The Heterotic String:

From Super-Geometry to the LHC

Burt Ovrut

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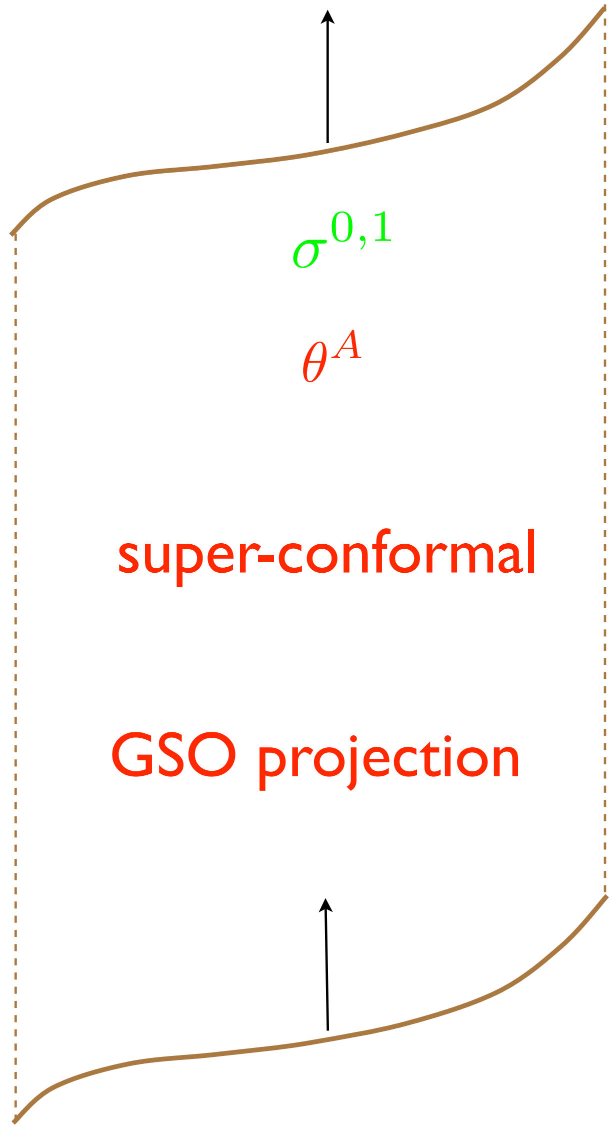
The Heterotic Superstring

