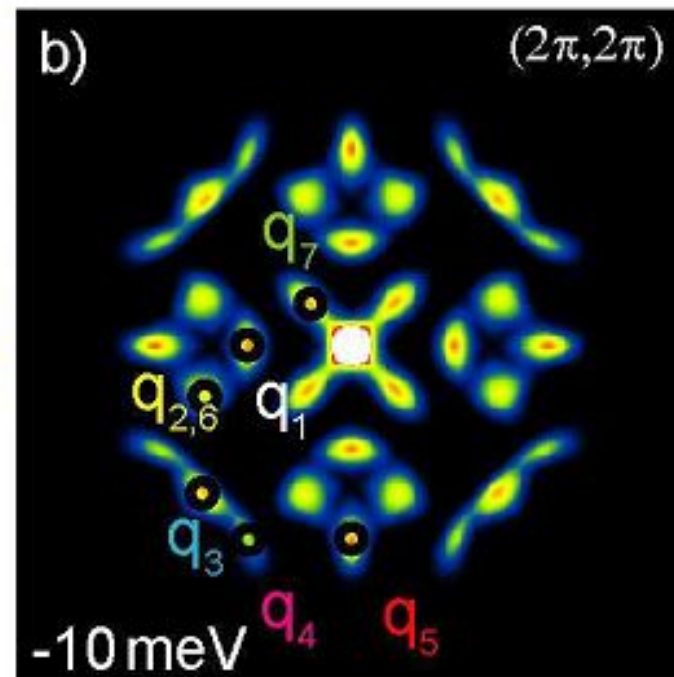


$$\text{JDOS}(\vec{q}, \omega) = \int A(\vec{k} + \vec{q}, \omega) A(\vec{k}, \omega) d^2k.$$

