Collective properties and coherent phases of graphene

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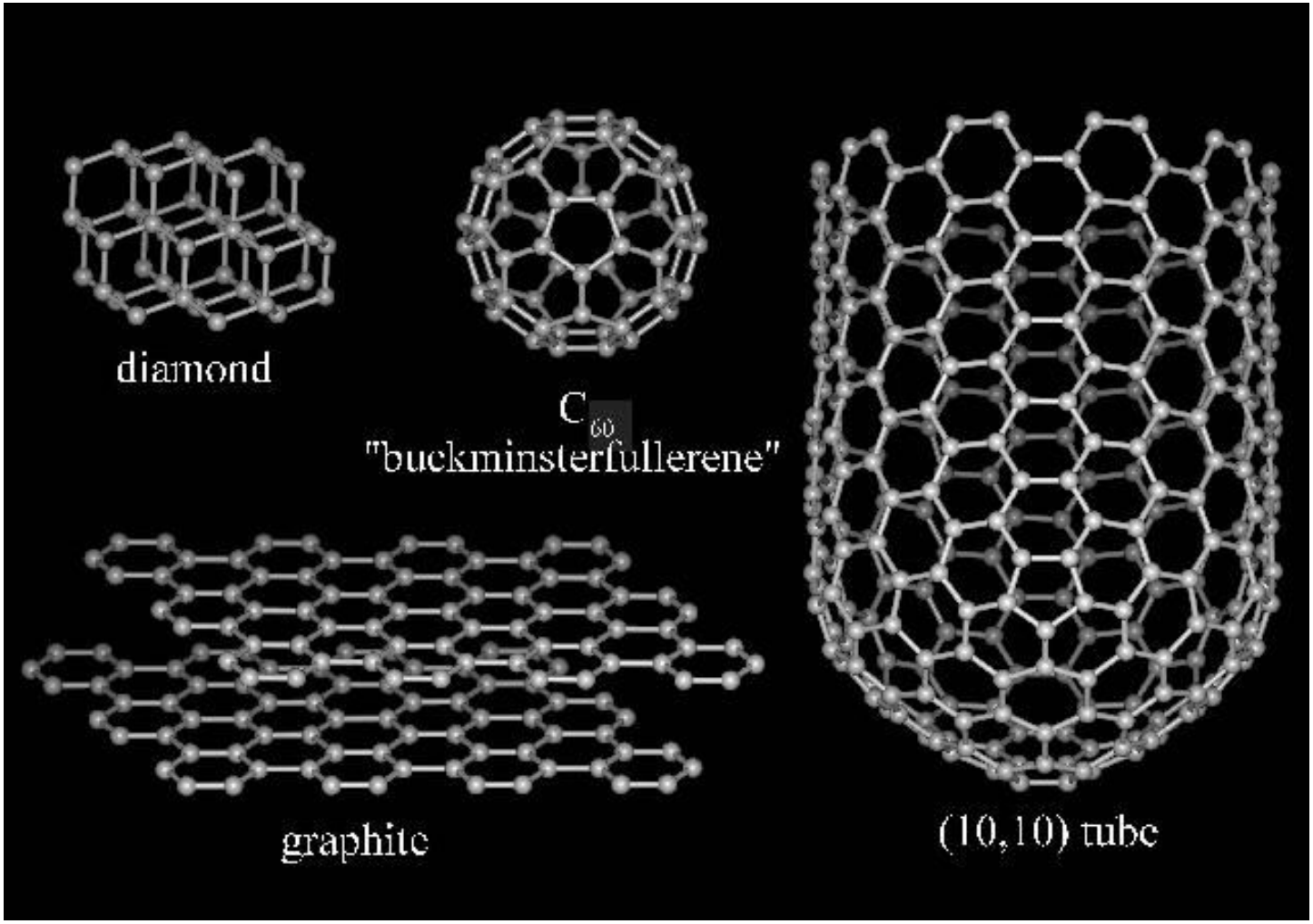
ISAN

Outline

- Allotrope forms of carbon
- Band structure and envelope function of graphene
- Pecularity of correlations in graphene
- Contolling by gate
- Coherent phases and collective properties
- Pecularities of coherent phase in graphene
- Landau quantization and anomalous QHE in graphene
- Magnentoexciton condensation
- Composite fermions

Allotropic forms of carbon

3D 0D 1D



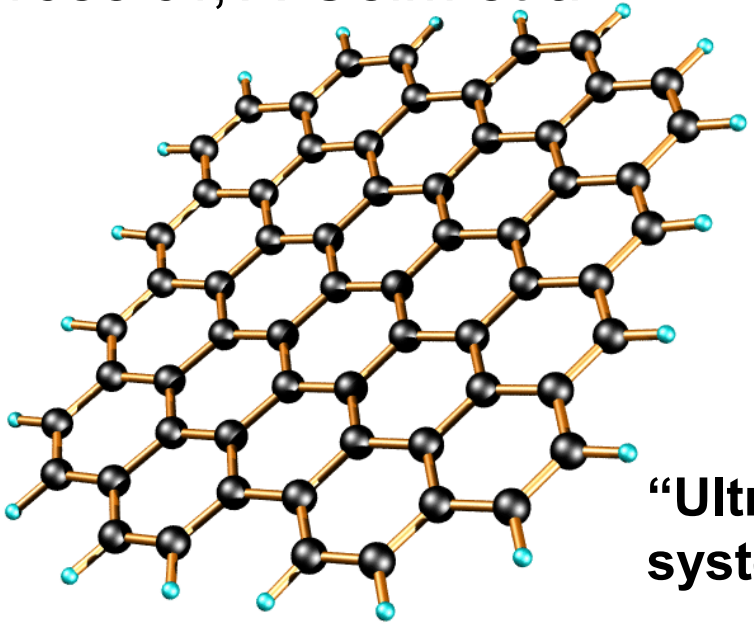
3D consisting from 2D

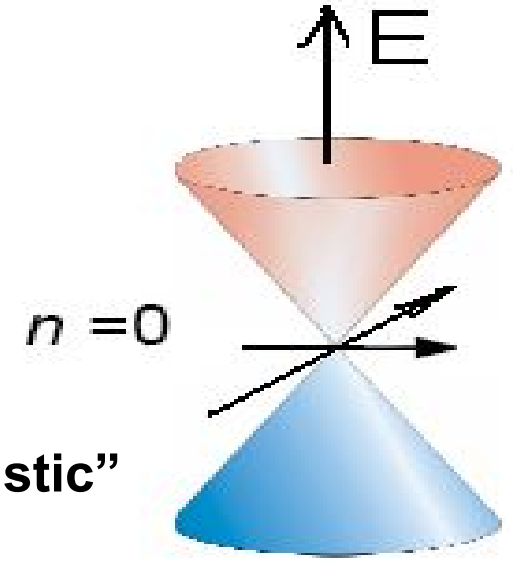
Why is graphene interesting?

- Absolute limit for Moore law – monolayer (one atom width)
- Compatibility with traditional plane transistor technology (e.g., in contrast to nanotubes)
- Graphene strength is essentially greater than that for steel

K.Novoselov, A.Geim et.al.

$E(p) = c\,p$ ($c = 10^8$ sm/sec)