D-dimensions

$g_{IJ}, \ B_{IJ}, \ \Phi$



$\sigma^{0,1}$

$\Psi \ \Rightarrow$ θ^{A} $\Rightarrow$ $A_I^a, \ E_8 \times E_8$

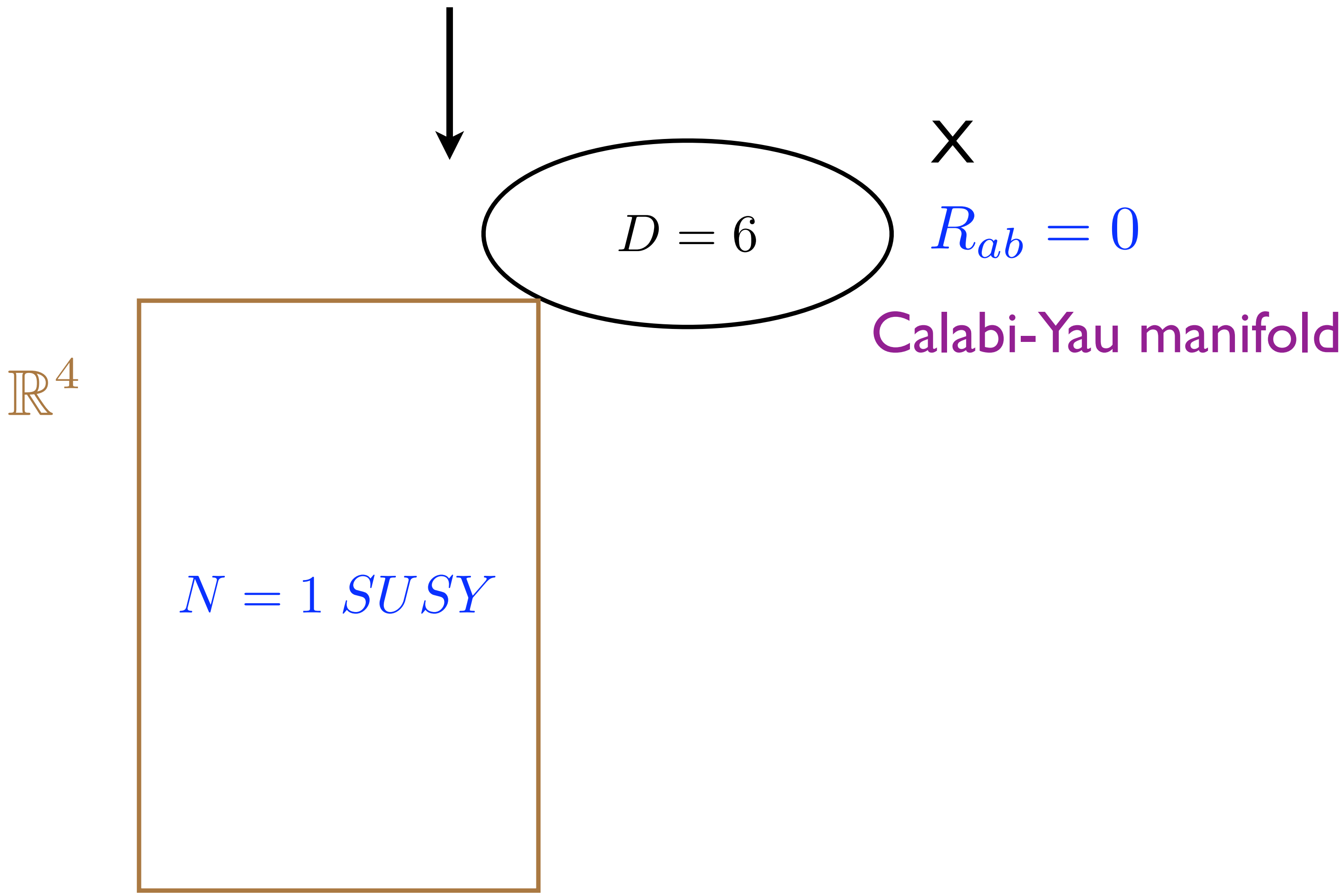
no "ghosts" $\Rightarrow$ super-conformal $\Rightarrow$ $D = 10$

no "tachyon" $\Rightarrow$ GSO projection $\Rightarrow$ $\underline{N = 1}$

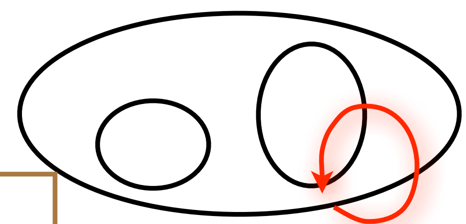
<u>supersymmetry</u>

Heterotic Compactifications

Spacetime: $D = 10\ ,\ g_{MN}$



Gauge Connection: $D = 10\ ,\ A^a_M\ , E_8$



V,G

"Slope" stable

W,F

$H = [E_8, G]$

$N = 1\ SUSY$

$\mathcal{H} = [H, F]$

$$H^1(V)^F \Rightarrow matter$$

$$H^1(V^*)^F \Rightarrow conjugate\ matter$$

$$H^1(\wedge^2 V)^F \Rightarrow Higgs$$

$$H^1(V \otimes V^*)^F \Rightarrow Bundle\ Moduli$$

- Heterotic Standard Model: $V\ , G = SU(4)\, ,\ W\ , F = \mathbb{Z}_3 \times \mathbb{Z}_3$

$\mathbb{R}^4$ Theory Gauge Group:

Gauge connection $G = SU(4) \Rightarrow$

$$E_8 \longrightarrow \quad H = Spin(10)$$

Wilson line $F = \mathbb{Z}_3 \times \mathbb{Z}_3 \Rightarrow$

$$Spin(10) \longrightarrow \quad \mathcal{H} = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$$

rank Spin(10)=5 plus F Abelian ⇒ extra gauged $U(1)_{B-L}$.

