Monodromies and Confinement in Supersymmetric QCD

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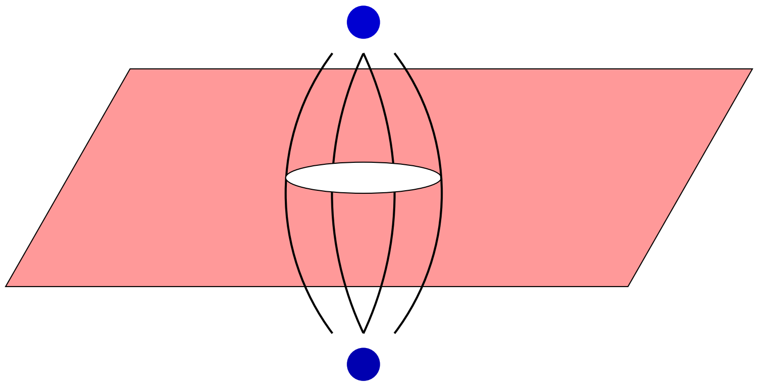
(based on joint work with Alexei Yung)

Meissner mechanism in superconductor: condensation of electric charge (red) kills magnetic field except for a tube, ensuring confinement of magnetic monopoles (blue).



To turn into problem of particle QFT one needs duality between electric and magnetic charges.

Dual picture - confenement of quarks?

Supersymmetric QCD: $\mathcal{N} = 2$ softly broken to $\mathcal{N} = 1$.

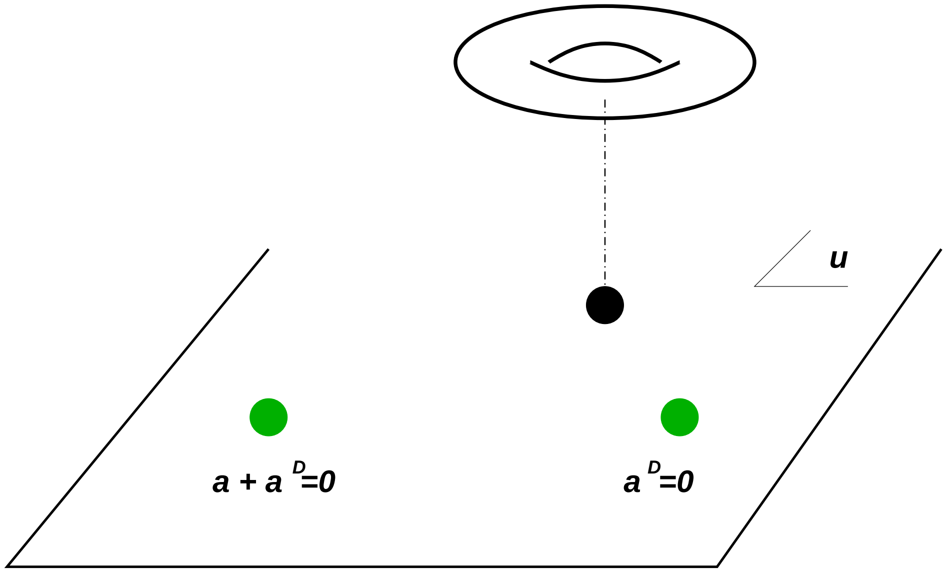
- Adjoint vector multiplet: Φ, A, . . .

- Fundamental (anti-fundamental) matter: Q^B, ($\tilde{Q}_B$) . . . with masses m_B; $B = 1, \ldots, N_f$

- Superpotential $\mathcal{W} = \tilde{Q}_B \Phi Q^B + \mu \mathrm{Tr} \Phi^2$

$SU(N_c)$ theory with $N_c < N_f < 2N_c$, broken down *minimally* to $SU(N_c - 1) \times U(1)$ (maximally to $U(1)^{N_c - 1}$).