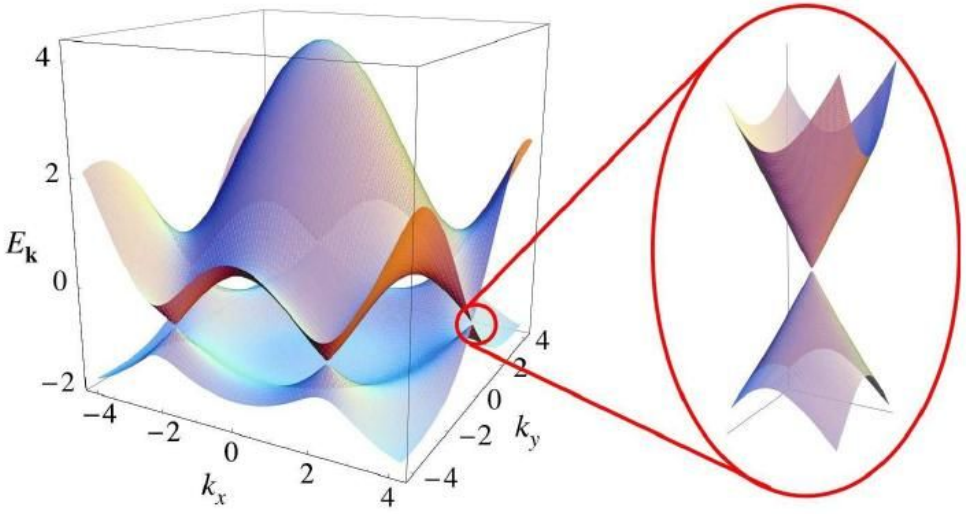


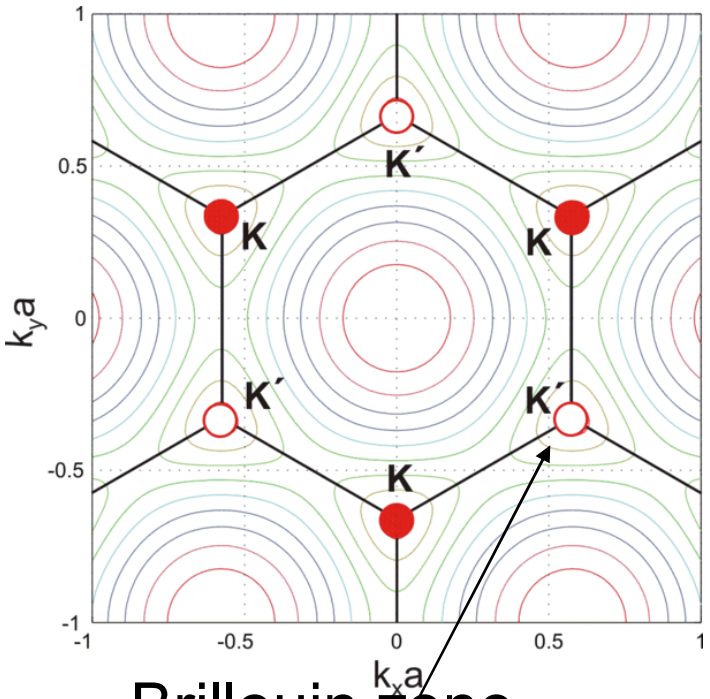


"Ultrarelativistic" system

- Effective masses of electrons and holes are equal to zero
- Energy gap is equal to zero
- Electronic properties are described by two-dimensional analogue of Dirac equation (quasiparticles are charged two-dimensional "neutrino"!)
- Bridge between solid state physics and quantum electrodynamics

Electron structure



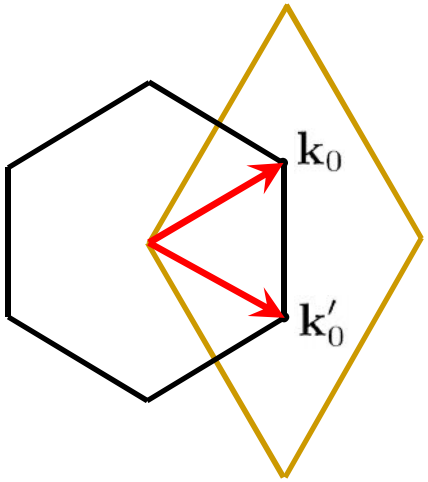


Brillouin zone

Triangle distortion

Valleys and isospin degeneration

Pseudospin operator $\frac{1}{2}\sigma$ acts in spaces of sublattices A и B.
It obeys the same commutation relations as spin operator.



First Brillouin zone has two nonequivalent points $\mathbf{k}_0$ and $\mathbf{k}_0'$ where conduction and valence band contact each other

These results to double degeneration of electronic states.
States with same energy corresponds to different valleys

The envelope wave function of electron in graphene is bispinor

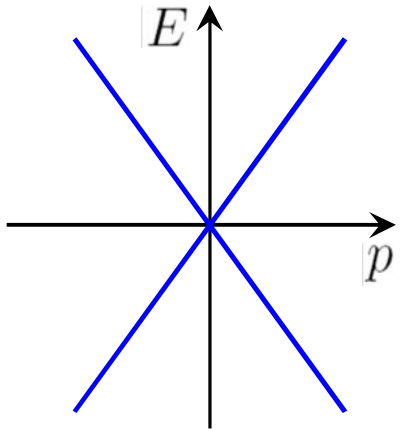
Equation for bispinor envelope wave function of electron in graphene

$$v_{\mathrm{F}}\mathbf{p}\begin{pmatrix}\sigma & 0\\ 0 & \sigma^*\end{pmatrix}\begin{pmatrix}u\\ v\end{pmatrix}=E\begin{pmatrix}u\\ v\end{pmatrix}$$

Dirac equation for massless particle in QED

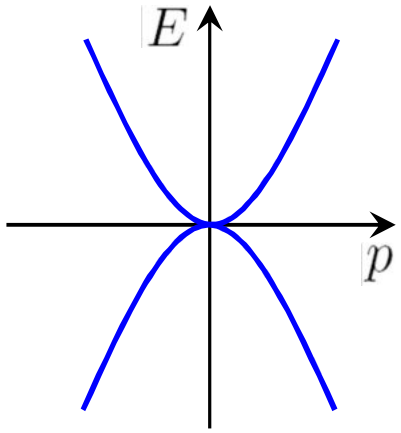
$$c\,\mathbf{p}\begin{pmatrix}0 & \sigma\\ \sigma & 0\end{pmatrix}\begin{pmatrix}u\\ v\end{pmatrix}=E\begin{pmatrix}u\\ v\end{pmatrix}$$

One-layer graphene versus many layers



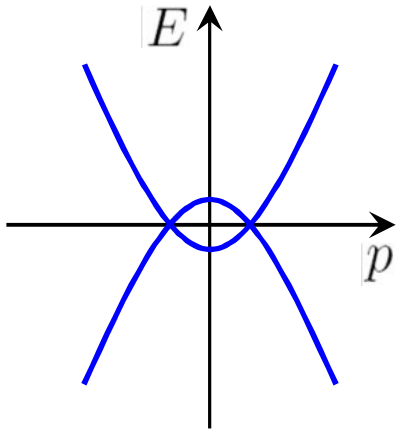
One-layer graphene

Gapless spectum
Linear dispersion



Bilayer graphene

Gapless spectum
Quadratic dispersion



Graphite

Semimetal with band intersection ~ 40 мэВ

Graphene: a bridge between QED and Cond-Mat?!

Chirality (electron pseudospin projection on the motion direction) should be conserved

$$\frac{1}{|\mathbf{p}|}(\sigma \cdot \mathbf{p}) = 1$$ for electrons and -1 for holes (compare with ν (left) and $\bar{\nu}$ (right))

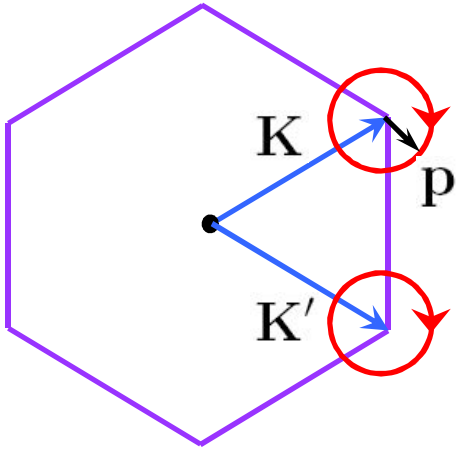
- Therefore there is no backscattering (for long-range potentials)
- Penetration through any high potential barrier
  Analogy with Klein paradox
- Absence of confinement in external potential
- Absence of isolated exciton state (without H!)

