Miracle of Conformal Bootstrap in 4-Dimensions

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Status of CFT in 4D

- **Abstract theory** – scaling operators, OPEs, conformal partial waves

- **Few (non-SUSY) concrete examples**

- E.g. **conformal windows** of $SU(N_c)$ gauge theories with N_f flavors. *Evidence from*
  - large N Belavin,Migdal'74
  - SUSY Seiberg'94
  - lattice simulations
    $N_c=3$, $N_f=12$: Appelquist et al, Deuzeman et al'09

- No theoretical control over these fixed points

Why want to know more?

- curiosity
- what if plays a role in Nature?
  - Unparticles Georgi'07
  - Conformal Technicolor Luty, Okui'04
  - Conformal SUSY-breaking sectors

    Roy,Schmaltz'08

CTC - Ideal theory of EWSB

Standard Model $y_{ij}\, H\, \bar{q}_i q_j$ unwanted flavor effects decouple $\frac{1}{\Lambda_{UV}^2}\bar{q}_i q_j \bar{q}_k q_l$

$[H] = 1$

very relevant operator $\Lambda_{UV}^2 |H|^2$

$\Lambda_{UV} \to \infty$ problematic

} **Hierarchy problem**

Usual Technicolor no relevant singlet scalar

$H_{TC} \sim \bar{\psi}\psi$

$[H_{TC}] = 3$

Yukawas $y_{ij}\frac{H_{TC}}{\Lambda_{\rm UV}^2}\bar{q}_i q_j$ as relevant as $\frac{1}{\Lambda_{UV}^2}\bar{q}_i q_j \bar{q}_k q_l$

«Conformal Technicolor»

Luty,Okui 2004

(based on earlier «walking TC» idea Holdom’81)

Flavor: $[H] \approx 1$

Hierarchy: $[H^+ H] \geq 4$

- **We do not know of any such theories**
- **Is this at all possible?**

$\Longrightarrow$ A concrete question

In *arbitrary* unitary CFT, take a Hermitean scalar operator, with OPE

$$\varphi \times \varphi = 1 + O + \ldots$$

leading scalar $O \sim "\varphi^2"$

Q:

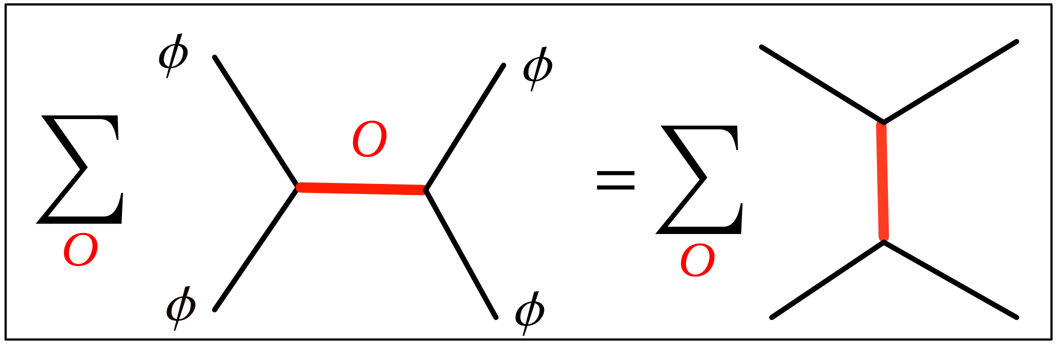
- **Is there any *upper bound* on [O], the dimension of O, in terms of [φ]?**
- **In particular, is it true that [O]$\to$2 as [φ] $\to$1?**

Recall unitarity bound:
[φ] $\geq$1 for any scalar operator in a unitary CFT
If [φ] =1, the scalar is free, and in particular O=:φ^2:, [O]=2

Conformal Boostrap – our only weapon

OPE + crossing symmetry

Polyakov'74
Belavin,Polyakov,Zamolodchikov'84



A germ of an idea:

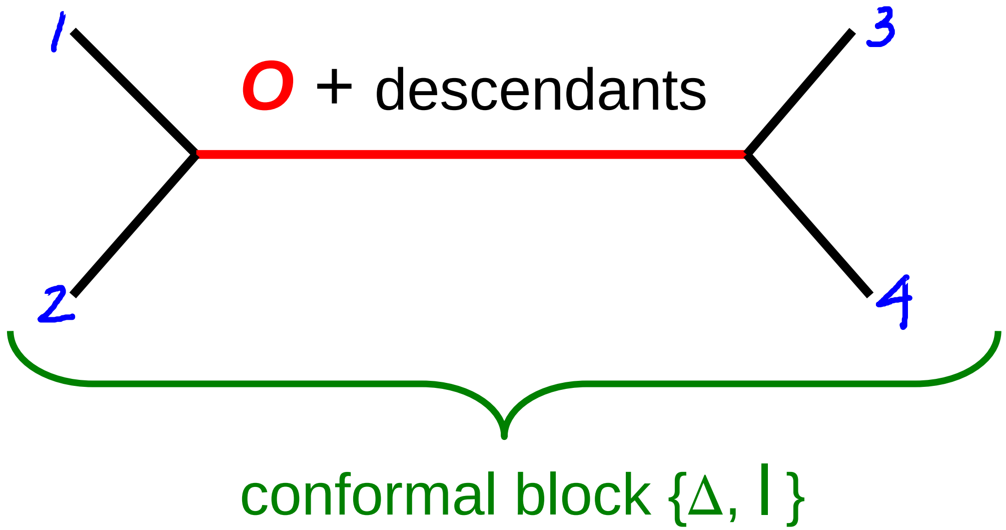
- **The Unit Operator exchange is NOT crossing-symmetric** $\frac{1}{x_{12}^{2d}\, x_{34}^{2d}} \neq \frac{1}{x_{14}^{2d}\, x_{23}^{2d}}$
- **Exchanges of scalars and higher spins should restore crossing**
- **Can it be that higher spins cannot do it by themselves, if all scalars are decoupled?**

Conformal block decomposition – divide and conquer

$$\langle \phi(x_1)\,\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{g(u,v)}{|x_{12}|^{2d}|x_{34}|^{2d}}$$

$d \equiv [\varphi]$

$$\langle \phi\phi\phi\phi \rangle = \sum_{O}$$



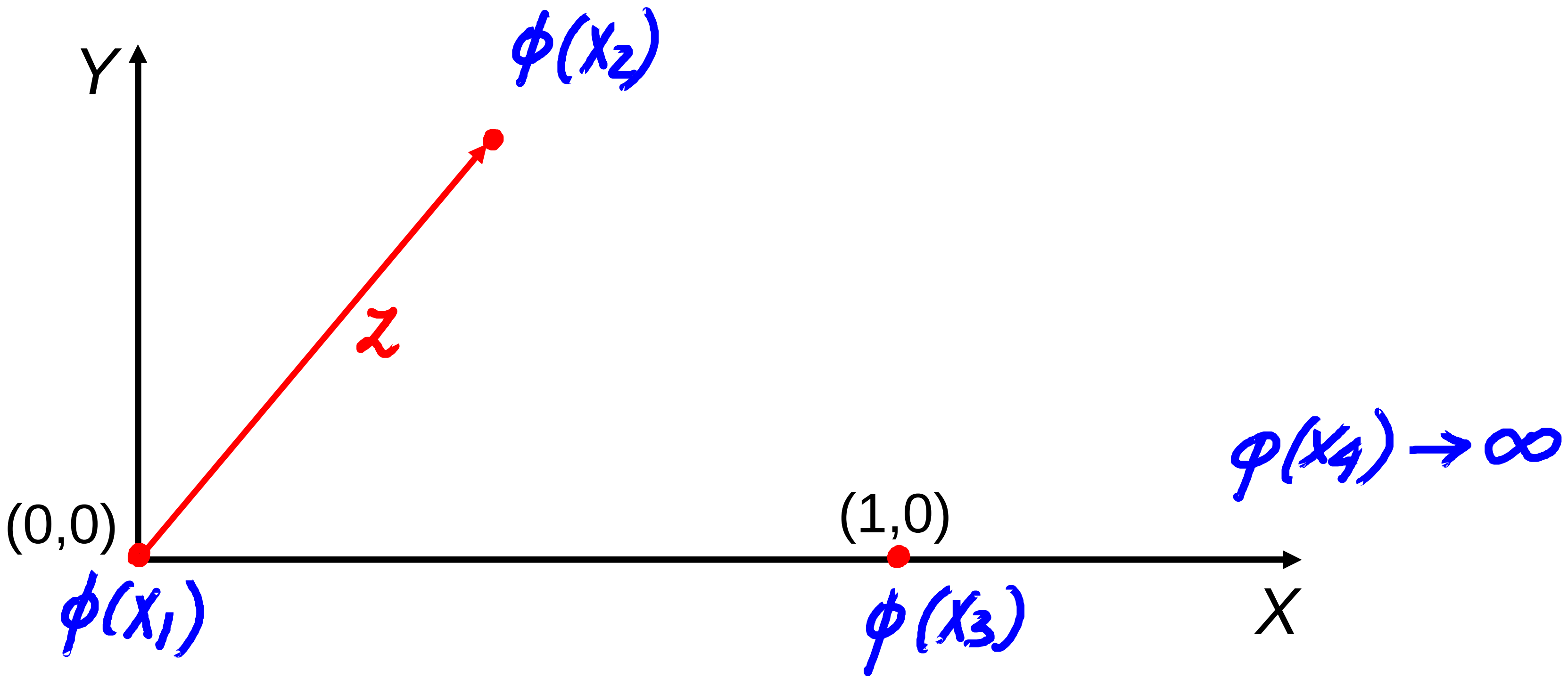
$$g(u,v) = 1 + \sum_{\Delta,l} \left(\lambda_{\Delta,l}\right)^2 g_{\Delta,l}(u,v)$$

