

# ISOTOPE EFFECT, PHONON AND MAGNETIC MECHANISM OF PAIRING IN HIGH-TC CUPRATES IN STRONG ELECTRON CORRELATION LIMIT

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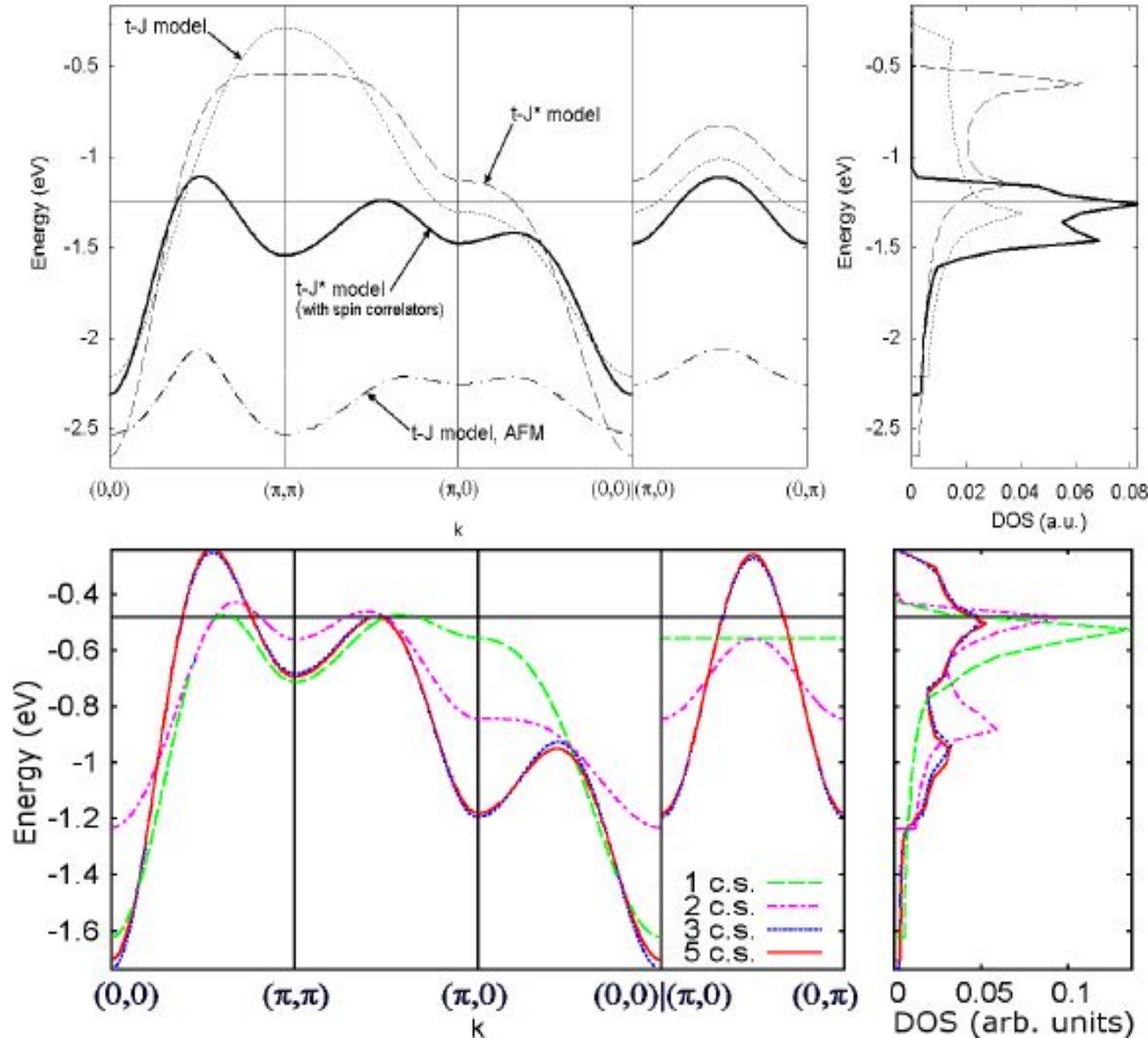
# Outline

- Microscopically derived  $t$ - $t'$ - $t''$ - $J^*$  model from ab initio multiband p-d model
- Strongly correlated electrons and spin-liquid with short AFM order
- Doping evolution of the Fermi surface and Lifshitz quantum phase transitions
- Magnetic and phonon contributions to d-pairing
- Isotope effect
- conclusions

Hybrid LDA+GTB scheme without fitting parameters  
(in collaboration with prof.V.I.Anisimov group, Ekaterinburg,  
(Korshunov, Ovchinnikov, etal, Phys.Rev.B 2005))

- Projection of LDA band structure and construction the Wannier functions for the multiband p-d –model
- *Ab initio* calculation of p-d –model parameters
- Quasiparticle band structure GTB calculations in the strongly correlated regime with *ab initio parameters*
- Comparison of  $\text{La}_2\text{CuO}_4$  and  $\text{Nd}_2\text{CuO}_4$  band structure with fitting and LDA+GTB parameters

## Quasiparticle dispersion in the t-J and t-J\* models



In the BCS-type theory:

$$T_c \propto \exp(-1/N(\varepsilon_F)V)$$

Appearance of the new Van-Hove singularity and corresponding maximum in  $T_c(x)$  at low  $x$ : [V.V. Val'kov and D.M. Dzebisashvili, JETP 100, 608 (2005)]

$v_F$  in nodal direction:  
 1.6-2.0 eV A - theory  
 1.8 $\pm$ 0.4 eV A - ARPES for  
 $0 < x < 0.2$ ,  
 Zhou et al, Nature, 2003

Low energy effective model is  
obtained to be the  
 $t$ - $t'$ - $t''$ - $J^*$  model  
with all parameters calculated from  
LDA+GTB method

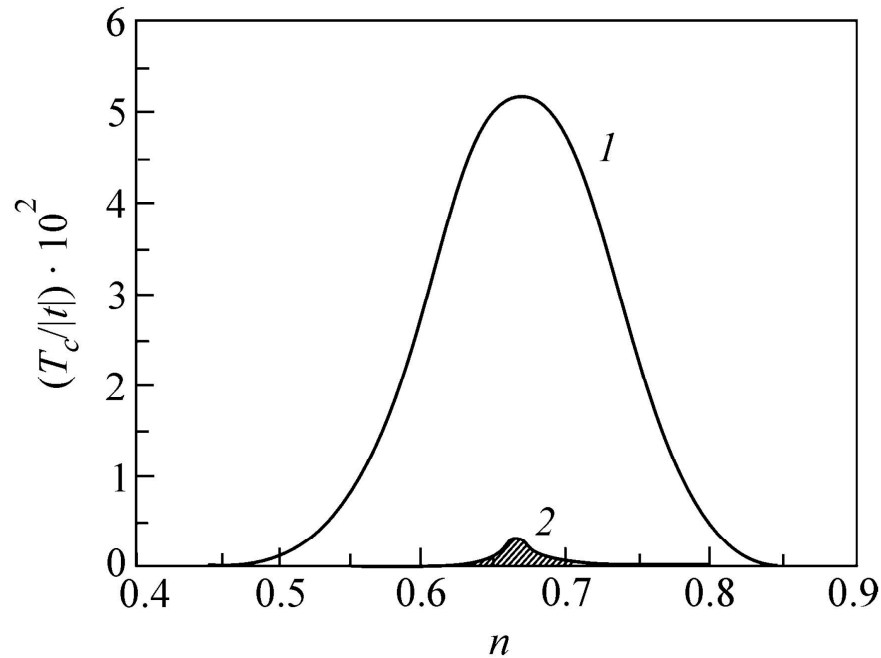
- Restriction to a single  $t$  or two  $t$  and  $t'$  hopping is not enough
- Negligible difference between 3 ( $t, t', t''$ ) and more hoppings is found

Effective Hamiltonian for *n*-type cuprates – *t*-*J*\* model:

$$H_{t-J^*} = H_{t-J} + H_3$$

$$H_{t-J} = \sum_{f,\sigma} \varepsilon_1 X_f^{\sigma\sigma} + \sum_{\langle f,g \rangle, \sigma} t_{fg}^{00} X_f^{\sigma 0} X_g^{0\sigma} + \sum_{\langle f,g \rangle} J_{fg} \left( \mathbf{s}_f \mathbf{s}_g - \frac{1}{4} \tilde{n}_f \tilde{n}_g \right),$$

$$H_3 = - \sum_{\langle f,g,m \rangle, \sigma} \frac{t_{fm}^{0S} t_{mg}^{0S}}{E_{ct}} \left( X_f^{\sigma 0} X_m^{\bar{\sigma}\bar{\sigma}} X_g^{0\sigma} - X_f^{\sigma 0} X_m^{\bar{\sigma}\sigma} X_g^{0\bar{\sigma}} \right).$$



**V.V. Val'kov, T.A. Val'kova, D.M. Dzebisashvili,  
S.G. Ovchinnikov, JETP Letters 75, 378 (2002)**

Hole dynamics in SCES at the short range order antiferromagnetic background. SCBA for Self-energy. At low T correlations are static (Barabanov et al, JETP 2001, Valkov and Dzebisashvili, JETP 2005, Plakida and Oudovenko JETP 2007, Korshunov and Ovchinnikov Eur.Phys.J.B, 2007)