Note that

$$\mathbb{Z}_2 \ (R - \text{parity}) \subset U(1)_{B-L}$$

⇒ no rapid proton decay. But must be spontaneously broken above the scale of weak interactions.

$\mathbb{R}^4$ Theory Spectrum:

$$E_8 \xrightarrow{V} Spin(10) \Rightarrow$$

$$248 = (1, 45) \oplus (4, 16) \oplus (\bar{4}, \bar{16}) \oplus (6, 10) \oplus (15, 1)$$

The Spin(10) spectrum is determined from

45 $\quad n_{45} = h^0(X, \mathcal{O}) = 1$

16 $\quad n_{16} = h^1(X, V) = 27$

$\overline{16}$ $\quad n_{\overline{16}} = h^1(X, V^*) = 0$

10 $\quad n_{10} = h^1(X, \wedge^2 V) = 4$

1 $\quad n_1 = h^1(X, V \otimes V^*) = 117$

$Spin(10) \xrightarrow{F} SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L} \Rightarrow$

a) Find representation of $\mathbb{Z}_3 \times \mathbb{Z}_3$ on $H^1(X, U_R(V))$.

Example: $n_{16} = h^1(X,V) = 27 \Rightarrow H^1(X,V) = RG^{\oplus 3}$ where

$$RG = 1 \oplus \chi_1 \oplus \chi_2 \oplus \chi_1^2 \oplus \chi_2^2 \oplus \chi_1\chi_2 \oplus \chi_1^2\chi_2 \oplus \chi_1\chi_2^2 \oplus \chi_1^2\chi_2^2$$

b) Find action of $\mathbb{Z}_3 \times \mathbb{Z}_3$ on representation R. Example:

$$16 = [\chi_1{\chi_2}^2(3,2,1,1) \oplus {\chi_2}^2(1,1,6,3) \oplus {\chi_1}^2{\chi_2}^2(\bar{3},1,-4,-1)]$$
$$\oplus[(1,2,-3,-3) \oplus {\chi_1}^2(\bar{3},1,2,-1)] \oplus \chi_2(1,1,0,3)$$

Tensoring and taking invariant subspace gives 3 families of quarks/leptons each transforming as

$$Q_L = (3,2,1,1), \quad u_R = (\bar{3},1,-4,-1), \quad d_R = (\bar{3},1,2,-1)$$

$$L_L = (1,2,-3,-3), e_R = (1,1,6,3), \quad \nu_R = (1,1,0,3)$$

under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$.

Similarly we get 1 pair of Higgs-Higgs conjugate fields

$$H = (1, 2, 3, 0), \quad \bar{H} = (1, \bar{2}, -3, 0)$$

That is, we get exactly the matter spectrum of the MSSM!

In addition, there are $n_1 = h^1(X, V \times V^*)^{\mathbb{Z}_3 \times \mathbb{Z}_3} = 13$ vector bundle moduli

$$\phi = (1, 1, 0, 0)$$

Supersymmetric Interactions:

The most general superpotential is

$$W = \sum_{i=1}^{3} (\lambda_{u,i} Q_i H u_i + \lambda_{d,i} Q_i \bar{H} d_i + \lambda_{\nu,i} L_i H \nu_i + \lambda_{e,i} L_i \bar{H} e_i)$$

Note B-L symmetry forbids dangerous B and L violating terms

$$LLe, \quad LQd, \quad udd$$

Can we evaluate the Yukawa couplings from first principles?