## *FT-STM intensity*



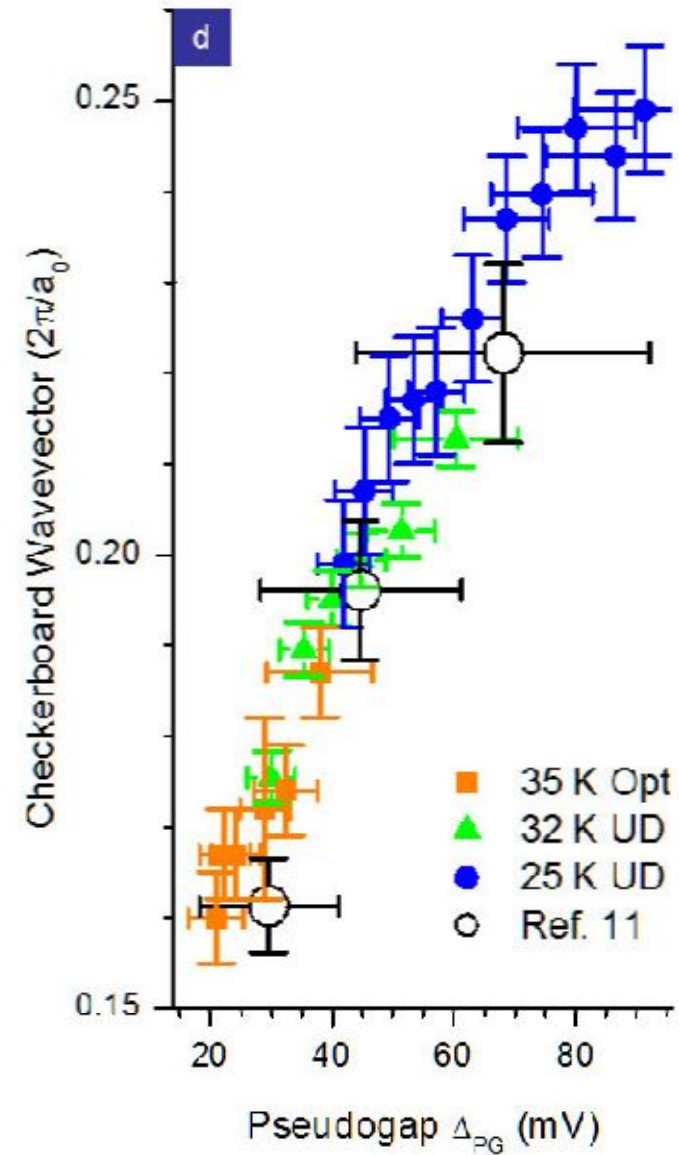
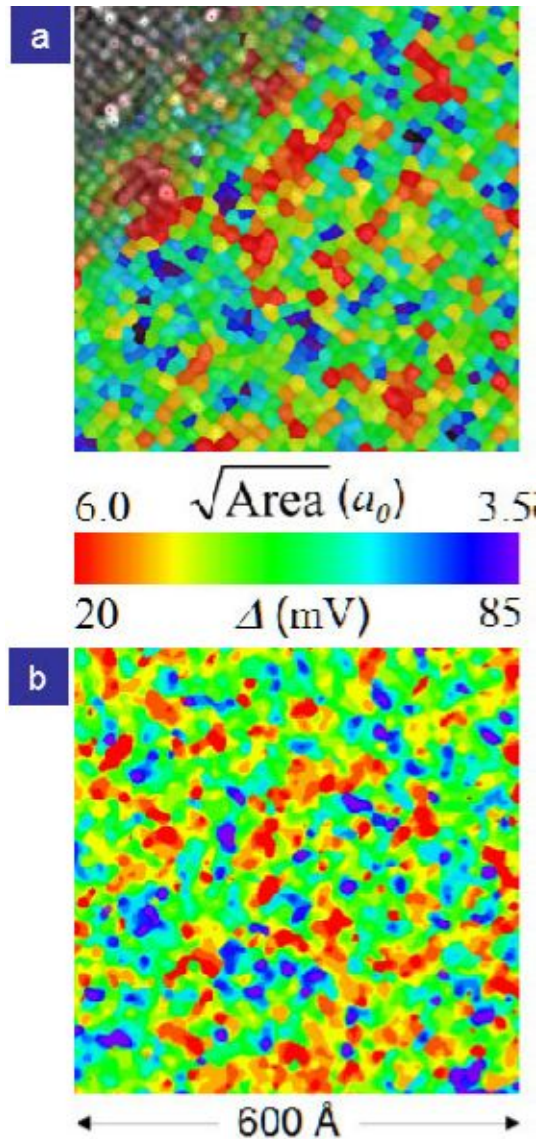
## *AC-ARPES*