4D Conformal Blocks in closed form [Dolan, Osborn, 2001]
It makes you feel powerful!

$$g_{\Delta,l}(u,v) = \frac{z\bar{z}}{z-\bar{z}}[\,f_{\Delta+l}(z)\,f_{\Delta-l-2}(\bar{z}) - (z \leftrightarrow \bar{z})]$$

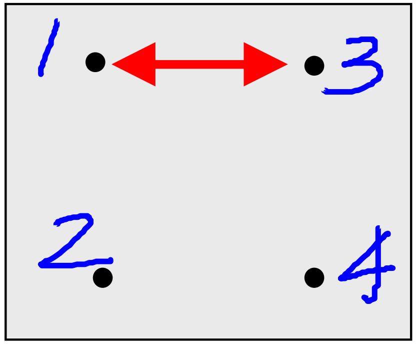
$$f_{\beta}(z) = z^{\beta/2}\,{}_2F_1(\frac{\beta}{2},\frac{\beta}{2},\beta;z)$$

$$u = z\bar{z}, \quad v = (1-z)(1-\bar{z})$$



Crossing Symmetry –
can we balance the budget deficit?

$$v^d g(u,v) = u^d g(v,u)$$



crossing deficit
from unit operator

$$u^d - v^d = \sum_{\Delta,l} \left(\lambda_{\Delta,l}\right)^2 \left[v^d g_{\Delta,l}(u,v) - u^d g_{\Delta,l}(v,u) \right]$$