$N_c = 2$ pure gauge theory: quantum moduli space ($u = \text{Tr}\Phi^2$)



contains $\mathcal{N} = 2$ singularities ($\mathcal{N} = 1$ vacua) with massless monopole at $a^D = 0$ and dyon at $a + a^D = 0$ (at strong coupling: instead of $u = 0$).
BPS masses $a(u)$, $a^D(u)$ - exactly computed SW period integrals, $a(u) \sim \sqrt{u}$, $a^D(u) \sim \sqrt{u} \log u$ at large u (semiclassical regime).

No vacuum at weak coupling, instead the effective theory at $a^D(u) = 0$, $u \sim \Lambda^2 \neq 0$

- $a^D = 0$, $a \neq 0$, $g_D(u) \sim \frac{1}{g(u)} \ll 1$ of $U(1)$ magnetic theory
  $\mathcal{L}_M = \frac{1}{g_D^2}\left(F^D_{\mu\nu}\right)^2 + |\nabla^D_\mu M|^2 + |\nabla^D_\mu \tilde{M}|^2 + \dots$;

- $\mathcal{N} = 1$ (super)potential $\mathcal{W} = \mu u(a^D) + \tilde{M} a^D M$ causes condensation of monopoles

  $$\langle \tilde{M} M \rangle \sim \mu\Lambda$$

  and confinement of electric charges (quarks (?!)) by ANO strings with tension $T \sim \mu\Lambda$.

No “real” quarks and no smooth interpolation ...

For the non-Abelian confinement ($SU(N_c-1)$ at scales between $\sqrt{\mu\Lambda}$ and Λ)

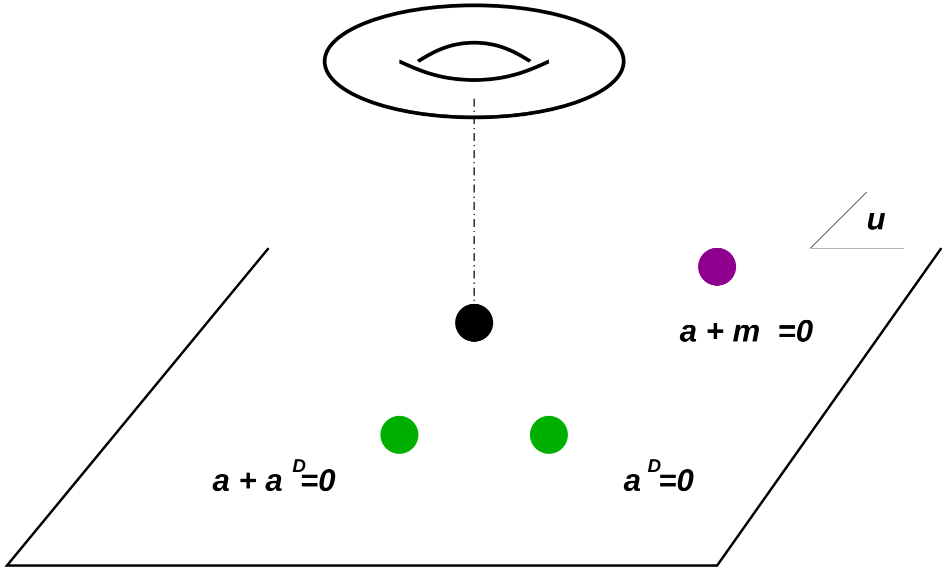
- Supersymmetric QCD with $N_c \geq 3$ and large fundamental masses: weak coupling $m \gg \Lambda$ and confinement of monopoles by ANO strings.

- Towards strong coupling: towards regime of dual theory, $m \ll \Lambda$, change of quantum numbers due to monodromies.