# Local variations of the Bi-2201 checkerboard.

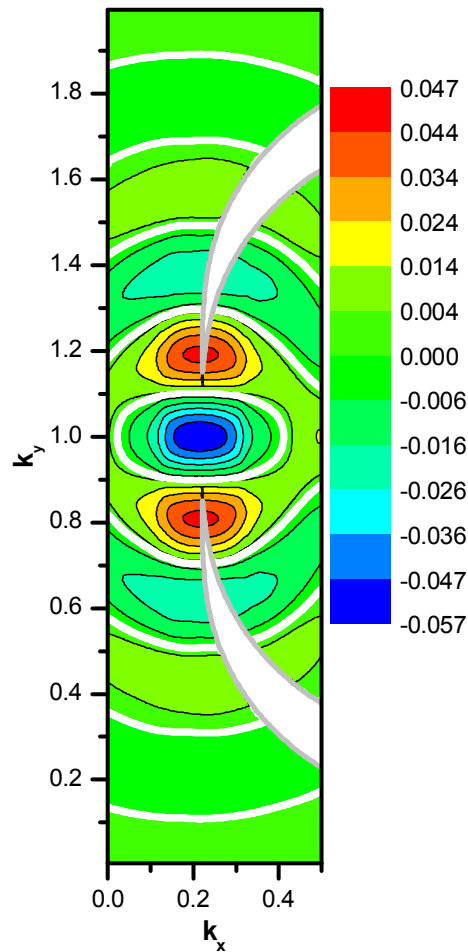
W. D. Wise et al, Cond-mat/0811.1585



# Superconducting antinodal state

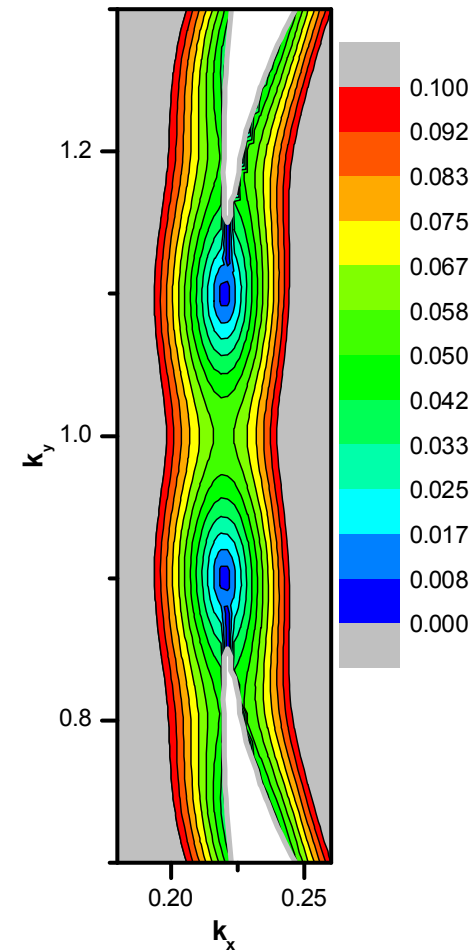
*Symmetric (with respect to  $k_y$ ) solution*

$$\Delta(k_x, k_y) = \Delta(k_x, -k_y)$$



*Isolines of quasi-particle dispersion*

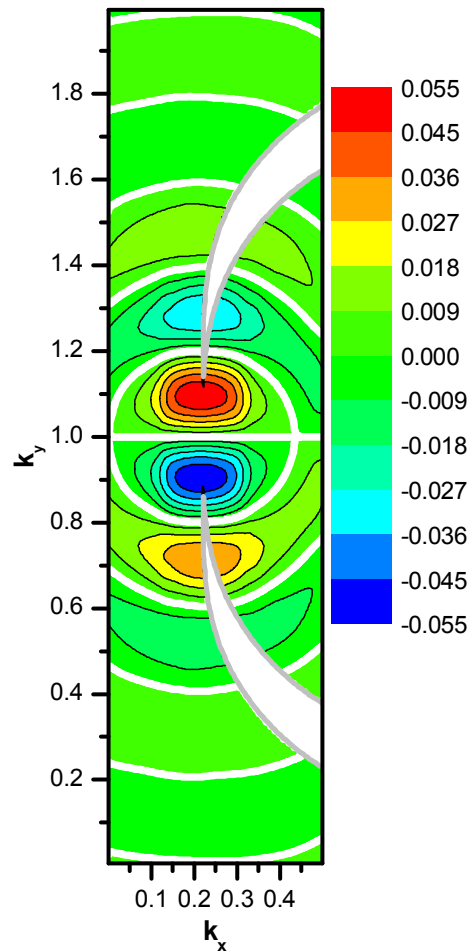
$$E(\mathbf{k}) = \sqrt{\xi^2(\mathbf{k}) + \Delta^2(\mathbf{k})}$$



# *Pseudogap antinodal state*

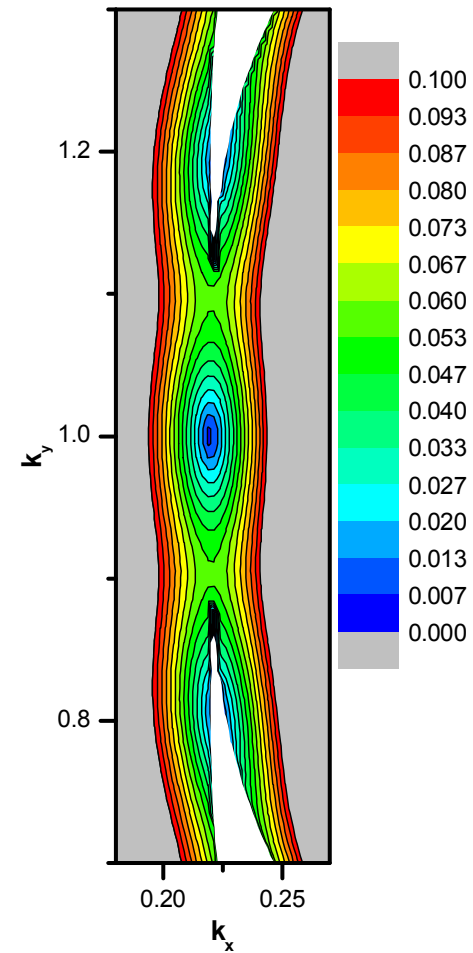
*Asymmetric (with respect to  $k_y$ ) solution*