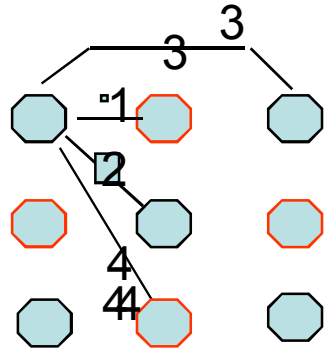
$$G_{\sigma}(\mathbf{k}, E) = \frac{(1+x)/2}{E - \varepsilon_0 + \mu - \frac{1+x}{2}t(\mathbf{k}) - \frac{1-x^2}{4} \cdot t_{01}^2(\mathbf{k})/U + \sum(\mathbf{k})}$$

$$\sum(\mathbf{k}) = \frac{2}{1+x} \frac{1}{N} \sum_{\mathbf{q}} \left\{ \left[ t(\mathbf{q}) - \frac{1-x}{2} J(\mathbf{k}-\mathbf{q}) - x \frac{t_{01}^2(\mathbf{q})}{U} - (1+x) \frac{t_{01}^2(\mathbf{k})t_{01}^2(\mathbf{q})}{U} \right] K(\mathbf{q}) + \left[ t(\mathbf{k}-\mathbf{q}) - \frac{1-x}{2} \left( J(\mathbf{q}) - \frac{t_{01}^2(\mathbf{k}-\mathbf{q})}{U} \right) - \frac{(1+x)t_{01}^2(\mathbf{k})t_{01}^2(\mathbf{k}-\mathbf{q})}{U} \right] \cdot \frac{3}{2} C(\mathbf{q}) \right\}$$

$$K(\mathbf{q}) = \sum_{\mathbf{f}-\mathbf{g}} e^{-i(\mathbf{f}-\mathbf{g})\mathbf{q}} \langle X_{\mathbf{f}}^{2\bar{\sigma}} X_{\mathbf{g}}^{2\bar{\sigma}} \rangle \quad C(\mathbf{q}) = \sum_{\mathbf{f}-\mathbf{g}} e^{-i(\mathbf{f}-\mathbf{g})\mathbf{q}} \langle X_{\mathbf{f}}^{\sigma\bar{\sigma}} X_{\mathbf{g}}^{\bar{\sigma}\sigma} \rangle = 2 \sum_{\mathbf{f}-\mathbf{g}} e^{-i(\mathbf{f}-\mathbf{g})\mathbf{q}} \langle S_{\mathbf{f}}^z S_{\mathbf{g}}^z \rangle$$

Correlation functions are calculated follow Valkov and Dzebisashvili, JETP 2005

# Beyond the Hubbard 1: short range magnetic order in spin liquid state up to 9-th neighbor

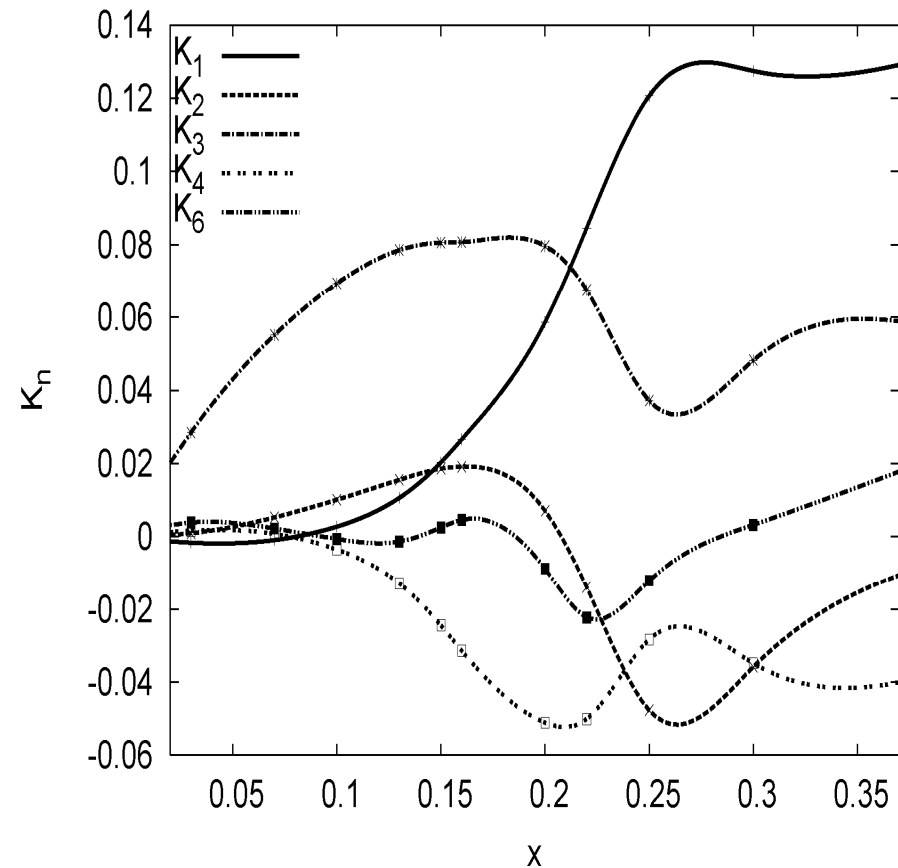
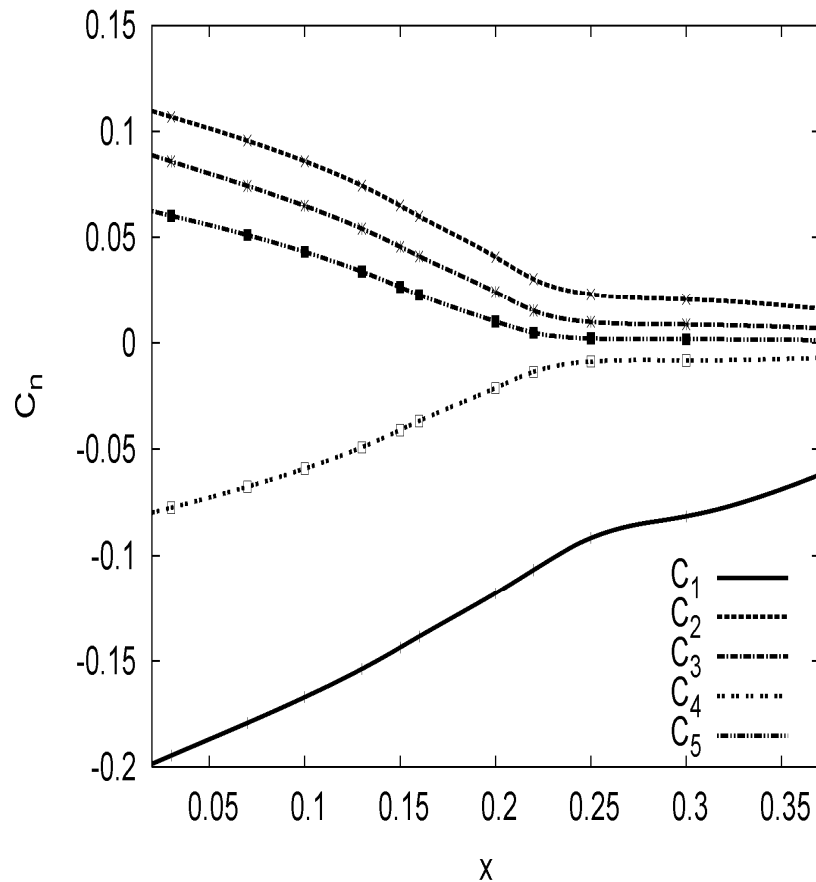


Self consistent spin and charge

Correlation functions in the t-J\* model

$$C_n = 2 \langle S_0^z S_n^z \rangle = \langle X_0^{\uparrow\downarrow} X_n^{\downarrow\uparrow} \rangle$$

