THE ISOTOPIC FOLDY-WOUTHUYSEN REPRESENTATION AS A POSSIBLE KEY TO UNDERSTANDING THE DARK MATTER

V.P.Neznamov

RFNC-VNIIEF, Institute of Theoretical and Computational Physics

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Introduction

Energy balance in the present-day Universe

Cosmological data of the XX-th century shows that the Universe has the following composition:

- **our world:**

  the baryon Universe - 0.05;

- **radiation - $5{\cdot}10^{-5}$**
- **«dark» matter- 0.25**
- **«dark» energy - 0.7**

Properties of dark matter

Features of “dark” matter

- **It doesn’t emit and absorb light;**
- **Weak interactions with the surrounding world;**
- **Non-relativistic motion.**

Candidates for particles of dark matter

Candidates	Mass
Gravitons	10^{-21} eV
Axions	10^{-5} eV
"Sterile" neutrinos	10 KeV
Specular material	1 GeV
Massive particles	100 GeV
Supermassive particles	10^{13} GeV
Monopoles and defects	10^{19} GeV
Primary black holes	$(10^{-16} - 10^{-7})\ M_{\odot}$

1. Main features of the Foldy-Wouthuysen representation

We know that the Foldy-Wouthuysen transformation is performed with the unitary operator U_{FW}.
In this case, the Dirac field operator and the Dirac equation's Hamiltonian are transformed as follows:

$$\psi_{FW} = U_{FW}\psi_D$$

$$H_{FW} = U_{FW} H_D U^{\dagger}_{FW} - iU_{FW}\frac{dU^{\dagger}_{FW}}{dt}$$

For free motion, $(H_0)_D = \boldsymbol{б}\mathbf{p} + \beta m$

$$(H_0)_{FW} = (U_0)_{FW}(H_0)_Д (U_0)^{+}_{FW};\ (U_0)_{FW} = R(1+L) = \sqrt{\frac{E+m}{2E}}\left(1+\frac{\beta \boldsymbol{б}\boldsymbol{p}}{E+m}\right);$$

$$E = \sqrt{m^2+\boldsymbol{p}^2};$$

The Foldy-Wouthuysen equation is

$$p_0\Psi_{FW}(x) = (H_0)_{FW}\Psi_{FW}(x) = \beta E\Psi_{FW}(x)$$

Asymmetry of the space and time coordinates is clearly seen in the Foldy-Wouthuysen equation, though it is Lorentz-invariant in itself.
The solutions to the free Foldy-Wouthuysen equation are plane waves of positive and negative energies

$$\Psi^{(+)}_{FW}(x,s) = \frac{1}{(2\pi)^{3/2}}U_s e^{-ipx};\Psi^{(-)}_{FW}(x,s) = \frac{1}{(2\pi)^{3/2}}V_s e^{ipx};\ p_0 \equiv E$$

$$U_s = \begin{pmatrix}\chi_s\\ 0\end{pmatrix};\quad V_s = \begin{pmatrix}0\\ \chi_s\end{pmatrix};$$

χ_s **are the two-component normalized Pauli's spin functions.**

Main features of the Foldy-Wouthuysen transformation and representation

1. In the Foldy-Wouthuysen representation, Hamiltonian H_{FW} is block-diagonal relative to the upper and lower components of the field operator.

2. The second necessary condition for transformation to the *FW representation, for free motion and motion in static external fields, is the requirement of zeroing either the upper, or lower components of* ψ_{FW} .

3. Because of the form of basic wave functions (field operators) in the *FW representation the FW transformation narrows the space of possible states of a Dirac particle during transformation from the Dirac representation*. Special measures are required (modifications of the *FW transformation*) to return to the Dirac space of states.

4. In the presence of the general boson field, the closed form of the FW transformation does not exist.

In his papers the author suggests a direct method to construct the Foldy-Wouthuysen transformation in case of fermions interacting with arbitrary boson fields. The transformation matrix U_{FW} and the relativistic Hamiltonian H_{FW} (6) have been obtained as powers series of the coupling constant:

$$U_{FW} = U_{FW}^{0}\left(1 + q\delta_1 + q^2\delta_2 + q^3\delta_3 + ...\right)$$

$$H_{FW} = \beta E + qK_1 + q^2K_2 + q^3K_3 + ...$$

***Here, q is the coupling constant, U_{FW}^{0} is the FW transformation matrix for free Dirac particles*,**

$E = \sqrt{\mathbf{p}^2 + m_f^2}$.

With Hamiltonian in the FW representation quantum electrodynamics and the Standard Model have been considered, a number of quantum-field effects have been calculated, and the *SU-2* – invariant formulation of the Standard Model with originally massive fermions and without Yukawa interactions between Higgs bosons and fermions has been offered.

2. Isotopic Foldy-Wouthuysen representation and chiral symmetry.

Consider the density of Hamiltonian of a Dirac particle of mass m_f **interacting with an arbitrary boson field** B^μ

$\mathscr{H}_D$ **=** $\psi^\dagger\left(\boldsymbol{\sigma}\mathbf{p}+\beta m_f+q\alpha_\mu B^\mu\right)\psi=\psi^\dagger\left(P_L+P_R\right)\left(\boldsymbol{\sigma}\mathbf{p}+\beta m_f+q\alpha_\mu B^\mu\right)\left(P_L+P_R\right)\psi=$

$$=\psi_L^\dagger\left(\boldsymbol{\sigma}\mathbf{p}+q\alpha_\mu B^\mu\right)\psi_L+\psi_R^\dagger\left(\boldsymbol{\sigma}\mathbf{p}+q\alpha_\mu B^\mu\right)\psi_R+\psi_L^\dagger\beta m_f\psi_R+\psi_R^\dagger\beta m_f\psi_L \quad \textbf{(1)}$$

In expression (1), $q-$ **is the interaction constant;** $P_L=\dfrac{1-\gamma_5}{2},\ P_R=\dfrac{1+\gamma_5}{2}-$ **are the left and right projective operators;** $\psi_L=P_L\psi,\ \psi_R=P_R\cdot\psi-$**are the left and right components of the Dirac field operator** ψ **.**

The Abelian case for boson field B^μ is considered for the sake of simplicity. The results and conclusions given below remain unchanged, if we consider a general case of a Dirac particle interacting with the non-Abelian boson field.

