Type IIA string in $AdS_4 \times CP^3$ superspace

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Based on:

ArXiv:0811.1566 Jaume Gomis, Linus Wulff and D.S.

ArXiv:0903.5407 P.A.Grassi, L.Wulff and D.S.

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AdS_4/CFT_3 correspondence

- Holographic duality between 3d supersymmetric Chern-Simons-matter theories (BLG & ABJM) and M/String Theory compactified on AdS_4 space

$$S_{CSM} = \int d^3x \left\{ -\frac{1}{2} D^\mu \Phi D_\mu \Phi + \frac{i}{2} \bar{\Psi} \Gamma^\mu D_\mu \Psi - V(\Phi, \Psi) \right\}$$

$$+ \frac{k}{4\pi} \int Tr \, (A \, dA + \frac{2}{3} A^3) - \frac{k}{4\pi} \int Tr \, (A' \, dA' + \frac{2}{3} A'^3)$$

Matter: 8 scalars Φ and 8 spinors Ψ in bi-fund. rep. of $U(N)_k \times U(N)_{-k}$

Chern—Simons gauge fields: A_μ and A'_μ

For a certain $V(\Phi, \Psi)$ the theory has $\mathcal{N}=6$ superconformal symmetry

$OSp(6|4) \supset SO(6) \times Sp(4) \sim SU(4) \times SO(2,3)$

$AdS_4 \times CFT_3$ correspondence

- In a limit $N^2 \gg \lambda^{5/2}$, where $\lambda = N/k$ is a `t Hooft coupling, the bulk description of the ABJM model is given by type IIA string theory on $AdS_4 \times CP^3$
  - this D=10 background preserves 24 of 32 susy, i.e. $\mathcal{N}=6$ from the AdS_4 perspective, and its isometry is $OSp(6|4)$

$$F_4 = \frac{3}{8}\,\mathrm{k}\,\lambda^{1/2}\, d^4x_{AdS_4}\,, \qquad F_2 = k\, J_{CP^3}, \qquad e^{2\phi} = \lambda^{5/2}/N^2 = g^2{}_{\mathrm{str}}$$

- To study this AdS/CFT duality, it is necessary to know an explicit form of the Green-Schwarz string action in the $AdS_4 \times CP^3$ superbackground.

Green-Schwarz superstring

in a generic supergravity background

$$S = -\int d^2\xi \sqrt{-\det g_{ij}} - \int B_2$$

$B_2(X^M, \Theta^{\underline{\alpha}})$ - worldsheet pullback of the NS-NS 2-form guage field

$M = 0,1,\ldots,9; \qquad \underline{\alpha} = 1,\ldots,32$

$g_{ij} = E_i^A E_j^B \eta_{AB}$ - induced worldsheet metric

$E_i^A = \partial_i Z^{\mathcal{M}}(\xi)\, E_{\mathcal{M}}{}^A(X,\Theta)$ - pullback of the vector supevielbein of $\mathrm{D}=10$ sugra

$A = 0,1,\ldots,9; \qquad Z^{\mathcal{M}} = (X^M, \Theta^{\underline{\alpha}})$

$E^{\underline{\alpha}} = dZ^{\mathcal{M}} E_{\mathcal{M}}{}^{\underline{\alpha}}(X,\Theta)$ - spinor supervielbein of $\mathrm{D}=10$ sugra

Type IIA sugra fields g_{MN}, Φ, B_{MN}, A_M, A_{LMN}, $\Psi_M{}^{\alpha}$, λ_{α} are contained in E^A and B_2

Our goal is to get the explicit form of B_2, E^A and $E^{\underline{\alpha}}$ as polinomials of Θ for the $AdS_4 \times CP^3$ case

Fermionic kappa-symmetry

Provided that the superbackground satisfies superfield supergravity constraints (or, equivalently, sugra field equations), the GS superstring action is invariant under the following local worldsheet transformations of the string coordinates $Z^{\mathcal{M}}(\xi)$:

$$\delta_\kappa Z^{\mathcal{M}} E_{\mathcal{M}}{}^{A}(X,\Theta)=0, \qquad \delta_\kappa Z^{\mathcal{M}} E_{\mathcal{M}}{}^{\underline{\alpha}}(X,\Theta)=\frac{1}{2}(\mathrm{I}+\Gamma)^{\underline{\alpha}}{}_{\underline{\beta}}\,\kappa^{\underline{\beta}}(\xi),$$

$$\Gamma=\frac{1}{2\sqrt{-\det g}}\varepsilon^{ij}E_i{}^{A}E_j{}^{B}\Gamma_{AB}\Gamma^{11}, \qquad \Gamma^2=\mathrm{I}, \qquad \mathrm{tr}\,\Gamma=0$$

Due to the projector, the fermionic parameter $\kappa^{\underline{\alpha}}(\xi)$ has only 16 independent components. They can be used to gauge away 1/2 of 32 fermionic worldsheet fields $\Theta^{\underline{\alpha}}(\xi)$

$AdS_4 \times CP^3$ superbackground

- Preserves 24 of 32 susy in type IIA D=10 superspace

  in contrast: $AdS_5 \times S^5$ background in type IIB string theory preserves all 32 supersymmetries