$$K_n = \langle X_0^{\sigma 0} X_n^{0\sigma} \rangle$$





# Change of Fermi surface topology with doping

*Korshunov,  
Ovchinnikov  
Eur.Phys.J.B 2007*

$x_{c1}=0.15=P_{opt}$  – maximum  $T_c(x)$

$x_{c2}=0.24=P^*$  - critical point of the pseudogap formation

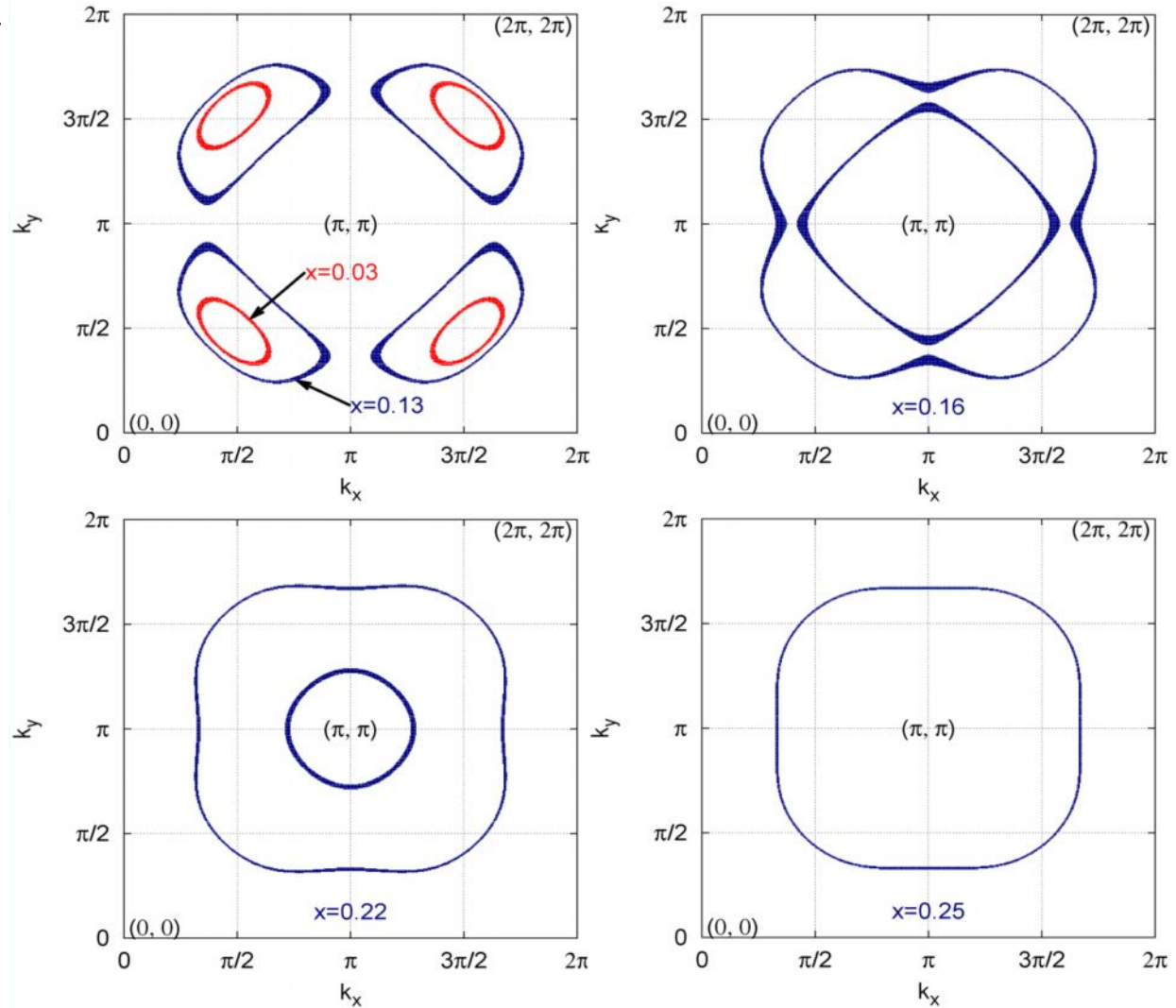


Fig. 1. Fermi surface evolution with doping (hole concentration)  $x$ .

## Effective mass from quantum oscillations measurements:

+ YBa<sub>2</sub>Cu<sub>3</sub>O<sub>6.5</sub>(p=0.1) Doiron-Leyraud | Nature, 2007

\* YBa<sub>2</sub>Cu<sub>4</sub>O<sub>8</sub> (p=0.125) Yelland et al, PRL 100, 2008

x YBa<sub>2</sub>Cu<sub>4</sub>O<sub>8</sub> Bangura et al, PRL 100, 047004, 2008

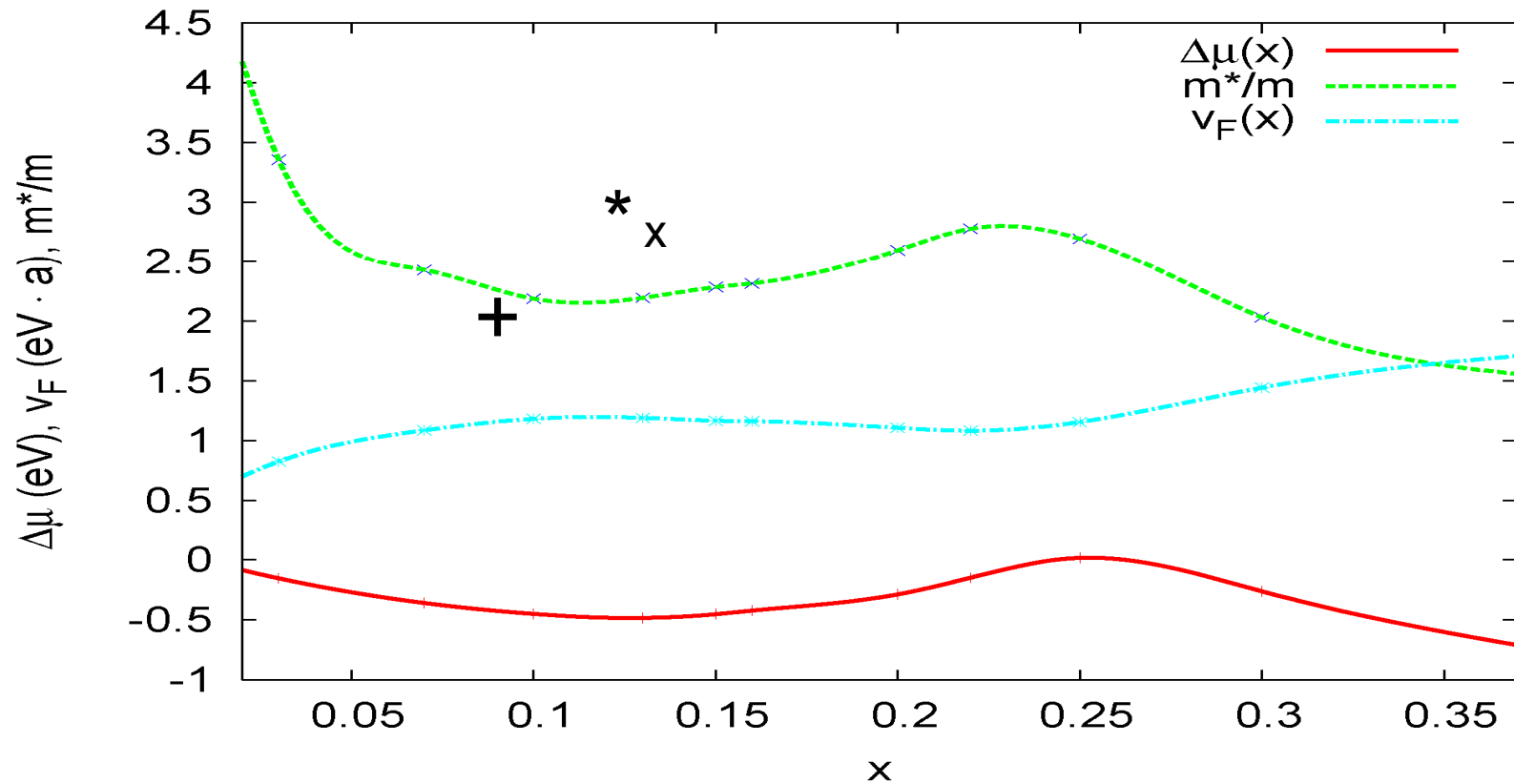


Fig. 2. Doping dependent evolution of the chemical potential shift, nodal Fermi velocity, and effective mass.

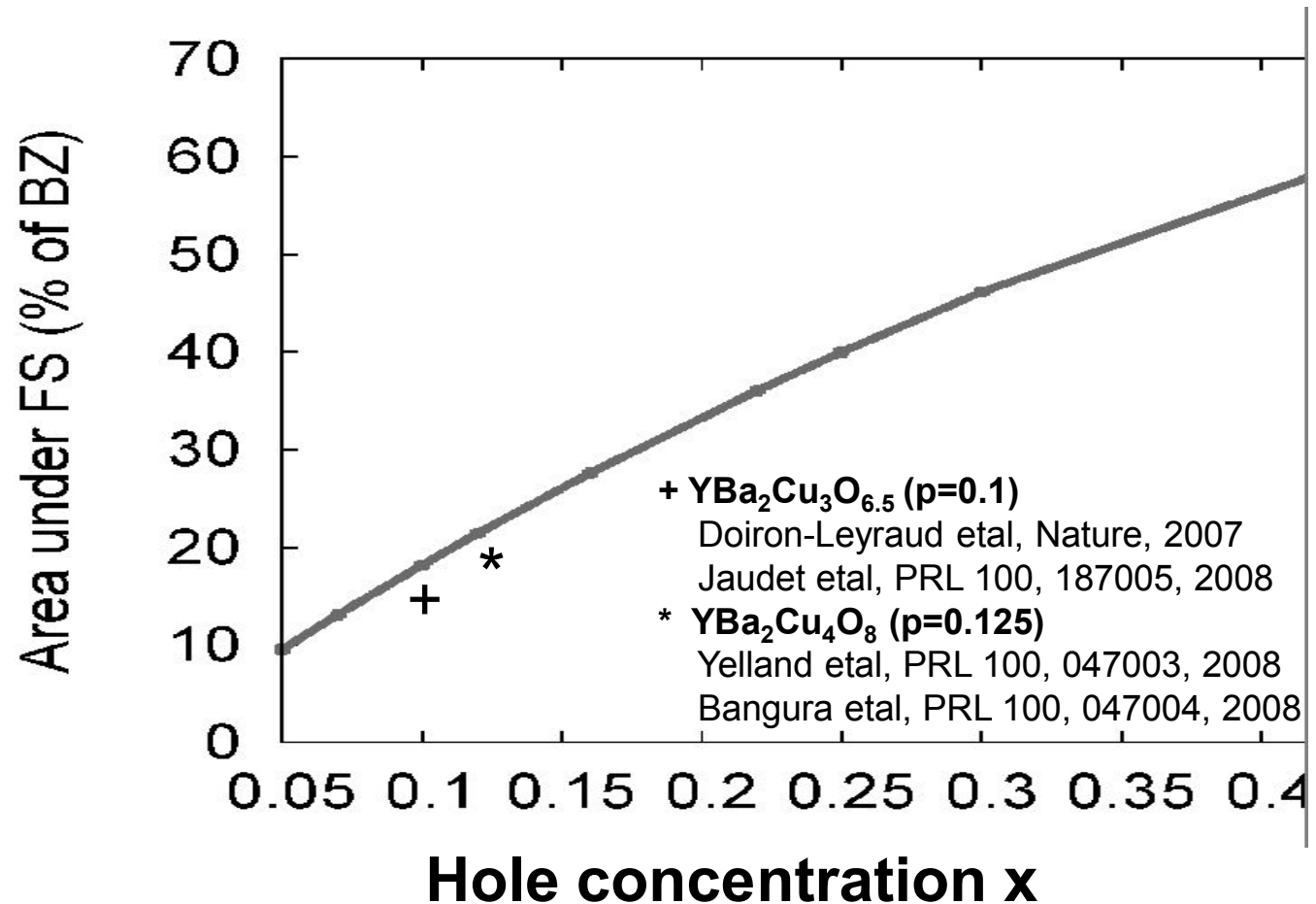
Fermi surface analysis: Hole concentration  $N_h=1+x$ ,  
 Electron concentration  $N_e=1-x$ . Number of occupied electron  
 states  $N_e(k)$ , spectral weight  $F=(1+x)/2$ . then  
 $N_e=2 \cdot F \cdot N_e(k)=1-x \rightarrow N_e(k)=(1-x)/(1+x)$