Berry phase in graphene

Diagonalizing the Hamiltonian,

$$H = v_{\mathrm{F}} \begin{pmatrix} 0 & p_x - ip_y \\ p_x + ip_y & 0 \end{pmatrix}$$

$$H\psi_{\mathbf{p},\gamma} = \gamma v_{\mathrm{F}} |\mathbf{p}| \psi_{\mathbf{p},\gamma} \qquad \gamma = \pm 1$$



one gets spinor wave function:

$$\psi_{\mathbf{p},\gamma} = \frac{e^{\frac{i}{\hbar}\mathbf{pr}}}{\sqrt{S}} \times \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-\frac{i}{2}\varphi_{\mathbf{p}}} \\ \gamma e^{\frac{i}{2}\varphi_{\mathbf{p}}} \end{pmatrix}$$

In the momentum space, while electron moves around Dirac point

$$\varphi_{\mathbf{p}} \to \varphi_{\mathbf{p}} + 2\pi$$

its wave function gains Berry phase π:

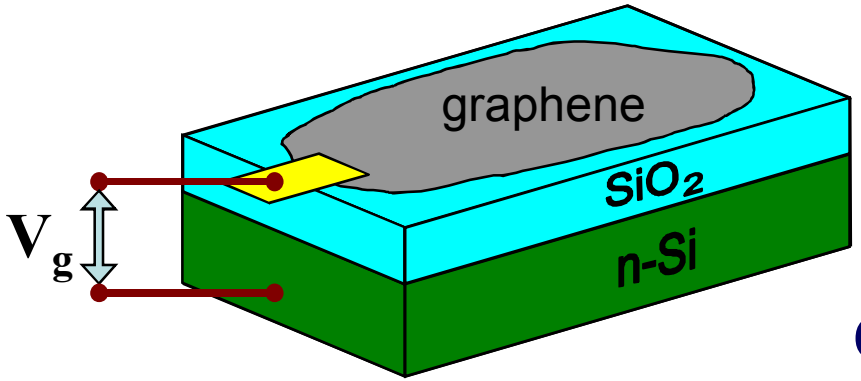
$$\psi_{\mathbf{p},\gamma} \to -\psi_{\mathbf{p},\gamma}$$

Role of Berry phase: Effects of backscattering impossibility in graphene

- **Weak localization is not possible. Positive magnetoresistance.**
- **Minimum of metallic conductivity exists; Ioffe-Regel criterium: $l >> \lambda_{Fermi}$** $\longrightarrow$

  $\sigma = ne\mu = 4e^2/h\ k_F l > 4e^2/h$ $\quad \sigma_{min} = 4e^2/h$
- **Creation of <u>controllable by voltage</u> quantum dots have peculiarity: at absence of magnetic field, the equation for massless particles has no localized solution in potential well.**
- **Pairing pecularity**

Controlling of carrier concentration



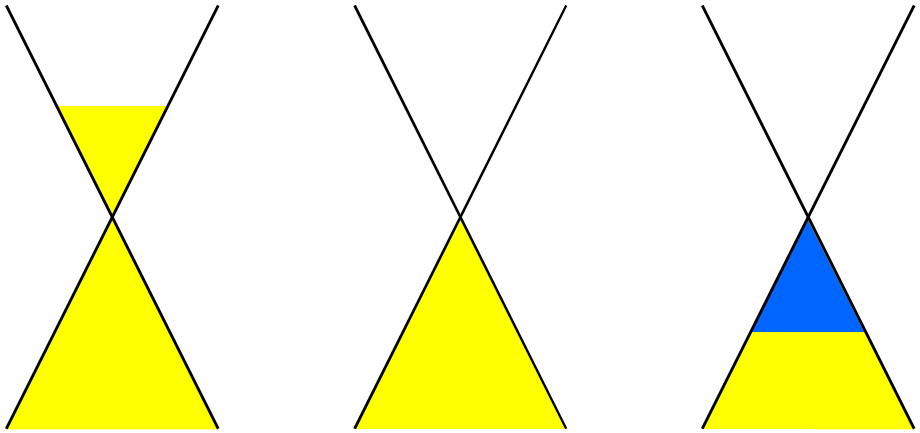
Capacitance of system:

$$C = \frac{Q}{V_g} = \frac{\varepsilon S}{4\pi d}$$

Concentration of charge carriers:

$$n = \frac{Q}{eS} = \frac{\varepsilon}{4\pi e d} V_g$$

$V_g > 0$ $V_g = 0$ $V_g < 0$



At $V_g = 100$ В $n = 7.2 \times 10^{12}$ см$^{-2}$

Varying gate voltage one can change charge carrier type and their concentration in graphene

"Quantum dots" and field-effect transistor calculation in graphene: density functional theory for envelope functions

Electron correlations in ordinary (particularly, quasi-two-dimensional) electron gas

Mean energy per electron

$$E = \left\langle \hat{T} \right\rangle + \left\langle \hat{U} \right\rangle \sim \frac{\left\langle \hat{\mathbf{p}}^2 \right\rangle}{2m} + \frac{e^2}{\varepsilon r} \sim \frac{\hbar^2}{m r^2} + \frac{e^2}{\varepsilon r}$$

$r \sim n^{-1/2}$ – mean interelectron separation, n- electron density

Role of correlations is described by the ratio

$$\frac{e^2 / \varepsilon r}{\hbar^2 / m r^2} = \frac{r}{a_0^*} \sim r_s^*$$

«Dense» gas ⇨ $r_s^* << 1$ – weak correlations, RPA

Rare gas ⇨ $r_s^* >> 1$ – strong correlations, Wigner crystallization (at $r_s^* > 37$ in 2D)

Electron correlations in graphene

$$E = \left\langle \hat{T} \right\rangle + \left\langle \hat{U} \right\rangle \sim v_{\mathrm{F}} p_{\mathrm{F}} + \frac{e^2}{\varepsilon r} \sim \frac{\hbar v_{\mathrm{F}}}{r} + \frac{e^2}{\varepsilon r}$$

Role of correlations is described by the ratio

$$\frac{e^2/\varepsilon r}{v_{\mathrm{F}} p_{\mathrm{F}}} \sim \frac{e^2}{\varepsilon \hbar v_{\mathrm{F}}} \sim \frac{2}{\varepsilon}$$

And does not depend on density!

- **Weak correlations at** $\varepsilon >> 1$
  and intermediate at $\varepsilon \sim 1$
  (ε **– dielectric susceptibility of graphene environment !)**

- **Strong correlation regime and Wigner crystal phase are absent in graphene (at** $H = 0$ **)**
- **Doped graphene is ordinary Fermi liquid (undoped graphene is marginal liquid (lack of Fermi surface)**

Role of number of components on correlations
(1/N expansion)
N=8 for graphene – 4 component envelope function
(2 sublattices x 2 valleys) x 2 real spins

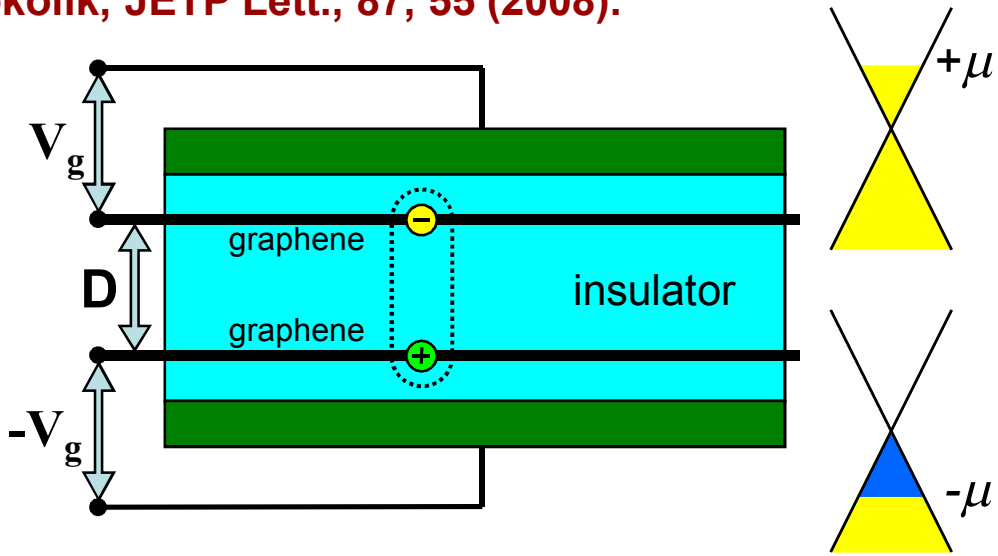
S. M. Apenko , D. A. Kirzhnits, Yu.E. Lozovik
Physics Letters A92, 103 (1982)

M.Yu. Kharitonov, K.B. Efetov, arXiv:0903.4445

Graphene bilayer

Yu.E.Lozovik, A.A.Sokolik et. al., UFN, 178, N 7 (2008).
Yu.E.Lozovik, A.A.Sokolik, JETP Lett., 87, 55 (2008).

Using opposite voltages one can produce some concentration of electrons in one graphene layer and that of holes in the other.