The density of Hamiltonian $\mathscr{H}_D$ can be used for derivation of the motion equations for ψ_L and ψ_R:

$$\begin{aligned} p_0\psi_L &= \left(\mathbf{\sigma p} + q\alpha_\mu B^\mu\right)\psi_L + \beta m_f \psi_R \\ p_0\psi_R &= \left(\mathbf{\sigma p} + q\alpha_\mu B^\mu\right)\psi_R + \beta m_f \psi_L \end{aligned} . \quad (2)$$

One can see that both the density of Hamiltonian $\mathscr{H}_D$ and motion equations have the form, in which the presence of fermion mass leads to mixing the left and right components of field operator ψ .

Here is the question:

Is it possible to write motion equations with chiral symmetry and their Hamiltonians for massive fermions?

It follows from equations (2) that

$$\psi_L = \left(p_0 - \mathbf{бp} - q\alpha_\mu B^\mu\right)^{-1} \beta m_f \psi_R$$
$$\psi_R = \left(p_0 - \mathbf{бp} - q\alpha_\mu B^\mu\right)^{-1} \beta m_f \psi_L \qquad (3)$$

Substitute (3) into the right-hand parts of equations (2) proportional to βm_f **and obtain the integro-differential equations for** ψ_R **and** ψ_L

$$\left[\left(p_0 - \mathbf{бp} - q\left(\alpha_0 B^0 - \mathbf{бB}\right)\right) - \beta m_f \left(p_0 - \mathbf{бp} - q\left(\alpha_0 B^0 - \mathbf{бB}\right)\right)^{-1} \beta m_f\right]\psi_L = 0$$
$$\left[\left(p_0 - \mathbf{бp} - q\left(\alpha_0 B^0 - \mathbf{бB}\right)\right) - \beta m_f \left(p_0 - \mathbf{бp} - q\left(\alpha_0 B^0 - \mathbf{бB}\right)\right)^{-1} \beta m_f\right]\psi_R = 0 \qquad (4)$$

It is clear that equations for ψ_R **and** ψ_L **have the same form and, in contrast to equations (2), the presence of mass** m_f **doesn't lead to mixing the right and left components of operator** ψ**.**

Equations (4) can be written as

$$\left[\left(p_0-\boldsymbol{\sigma}\mathbf{p}-q\alpha_\mu B^\mu\right)-\left(p_0+\boldsymbol{\sigma}\mathbf{p}-q\bar{\alpha}_\mu B^\mu\right)^{-1}m^2\right]\psi_{L,R}=0 \qquad \textbf{(5)}$$

In expression (5), $\psi_{L,R}$ indicates that equations for ψ_L and ψ_R have the same form; $\bar{\alpha}_\mu=\begin{cases}1\\-\alpha^i\end{cases}$.

If equation (5) is multiplied (on the left) by factor $p_0+\boldsymbol{\sigma}\mathbf{p}-q\bar{\alpha}_\mu B^\mu$, then we obtain the second-order equation with respect to p^μ

$$\left[\left(p_0+\boldsymbol{\sigma}\mathbf{p}-q\bar{\alpha}_\mu B^\mu\right)\left(p_0-\boldsymbol{\sigma}\mathbf{p}-q\alpha_\mu B^\mu\right)-m^2\right]\psi_{L,R}=0 \qquad \textbf{(6)}$$

For quantum electrodynamics $\left(q=e,\, B^\mu=A^\mu\right)$,equations (6) have the form

$$\left[\left(p_0-eA_0\right)^2-\left(\mathbf{p}-e\mathbf{A}\right)^2-m^2+e\boldsymbol{\sigma}\mathbf{H}+i\boldsymbol{\sigma}\mathbf{E}\right]\psi_{L,R}=0 \qquad \textbf{(7)}$$

In (7), $\mathbf{H}=rot\,\mathbf{A}$ – is magnetic field, $\mathbf{E}=-\dfrac{\partial\mathbf{A}}{\partial t}-\nabla A_0$ – is electrical field,

$\vec{\sigma}=\begin{pmatrix}\boldsymbol{\sigma}' & 0\\ 0 & \boldsymbol{\sigma}'\end{pmatrix}$, σ'^i are Pauli matrices.

Equations (7) coincide with the second-order equation derived d by Dirac in the 1920's. Equations (7) have no "excess" solutions. Operator γ_5 commutes with equations (7). Hence, $\gamma_5\psi = \delta\psi \left(\delta^2 = 1; \delta = \pm 1\right)$. The case of $\delta = -1$ corresponds to the equations (7) solution for ψ_L and $\delta = +1$ corresponds to the equations (7) solution for ψ_R.

Equations (5), (6) are invariant relative to $SU(2)-$transformations, however, they are nonlinear relative to operator $p_0 = i\dfrac{\partial}{\partial t}$. The linear form of $SU(2)-$invariant equations for fermion fields relative to p_0 can be obtained using the Foldy-Wouthuysen transformation in a specially introduced isotopic space.

$$p_0\psi_L = \left(\mathbf{б p} + q\alpha_\mu B^\mu\right)\psi_L + \beta m_f \psi_R$$
$$p_0\psi_R = \left(\mathbf{б p} + q\alpha_\mu B^\mu\right)\psi_R + \beta m_f \psi_L \qquad \textbf{(2)}$$

Introduce the 8-component field operator $\Phi_1 = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix}$

and isotopic matrices $\tau_3 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \tau_1 = \begin{pmatrix} 0 & I \\ I & ,0 \end{pmatrix}$ **that act on the four upper and**

the four lower components of operator Φ_1
Then, equations (2) can be written as

$$p_0\Phi_1 = \left(\mathbf{б p} + \tau_1\beta m_f + q\alpha_\mu B^\mu\right)\Phi_1 \qquad \textbf{(3)}$$

As far as τ_1 **commutes with the right-hand part of (3), the field** $\Phi_2 = \tau_1\Phi_1 = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$ **is also a solution to equation (3).**

$$p_0\Phi_2 = \left(\mathbf{б p} + \tau_1\beta m_f + q\alpha_\mu B^\mu\right)\Phi_2 \qquad \textbf{(4)}$$

Finally, use equality $\Phi_2 = \tau_1\Phi_1 = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$ **and write equations (3), (4) in the form**

$$p_0\Phi_1 = \left(\mathbf{б p} + \tau_1\beta m_f + \frac{1}{2}q\alpha_\mu B^\mu\right)\Phi_1 + \frac{1}{2}q\tau_1\alpha_\mu B^\mu\Phi_2$$

$$p_0\Phi_2 = \left(\mathbf{б p} + \tau_1\beta m_f + \frac{1}{2}q\alpha_\mu B^\mu\right)\Phi_2 + \frac{1}{2}q\tau_1\alpha_\mu B^\mu\Phi_1 \qquad \textbf{(5)}$$

It will be shown below that being actually equivalent in the Dirac representation equations (3), (4), (5) lead us to different physical pictures of combinations and interactions of elementary particles in the isotopic Foldy-Wouthuysen representation.