Hole occupied  
 states

$$N_h(k)=1-N_e(k)$$

$$= \frac{2x}{1+x}$$

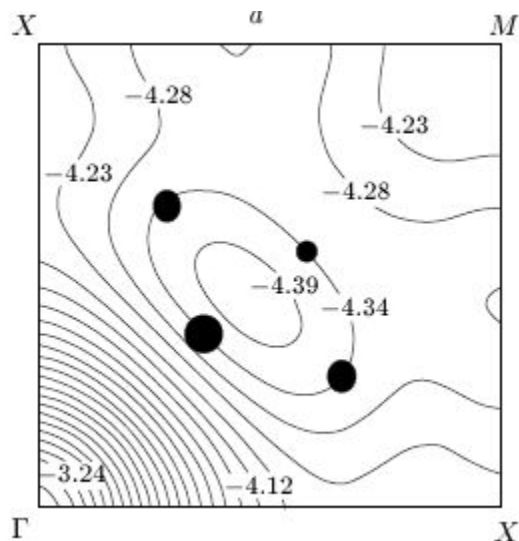
Generalised  
 Luttinger theorem:  
 Korshunov,  
 Ovchinnikov,  
 Sol.St.Phys 2003



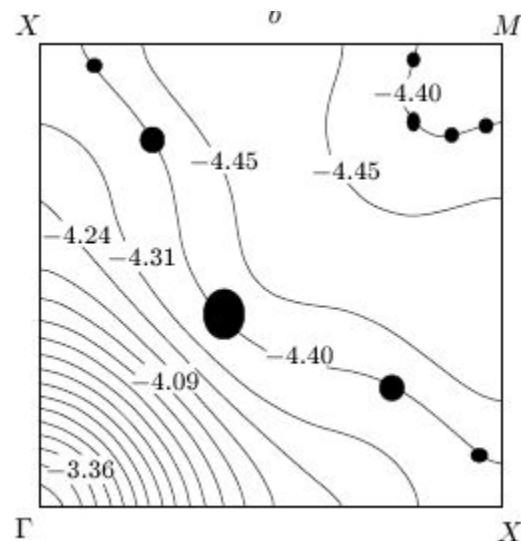
# Barabanov et al, JETP 119, вып.4 (2001)

Spin-polaron approach, *Mori-type projection method for spin-fermion model*

X=0.06



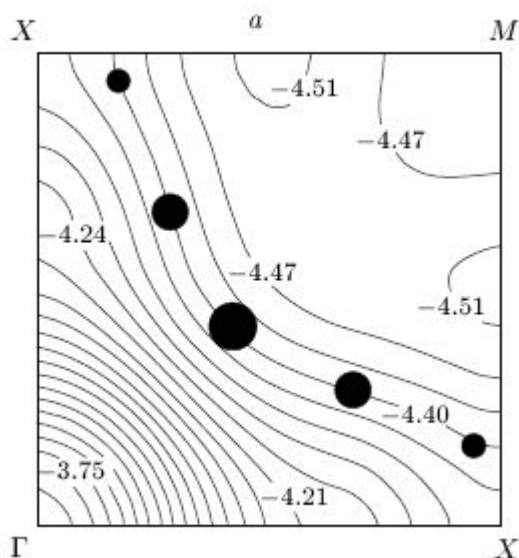
X=0.11



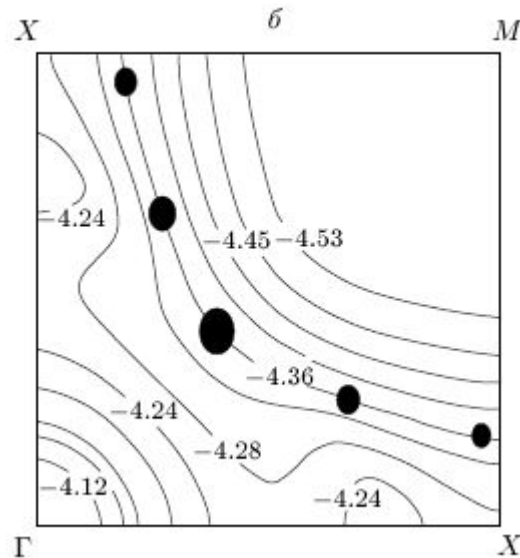
Shadow band intensity is smaller due to spectral weight redistribution.

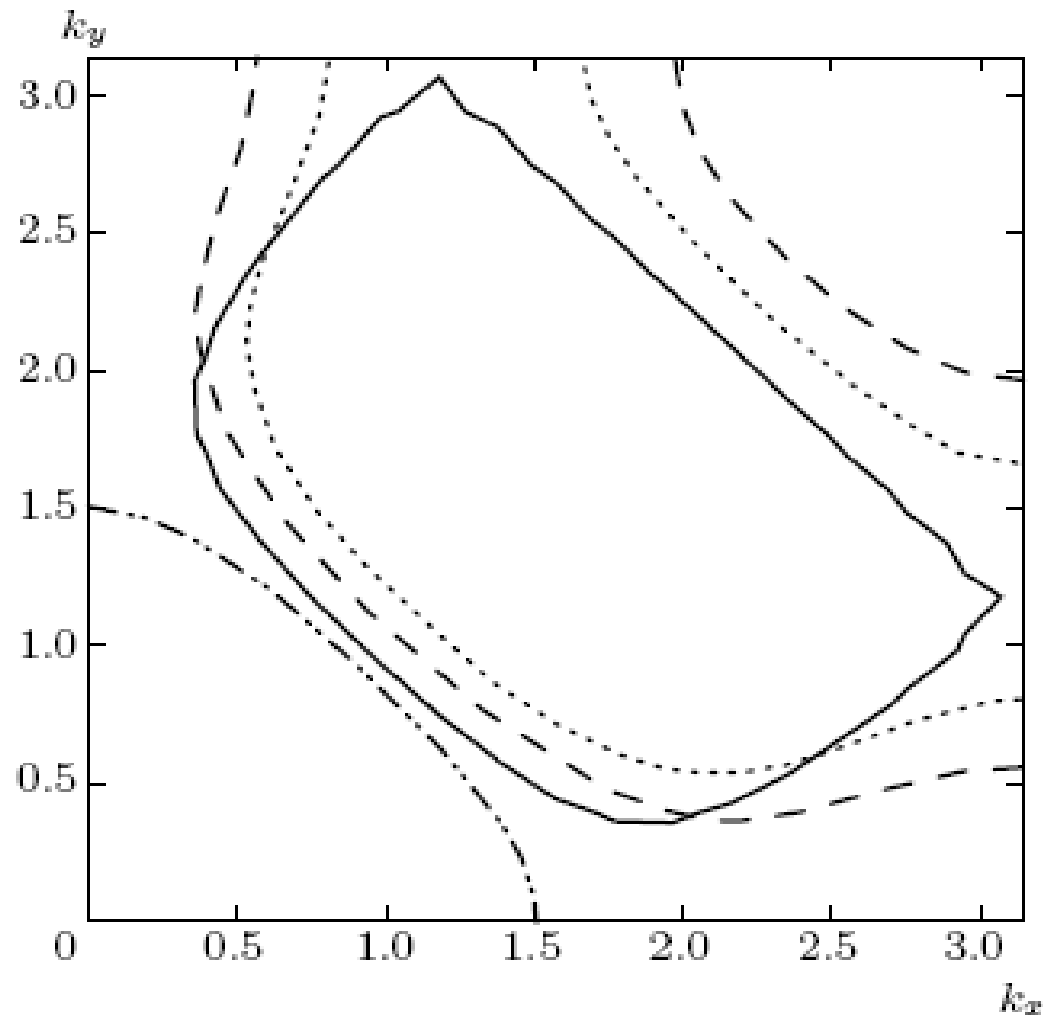
No QP damping

X=0.14



X=0.19





**Fig. 15.** Doping dependence of the FS for  $\delta = 0.1$  (solid line at  $T = 0.03t$  and dotted line at  $T = 0.3t$ ),  $\delta = 0.2$  (dashed line), and  $\delta = 0.3$  (dot-dashed line) for  $\Delta = 4t$