- Exact solution based on studying (degenerations of) the curve

$$W + \frac{\Lambda^{2N_c}}{W} = \frac{P_{N_c}(x)}{\sqrt{Q_{N_f}(x)}}$$

Supersymmetric QCD (e.g. $N_c = 2$) with matter of mass m



contains also (weakly coupled for $m \gg \Lambda$) quark singularity (vacuum) at $a + m = 0$. Here

$$a = \oint_A x \frac{dW}{W}, \quad a^D = \oint_B x \frac{dW}{W}, \quad m = \text{res } x \frac{dW}{W}$$

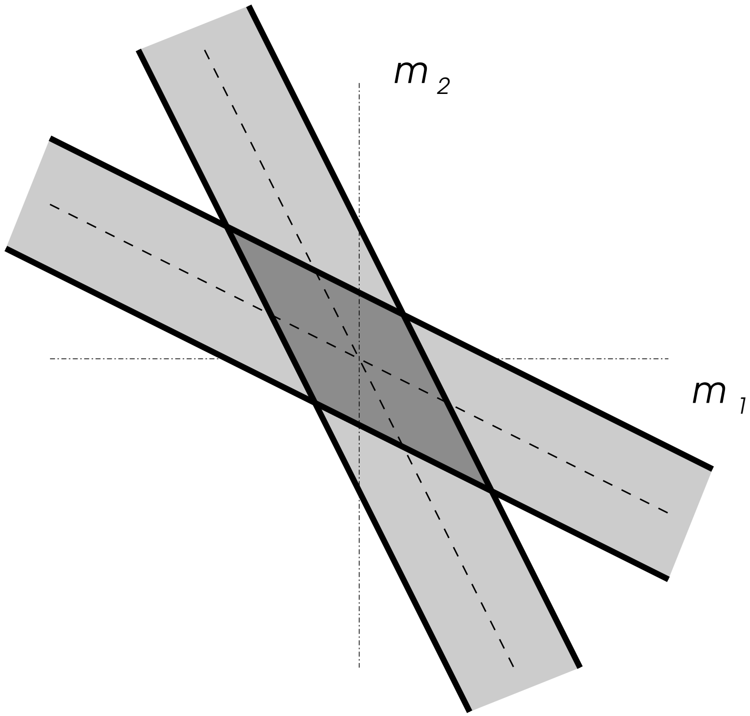
Regime of weak coupling:
$\beta_{\mathsf{UV}} = 2N_c - N_f > 0$: asymptotic freedom in UV;
$\beta_{\mathsf{eff}} = 2(N_c - 1) - N_f = \beta_{\mathsf{UV}} - 2 \leq 0$: IR free or conformal theory: allows semiclassical analysis!

Hence, $2N_c - 2 \leq N_f < 2N_c$, e.g. $N_c = 3$, $N_f = 4, 5$ with non Abelian confinement in IR: at least $SU(2) \times U(1)$ non Abelian gauge symmetry.

$r = 2$ vacuum ($r = N_c - 1$ for $SU(N_c - 1)$): at least two (light) flavors condense, one gets the non Abelian structure for the (e.g. doublets) of confined states.