$$\Delta(k_x, k_y) = -\Delta(k_x, -k_y)$$



*Isolines of quasi-particle dispersion*

$$E(\mathbf{k}) = \sqrt{\xi^2(\mathbf{k}) + \Delta^2(\mathbf{k})}$$



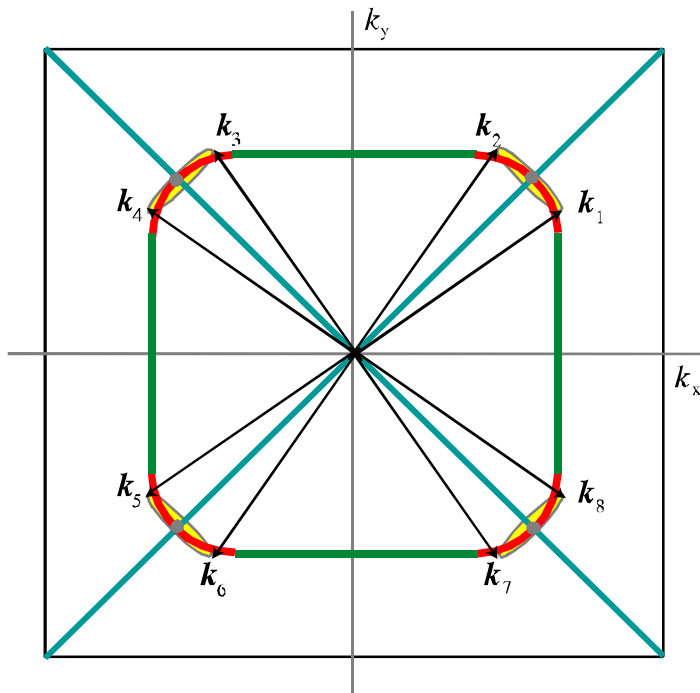
*Closed isolines of the quasi-particles spectrum in the nodal and antinodal regions*

$$E(\mathbf{k}) = \sqrt{\xi^2(\mathbf{k}) + \Delta^2(\mathbf{k})}$$

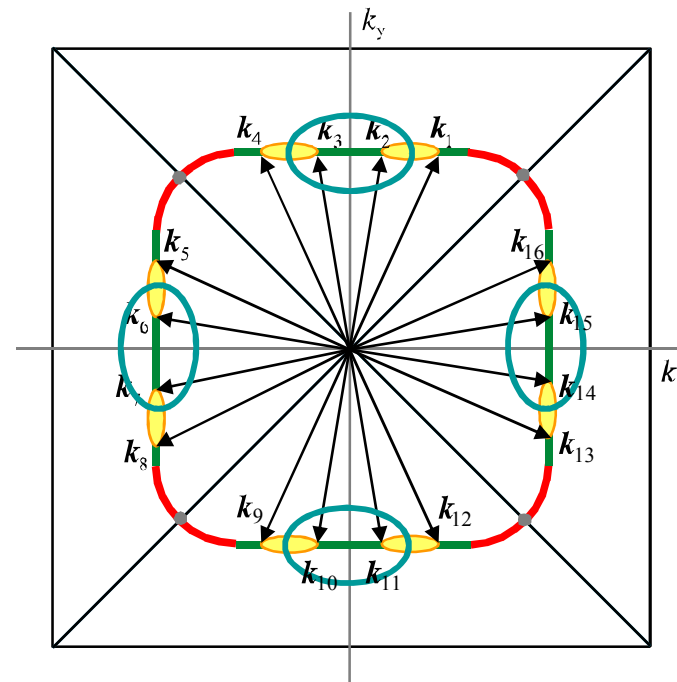
*Singular points of quasi-particle dispersion:*