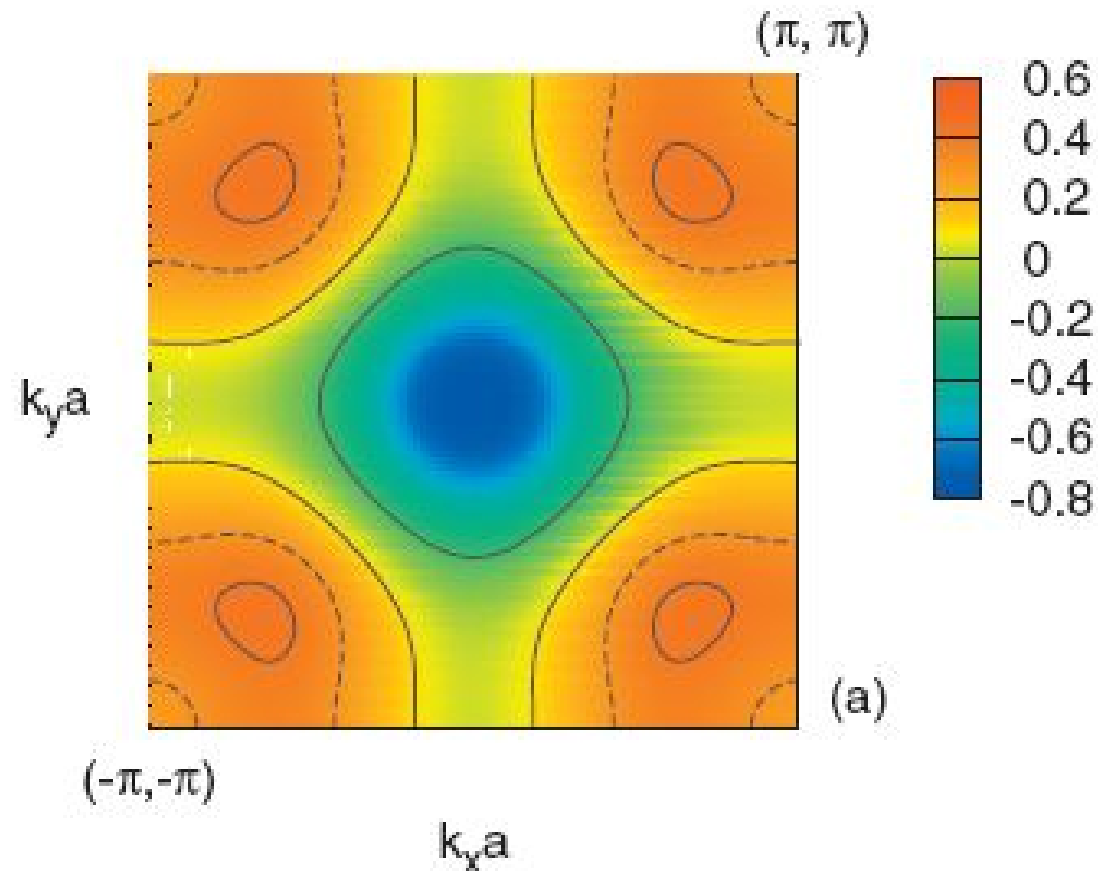
Fermi surface  
in the p-d Hubbard  
model,

SCBA (non crossing  
approximation) for the  
self-energy,

$$\Sigma(\kappa, \omega) = \Sigma_{\text{Re}} + i\Sigma_{\text{Im}}$$

Plakida, Oudovenko  
JETP 131, 259 (2007)

**Ab initio variational CASSSF method gives similar results for the doping evolution of the Fermi surface: from small hole pockets to large hole FS with both hole and electron FS in the intermediate region**  
(Hozoi, Laad, Fulde, PRB 2008)



# Effect of short antiferromagnetic order on the electron spectrum

*(Kuchinskii, Nekrasov, Sadovskii,*

*JETP Lett. 2005, JETP 2006, JETP Lett. 88, 224, 2008)*

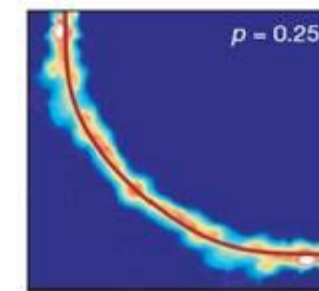
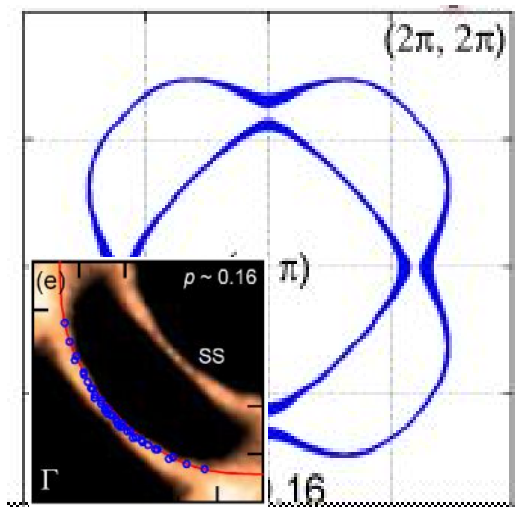
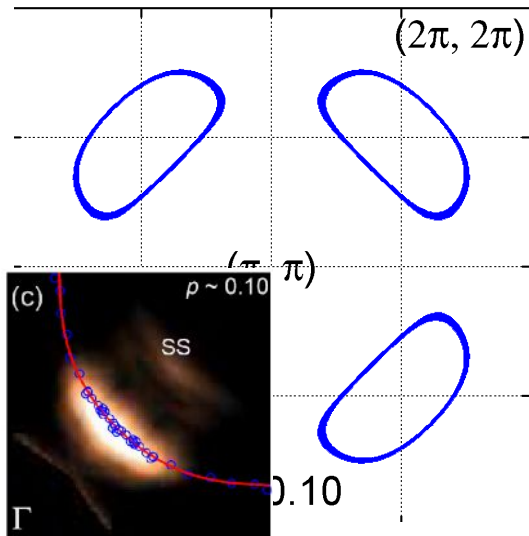
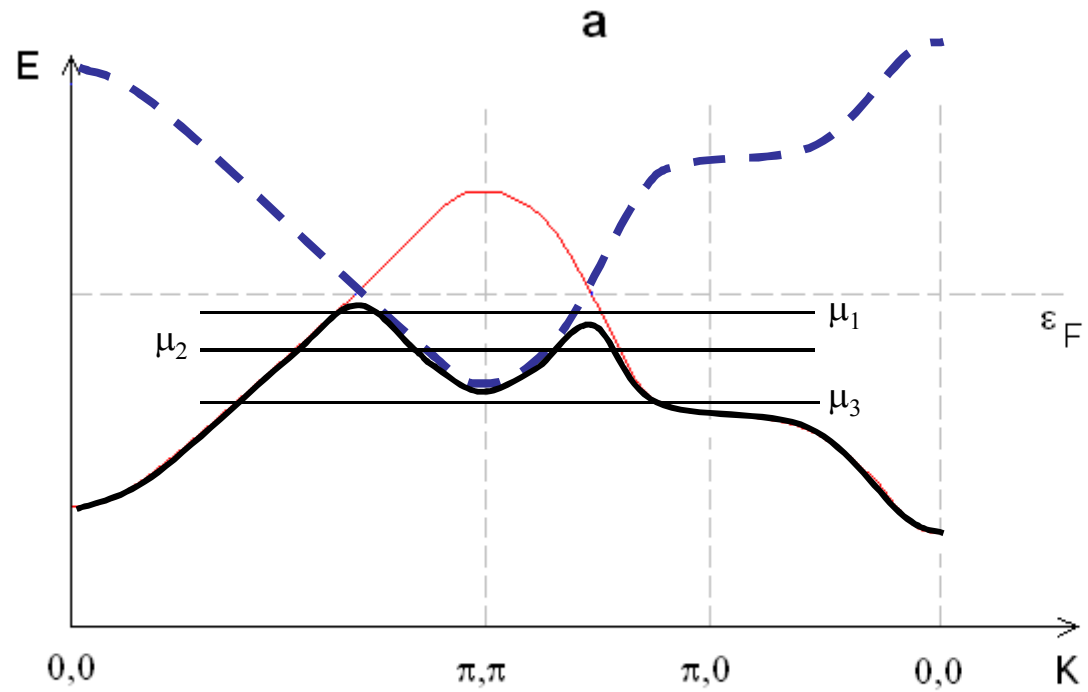
Green function for electron interacting with a random Gaussian spin fluctuation field is given by

$$G_D(\mathbf{k}, \varepsilon) = \frac{\varepsilon - \varepsilon(\mathbf{k} + \mathbf{Q}) + i\nu k}{(\varepsilon - \varepsilon(\mathbf{k}))(\varepsilon - \varepsilon(\mathbf{k} + \mathbf{Q}) + i\nu k) - |D|^2}$$

Here  $D$  is the amplitude of the fluctuating AFM order,  $\varepsilon(\mathbf{k})$  is the electron dispersion in the paramagnetic state