Find the Foldy-Wouthuysen transformation in the introduced isotopic space for the free-motion Dirac equations (3), (4) without terms of interaction with boson fields using the Eriksen transformation.

$$U_{FW}^{0} = U_{Er} = \frac{1}{2}\left(1+\tau_3\lambda\right)\left(\frac{1}{2}+\frac{\tau_3\lambda+\lambda\tau_3}{4}\right)^{-1/2} \qquad \textbf{(6)}$$

In expression (6) $\lambda = \dfrac{\boldsymbol{\sigma}\mathbf{p}+\tau_1\beta m_f}{E}$; $E=\left(\mathbf{p}^2+m^2\right)^{1/2}$**. Since we have** $\left(\boldsymbol{\sigma}\mathbf{p}+\tau_1\beta m_f\right)^2 = E^2$**, then** $\lambda^2 = 1$**.**

Expression (6) can be transformed to the following form:

$$U_{FW}^{0} = U_{Er} = \frac{1}{2}\left(1+\frac{\tau_3\boldsymbol{\sigma}\mathbf{p}+\tau_3\tau_1\beta m}{E}\right)\left(\frac{1}{2}+\frac{\tau_3\boldsymbol{\sigma}\mathbf{p}}{2E}\right)^{-1/2} =$$

$$= \sqrt{\frac{E+\tau_3\boldsymbol{\sigma}\mathbf{p}}{2E}}\left(1+\frac{1}{E+\tau_3\boldsymbol{\sigma}\mathbf{p}}\tau_3\tau_1\beta m\right) \qquad \textbf{(7)}$$

Transformation (7) is a unitary transformation $\left(U_{FW}^{0}\left(U_{FW}^{0}\right)^{\dagger}=1\right)$ **and**

$$H_{FW} = U_{FW}^{0}\left(\boldsymbol{\sigma}\mathbf{p}+\tau_1\beta m_f\right)\left(U_{FW}^{0}\right)^{\dagger} = \tau_3 E \qquad \textbf{(8)}$$

Thus, in the Foldy-Wouthuysen representation equations (3), (4) has the form

$$p_0\left(\Phi_{1,2}\right)_{FW} = \tau_3 E\left(\Phi_{1,2}\right)_{FW} \qquad \textbf{(9)}$$

Basic orthonormalized functions $\Phi_{1FW}(x), \Phi_{2FW}(x)$ for free motion of fermions are expressed via the left and right components of the Dirac field and have the following form:

$$\Phi_{1FW}^{(+)}(\mathbf{x},t) = U_{1FW}^{0}\Phi_{1}^{(+)}(\mathbf{x},t) = e^{-iEt}\begin{pmatrix} \sqrt{\dfrac{2E}{E+\mathbf{yp}}}\psi_R^{(+)}(\mathbf{x}) \\ 0 \end{pmatrix}$$

$$\Phi_{1FW}^{(-)}(\mathbf{x},t) = U_{FW}^{0}\Phi_{1}^{(-)}(\mathbf{x},t) = e^{iEt}\begin{pmatrix} 0 \\ \sqrt{\dfrac{2E}{E+\mathbf{yp}}}\psi_L^{(-)}(\mathbf{x}) \end{pmatrix}$$

$$\Phi_{2FW}^{(+)}(\mathbf{x},t) = U_{FW}^{0}\Phi_{2}^{(+)}(\mathbf{x},t) = e^{-iEt}\begin{pmatrix} \sqrt{\dfrac{2E}{E-\mathbf{yp}}}\psi_L^{(+)}(\mathbf{x}) \\ 0 \end{pmatrix}$$

$$\Phi_{2FW}^{(-)}(\mathbf{x},t) = U_{FW}^{0}\Phi_{2}^{(-)}(\mathbf{x},t) = e^{iEt}\begin{pmatrix} 0 \\ \sqrt{\dfrac{2E}{E-\mathbf{yp}}}\psi_R^{(-)}(\mathbf{x}) \end{pmatrix}$$

The continuity equation and current of particles

$$
-\begin{cases} i\dfrac{\partial \Phi_{1FW}}{\partial t} = \tau_3 E \Phi_{1FW} & \text{On the left, multiply by } \Phi^{\dagger}_{1FW} \\[2ex] -i\dfrac{\partial \Phi^{\dagger}_{1FW}}{\partial t} = (E\Phi^{\dagger}_{1FW})\tau_3 & \text{On the right, multiply by } \Phi_{1FW} \end{cases}
$$

$$\frac{\partial}{\partial t}(\Phi^{\dagger}_{1FW}\Phi_{1FW}) = -i(\Phi^{\dagger}_{1FW}\tau_3 E\Phi_{1FW} - (E\Phi^{\dagger}_{1FW})\tau_3\Phi_{1FW})$$

$$\frac{\partial}{\partial t}(\Phi^{\dagger}_{1FW}\Phi_{1FW}) = -div\,\mathbf{j}$$

$$j^i = -\frac{1}{2m}(\Phi^{\dagger}_{1FW}p^i\tau_3\Phi_{1FW} - (p^i\Phi^{\dagger}_{1FW})\tau_3\Phi_{1FW}) + \frac{1}{8m^3}p^2(\Phi^{\dagger}_{1FW}p^i\tau_3\Phi_{1FW} - (p^i\Phi^{\dagger}_{1FW})\tau_3\Phi_{1FW}) -$$
$$-\frac{1}{16m^5}p^2(\Phi^{\dagger}_{1FW}p^i p^2\tau_3\Phi_{1FW} - (p^i p^2\Phi^{\dagger}_{1FW})\tau_3\Phi_{1FW}) + \ldots$$