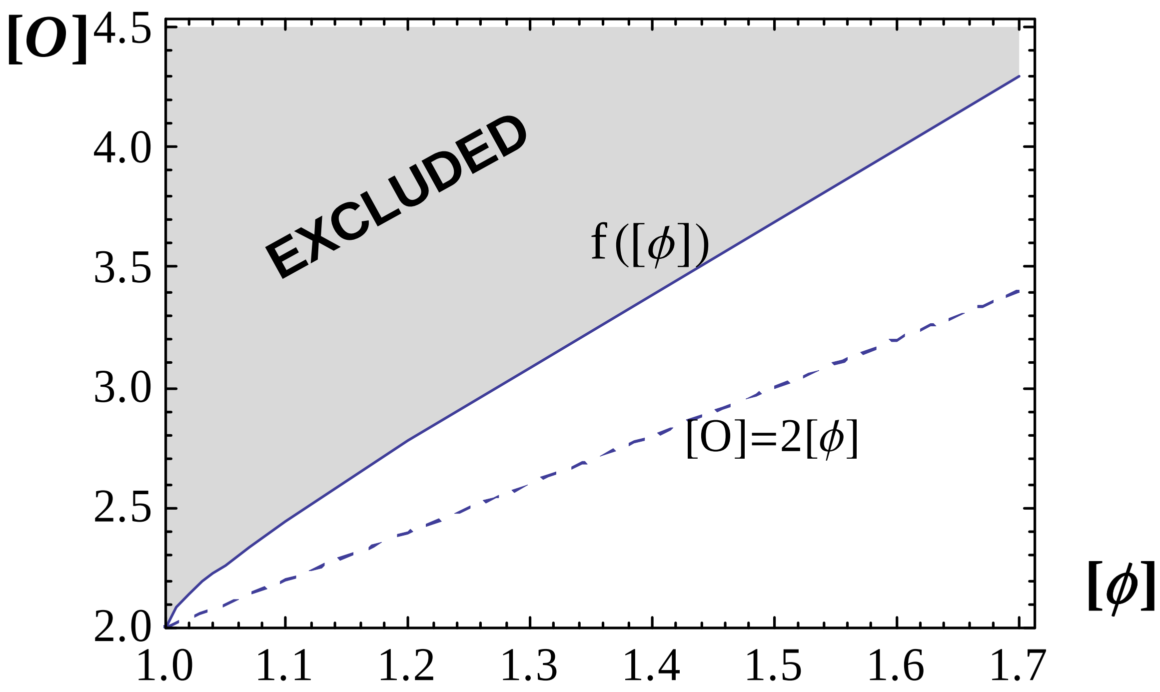
Sum Rule:

$$1 = \sum_{\Delta,l} \lambda_{\Delta,l}^2 F_{d,\Delta,l}(u,v)$$

$$F_{d,\Delta,l}(u,v) := \frac{v^d g_{\Delta,l}(u,v) - u^d g_{\Delta,l}(v,u)}{u^d - v^d}$$

Sum rule $\Longrightarrow$ Results

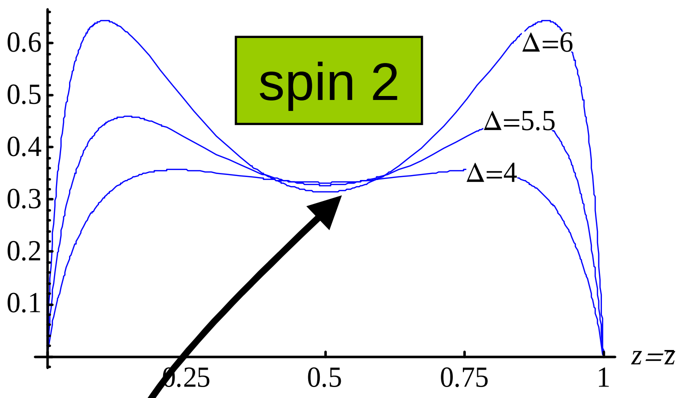
Theorem (with Rattazzi, Tonni, Vichi):
OPE $\varphi\times\varphi$ must contain at least one scalar O of dimension [O]<f([φ])

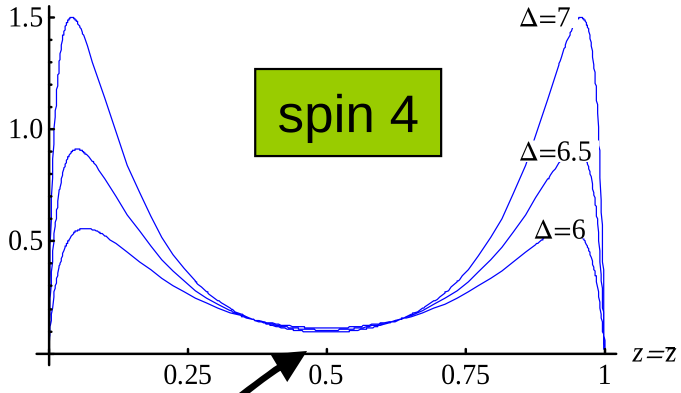


- Numerical fit: $$f(d) \approx 2 + 0.7\sqrt{d-1} + 2.1(d-1) + 0.43(d-1)^{3/2}$$
- f(d)→2 as d→1