$\xi(\mathbf{k})=0$  (*Fermi contour*),  $\Delta(\mathbf{k})=0$  (*nodal line*)

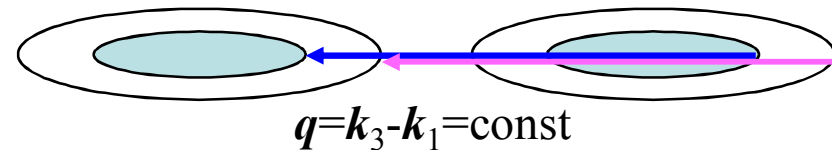
*Nodal region: “banana”-like isolines*



*Antinodal region: pairs of closed isolines*



*Fixed (energy independent) scattering momentum*

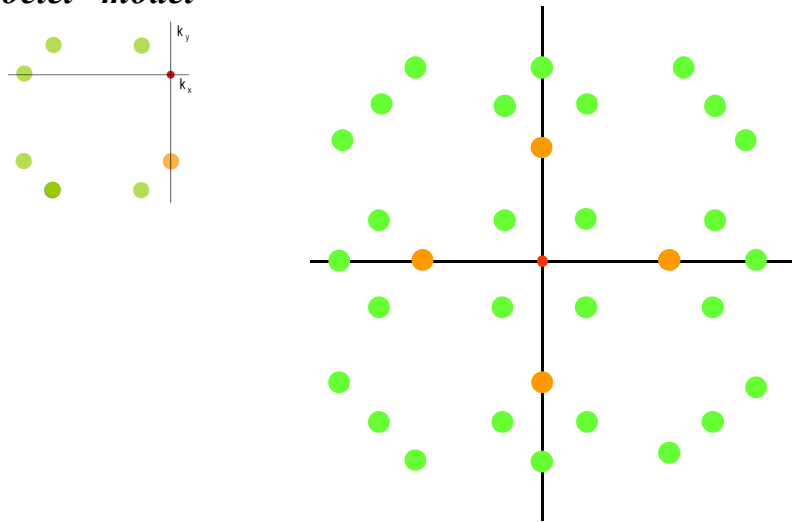


# Nodal and antinodal quasi-particle interference patterns in the momentum space

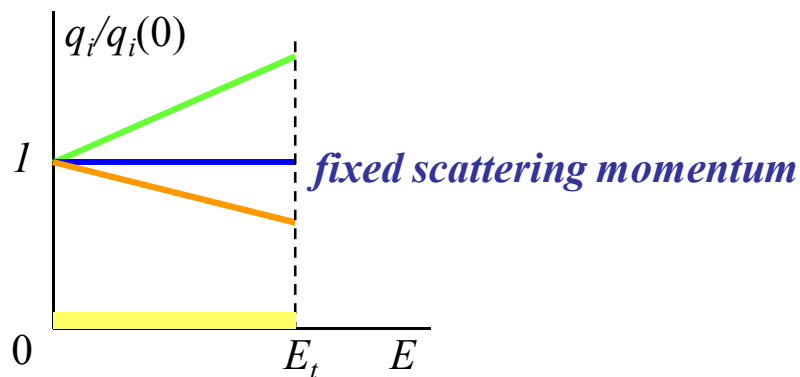
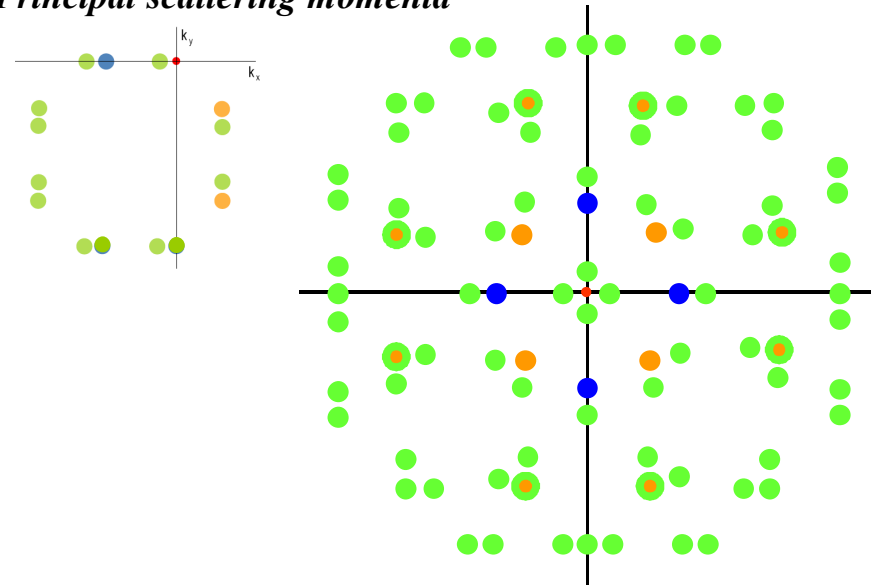
**Nodal region:**  
no fixed scattering momenta