In the general case of interactions with arbitrary boson fields it is possible to use the direct method (developed by the author) of transformation from the isotopic Dirac equation representation to the isotopic Foldy-Wouthuysen representation. Upon completion of one and the same isotopic Foldy-Wouthuysen transformation of equations (3), (4), (5) we obtain

$$p_0\Phi_{1FW} = \left(\tau_3 E + qK_1 + q^2K_2 + q^3K_3 + ...\right)\Phi_{1FW} \qquad \textbf{(10)}$$

$$H_{FW}^{I} = \Phi_{1FW}^{\dagger}\left(\tau_3 E + qK_1 + q^2K_2 + q^3K_3 + ...\right)\Phi_{1FW}$$

$$p_0\Phi_{2FW} = \left(\tau_3 E + qK_1 + q^2K_2 + q^3K_3 + ...\right)\Phi_{2FW} \qquad \textbf{(11)}$$

$$H_{FW}^{II} = \Phi_{2FW}^{\dagger}\left(\tau_3 E + qK_1 + q^2K_2 + q^3K_3 + ...\right)\Phi_{2FW}$$

$$p_0\Phi_{1FW} = \left(\tau_3 E + \frac{q}{2}K_1 + \left(\frac{q}{2}\right)^2 K_2 + \left(\frac{q}{2}\right)^3 K_3 + ...\right)\Phi_{1FW} + \left(\frac{q}{2}K_{1\tau_1} + \left(\frac{q}{2}\right)^2 K_{2\tau_1} + \left(\frac{q}{2}\right)^3 K_{3\tau_1} + ...\right)\Phi_{2FW}$$

$$p_0\Phi_{2FW} = \left(\tau_3 E + \frac{q}{2}K_1 + \left(\frac{q}{2}\right)^2 K_2 + \left(\frac{q}{2}\right)^3 K_3 + ...\right)\Phi_{2FW} + \left(\frac{q}{2}K_{1\tau_1} + \left(\frac{q}{2}\right)^2 K_{2\tau_1} + \left(\frac{q}{2}\right)^3 K_{3\tau_1} + ...\right)\Phi_{1FW}$$

$$\begin{aligned} H_{FW}^{IV} = {} & \Phi_{1FW}^{\dagger}\left(\tau_3 E + \frac{q}{2}K_1 + \left(\frac{q}{2}\right)^2 K_2 + \left(\frac{q}{2}\right)^3 K_3 + ...\right)\Phi_{1FW} + \Phi_{1FW}^{\dagger}\left(\frac{q}{2}K_{1\tau_1} + \left(\frac{q}{2}\right)^2 K_{2\tau_1} + \left(\frac{q}{2}\right)^3 K_{3\tau_1} + ...\right)\Phi_{2FW} + \\ & + \Phi_{2FW}^{\dagger}\left(\tau_3 E + \frac{q}{2}K_1 + \left(\frac{q}{2}\right)^2 K_2 + \left(\frac{q}{2}\right)^3 K_3 + ...\right)\Phi_{2FW} + \Phi_{2FW}^{\dagger}\left(\frac{q}{2}K_{1\tau_1} + \left(\frac{q}{2}\right)^2 K_{2\tau_1} + \left(\frac{q}{2}\right)^3 K_{3\tau_1} + ...\right)\Phi_{1FW} \end{aligned} \qquad \textbf{(12)}$$

Apparently, there is a possibility to construct Hamiltonian H_{FW}^{III} with equations for fields Φ_{1FW}, Φ_{2FW} from (10), (11)

$$H_{FW}^{III} = H_{FW}^{I} + H_{FW}^{II} \qquad \textbf{(13)}$$

Expressions for operators, which are the basis for the interaction Hamiltonian in the isotopic Foldy-Wouthuysen representation (i.e. terms $K_1, K_2, K_3..., K_{1\tau_1}, K_{2\tau_1}, K_{3\tau_1}...$), can be written as

$$C = \left[U_{FW}^0 q\alpha_\mu B^\mu \left(U_{FW}^0\right)^\dagger \right]^{even} = qR\left(B^0 - LB^0L\right)R - qR\left(\boldsymbol{\sigma}\mathbf{B} - L\boldsymbol{\sigma}BL\right)R$$

$$N = \left[U_{FW}^0 q\alpha_\mu B^\mu \left(U_{FW}^0\right)^\dagger \right]^{odd} = qR\left(LB^0 - B^0L\right)R - qR\left(L\boldsymbol{\sigma}\mathbf{B} - \boldsymbol{\sigma}\mathbf{B}L\right)R$$

$$C_{\tau_1} = \left[U_{FW}^0 \tau_1 q\alpha_\mu B^\mu \left(U_{FW}^0\right)^\dagger \right]^{odd} = qR\left(\tau_1 B^0 - L\tau_1 B^0 L\right)R - qR\left(\tau_1\boldsymbol{\sigma}\mathbf{B} - L\tau_1\boldsymbol{\sigma}\mathbf{B}L\right)R$$

$$N_{\tau_1} = \left[U_{FW}^0 \tau_1 q\alpha_\mu B^\mu \left(U_{FW}^0\right)^\dagger \right]^{even} = qR\left(L\tau_1 B^0 - \tau_1 B^0 L\right)R - qR\left(L\tau_1\boldsymbol{\sigma}\mathbf{B} - \tau_1\boldsymbol{\sigma}\mathbf{B}L\right)R$$

$$R = \sqrt{\frac{E + \tau_3\boldsymbol{\sigma}\mathbf{p}}{2E}}; \; L = \frac{1}{E + \tau_3\boldsymbol{\sigma}\mathbf{p}}\tau_3\tau_1\beta m$$