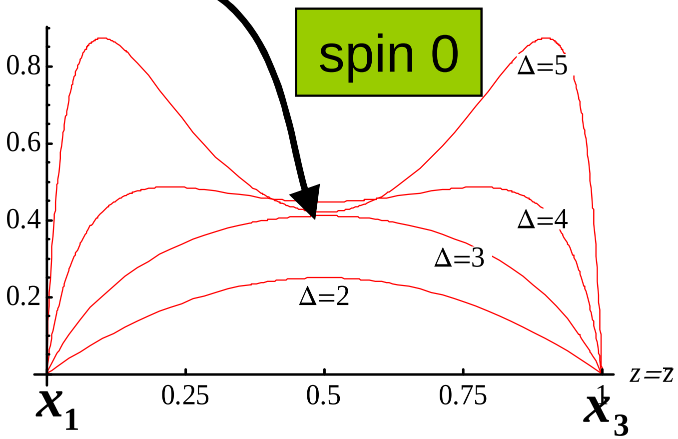
How can this be at all possible?

Example:
d=1.01

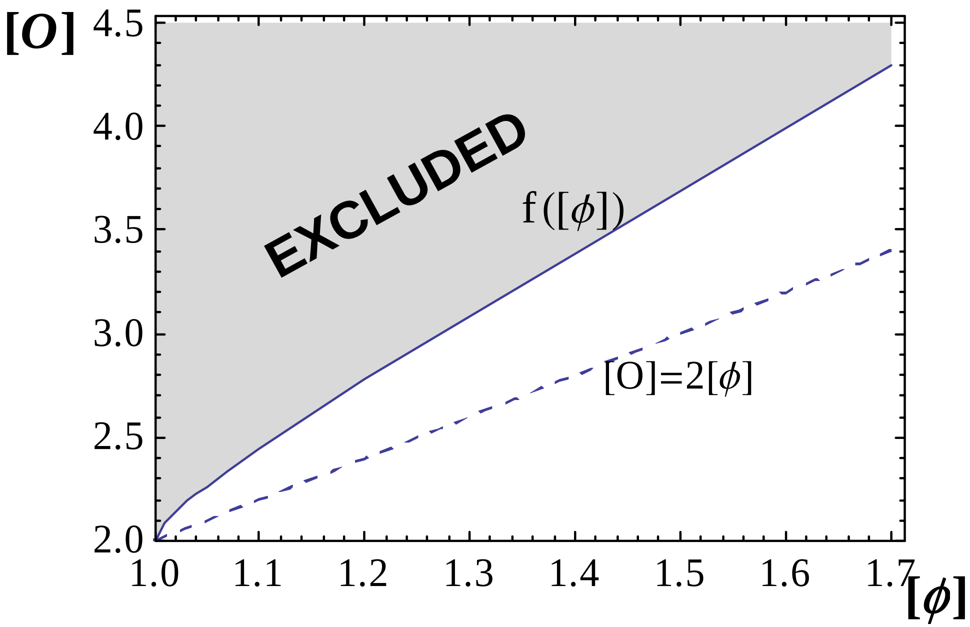


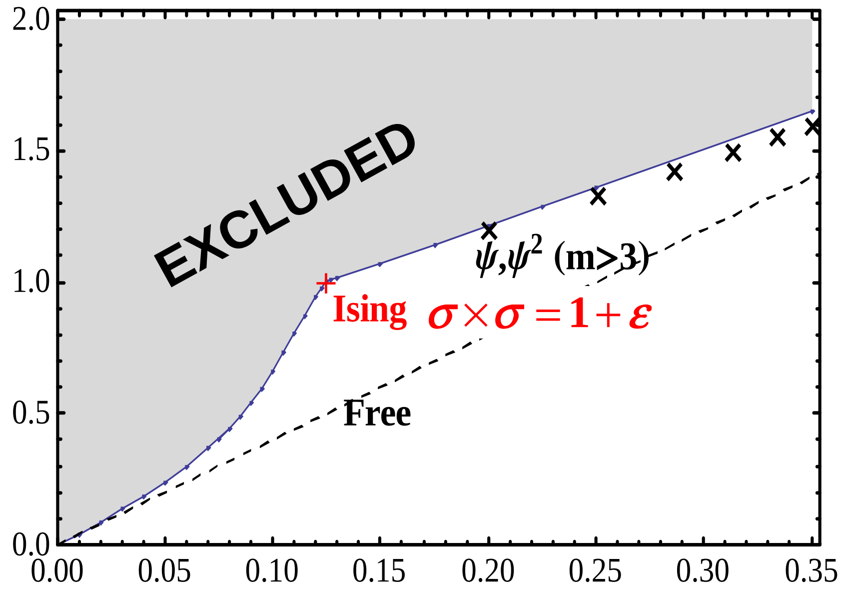


- **Must sum up to ≡1 with positive coefficients**
- **F''(1/2)>0 for all higher spins and for all scalars with small Δ**