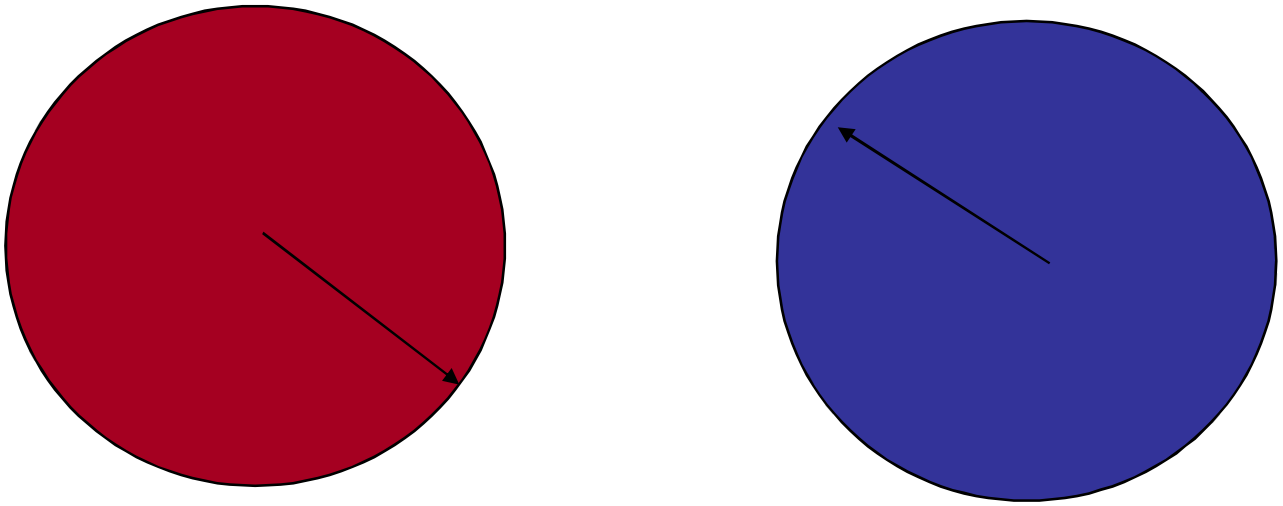
Due to Coulomb attraction electrons and holes can form pairs, resulting in decreasing of total energy of the system.

Motion of these pairs might appear as existence of nondissipative supercurrents moving on different graphene layers in opposite directions.

TWO-LAYER ELECTRON-HOLE SYSTEM. HIGH DENSITY. WEAK COUPLING.

Keldysh, Kopaev 3D equilibrium system

Lozovik, Yudson CQWs



Fermi "sphere" of electron system

Fermi "sphere" of hole system

BCS- type coherent state with the gap in spectra

ODLRO in e-h system

Condition: (Almost) congruent Fermi surfaces

Gap Δ larger than e-h Fermi surfaces difference (anisotropy, imbalance)

Why is graphene?

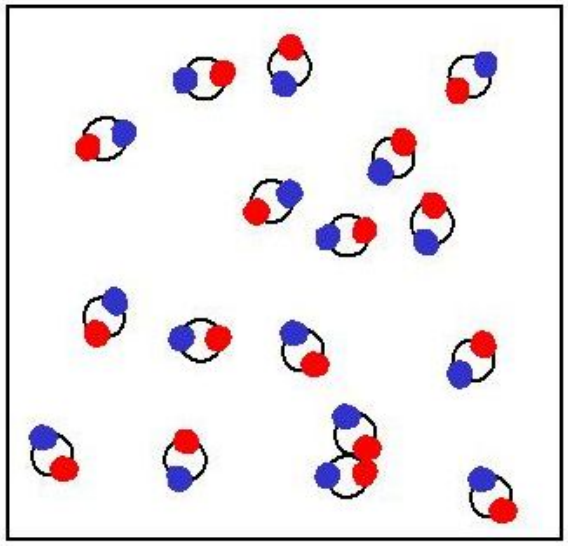
- **BCS – pairing in effective ultrarelativistic regime – analogy with the problem of** → **quark matter (quark- antiquark or quark-quark pairing ;colour superconductivity)**
- **Electron-hole symmetry - linear dispersion law are identical for electron and holes**

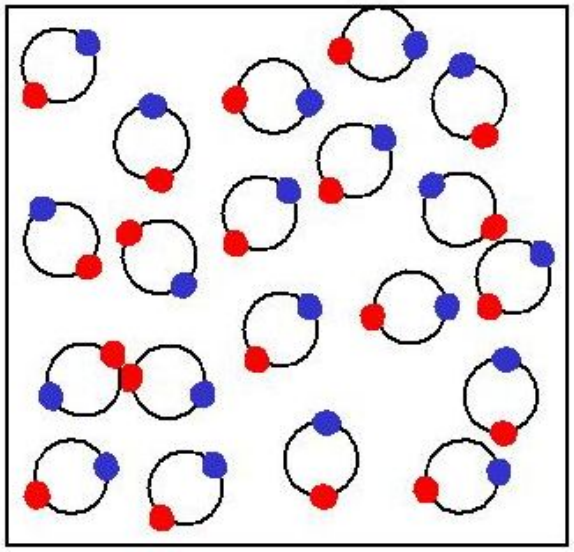
Fermi surfaces of electron and holes are congruent (which is necessary for BCS regime)

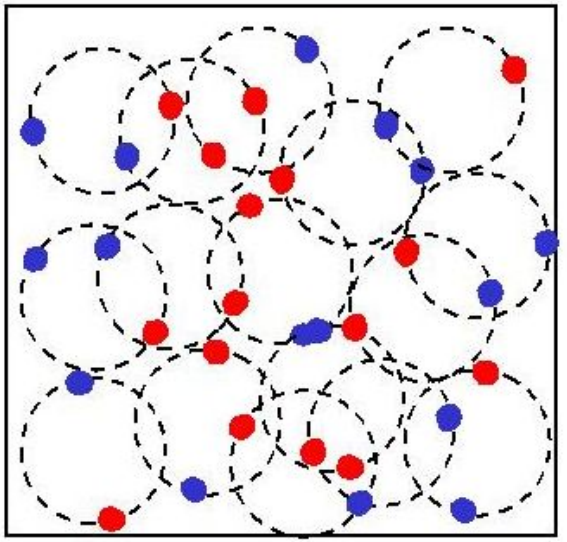
- **Weaker influence of disorder on electron-hole pairing due to Berry phase.**

BCS-BEC crossover

- for Fermi atoms
- for electron-hole pairs in semiconductors
- but not for electron-hole pairs in graphene!







BEC regime

BEC –BCS crossover

BCS regime

Local Bose pairs-
radius<<
mean interparticle separation

Cooper pairs
radius >>
mean separation

Differences from coupled semiconductor quantum wells

- Presence of Berry phase. Interaction is not isotropic - different types of pairing are possible.
- The density of states in 2D at small energies is not constant and is proportional to E
- Other screening of Coulomb interaction
- There are no excitons (spatially localized local pairs) - there is no Bose condensation of excitons
- Absence of BCS-BEC crossover at decreasing of concentration of carriers

Yu.E.Lozovik, A.A.Sokolik, JETP Lett., 87, 55 (2008).
Physics: Uspekhi ,178, N 7 (2008).

Bardeen-Cooper-Schrieffer–type pairing for coupled two-layer graphene

Hamiltonian describing electron-hole pairing

$$H = g_s g_v \sum_{\mathbf{k}} (\hbar v_F |\mathbf{k}| - \mu)(a^+_{\mathbf{k}} a_{\mathbf{k}} + b^+_{\mathbf{k}} b_{\mathbf{k}}) + \frac{g_s^2 g_v^2}{S} \sum_{\mathbf{k},\mathbf{q}} V(\mathbf{q},\omega) \frac{1+\cos\varphi}{2}\, a^+_{\mathbf{k}+\mathbf{q}} b^+_{-\mathbf{k}-\mathbf{q}} b_{-\mathbf{k}} a_{\mathbf{k}}$$

Equation for energy gap:

$$\Delta_{\mathbf{k}} = -\frac{g_s g_v}{4\pi^2} \int d\mathbf{q} V(\mathbf{q},\omega) \frac{1+\cos\varphi}{2} \frac{\Delta_{\mathbf{k}+\mathbf{q}}}{2\sqrt{\xi^2_{\mathbf{k}+\mathbf{q}} + \Delta^2_{\mathbf{k}+\mathbf{q}}}}$$

$$\xi_{\mathbf{k}} = \hbar v_F |\mathbf{k}| - \mu$$

Yu.E.Lozovik, A.A.Sokolik, JETP Lett., 87, 55 (2008).
Physics: Uspekhi ,178, N 7 (2008).