For free motion, write the density of Hamiltonian (13), for example, with respect to basic functions (16) H_{FW}^{III}

$$H_{FW}=(\Phi_1)_{FW}^{\dagger}\tau_3 E(\Phi_1)_{FW}+(\Phi_2)_{FW}^{\dagger}\tau_3 E(\Phi_2)_{FW}=(\Phi_1^{(+)})_{FW}^{\dagger}E(\Phi_1^{(+)})_{FW}-(\Phi_1^{(-)})_{FW}^{\dagger}E(\Phi_1^{(-)})_{FW}+$$
$$+(\Phi_2^{(+)})_{FW}^{\dagger}E(\Phi_2^{(+)})_{FW}-(\Phi_2^{(-)})_{FW}^{\dagger}E(\Phi_2^{(-)})_{FW}=\left(\psi_R^{(+)}\right)^{\dagger}\frac{2E}{E+\mathbf{yp}}E\psi_R^{(+)}-\left(\psi_L^{(-)}\right)^{\dagger}\frac{2E}{E+\mathbf{yp}}E\psi_L^{(-)}+ \quad \mathbf{(17)}$$
$$+\left(\psi_L^{(+)}\right)^{\dagger}\frac{2E}{E-\mathbf{yp}}E\psi_L^{(+)}-\left(\psi_R^{(-)}\right)^{\dagger}\frac{2E}{E-\mathbf{yp}}E\psi_R^{(-)}$$

With regard to (16), the density of Hamiltonian (17) bracketed between two-component spinors $\varphi^{(+)}(\mathbf{x})$ and $\chi^{(-)}(\mathbf{x})$ has the form, which is the ordinary one for the quantum field theory:

$$H_{FW}=2\left(\varphi^{(+)\dagger}E\varphi^{(+)}-\chi^{(-)\dagger}E\chi^{(-)}\right)$$

It is clear that Hamiltonian (17) has chiral symmetry, regardless of whether the fermions are massive, or massless.

In the presence of static and dynamic boson fields, basic functions in the Foldy-Wouthuysen representation are similar to those in (16) in their isotopic structure. When solving applied problems in the quantum field theory using the perturbation theory, fermion fields are expanded in series with respect to solutions of Dirac equations for free motion, or motion in static external fields. In our case of the isotopic Foldy-Wouthuysen representation we can also expand in series fermion fields with respect to the basis of solutions to (16), or basis of solutions to Foldy-Wouthuysen equations in static external fields.

As far as the earlier considered Hamiltonians in (10)÷(12) are diagonal, by definition, relative to the upper and lower isotopic components, they have chiral symmetry at the same time, regardless of whether the fermions are massive, or massless.

3. Isotopic Foldy-Wouthysen representation and dark mater.

Consider the Hamiltonian densities and equations (10)-(12) from the viewpoint of possible combinations and interactions of elementary particles. Remember that equations (10)-(12) have been derived from one and the same Dirac equation written in various forms in expressions (3)-(5). Hence, the physical pictures corresponding to equations (10)-(12) can take place in our universe.

First, consider Hamiltonian density $\mathscr{H}_{FW}^{IV}$ and equations (12). The appropriate physical picture is symbolically represented in Fig.1.

E $\qquad\qquad$ H_{FW}^{IV} $\qquad\qquad$ $E<0$

$T_3=+\frac{1}{2}$ $\qquad\qquad$ $T_3=-\frac{1}{2}$

$$\Phi_{1FW}^{(+)}(\mathbf{x},t)=e^{-iEt}\begin{pmatrix}A\psi_R^{(+)}(\mathbf{x})\\0\end{pmatrix} \quad \longleftarrow\text{-}\text{-}\text{-}\longrightarrow \quad \Phi_{1FW}^{(-)}(\mathbf{x},t)=e^{-iEt}\begin{pmatrix}0\\A_1\psi_L^{(-)}(\mathbf{x})\end{pmatrix}$$

$$\Phi_{2FW}^{(+)}(\mathbf{x},t)=ie^{-iEt}\begin{pmatrix}A_1\psi_L^{(+)}(\mathbf{x})\\0\end{pmatrix} \quad \longleftarrow\longrightarrow \quad \Phi_{2FW}^{(-)}(\mathbf{x},t)=ie^{-iEt}\begin{pmatrix}0\\A\psi_R^{(-)}(\mathbf{x})\end{pmatrix}$$

Fig. 1

The left half-plane in Fig.1 represents states of basic functions (12) with isotopic spin $T_3=+\frac{1}{2}$ and positive value of energy $E>0$; the right half-plane represents states with $T_3=-\frac{1}{2}$ and negative value of energy $E<0$.

Hamiltonian H^{IV} contains the states of the left and right fermions, as well as the states of the left and right antifermions; particles and antiparticles interact with each other both really (solid line with arrows in Fig.1) and virtually (dashed line with arrows in Fig.1). The physical picture in Fig.1 represents the real world around us.

Now, consider Hamiltonian density $\mathscr{H}_{FW}^{IV}$ with equations from (10), (11). The symbolic representation of combinations and interactions of elementary particles is given in Fig.2.

$$H_{FW}^{III}$$

$E > 0$

$$T_3 = +\frac{1}{2}$$

$$\Phi_{1FW}^{(+)}(\mathbf{x},t) = e^{-iEt}\begin{pmatrix} A\psi_R^{(+)}(\mathbf{x}) \\ 0 \end{pmatrix}$$

$$\Phi_{2FW}^{(+)}(\mathbf{x},t) = ie^{-iEt}\begin{pmatrix} A_1\psi_L^{(+)}(\mathbf{x}) \\ 0 \end{pmatrix}$$

$E < 0$

$$T_3 = -\frac{1}{2}$$

$$\Phi_{1FW}^{(-)}(\mathbf{x},t) = e^{-iEt}\begin{pmatrix} 0 \\ A_1\psi_L^{(-)}(\mathbf{x}) \end{pmatrix}$$

$$\Phi_{2FW}^{(-)}(\mathbf{x},t) = ie^{-iEt}\begin{pmatrix} 0 \\ A\psi_R^{(-)}(\mathbf{x}) \end{pmatrix}$$

Fig.2

The world in Fig.2 is poorer than that in Fig.1. The physical picture in Fig.2 has both left and right fermions and left and right antifermions. However, there are no interactions between real fermions and antifermions; there is only virtual interaction between them (a dashed line with arrows in Fig.2). Fig.2 allows strong and electromagnetic interactions between particles and antiparticles without any real interactions between them. There are no processes of production and absorption of real particle-antiparticle pairs and there are no coupled states of real particles and antiparticles, etc. Weak interactions are significantly poorer, as well, because there are no processes of simultaneous production and absorption of real particles together with antiparticles.