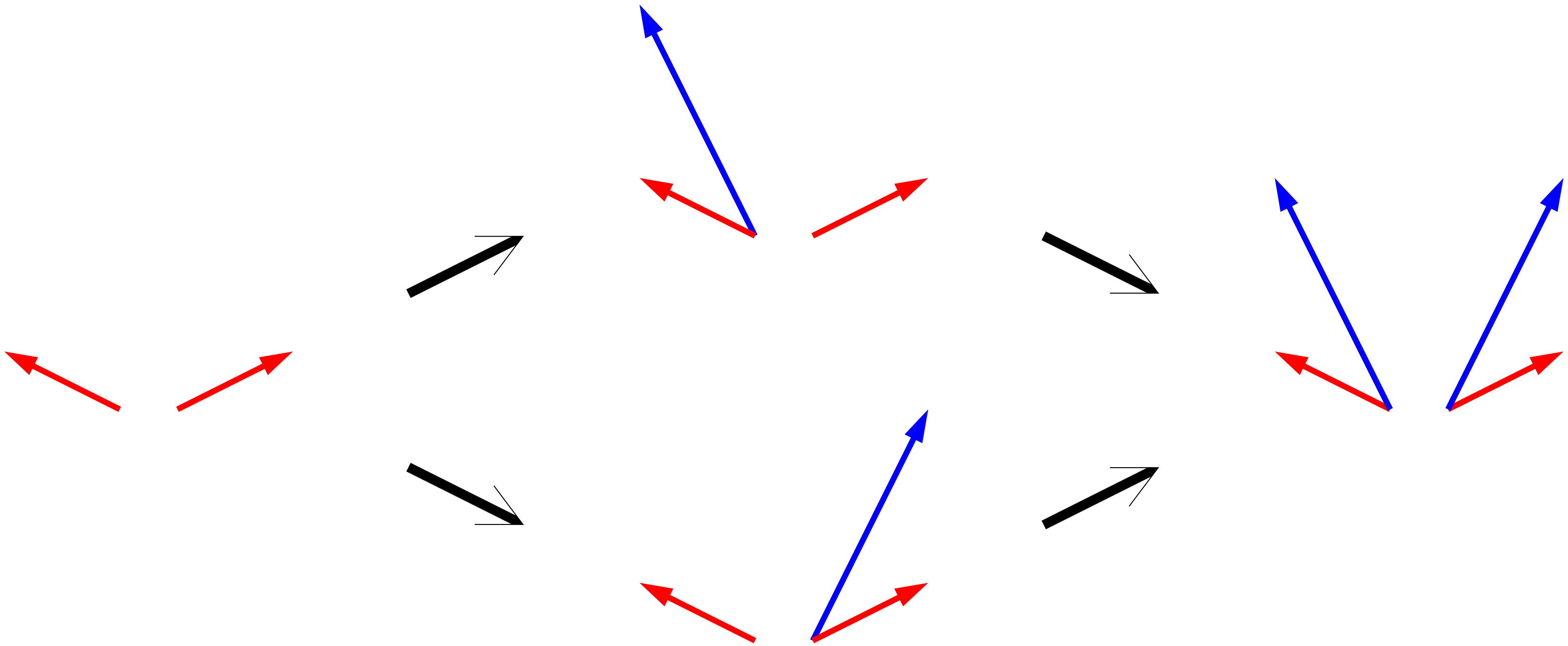
Different effective theories (light fields, Lagrangians,...) at different regions of moduli space.

$N_c = 3$, $N_f = 4$ theory with pairwise coinciding masses $m_1 = m_3$, $m_2 = m_4$.



weakly-coupled (white), intermediate (grey) and strongly-coupled (dark) regions - the real slice of complex mass picture.

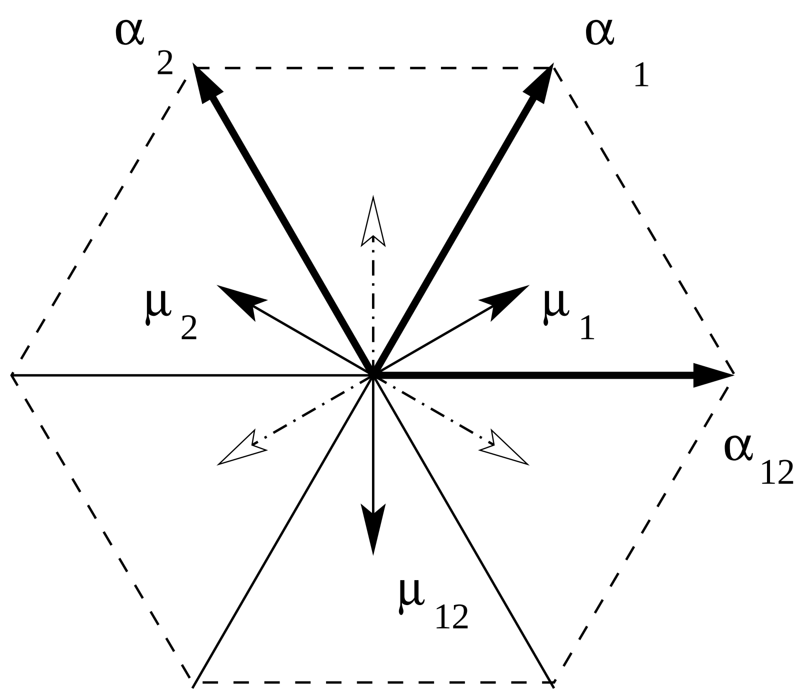
Change of the quantum numbers due to monodromies:



the doublet of quarks (weak coupling region) turns finally into the doublet of dyons (at strong coupling).

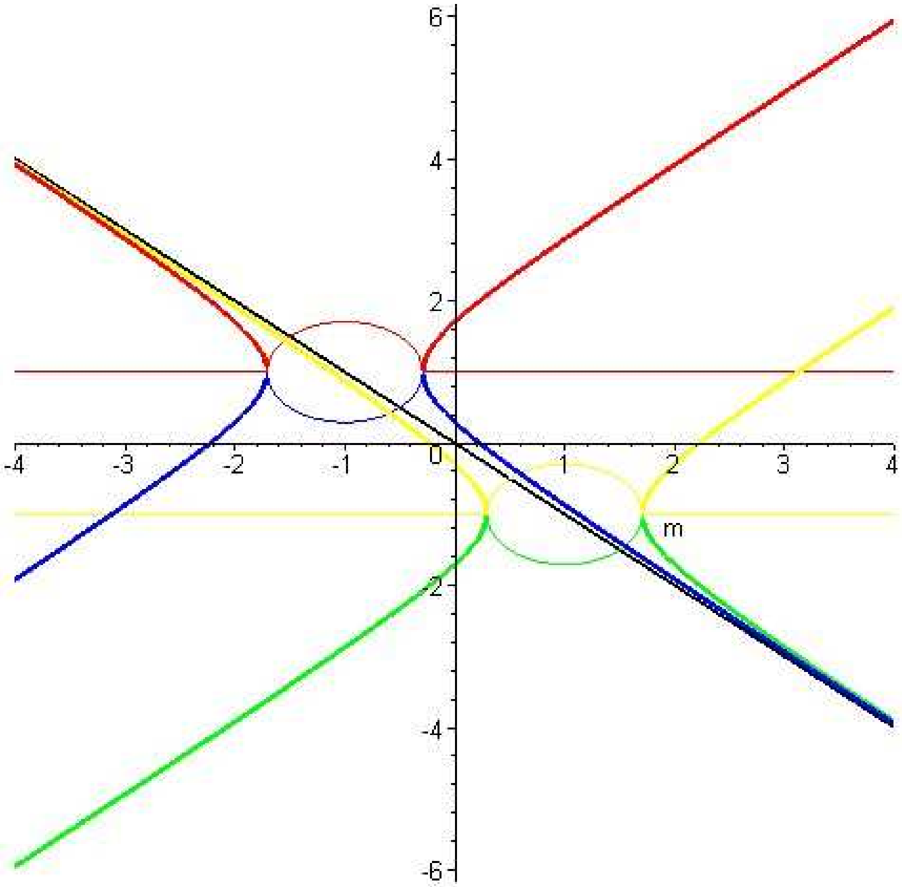
Colliding of $r = 2$ vacua with $r = 1$ vacua at border lines.

Quark charges (red) are weights and monopole charges (blue) are roots of the $SU(3)$ gauge group



$\boldsymbol{\mu}_i \cdot \boldsymbol{\alpha}_j = \delta_{ij} \Rightarrow A_i \circ B_j = \delta_{ij}$ intersection of the cycles: physical charges different from "homological charges".