Yes!

a) Texture:

$$W = \ldots \lambda L H r + \ldots$$

$\Rightarrow$ a Yukawa coupling is the triple product

$$H^1(X,V)^{\mathbb{Z}_3\times\mathbb{Z}_3} \otimes H^1(X,\wedge^2 V)^{\mathbb{Z}_3\times\mathbb{Z}_3} \otimes H^1(X,V)^{\mathbb{Z}_3\times\mathbb{Z}_3} \longrightarrow \mathbb{C}$$

Internal super-geometry (X elliptically fibered over dP9 base) $\Rightarrow$ in flavor diagonal basis for each of u, d, ν, e

$$\lambda_1 = 0, \quad \lambda_2, \lambda_3 \neq 0$$

That is, naturally light first family and heavy second/third families.

b) Explicit Calculation:

The triple product ⇒

$$\lambda = \int_X \sqrt{g_{\mu\nu}} \psi_L^a \psi_H^{[b,c]} \psi_r^d \epsilon_{abcd} d^6 x$$

where

$$\nabla_{**}^2 \psi^* = \lambda \psi^* \, , \lambda = 0$$

⇒ need to calculate the metric and eigenfunctions of the Laplacian. Unfortunately, a Calabi-Yau manifold does not admit a continuous symmetry. ⇒ the metric, gauge connection and, hence, the Laplacian are unknown! Remarkably, these can be well-approximated by numerical methods.

Ricci-Flat Metrics and Scalar Laplacians on Calabi-Yau Threefolds

Let $s_\alpha, \alpha = 0, \ldots, N_k - 1$ be degree-k polynomials on the CY and $h_{\text{bal}}^{\alpha\bar{\beta}}$ a specific matrix. Defining

$$g_{(\text{bal})i\bar{j}}^{(k)} = \frac{1}{k\pi}\partial_i\partial_{\bar{j}} \ln \sum_{\alpha,\bar{\beta}=0}^{N_k-1} h_{\text{bal}}^{\alpha\bar{\beta}} s_\alpha \bar{s}_{\bar{\beta}}$$

.