Consider Hamiltonian densities H^{I}_{FW}, H^{II}_{FW} with the appropriate equations from (10), (11). The symbolic picture is given in Figs. 3, 4.

H^{I}_{FW}

$E>0$		$E<0$
$T_3=+\frac{1}{2}$		$T_3=-\frac{1}{2}$
$\Phi^{(+)}_{1FW}(\mathbf{x},t)=e^{-iEt}\begin{pmatrix}A\psi^{(+)}_R(\mathbf{x})\\0\end{pmatrix}$		$\Phi^{(-)}_{1FW}(\mathbf{x},t)=e^{-iEt}\begin{pmatrix}0\\A_1\psi^{(-)}_L(\mathbf{x})\end{pmatrix}$

Fig. 3

H^{II}_{FW}

$E>0$		$E<0$
$T_3=+\frac{1}{2}$		$T_3=-\frac{1}{2}$
$\Phi^{(+)}_{2FW}(\mathbf{x},t)=ie^{-iEt}\begin{pmatrix}A_1\psi^{(+)}_L(\mathbf{x})\\0\end{pmatrix}$		$\Phi^{(-)}_{2FW}(\mathbf{x},t)=ie^{-iEt}\begin{pmatrix}0\\A\psi^{(-)}_R(\mathbf{x})\end{pmatrix}$

Fig. 4

It follows from Figs.3, 4 that Hamiltonian densities $\mathscr{H}^{I}_{FW}$, $\mathscr{H}^{II}_{FW}$ stipulate existence of either right fermions and left antifermions ($\mathscr{H}^{I}_{FW}$), or left fermions and right antifermions ($\mathscr{H}^{II}_{FW}$). In both cases, there are no interactions between real particles and antiparticles.

The physical world in Fig.3, or in Fig.4 has no electromagnetic and no strong interactions, since because of parity preservation in such interactions both left and right fermions are required to be present. The same requirement is valid for processes with neutral-current weak interactions and so, they are also absent in Figs.3, 4. The processes with weak currents and with participation of either left fermions (Fig.4), or left antifermions (Fig.3) also appear to be suppressed because of impossibility to emit, or absorb particles and antiparticles. The vacuum state in Fig.3 and Fig.4 significantly differs from that in Fig.1 and Fig.2. Since no interactions are allowed (except for the gravitational interaction) in the vacuums in Fig.3 and Fig.4, no “bouillon” of particle-antiparticle virtual pairs and virtual carriers of interactions is found.

Here is a summary of the physical pictures given in Figs. 1, 2, 3, 4.

As the strong and electromagnetic interactions are prohibited and weak interactions are virtually absent, the world shown in Figs.3, 4 should have the following properties:

- it does not emit/absorb light;
- free motion of fermions is non-relativistic;
- it weakly interacts with the outer world.

The aforementioned properties are those of "dark matter." Hence, we may assume that "dark matter" is an implementation of the physical picture in Figs.3, 4. It consists of either right fermions and left antifermions, or left fermions and right antifermions. A set of fermions and antifermions requires no changes in the composition of particles in the Standard Model. In this picture, all particles (including quarks and antiquarks) demonstrate free motion, without any interaction with each other.

As it has been mentioned earlier, the physical picture in Fig.1 represents the world around us in our part of the universe. The baryon matter (“light” matter) constitutes ~ 4% of the universe structure. If we assume that in the past transformation of the physical picture shown in Fig.1 to the physical picture in Figs.3, 4 occurred in a part of the universe, then in addition to generation of “dark” matter, vacuum would have been significantly restructured owing to the disappearance of quark, gluon, and electroweak condensates.

A principal question still requires an answer:

- How and why does the universe, if it does at all, transform to various physical pictures of its elementary particle compositions and interactions?

Note once again that different physical pictures of the composition and interactions of elementary particles have been derived from one equation of the Dirac field interacting with the boson fields described by (3)-(5) using the isotopic Foldy-Wouthuysen transformation that allows writing field equations with massive fermions and their Hamiltonians in the chiral-symmetry form invariant relative to *SU*-2 - transformations.

The composition of elementary particles in the physical pictures above falls within the set of particles of the Standard Model.