**Antinodal region:**  
four fixed scattering momenta →  
Checkerboard spatial ordering

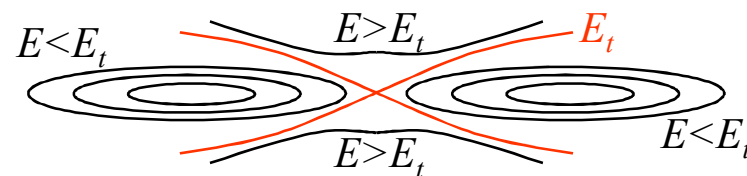
Principal scattering momenta:  
“octet” model



Principal scattering momenta



**Topological transition**



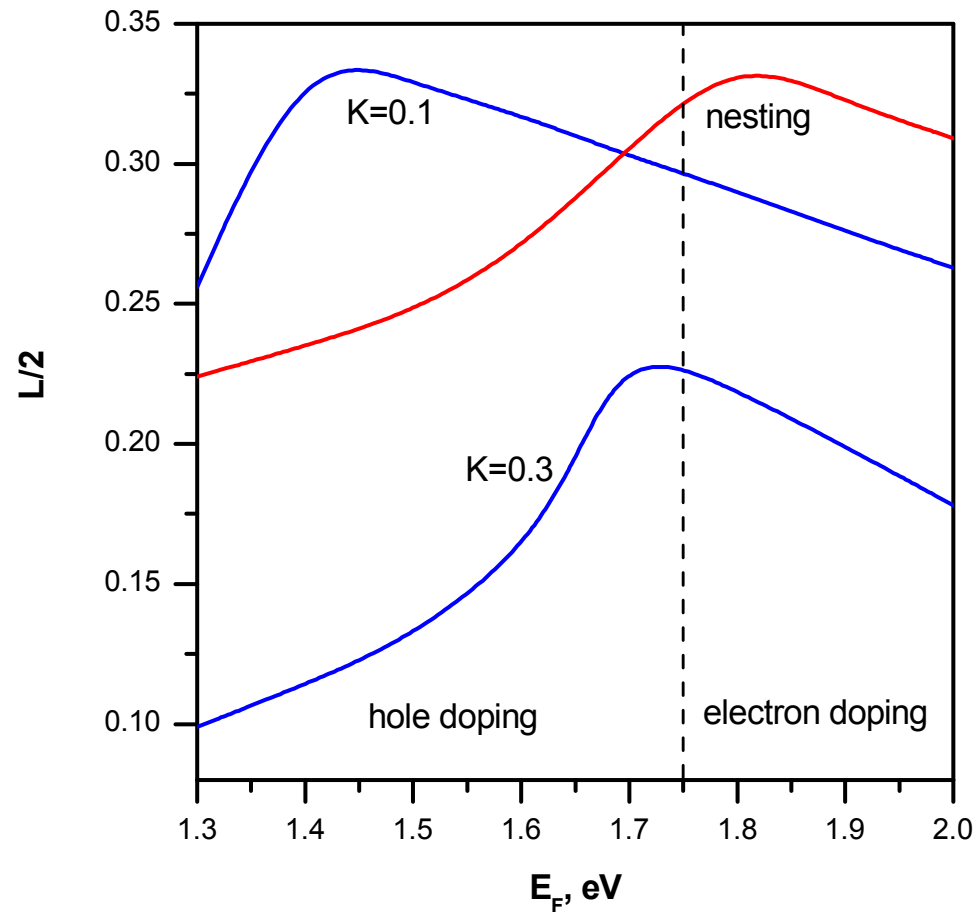
# Length of **electron-electron** and **electron-hole** pair fermi contours