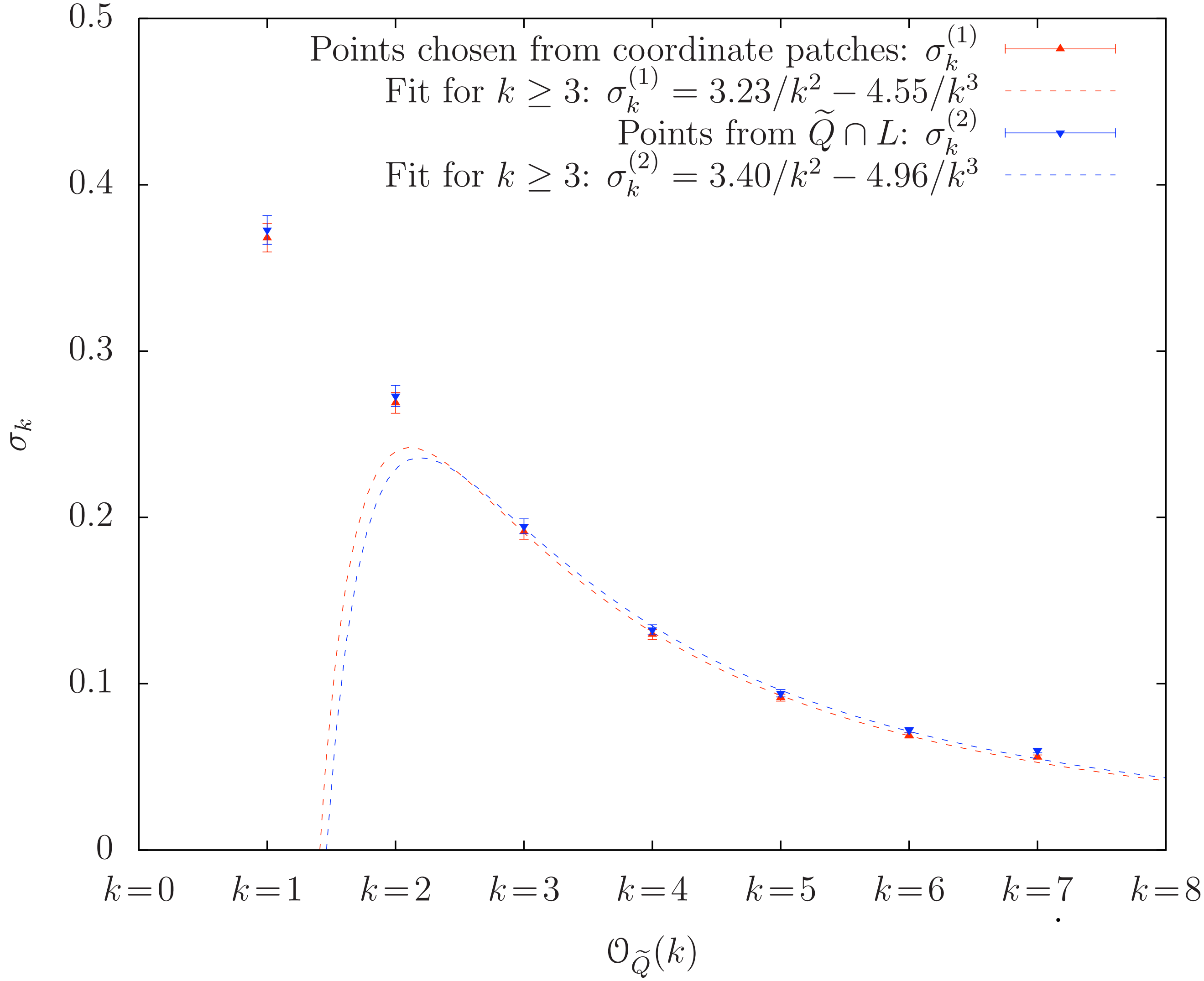
then

$$g_{(\text{bal})i\bar{j}}^{(k)} \xrightarrow{k\to\infty} g_{i\bar{j}}^{CY}$$

Expressed this way, $g_{(\text{bal})i\bar{j}}^{(k)}$ at any finite k is not very enlightening. More interesting is how closely they approach $g_{i\bar{j}}^{CY}$ for large k. This can be estimated using

$$\sigma_k(\tilde{Q}) = \frac{1}{Vol_{CY}(\tilde{Q})} \int_{\tilde{Q}} \left| 1 - \frac{\omega_k^3 \Big/ Vol_K(\tilde{Q})}{\Omega \wedge \bar{\Omega} \Big/ Vol_{CY}(\tilde{Q})} \right| dVol_{CY}$$

Fermat quintic:



The error measure σ_k for the metric on the Fermat quintic, computed with the two different point generation algorithms

Scalar Laplacians:

Given a metric $g_{\mu\nu}$ $\Rightarrow$

$$\Delta = -\frac{1}{\sqrt{g}}\partial_\mu(g^{\mu\nu}\sqrt{g}\partial_\nu)$$

Solve the eigen-equation

$$\Delta\phi_{m,i} = \lambda_m\phi_{m,i}\ ,\ i = 1,\ldots\mu_m$$

where μ_m is the multiplicity from continuous/finite symmetry.

Choose a basis $\{f_a\}$ $\Rightarrow$ the eigen-equation becomes

$$\sum_b \langle f_a|\Delta|f_b\rangle\langle f_b|\tilde{\phi}_{m,i}\rangle = \sum_b \lambda_m\langle f_a|f_b\rangle\langle f_b|\tilde{\phi}_{m,i}\rangle$$

Numerical Solution:

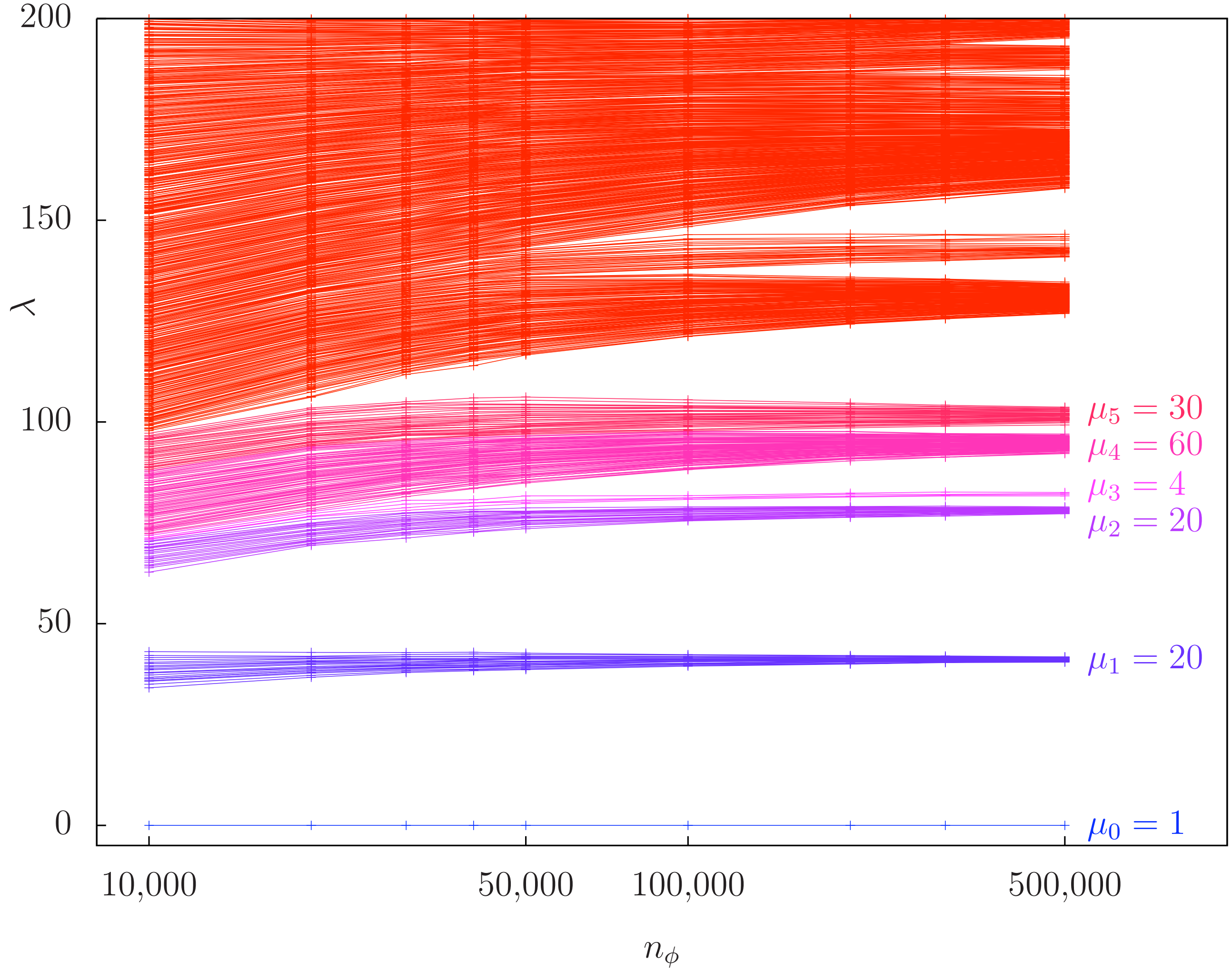
1) Choose a finite sub-basis $\{f_a|a=1,\ldots,k\}$

2) Calculate the finite-dimensional matrices $(\Delta_{ab})_{1\leq a,b\leq k}$ and $\langle f_a|f_b\rangle_{1\leq a,b\leq k}$

3) Solve numerically for λ_n and ϕ_n

4) For fixed k let $n_\phi\to\infty$ / for fixed n_ϕ let $k\to\infty$

Fermat quintic:



Eigenvalues of the scalar Laplace operator on the Fermat quintic. The metric is computed at degree $k_h = 8$, using $n_h = 2{,}166{,}000$ points. The Laplace operator is evaluated at degree $k_\phi = 3$ using a varying number n_ϕ of points.