4. Isotopic Foldy-Wouthuysen representation and the Universe evolution prospects

1

Left particles	Left antiparticles
Right particles	Right antiparticles

Gravitational interactions +

Strong interactions +

Electromagnetic interactions +

Weak interactions +

2

Left particles	Right antiparticles

3

Right particles	Left antiparticles

Gravitational interactions +

Strong interactions -

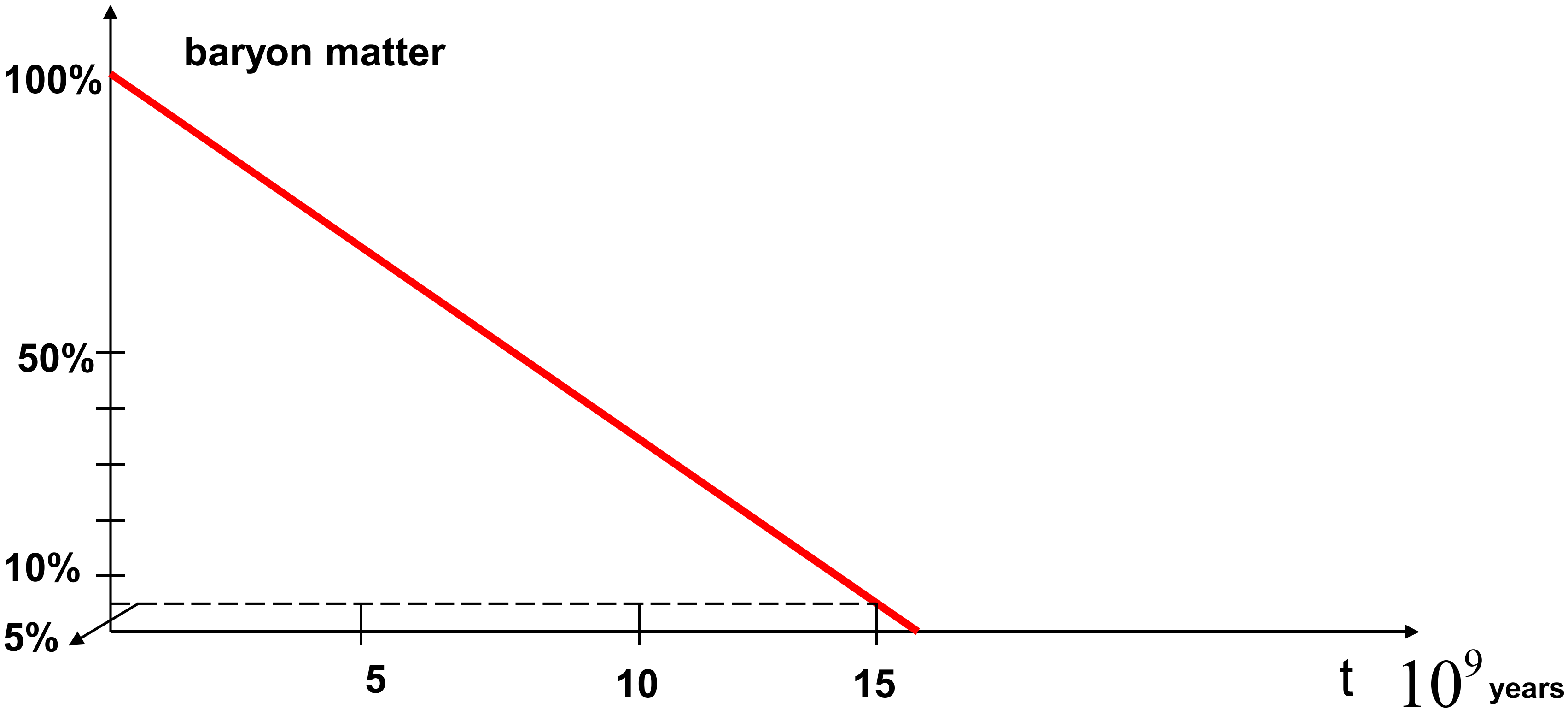
Electromagnetic interactions -

Weak interactions -

The Universe evolution prospects

1) **Static option**

2) **Dynamic option**



$$\Delta t \approx 15 \cdot 0.05 \cdot 10^9 \approx 0.8 \cdot 10^9 \text{ years}$$

Conclusion

1. Hamiltonians and the corresponding equations of fermion fields obtained in the isotopic Foldy-Wouthuysen representation are invariant with respect to $SU(2)$ transformations, regardless of whether the fermions are massive or massless.
2. This allows constructing the Standard Model without Higgs bosons in the fermion sector.
3. Many processes of interactions between Higgs boson and fermions do not take place in this case. For example, there are no processes of scalar boson decay to fermions $\left(H \to f\,\bar{f}\right)$, quarkonium states ψ, Υ, θ including Higgs boson are absent, there are no boson interactions with gluons $\left(ggH\right)$ and photons $\left(\gamma\gamma H\right)$ via fermion loops, etc.
4. The four resultant Hamiltonians correspond to different physical pictures of composition and interactions of elementary particles in the isotopic Foldy-Wouthuysen representation. Two physical pictures are close in their features to the observed features of the “dark matter”. In all physical pictures, compositions of elementary particles are not beyond the set of particles of the Standard Model

.


Thank you for attention

The Dirac equation for electron-positron field in the FW representation can be written as

$$p_0 \Psi_{FW}(x) = H_{FW}\Psi_{FW}(x) = (\beta E + K_1 + K_2 + K_3 + ...)\Psi_{FW}(x);$$

$$K_1 \sim e, K_2 \sim e^2, K_3 \sim e^3 ...$$

The Feynman propagator of the Dirac equation in the Foldy-Wouthuysen representation is

$$S_{FW}(x-y) = \frac{1}{(2\pi)^4}\int d^4 p \frac{e^{-ip(x-y)}}{p_0 - \beta E} = \frac{1}{(2\pi)^4}\int d^4 p e^{-ip(x-y)} \frac{p_0 + \beta E}{p^2 - m^2 + i\varepsilon}$$

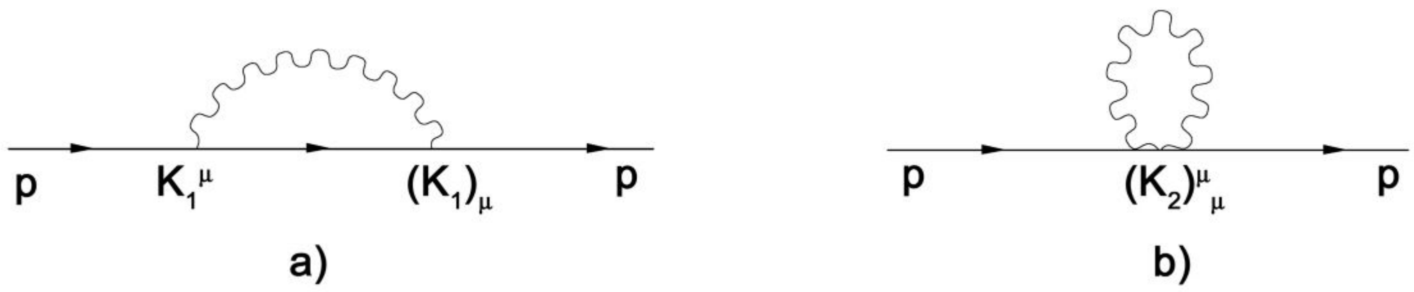
The integral equation for $\Psi_{FW}(x)$ has the form

$$\Psi_{FW}(x) = \Psi_0(x) + \int d^4 y \, S_{FW}(x-y)(K_1 + K_2 + ...)\Psi_{FW}(y)$$

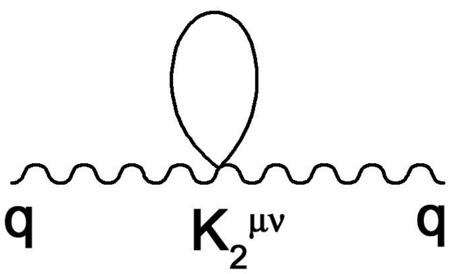
$\Psi_0(x)$ **is solution to the Dirac equation in the FW representation in the absence of electromagnetic field** $(A^\mu = 0)$.