- **⟹ a scalar with small Δ must be present**



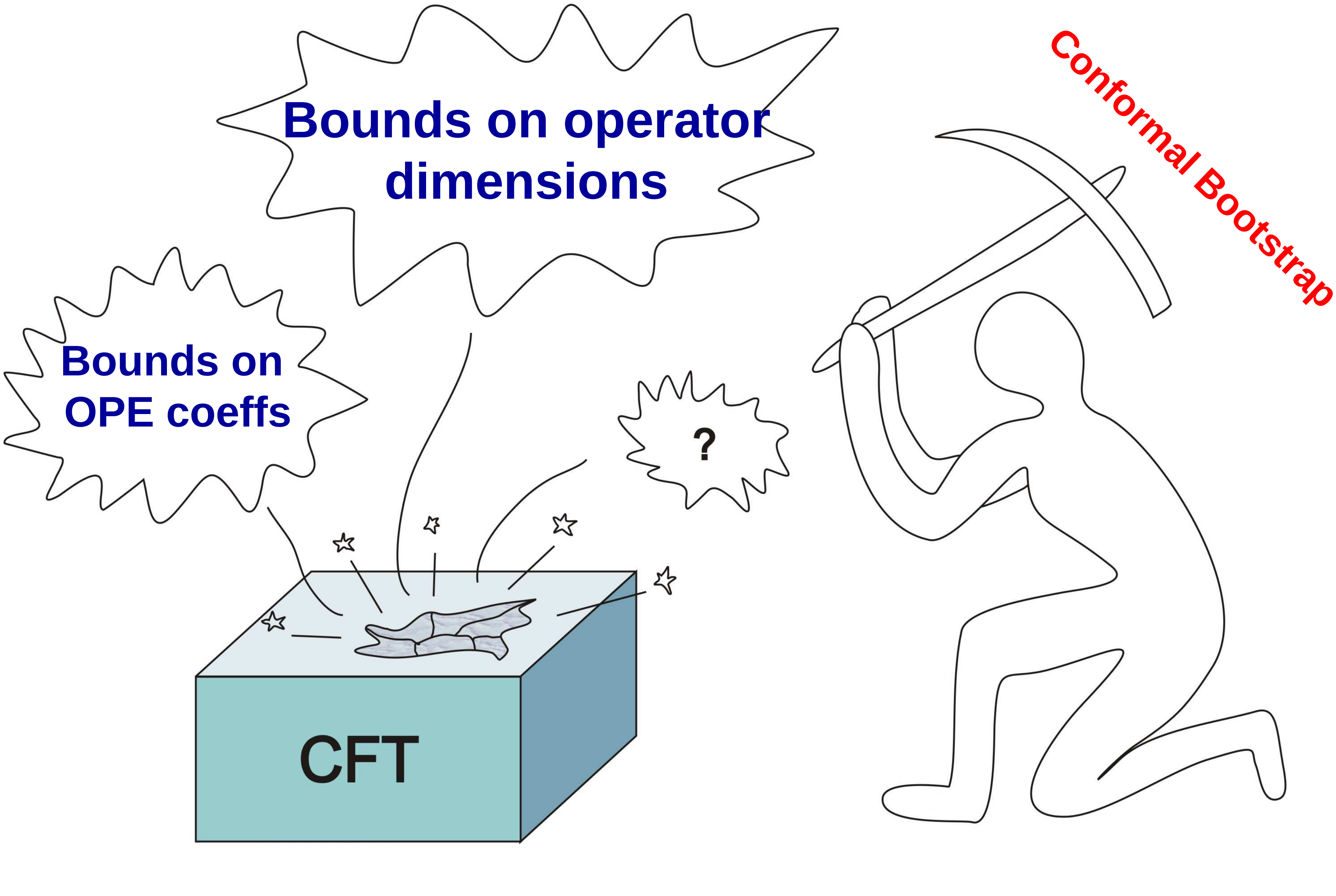
4-D versus 2-D





- **In 2D, the bound is saturated by Minimal Models and Free Scalar, interpolating in between**
- **We hope that also in 4D CFTs near-saturating the bound must exist**