M.Yu. Kharitonov, K.B. Efetov, arXiv:0903.4445

Dynamical screening

Bare interaction

$$v_{\mathbf{q}} = \frac{2\pi e^2}{\varepsilon q}$$

Screen ineraction between graphene layers

$$V(\mathbf{q},\omega) = \frac{-v_{\mathbf{q}} e^{-qD}}{1 - v_{\mathbf{q}}(\chi_1 + \chi_2) + v_{\mathbf{q}}^2 \chi_1 \chi_2 (1 - e^{-2qD})}$$

Dedominator=0 →2 modes plasma oscillations:

Anti-phase

$$\omega_-(q) = (k_0 D/\alpha)^{1/2} v_{\mathrm{F}} q$$

In phase

$$\omega_+(q) = (2k_0 q/\alpha)^{1/2} v_{\mathrm{F}}$$

$$k_0 = \frac{\mu}{\hbar v_{\mathrm{F}}}$$ – Fermi momentum

$$\alpha = \frac{2\varepsilon \hbar v_{\mathrm{F}}}{g_{\mathrm{s}} g_{\mathrm{v}} e^2} \approx 0.23 \times \varepsilon$$

$$\tilde{\omega} = \omega_-(\tilde{q})$$

$$\hbar\tilde{\omega} = \frac{4\mu (k_0 D)^{1/2}}{\alpha^{3/2}}$$

Controlling parameters

$$\alpha = \frac{\varepsilon \hbar v_F}{2e^2} \approx 0.23 \times \varepsilon$$

Ratio of Coulomb and quantum kinetic energy

$$k_0 = \frac{\mu}{\hbar v_F}$$

Fermi momomentum

Three characteristic lengthes:

- interlayer distance
- interparticle distance
- radius of Thomas- Fermi screening

α and $k_0 D$

Energy gap

Yu.E.Lozovik, A.A.Sokolik, JETP Lett., 87, 55 (2008).
Physics: Uspekhi ,178, N 7 (2008).

Asimptotical expressions for energy gap:

$$\alpha << k_0 D \quad \Rightarrow \quad \Delta = \frac{4\mu}{(k_0 D\alpha)^{1/2}} \exp\left\{-8\pi k_0 D\left(1+\frac{k_0 D}{\alpha}\right)\right\}$$

$$k_0 D << 1 << \alpha \quad \Rightarrow \quad \Delta = \frac{8\mu}{\alpha} \exp\left\{-\frac{2\pi\alpha}{\ln(1+\alpha/2)}\right\}$$

$$1 << k_0 D << \alpha \quad \Rightarrow \quad \Delta = \frac{8\mu}{\alpha} \exp\left\{-\frac{2\pi\alpha}{\ln(\alpha/4k_0 D)-\gamma}\right\}$$

Landau quantization in graphene at room temperature

The equation for electrons In magnetic field:

$$v_{\mathrm{F}}\left(\mathbf{p}+\frac{e}{c}\mathbf{A}\right)\sigma\psi = E\psi \qquad \psi = \begin{pmatrix} |\psi_1\rangle \\ |\psi_2\rangle \end{pmatrix}$$

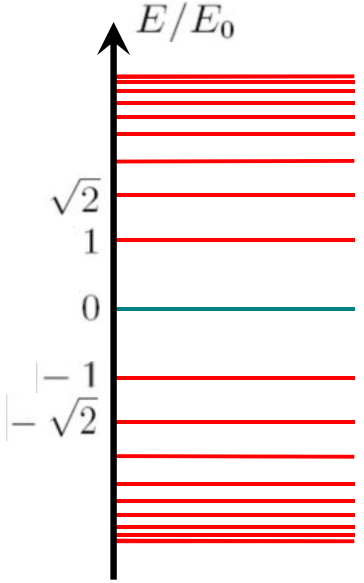
In gauge $\mathbf{A} = \frac{1}{2}[\mathbf{H}\times\mathbf{r}]$ **it can be reduced to**

$$\begin{cases} \hat{a}^{+}|\psi_2\rangle = \dfrac{E}{E_0}|\psi_1\rangle \\ \hat{a}|\psi_1\rangle = \dfrac{E}{E_0}|\psi_2\rangle \end{cases}$$

$$\left[\hat{a},\hat{a}^{+}\right] = 1 \qquad E_0 = v_{\mathrm{F}}\frac{\sqrt{2}\,\hbar}{l} \qquad l = \sqrt{\frac{\hbar c}{eH}}$$

Energy levels and corresponding vectors of states:

$$E_n^{\pm} = \pm\sqrt{n}\,E_0 \qquad \psi_n^{\pm} = \begin{pmatrix} |n\rangle \\ \pm|n-1\rangle \end{pmatrix}$$



Degeneracy of each level $E_n^{\pm}$ **with** $n \neq 0$ **twice is in two time greater, than for a level** $E = 0$

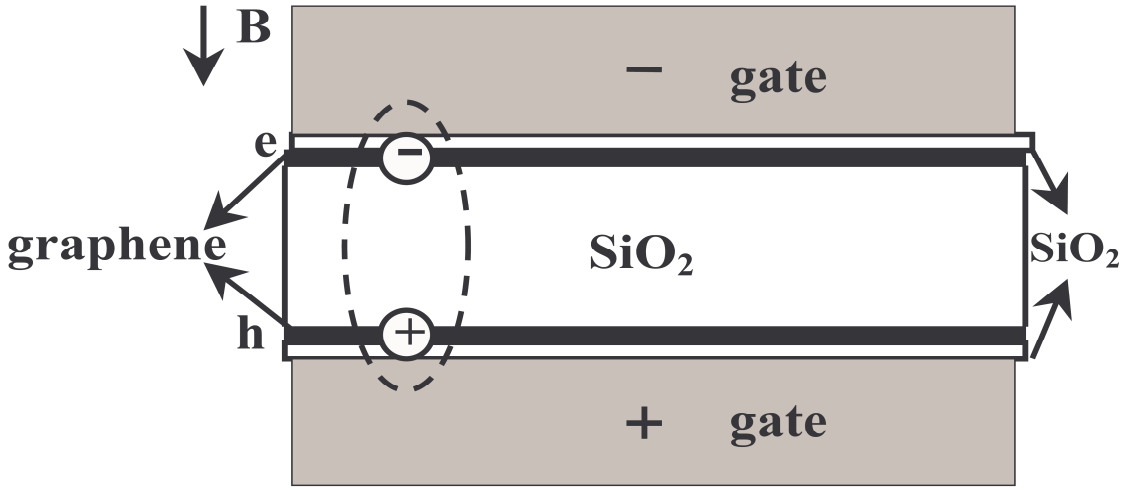
The interlevel distance is of order 1000 K at fields 10 Tesla !!

Indirect magnetoexcitons in two graphene layers

1. **equilibrium:**

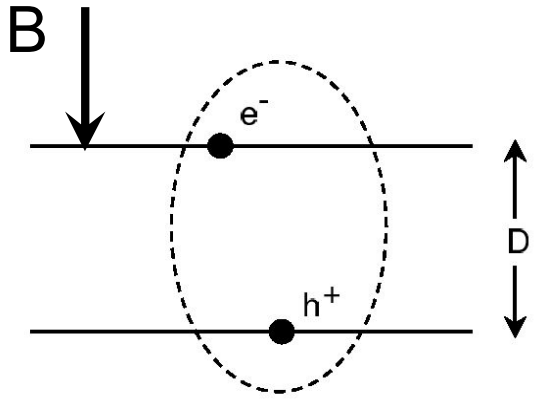
Landau level: $e \rightarrow n_L = +1$

$h \rightarrow n_L = -1$



2. **Quasiequilibrium (laser pumping):**

Indirect magnetoexcitons:



Excitons are absent in graphene but

Magnetoexcitons in graphene

Hamiltonian

$$\hat{H} = v_F \begin{pmatrix} 0 & p_x^{(e)} + ip_y^{(e)} & 0 & 0 \\ p_x^{(e)} - ip_y^{(e)} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_x^{(h)} - ip_y^{(h)} \\ 0 & 0 & p_x^{(h)} + ip_y^{(h)} & 0 \end{pmatrix}$$

Berman,Lozovik,Gumbs Phys. Rev. B 77, 155433 (2008).
Physics: Uspekhi ,178, N 7 (2008).

$$\mathrm{p}^{(e)} = -i\hbar\nabla_e + \frac{e}{c}\mathbf{A}_e; \qquad \mathrm{p}^{(h)} = -i\hbar\nabla_h - \frac{e}{c}\mathbf{A}_h$$

Integral of motion: Magnetic momentum

$$\hat{\mathbf{P}} = -i\hbar\nabla_e - i\hbar\nabla_h + \frac{e}{c}(\mathbf{A}_e - \mathbf{A}_h) - \frac{e}{c}[\mathbf{B} \times (\mathbf{r}_e - \mathbf{r}_h)]$$