The expressions given above allow formulating Feynman laws to write elements of scattering matrix S_{fi} and calculate QED processes. In contrast to the Dirac representation, the Foldy-Wouthuysen representation has an infinite set of various types of photon-interaction vertices depending on the perturbation theory order: factor $(-iK_{1\mu})$ corresponds to a single-photon interaction vertex, factor $(-iK_{2\mu\nu})$ corresponds to a two-photon interaction vertex, etc. For convenience, $K_{1\mu}, K_{2\mu\nu},...$ denote the corresponding parts of the interaction Hamiltonian terms $K_1, K_2...$ without electromagnetic potentials $A^{\mu}, A^{\mu}A^{\nu},.....$

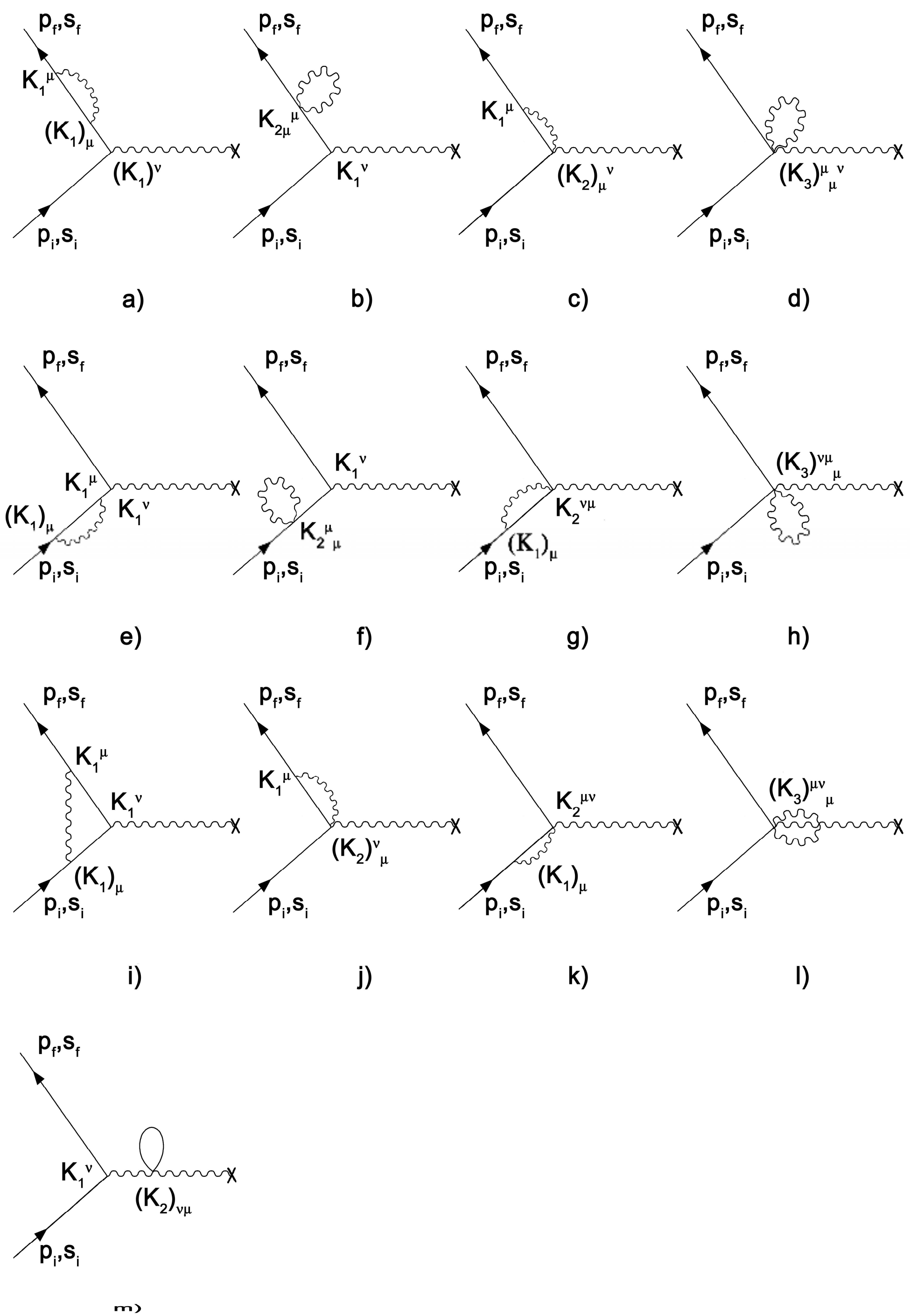
One of the functions $\Psi_0(x)$ corresponds to each external fermion line. As usual, positive-energy solutions correspond to particles and negative-energy solutions correspond to antiparticles. The rest Feynman laws are the same as in the spinor electrodynamics in the Dirac representation.



Self-energy of electron



Vacuum polarization



Radiation corrections to the electron scattering in external field

Final results of calculations of the QED processes, which diagrams are shown in the figures, coincide with the similar values calculated in the Dirac representation. Radiation corrections to the electron scattering in an external field give us, upon renormalization of mass and charge, the correct value of abnormal magnetic electron and Lamb shift of energy levels.

A specific feature of the theory is in the presence of an even number N of odd operators in terms K_n (except K_1) of the interaction Hamiltonian that allow establishing relations between the initial and final positive-energy states and the intermediate negative-energy states, and vice versa. Owing to this fact, we have a diagram related to the electron-positron vacuum polarization. An ordinary diagram of vacuum polarization with two vertices of the first order with respect to e is absent in the given theory because of the evenness of operator K_1.

Possible consequences of the physical vacuum reconstruction

1. **When passing from picture 1 to pictures 2 and 3, reconstruction of physical vacuum should take place.**

Example:

***In picture 1, the mass of coupled light quarks u, d with a "coat" of gluons and quark-antiquark pairs is ~ 300 Mev*.**

In pictures 2, 3 the mass of free light quarks *u, d is ~ 10 Mev*.

2. Hypothesis: «dark» energy occurs due to reconstruction of physical vacuum.