Tabulating the results

m	0	1	2	3	4	5
$\hat{\lambda}_m$	1.18×10^{-14}	41.1 ± 0.4	78.1 ± 0.5	82.1 ± 0.3	94.5 ± 1	102 ± 1
μ_m	1	20	20	4	60	30

The non-trivial multiplicity $\Longrightarrow$ there must be a symmetry.
CY manifolds have no continuous symmetry, but they can have a finite isometry. For the Fermat quintic this is

$$\overline{\mathrm{Aut}}(\tilde{Q}_F) = (S_5 \times \mathbb{Z}_2) \ltimes (\mathbb{Z}_5)^4$$

with irreducible representations

d	1	2	4	5	6	8	10	12	20	30	40	60	80	120
# of irreps in dim d	4	4	4	4	2	4	4	2	8	8	12	18	4	2

Match perfectly!

Supersymmetry Breaking, the Renormalization Group and the LHC

Soft Supersymmetry Breaking:

N=1 Supersymmetry is spontaneously broken by the moduli during compactification ⇒ soft supersymmetry breaking interactions. The relevant ones are

$$V_{2s} = m_{\nu_3}^2 |\nu_3|^2 + m_H^2 |H|^2 + m_{\bar{H}}^2 |\bar{H}|^2 - (BH\bar{H} + hc) + \ldots$$

$$V_{2f} = \frac{1}{2} M_3 \lambda_3 \lambda_3 + \ldots$$

At the compactification scale $M_C \simeq 10^{16} GeV$ these parameters are fixed by the vacuum values of the moduli. For example

$$m_{\nu_3}^2 = m_{\nu_3}^2(\langle \phi \rangle)$$

However, at a lower scale μ measured by $t = ln(\frac{\mu}{M_C})$ these parameters change under the renormalization group.

For example,

$$16\pi^2 \frac{dm_{\nu_3}^2}{dt} \simeq \frac{3}{4} {g_4}^2 \mathcal{S}_1^{'}, \quad \mathcal{S}_1^{'}(0) = 61.5 \; m_\nu(0)^2$$

Solving this, at a scale $\mu \simeq 10^4 GeV \Rightarrow t_{B-L} \simeq -25$

$$m_{\nu_3}(t_{B-L})^2 = m_\nu(0)^2 - (3.10 \times 10^{-2}) \mathcal{S}_1^{'}(0)$$

Including another effect

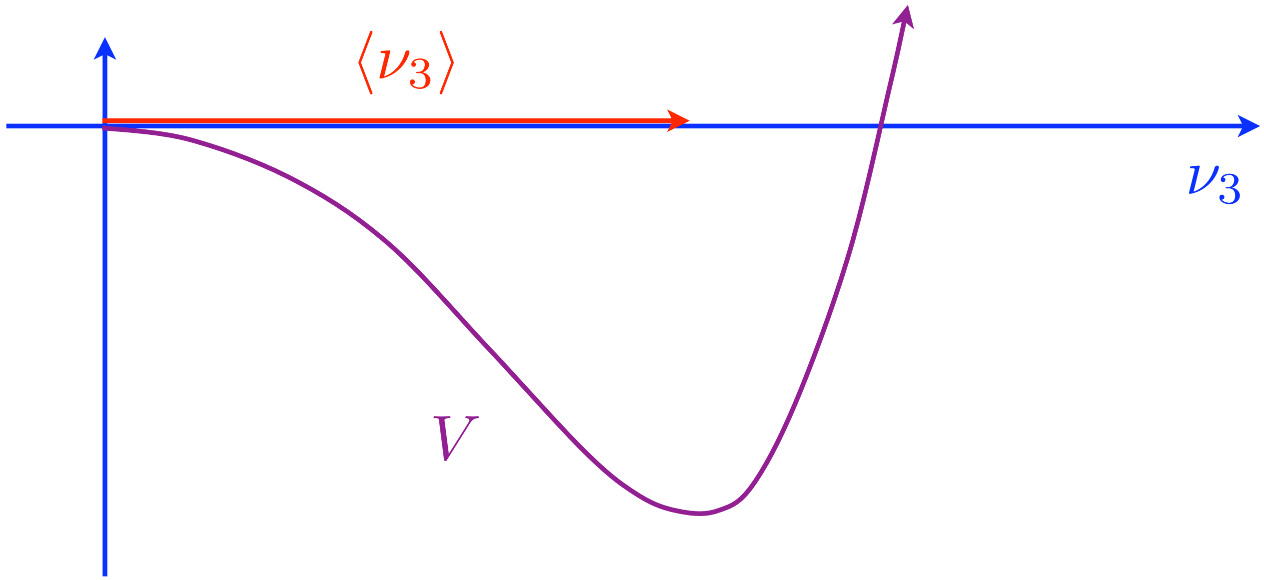
$$m_{\text{eff}\,\nu_3}(t_{B-L})^2 = m_{\nu_3}(t_{B-L})^2 + \sqrt{\frac{3}{4}} g_4 \xi_{B-L}$$