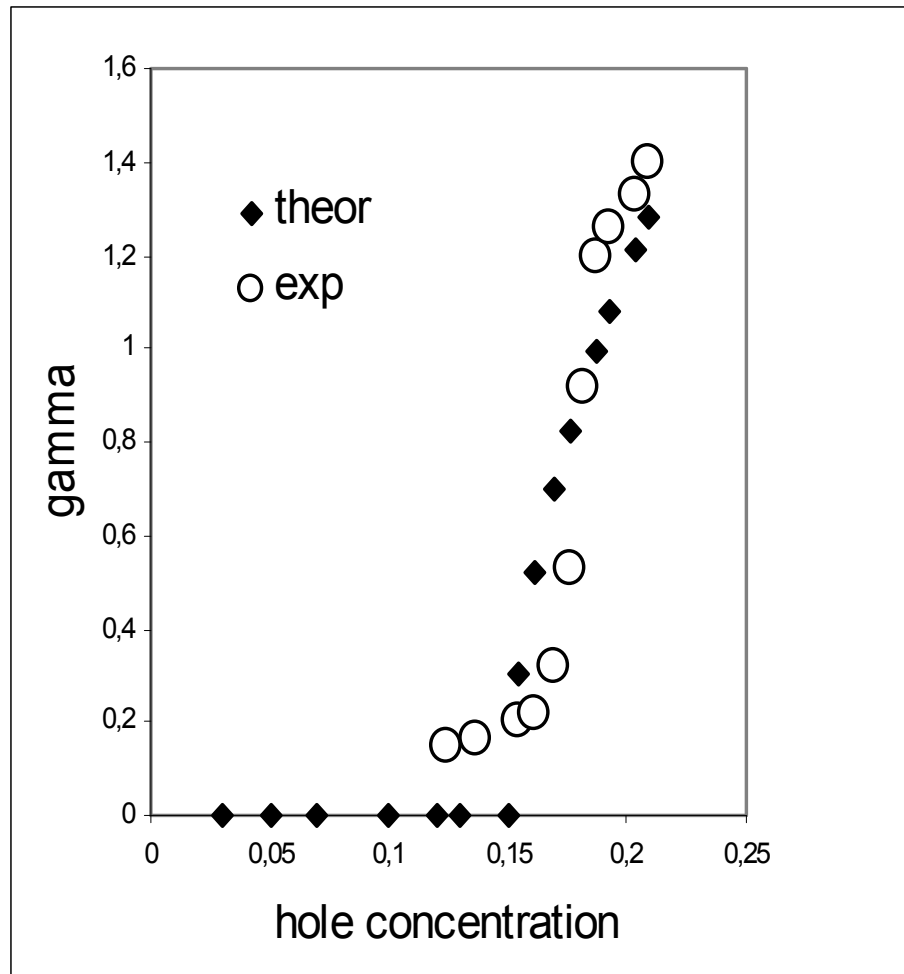
$$\nu = \left| v_x(\mathbf{k} + \mathbf{Q}) \right| + \left| v_y(\mathbf{k} + \mathbf{Q}) \right|, \quad v_{x,y}(\mathbf{k}) = \partial \varepsilon(\mathbf{k}) / \partial k_{xy}$$

In the limit of zero Im part the long range AFM (SDW) is restored





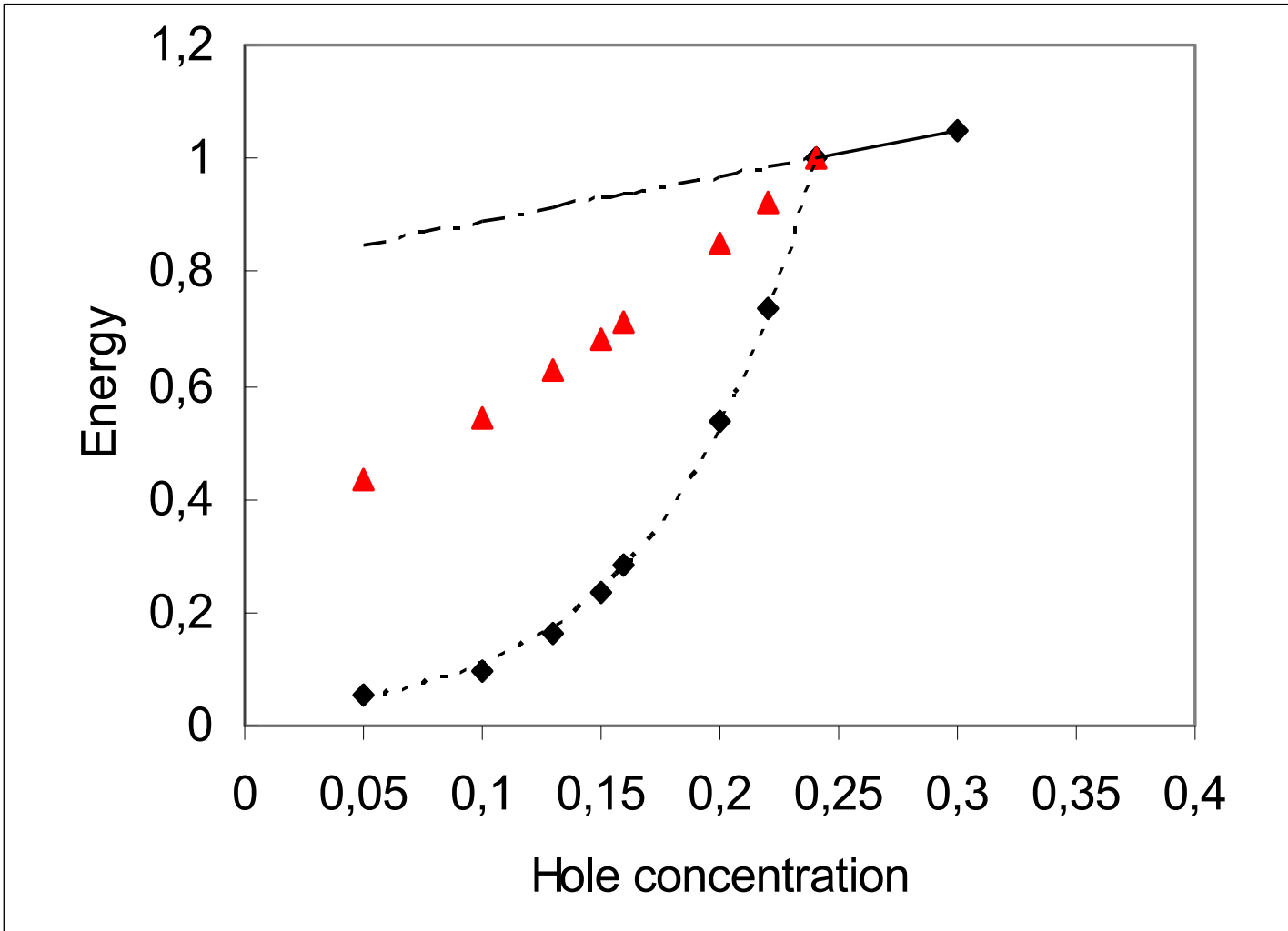
# Electronic heat capacity near the Lifshitz QPT



$$\delta\gamma = \delta(C_e/T) \sim (x - x_{c1})^{0.5}$$

Experiment from Loram et al,  
J.Phys.Chem.Sol.2001,  
T=10K

$$\gamma_{\text{exp}} = \gamma_0 + \delta\gamma$$



$$E_g(x) = J(1-x/p^*)$$

Kinetic energy  $E_{kin}(x)/E_{kin}(p^*)$  as function of doping. Above  $p^*$  dependence  $\sim(1+x)$  is expected for 2D electron gas. Below  $p^*$  its extrapolation reveals the depletion of kinetic energy due to pseudogap. Red triangles-fitting with Loram-Cooper triangular pseudogap model. Dash line – exponential fitting  $E/E^* \sim \exp(-4E_g(x)/J)$

# Magnetic mechanism of superconductivity in the Hubbard and t-J models

- Anderson RVB 1987, Baskaran, Zhou, Anderson 1987
- Cluster DMFT: Maier et al 2000; Lichtenstein and Katznelson 2000
- What is the superconducting glue: a combination of static and dynamical contributions (Scalapino).
- Dahm et al Nature Phys.2009:  $U$  and  $\chi(q,E)$  found from INS and ARPES, then  $T_c=150K$  for optimal doping
- Plakida et al, 1999: X-operators perturbation theory, Self energy in SCBA, both static  $J$  (85%) and dynamical  $\chi(q,E)$  (15%) contributions
- QMC gives controversial conclusions some pro and some contra, the well-known paper by Aimi and Imada, JPSJ 2007 by a new QMC reject d-type superconductivity. Is it the final answer?
- We will use Plakida-type formulation of the mean field theory with “no-double occupation” constraint and short AFM correlations

# Electron-phonon interaction in GTB method

S.G.Ovchinnikov and E.I.Shneyder, JETP 101, 844 (2005)

- $H_{el} = \sum_{fnp} (E_n - n\mu) X_f^{p,p} + \sum_{fgmm'} t_{fg}^{mm'} X_f^+ X_g^{m'}$  GTB Hamiltonian

- *Due to atomic displacements*  $\vec{R}_f = \vec{R}_f^{(0)} + \vec{u}_f$

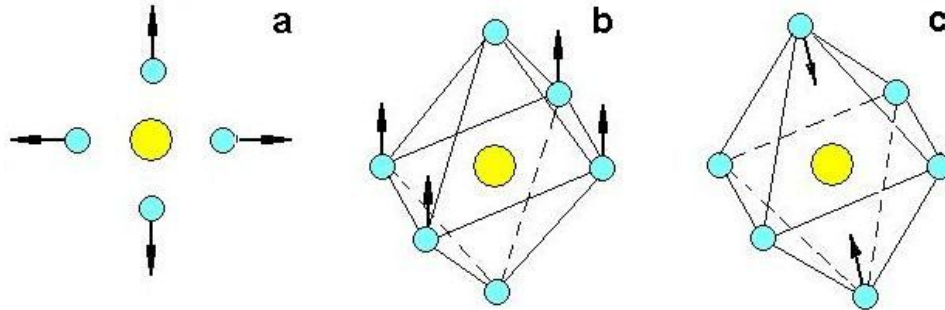
$$E_n \rightarrow E_n(f) = E_n(0) + \vec{g}_n \vec{u}_f, \quad t_{fg}^{mm'} \rightarrow t_{fg}^{mm'} = t_{fg}^{mm'}(0) + \vec{V}^{mm'} \vec{u}_{fg}$$

- *Electron – phonon interaction*

$$H_{el-ph} = \sum_{kqvmm'} g_{mm'}^{(v)}(\vec{k}, \vec{q}) X_k^+ X_{k+q}^{m'} (b_{q,v} + b_{-q,v}^+),$$

$$g_{mm'}^{(v)}(\vec{k}, \vec{q}) = \delta_{mm'} g_{m,dia}(q) + g_{mm',off}(\vec{k}, \vec{q})$$

## Oxygen displacements



- Breathing mode (a),
- Buckling mode (b),
- Apical breathing mode (c).