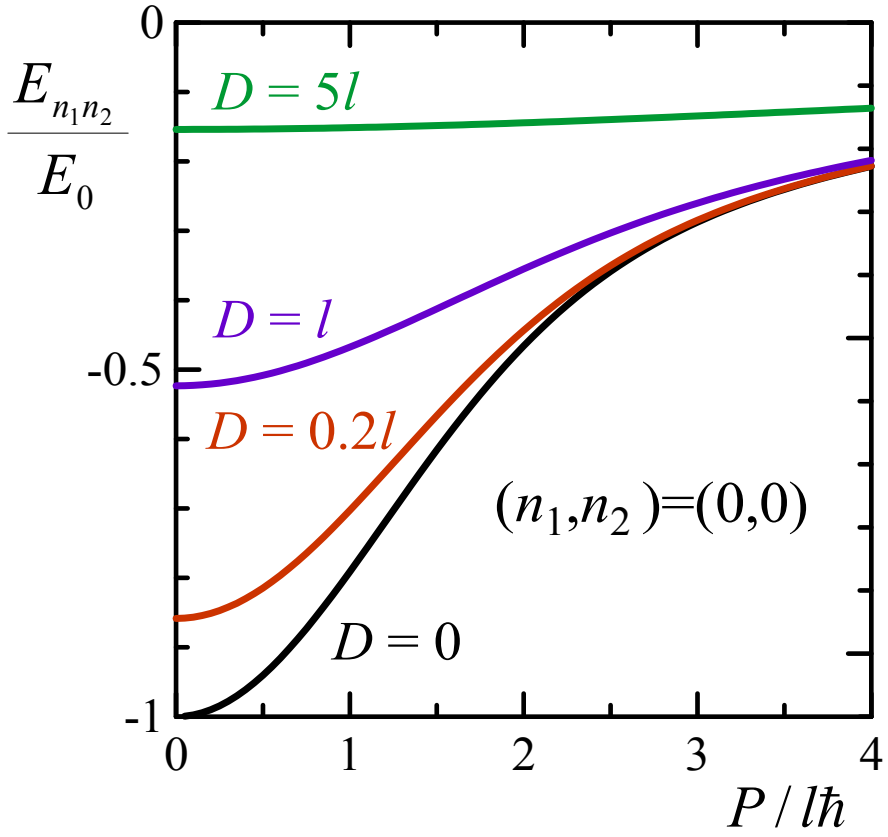
Wave function of e-h pair

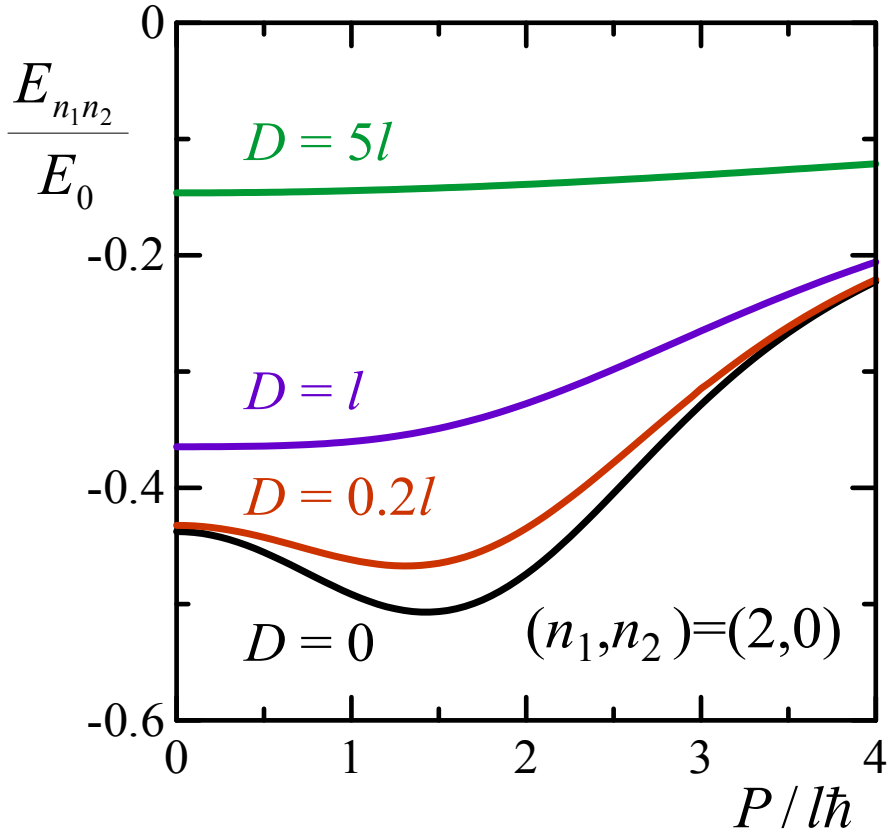
$$\tilde{\Phi}_{n_+,n_-}(\mathbf{r}) = \left(\sqrt{2}\right)^{\delta_{n_+,0}+\delta_{n_-,0}-2} \begin{pmatrix} s_+ s_- \Phi_{|n_+|-1,|n_-|-1}(\mathbf{r}) \\ s_+ \Phi_{|n_+|-1,|n_-|}(\mathbf{r}) \\ s_- \Phi_{|n_+|,|n_-|-1}(\mathbf{r}) \\ \Phi_{|n_+|,|n_-|}(\mathbf{r}) \end{pmatrix}$$

Lerner, Lozovik – for 2D magnetoexcitons in semiconductor leyers

Dispersion dependence of magnetoexcitons

$$E_{n_1 n_2}(P,D) = \left\langle \Phi_{n_1 n_2} \left| -\frac{e^2}{\varepsilon\,|\,\mathbf{r}+\mathbf{r}_0\,|} \right| \Phi_{n_1 n_2} \right\rangle \qquad E_0 = \frac{e^2}{\varepsilon l}\sqrt{\frac{\pi}{2}}$$