$\Rightarrow$

$$m_{\text{eff}\,\nu_3}(t_{B-L})^2 = -4 m_\nu(0)^2$$

Therefore, we expect the spontaneous breaking of B-L at t_{B-L}.

Result:



The vacuum expectation value at t_{B-L} is

$$\langle \nu_3 \rangle = \frac{2m_\nu(0)}{\sqrt{\frac{3}{4}g_4}}$$

⇒ a B-L vector boson mass of

$$M_{A_{B-L}} = 2m_\nu(0)$$

At this scale, no other symmetry is broken.

Similarly, under the renormalization group

$$m_H(t)^2 \simeq m_H(0)^2 e^{-\frac{3}{4\pi^2}\int_t^0 \lambda_3^2 (1+[\frac{-\frac{2}{3\pi^2}\int_0^{t'} g_3^2 |M_3|^2}{m_H^2}])}$$

$$m_{\bar{H}}(t)^2 \simeq m_{\bar{H}}(0)^2$$

At the electroweak scale $\mu \simeq 10^2 GeV \Rightarrow t_{EW} \simeq -29.6$

$$m_{\rm effH'}(t_{EW})^2 \simeq -\frac{\epsilon^2 m_H(0)^2}{tan\beta^2} \quad , \quad m_{\bar{H}'}(t_{EW})^2 \simeq m_H(0)^2$$

where $tan\beta = \frac{\langle H \rangle}{\langle \bar{H} \rangle}$ and $\epsilon < 1$ is related to $M_3(0)$. Therefore, at t_{EW} electroweak symmetry is broken by the expectation value

$$\langle H'^0 \rangle = \frac{2\epsilon m_H(0)}{tan\beta\sqrt{\frac{3}{5}g_1^2 + g_2^2}}$$

$\Rightarrow$ a Z-boson mass of

$$M_Z = \frac{2\epsilon m_H(0)}{tan\beta} \simeq 91 GeV$$

It follows that there is a B-L/EW gauge hierarchy given by

$$\frac{M_{A_{B-L}}}{M_Z} \simeq \frac{tan\beta}{\epsilon}$$

Our approximations are valid for the range $6.32 \leq tan\beta \leq 40$.

For $\epsilon = \frac{1}{2.5}$, the B-L/EW hierarchy in this range is

$$15.8 \lesssim \frac{M_{A_{B-L}}}{M_Z} \lesssim 100$$

We conclude that this vacuum exhibits a natural hierarchy of $\mathcal{O}(10)$ to $\mathcal{O}(100)$ ⇒

$$1.42 \times 10^3 GeV \lesssim M_{A_{B-L}} \lesssim 0.91 \times 10^4 GeV$$

All super-partner masses are related through intertwined renormalization group equations. ⇒ Measuring some masses predicts the rest!

For example, if

$$tan\beta \simeq 6.32, \quad \frac{M_{A_{B-L}}}{M_Z} \simeq 15.2 \ \Rightarrow \epsilon \simeq \frac{1}{2.5}$$

This then requires

$$M_3(0) = .216 \ m_H(0), \quad m_H(0) \simeq 7.19 \times 10^2 GeV$$

which, using the scaling equation for $M_3(t)$ predicts

$$M_3(t_{EW}) \simeq 3.83 \times 10^2 GeV$$