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N.S. Nunner et.al., PRB, 8859 (1999).

### Matrix

$$V_{dia,m}^{(1)}(\mathbf{q}) = \frac{2ig_{dia,m}^{(1)}}{\sqrt{2M_O\omega_{q,v=1}}} \left( e_x(O) \sin \frac{q_x a}{2} + e_y(O) \sin \frac{q_y a}{2} \right),$$

$$V_{off,mm'}^{(1)}(\mathbf{k}, \mathbf{q}) = \frac{8ig_{off,mm'}^{(1)}}{\sqrt{2M_O\omega_{q,1}}} \left[ e_x(O_x) \sin \frac{q_x a}{2} + e_y(O_y) \sin \frac{q_y a}{2} \right] [\gamma(\mathbf{k}) + \gamma(\mathbf{k} + \mathbf{q})],$$

$$\text{zde } \gamma(\mathbf{q}) = (\cos q_x a + \cos q_y a) / 2.$$

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Sumio Ishihara and Naoto Nagaosa, PRB,144520 (2004).

# Mean field theory of d-type superconductivity with magnetic and phonon pairing with “no-double occupation” constraint

(E.Shneyder, S.Ovchinnikov, JETP Lett. 83,394(2006))

Electronic contribution to pairing in t-J\* model

Phonon contribution

$$\Delta_{\mathbf{k}}^{tJ^*} = \frac{1}{N} \sum_{\mathbf{q}} \left( -\frac{4}{1+x} t_{\mathbf{q}} - \frac{1-x}{1+x} (J_{\mathbf{k}+\mathbf{q}} + J_{\mathbf{k}-\mathbf{q}}) - 4t_{\mathbf{k}}t_{\mathbf{q}} + \frac{1-x}{1+x} \frac{t_{\mathbf{q}}^2}{E_{ct}} \right) B_{\mathbf{q}},$$

$$\Delta_{\mathbf{k}}^{el-ph} = \frac{1}{N} \sum_{\mathbf{q}} \frac{1+x}{4} \{V_{-\mathbf{q},\mathbf{q},\mathbf{q}+\mathbf{k}} + V_{-\mathbf{q},\mathbf{q},\mathbf{q}-\mathbf{k}}\} B_{\mathbf{q}} - \frac{1}{N^2} \sum_{\mathbf{q},\mathbf{p}} \frac{3}{2(1+x)} \{V_{-\mathbf{q},\mathbf{q},\mathbf{p}+\mathbf{k}} + V_{-\mathbf{q},\mathbf{q},\mathbf{p}-\mathbf{k}}\} B_{\mathbf{q}} c_{\mathbf{q}-\mathbf{p}}.$$

$$B_{\mathbf{q}} = \langle X_{-\mathbf{q}}^{0,-\sigma} X_{\mathbf{q}}^{0,\sigma} \rangle$$

$c_{\mathbf{q}}$  - spin correlation function,

$$\Delta_{\mathbf{k}}^{\text{tot}} = \Delta_{\mathbf{k}}^{tJ^*} + \Delta_{\mathbf{k}}^{el-ph}.$$

$$\Delta_{\mathbf{k}}^{(d_x^2-y^2)} = -\frac{1}{N} \varphi(\mathbf{k}) \sum_{\mathbf{q}} \left\{ \frac{1-x}{1+x} J + \left( \frac{3(-c_{01}) + 0.5(1+x)^2}{2(1+x)} \right) \times \theta(|\xi_{\mathbf{q}} - \mu| - \omega_D) G \right\} \times B_{\mathbf{q}} \varphi(\mathbf{q}),$$

$$G = \left( \frac{v_{\text{dia},\nu=2}^2}{\omega_{\nu=2}} - \frac{v_{\text{dia},\nu=1}^2}{\omega_{\nu=1}} \right)$$

$$\varphi(\mathbf{k}) = (\cos(k_x) - \cos(k_y)), \theta(x) = 0$$

G =Buckling- Breathing modes EPI, no contribution from the apical breathing mode

# Oxygen isotope effect: experiment and theory

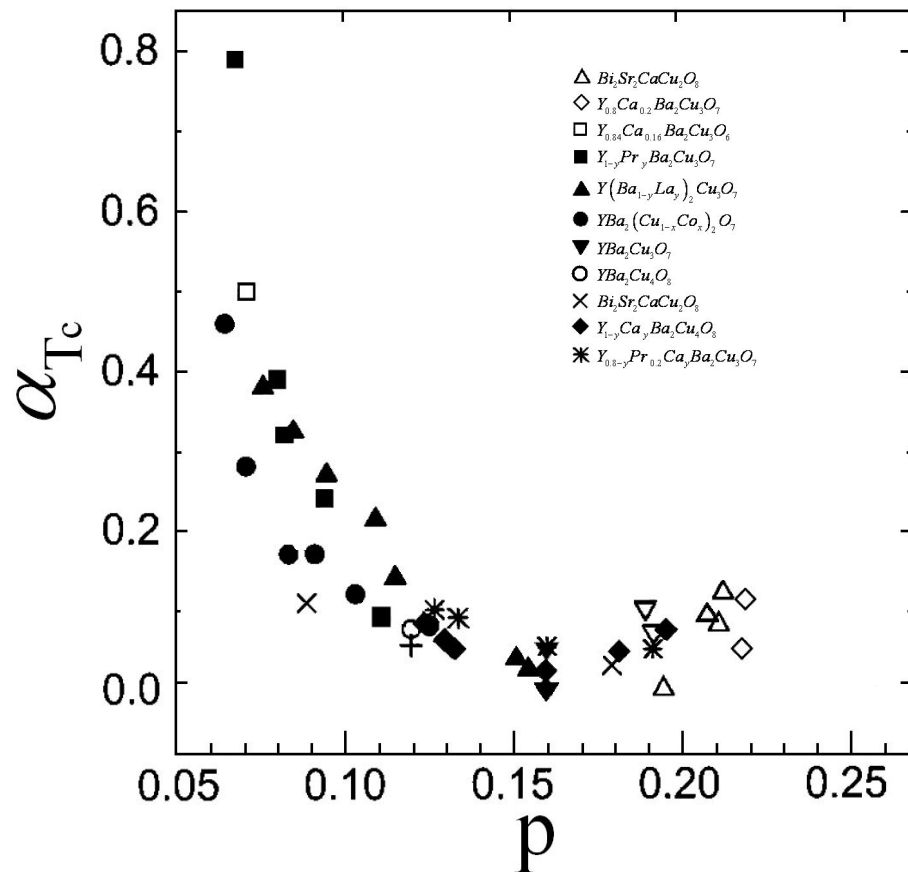
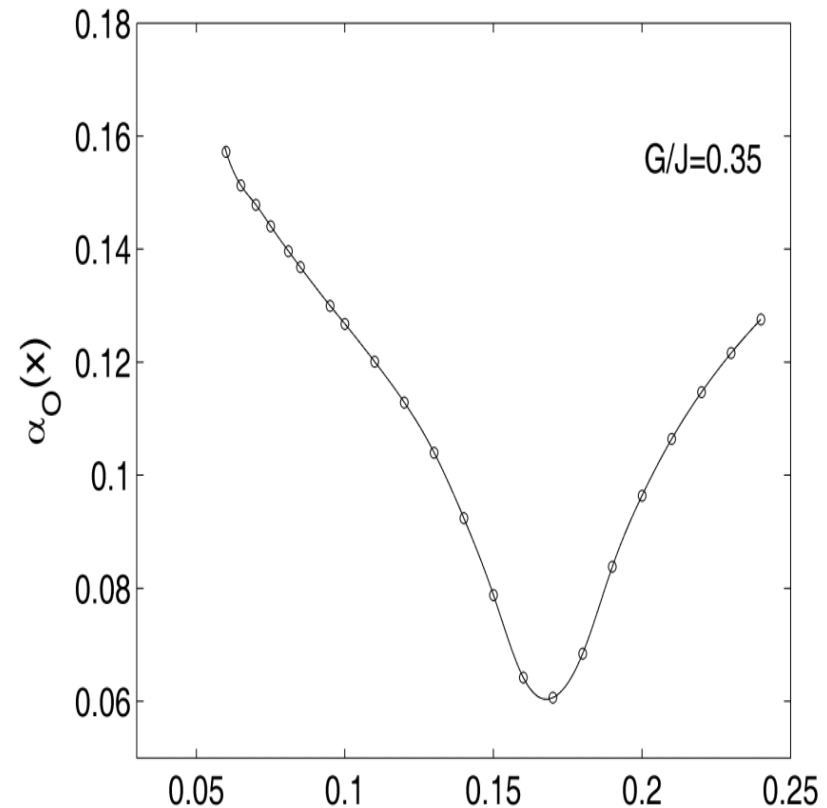
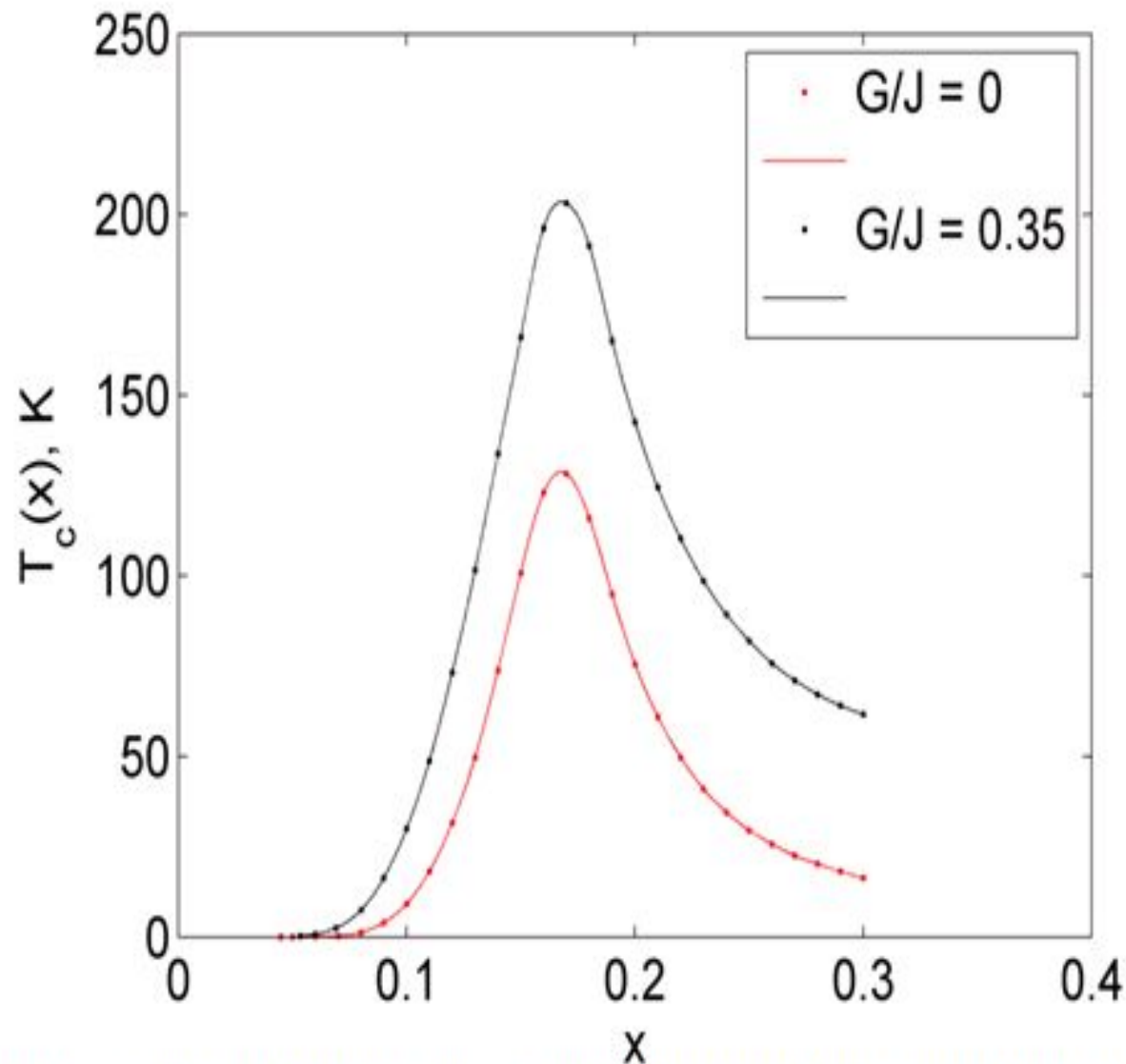