Binding energy and effective magnetic mass

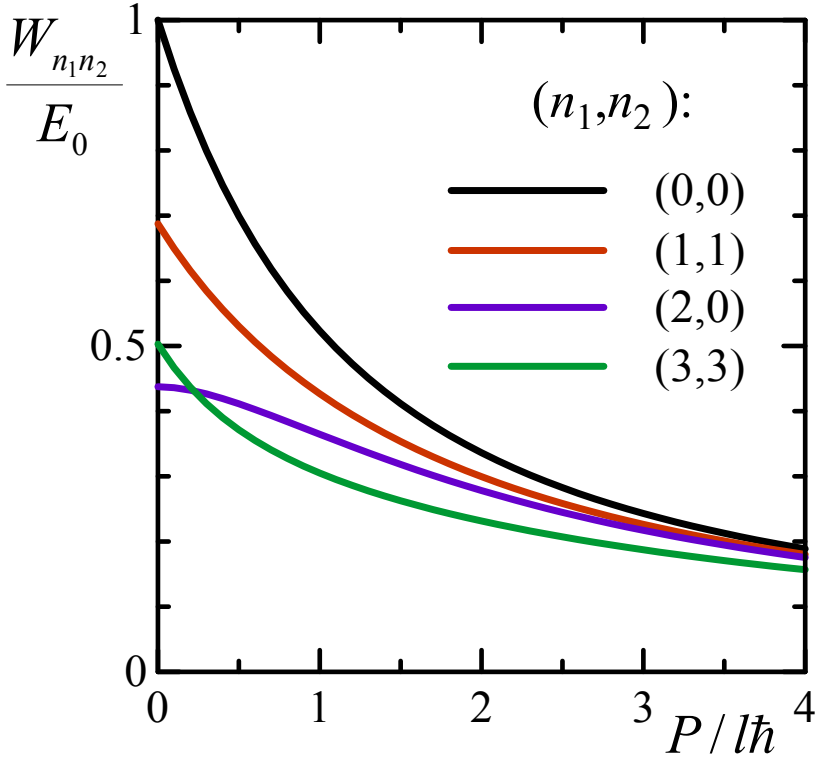
At small P

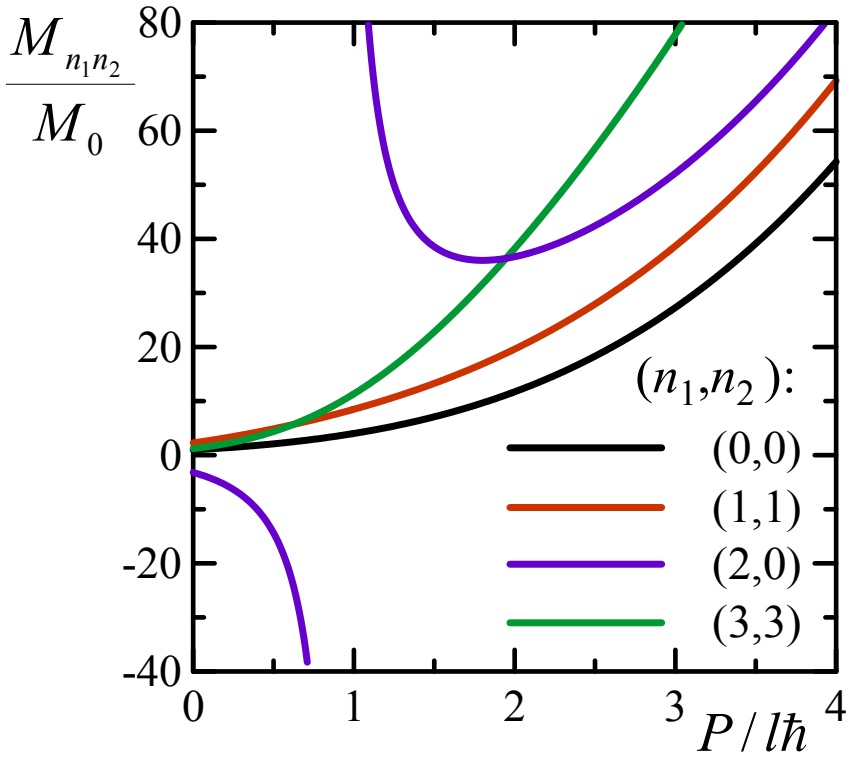
$$E_{n_1 n_2} \approx -W_{n_1 n_2} + \frac{P^2}{2 M_{n_1 n_2}}$$

Asymptotcs at

$$\begin{cases} W \approx \dfrac{e^2}{\varepsilon D} \\ M \approx \dfrac{M_0}{2}\sqrt{\dfrac{\pi}{2}}\left(\dfrac{D}{l}\right)^3 \end{cases} \qquad D >> l, \quad D >> Pl^2/\hbar$$

$$M_0 = \frac{2\hbar^2}{E_0 l^2}$$





Berman, Gumbs,Lozovik, Phys.Rev. B77,155433(2008)and Refs.

Mapping:

The problem of excitons superfluidity in high magnetic fields B $\longrightarrow$

$\longrightarrow$ The problem of excitons without B but with effective (magnetic mass) defined from isolated magnetoexciton dispersion law.

For one graphene layer or isolated quantum well in high B :

EXACT SOLUTION – Bose-condensate of noninteracting magnetoexcitons

For coupled graphene layers or CQW – superfluid Kosterlitz-Thouless transition.