Conclusions/Подведем итоги



OPE

Primaries

Descendants (fixed by symmetry)

$$\phi(x)\phi(0) \sim \frac{1}{|x|^{2d}} \left\{ \mathbb{1} + \sum_{l=2n} c_{\Delta,l} \left[|x|^{-\Delta} K_l(x) \cdot O_{\Delta,l}(0) + \cdots \right] \right\}$$

$d \equiv [\varphi]$

by Bose symmetry

$$K_l(x) = \frac{x^{\mu_1} \cdots x^{\mu_l}}{|x|^l}$$

$\Delta \geq 1$ (l=0)

$\Delta \geq l+2$ (l=2,4,6…)

Unitarity bounds

Mack'77

2D and 3D examples

show that $\gamma_{\phi^2} >> \gamma_\phi$ is not impossible.

Ising model: $\sigma \times \sigma = \mathbf{1} + \varepsilon$

2-dimensions (Onsager)	$[\sigma] = \mathbf{1/8}, \quad [\varepsilon] = \mathbf{1}$
3-dimensions (ϵ- and high-T expansions, Monte-Carlo)	$\gamma_\sigma \approx \mathbf{0.02}, \quad \gamma_\varepsilon \approx \mathbf{0.4}$

OPE

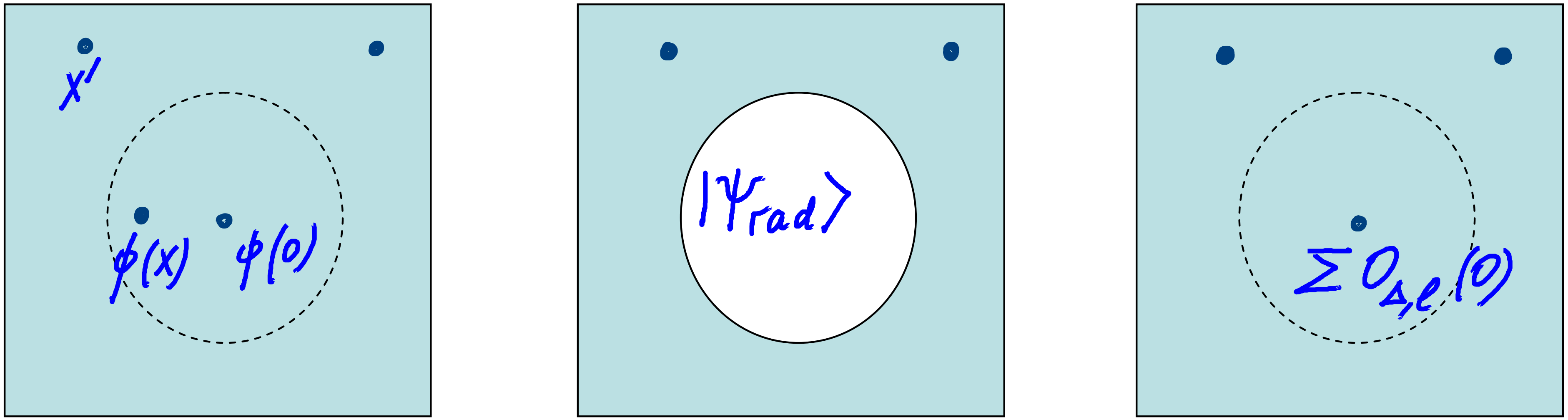
$$\phi(x)\phi(0) = \frac{1}{x^{2d}} + \sum_{l=0,2,4...} \sum_{\Delta} \lambda_{\Delta,l} [C(x)O(0) + ...]$$