Fig. 11. Plot of the oxygen isotope effect coefficient in Tc against hole concentration for various SC cuprates (D.J. Pringle et al., PRB 62 and references therein).



E.Shneyder, S.Ovchinnikov  
FPS, 2008



S.Ovchinnikov,  
E.Shneyder, 2008

Figure 2. The critical temperature as function of the doping concentration in  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ . The maximum corresponds to the optimal doping  $x=0.17$ . The pure magnetic mechanism takes place at  $G=0$  and both magnetic and phonon mechanisms are considered with parameter  $G/J=0.35$  determined from the isotope effect. The phonon-mediated pairing works together with the magnetic one increasing each other.



# Conclusion

- Both “normal pseudogapped”, and d-type superconducting state can be obtained from hole dynamics at the short range AFM background in the self consistent 2D electronic and spin systems in the strong electron correlation regime
- Fermi surface topology changes result in QPT with doping
- Phonon contribution to pairing may decrease or increase magnetic one. The only fitting parameter  $G > 0$  of the EPI was found from the isotope effect. The EPI and magnetic mechanism support each other and are of the same order of magnitude

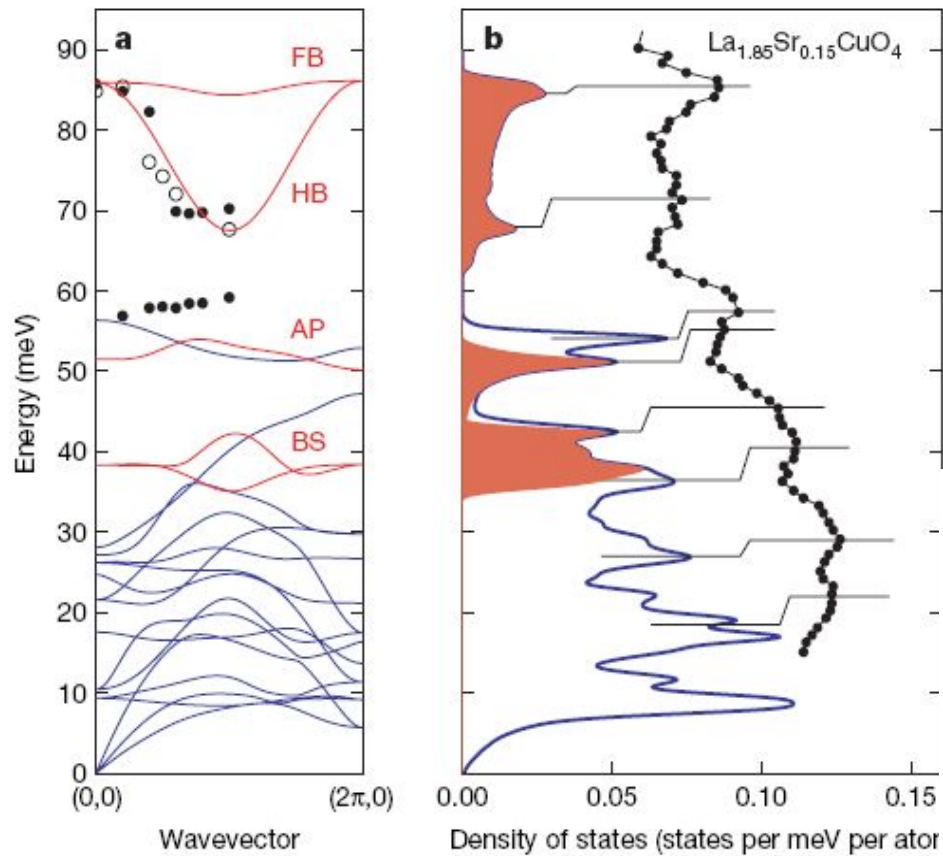


Figure 1 | Phonons of  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  at optimal doping ( $x = 0.15$ ).

**Breathing modes 70meV + buckling / stretching in plane O-O modes 40meV provide 80% (green) of the total self-energy**

**Small phonon contribution to ARPES kink**  
 F.Giustino, M.Cohen, S.Louie,  
 NATURE 06874, 2008  
 (GGA+DF perturbation theory)

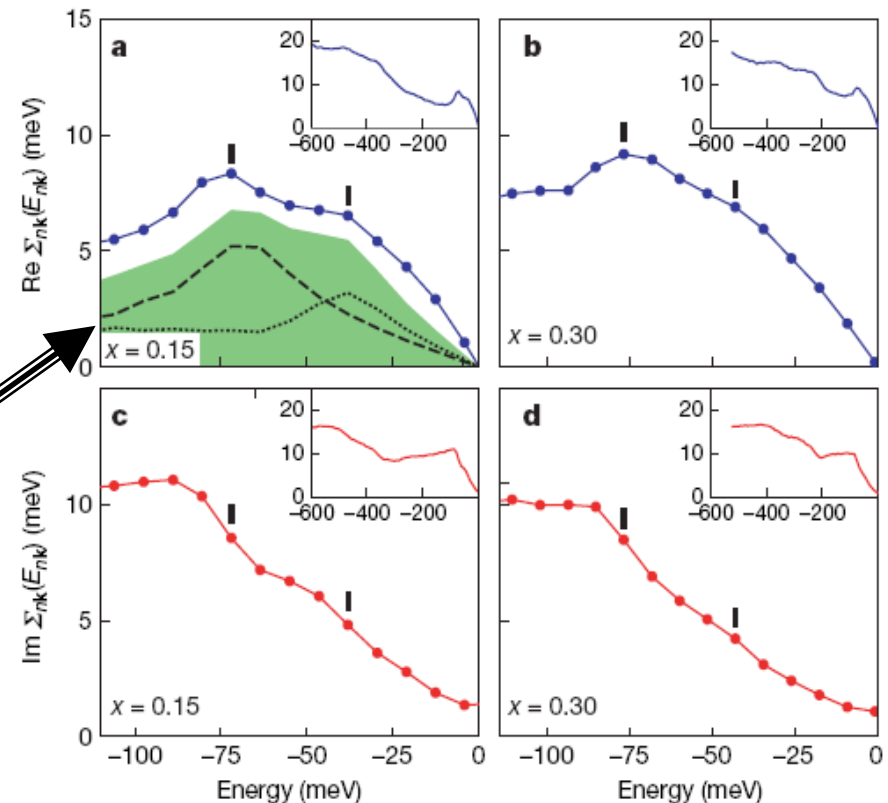


Figure 2 | Calculated electron self-energy in LSCO due to the electron-phonon interaction. a, b, Real parts of the self-energy for optimally