Kosterlitz-Thouless transition temperature for 2D magnetoexcitons vs. density

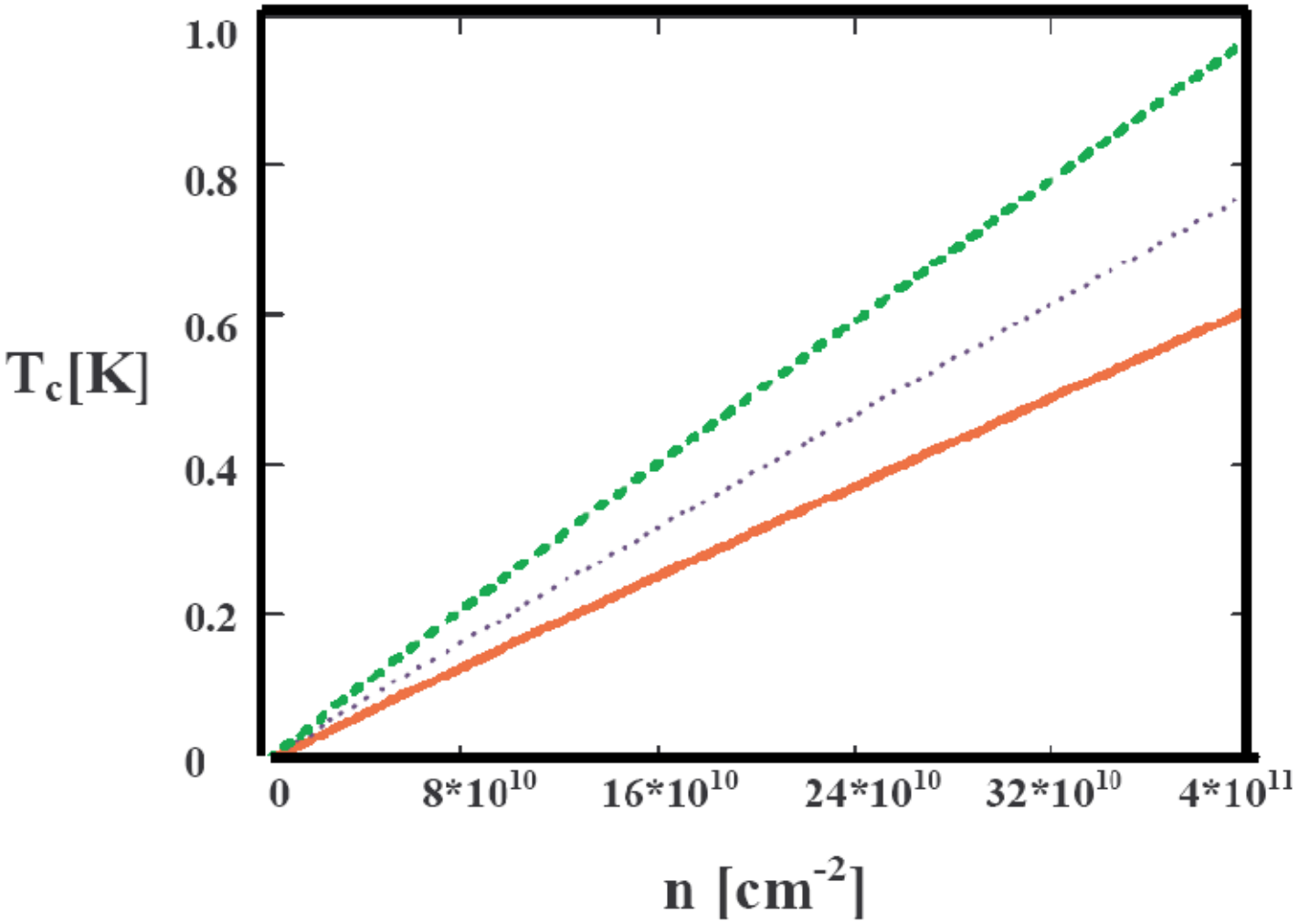


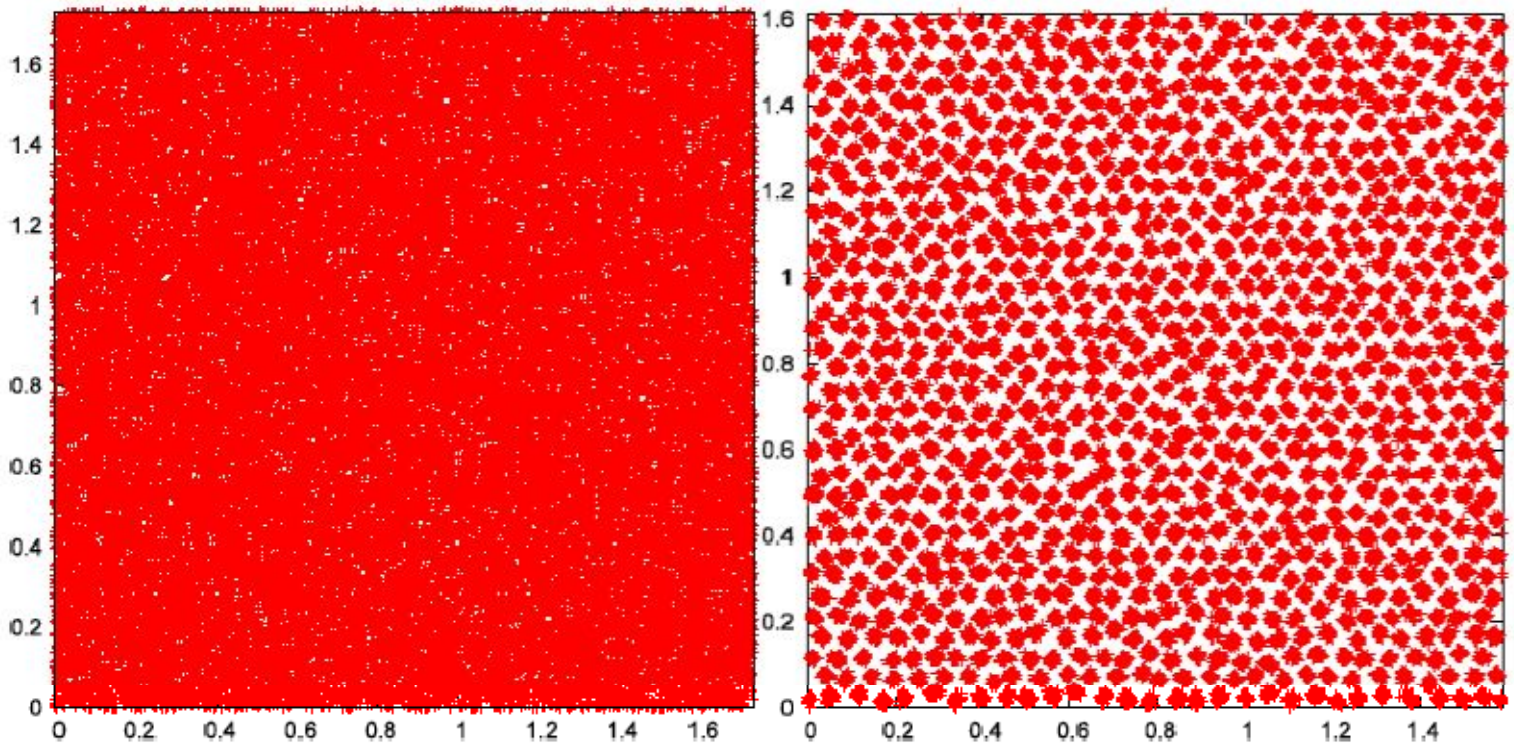
FIG. 3: Dependence of temperature of Kosterlitz-Thouless transition $T_c = T_c(B)$ (in units K; for bilayer graphene layers separated by the layer of SiO_2 ($\epsilon = 4.5$) on the magnetoexciton density n (in units cm^{-2}) at the interlayer separation $D = 25nm$ at different magnetic fields B: $B = 23T$ – solid curve; $B = 20T$ – dotted curve; $B = 17T$ – dashed curve.

Berman,Lozovik et.al.,
Phys. Rev. B 77, 155433 (2008).).

Dipole magnetoexciton crystallization

Lozovik,Berman instability to CDW in-phase
Astrakharchik, Lozovik et.al. PRL 98,060405 (2007) quantum phase transition

liquid



crystal

Quantum (diffusional)Monte Carlo: large d -strong correlated regime. Crystallization at Lindemann parameter =0.23, nr_0^2=290
Supersolid?

Effects of strong correlations in magnetoexciton system

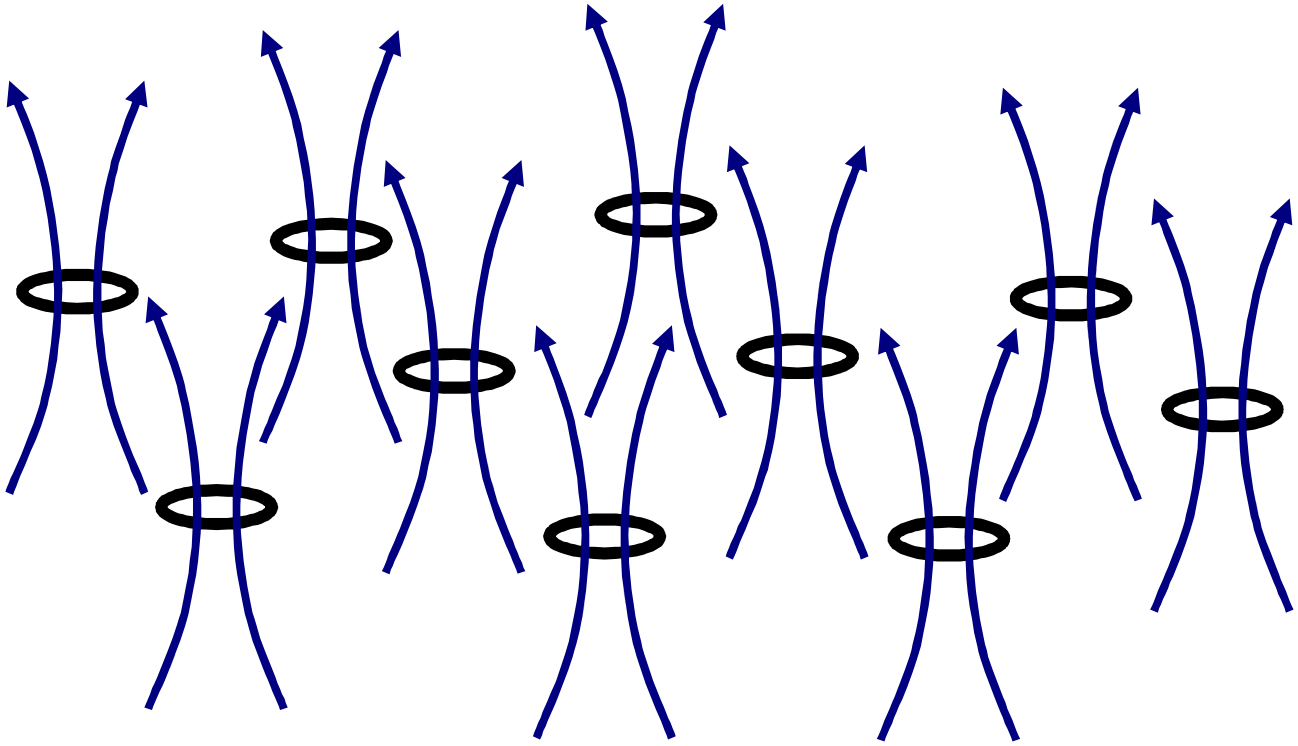
- Depletion of condensate
- Increasing of depletion region in real space
- Appearance of short range order
- Essential modification of excitation spectra

and appearance of roton minimum

2D electrons in the strong magnetic field.

Landau level filling is $\nu = 1/2$

2 flux quanta attached to each electron = composite fermions



Double - layer electron or electron-hole system

$$\nu_e^{(1)} = \nu_e^{(2)} = 1/2$$

2 flux quanta per each quasiparticle

Attach 2 flux quanta to each electron(composite fermion)

Jain; Halperin, Lee, Read

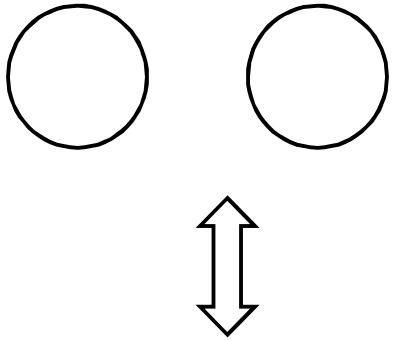
$$B_{eff} = B_{ext} - B_{gauge} = 0 \rightarrow$$

Fermi surfaces for composite fermions in both layers.

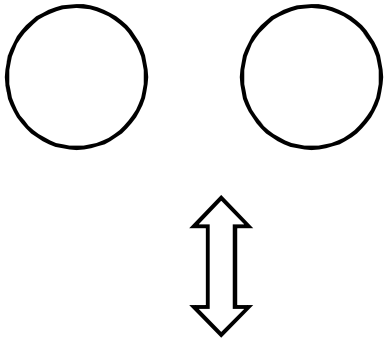
Composite fermions in zero mean magnetic field.

Mapping

- Two layers $\nu=1/2+1/2 \rightarrow$ Two Fermi surfaces of composite fermions with attraction between particles (electrons and holes) near the Fermi surface



- BCS instability in system with two Fermi surfaces with attraction (the role of a spin plays an index of the layer)



Two-layer system of electrons and hole with BCS pairing
(Lozovik, Yudson, 1975; Lozovik, Berman; Lozovik, Klyuchnik)

Conclusion

- “Ultrarelativistic” e-h pairing in graphene was considered
- Difference and similarity in “nonrelativistic” (nonzero effective mass) and “ultrarelativistic” graphene cases were analyzed
- Strong magnetic fields for e-h bilayers
- Theory of 2D dipole rarified Bose systems of magnetoexcitons
- 2D dipole quantum crystal of magnetoexcitons

Thanks