Mirror nesting

$$\varepsilon(K/2 + k) = \varepsilon(K/2 - k)$$

Nesting

$$\varepsilon(k) = -\varepsilon(k + Q_1)$$



# Nesting and mirror nesting:

## a competition between insulating and superconducting states

**Biordered superconducting state**  
(coexistence of Cooper and  
**K-pairing channels**)

- Cooper channel ( $K=0$ ): phonon mediated pairing (NR)
- K-pairing channel ( $K \neq 0$ ): Coulomb pairing (ANR)

Insulating-pairing channel ( $Q_1 \neq 0$ ): CDW, DDW (ANR)

Mirror nesting  $\leftarrow K \approx Q_1$  Nesting  $\rightarrow$

Cooper and K-pairing.  
Weak CDW (DDW) as Cooper  
and K-pairing induced order

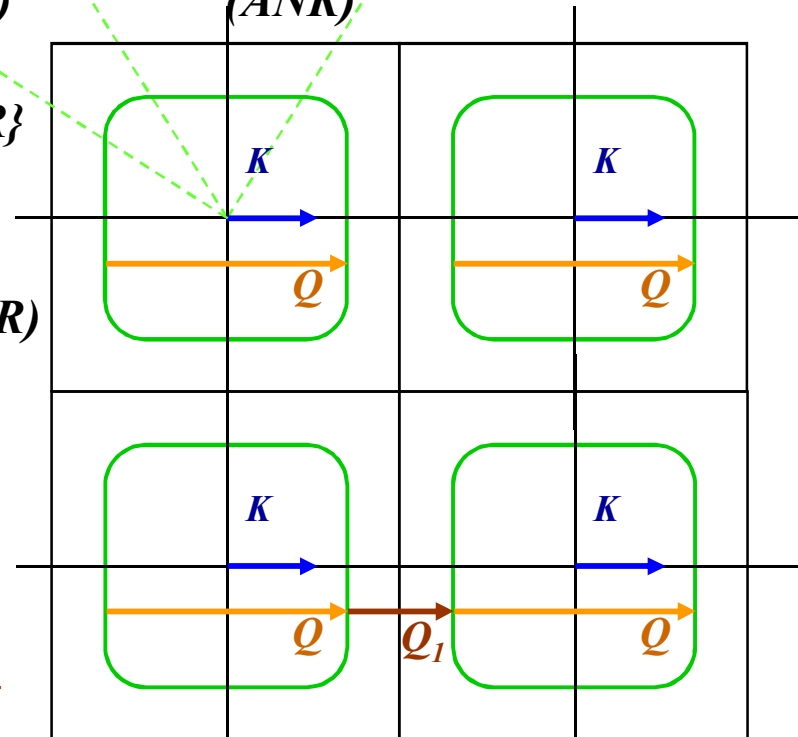
CDW (DDW) and Cooper pairing.  
K-pairing as CDW (DDW) and  
Cooper pairing induced order

**Strongly mixed biordered superconducting  
and insulating states**

V.I. Belyavsky, Yu.V. Kopaev, Nguyen Ngoc Tuan,  
*JETP* **105**, 726 (2007)

Nodal region  
(NR)

Antinodal region  
(ANR)



$Q+Q_1 =$  reciprocal lattice vector

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