Monodromies, say, in the effective theory with $N_c - 1 = 2$ and $\hat{N}_f = 2$ with equal (effective) mass, e.g. $\hat{m} = m_1 + \frac{1}{2}m_2$ (real slice)



Branch points of the SW curve

$$y^2 = \left(x^2 - \widehat{u} + \Lambda^2\right)^2 - 4\Lambda^2(x + \widehat{m})^2$$

with

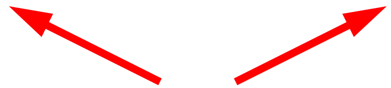
$$\widehat{m} = m_1 + \frac{m_2}{2}, \qquad \widehat{u} = u - \frac{3}{4}m_2^2$$

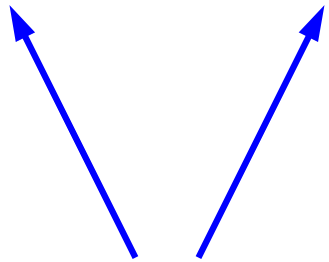
and studying the periods and residues of

$$dS = x\frac{dW}{W} = \frac{2x^2dx}{y} - x\frac{x - \widehat{u}}{y}\frac{dx}{x + \widehat{m}}$$

in the vicinity of "permutation of the branch points".

Alternative: the Picard-Fuchs equations for the periods and their monodromies (possible relation with qKZ equation in some models).

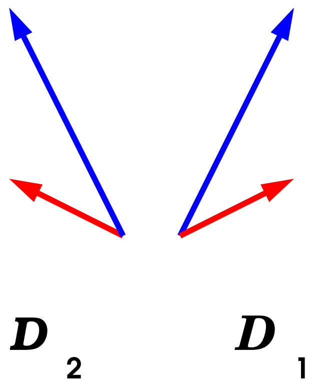
Confinement in $r = 2$ vacuum: condensate $Q_{1,2}$ 

The doublet of magnetic strings $S_{1,2}$  which confine monopoles.

Generalization of common ANO strings from relativistic Landau-Ginzburg theory: the first order BPS vortex equations, etc.

The doublet of confined monopoles, w.r.t. $\boldsymbol{\alpha}_{12} = \boldsymbol{\alpha}_1 - \boldsymbol{\alpha}_2$ $SU(N_c - 1) = SU(2)$ subgroup.

Confinement with condensed dyons:



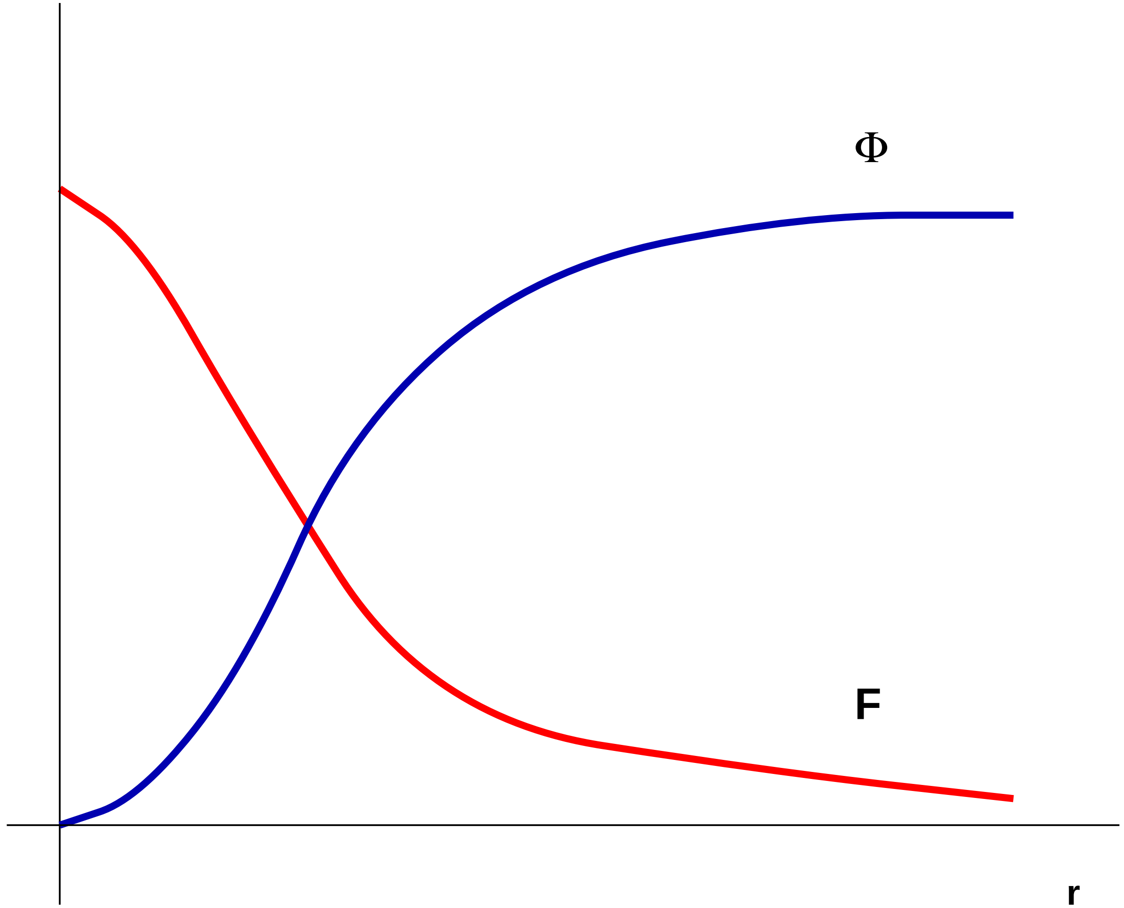
Condensate is doublet of dyons:

The fundamental strings: $\boldsymbol{\mu}_i \cdot \mathbf{A} + \boldsymbol{\alpha}_i \cdot \mathbf{A}^D \sim \delta_{iK} d\theta$, $K = 1, 2$ from the effective Lagranian for light dyons: $\mathcal{L} \sim \sum_{K=1,2} \left| \nabla \mathcal{D}_K \right|^2 + \ldots$,

$$\nabla_\mu \mathcal{D}_K = \left(\partial_\mu - i(\boldsymbol{\mu}_K \cdot \mathbf{A} + \boldsymbol{\alpha}_K \cdot \mathbf{A}^D) \right) \mathcal{D}_K$$

and θ is angle in the plane transversal to the direction of string.

String or flux tube in $(x, y) = (r, \theta)$ plane:



profile functions for the condensate $\Phi \to \mathcal{D}$ and gauge field $F \to F + F^D$.

From these equations $\boldsymbol{\alpha}_{12} \cdot (\mathbf{A} + \mathbf{A}^D) \sim d\theta$, for the dual component $\boldsymbol{\alpha}_{12} \cdot (\mathbf{A} - \mathbf{A}^D) \sim 0$.

Dymamics in the only non-Abelian direction $\boldsymbol{\alpha}_{12}$ is determined by "difference dyon" $\mathcal{D}_1 - \mathcal{D}_2 =$ 
and "difference string" $S_1 - S_2 =$ .

Screening of the electric charge of "difference string" by the condensate of the "difference dyon"

$$S_1 - S_2 + \frac{1}{2}(\mathcal{D}_1 - \mathcal{D}_2) =$$ 

causes still the *confinement of monopoles at strong coupling*!

Conclusions

- All technical details were omitted: studying the periods, carefull analysis of monodromies, etc

- Even more principal issues: existence of the Higgs branch in case of many flavors, "fat strings" (semi-local, non-BPS, ...) deserve additional study.

- Main principal outcome: transporting the picture from weak to strong coupling the quantum number of condensates change, but one does not get the confinement of quarks - the confined objects are still monopoles!