descendants

NB. Only even spins appear

Convergence at finite separation:

should converge if no other operators with |x'|<|x|

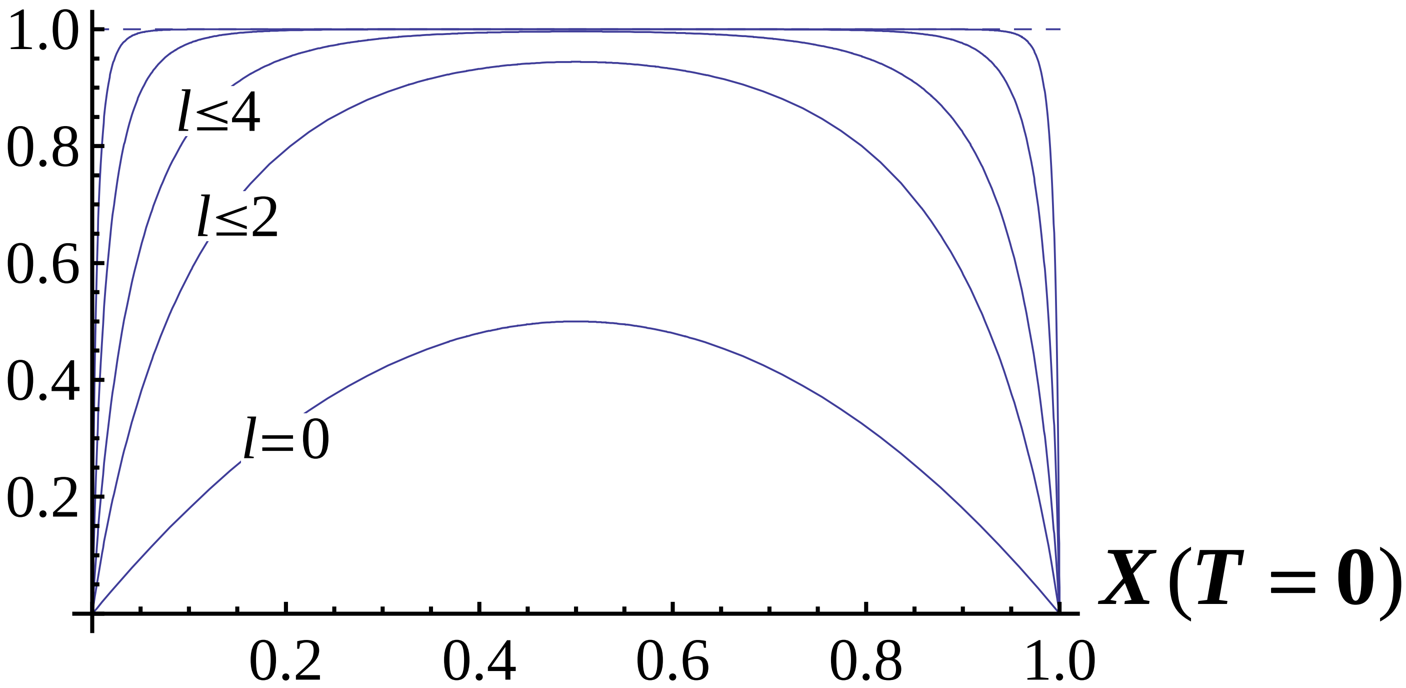


Sum rule convergence in free scalar theory

$$\phi \times \phi = \sum_{l=2n} \phi \overleftrightarrow{\partial}^{2n} \phi$$

twist 2 fields only

$$\lambda_l^2 = 2^{l+1} \frac{(l!)^2}{(2l!)^2}$$



Monotonic convergence

-Paradox in 4-epsilon dimensions

Naive extrapolation of our 4d bound:

$$\gamma_{\phi^2} \leq 0.7\sqrt{\gamma_\phi} \qquad (\gamma_\phi << 1)$$

to 4-epsilon is in contradiction with Wilson-Fischer fixed point anomalous dimensions for N=1,2:

$$\gamma_\phi = \frac{N+2}{4(N+8)^2}\varepsilon^2$$

$$\gamma_{\phi^2} \equiv \gamma_T = \frac{2}{N+8}\varepsilon$$