- fermionic modes of the $AdS_4 \times CP^3$ superstring are of different nature: $\Theta_{32}(\xi) = (\vartheta_{24}, \upsilon_8)$

  unbroken broken susy

$\vartheta_{24} = \mathcal{P}_{24}\,\Theta, \qquad \upsilon_8 = \mathcal{P}_8\,\Theta, \qquad \mathcal{P}_{24} + \mathcal{P}_8 = I$

$\mathcal{P}_{24} = 1/8\,(6 - \Gamma^{a'b'} J_{a'b'} \Gamma_7), \qquad \Gamma_7 = \Gamma_1 \cdots \Gamma_6$

a',b'=1,...,6 - CP^3 indices, $J_{a'b'} \sim F_{a'b'}$ Kaehler form on CP^3

$OSp(6|4)$ supercoset sigma model

It is natural to try to get rid of the eight "broken susy" fermionic modes υ_8 using kappa-symmetry

$\upsilon_8 = 0$ - partial kappa-symmetry gauge fixing

Remaining string modes are:
10 ($AdS_4 \times CP^3$) bosons $x^a(\xi)$ (a=0,1,2,3), $y^{a'}(\xi)$ (a'=1,2,3,4,5,6)
24 fermions $\vartheta(\xi)$ corresponding to unbroken susy

they parametrize coset superspace $OSp(6|4)/U(3)xSO(1,3) \supset AdS_4 x CP^3$

$$K_{10,24}(x,y,\vartheta) = e^{x^a P_a}\, e^{y^{a'} P_{a'}}\, e^{\vartheta Q} \in OSp(6|4)/U(3)xSO(1,3)$$

$OSp(6|4)$ superalgebra:

$[P,P]=M$, $[P,M]=P$, $P \subset AdS_4 x CP^3$ $M \subset SO(1,3) \times U(3)$

$\{Q,Q\}=P+M$, $[Q,P]=Q$, $[Q,M]=Q$

$OSp(6|4)$ supercoset sigma model

Cartan forms:

$$K^{-1}dK = E^{a}(x,y,\vartheta)\, P_a + E^{a'}(x,y,\vartheta)\, P_{a'} + E^{\alpha\alpha'}(x,y,\vartheta)\, Q_{\alpha\alpha'} + \Omega\,(x,y,\vartheta)\, M$$

Superstring action on $OSp(6|4)/U(3)\times SO(1,3)$ – classically integrable

(Arutyunov & Frolov; Stefanskij; D'Auria, Frè, Grassi & Trigiante, 2008)

$$S = \int d^2\xi\, (-\det E_i{}^{A} E_j{}^{B}\, \eta_{AB})^{1/2} + \int E^{\alpha\alpha'} \wedge E^{\beta\beta'}\, J_{\alpha'\beta'}\, C_{\alpha\beta}$$

A problem with this model is that it does not describe all possible string configurations. E.g., it does not describe a string moving in AdS_4 only.

<u>Reason</u> – kappa-gauge fixing $\upsilon_8 = \mathcal{P}_8 \Theta = 0$ is <u>inconsistent</u> in the AdS_4 region

$[(1+\Gamma_\kappa), \mathcal{P}_8] = 0$ ⟹ only ½ of υ_8 can be eliminated

Towards the construction of the complete $AdS_4 \times CP^3$ superspace (with 32 Θ)

- We shall construct this superspace by performing the dimension reduction of D=11 coset superspace $OSp(8|4)/SO(7)\times SO(1,3)$ it has 32 Θ and subspace $AdS_4\times S^7$, $SO(2,3)\times SO(8)\subset OSp(8|4)$

- $AdS_4\times CP^3$ sugra solution is related to $AdS_4\times S^7$ (with 32 susy) by dimenisonal reduction (*Nilsson and Pope; D.S., Tkach & Volkov 1984*)

Geometrical ground: S^7 is the Hopf fibration over CP^3 with S^1 fiber:

S^7 vielbein: $e_m{}^a = \begin{pmatrix} e_{m'}{}^{a'}(y) & A_{m'}(y) \\ 0 & 1 \end{pmatrix}$ ⟸ CP^3 vielbein and $U(1)$ connection

(does not depend on S^1 fiber coordinate z)

$$F_2 \sim dA = J_{CP^3}$$

Our goal is to extend this structure to $OSp(8|4)/SO(7)\times SO(1,3)$

Hopf fibration of S^7 as a coset space $\frac{SU(4)\times U(1)}{SU(3)\times U(1)}$

$S = \frac{SO(8)}{SO(7)}$ - symmetric space

$SU(4)\times U(1) \subset SO(8)$

As a Hopf fibration, locally, $S^7 = CP^3 \times U(1)$

$CP^3 =$ SU(4)/SU(3)×U'(1) – symmetric space

$\frac{SU(4)\times U(1)}{SU(3)\times U_d(1)}$

Coset representative and Cartan forms:

$K_{S^7}(y,z) = K_{CP^3}(y)\, e^{zT_1} = e^{y^{a'}P_{a'}}\, e^{zT_1}$ (T_1 is U(1) generator)

$K^{-1}{}_{S^7}\, dK_{S^7} = e^{a'}(y)\, P_{a'} + \Omega(y)\, M_{SU(3)} + A(y)\, T_2 + dz\, T_1$ ($T_2 \subset U'(1)$)

$= dy^{m'} e_{m'}{}^{a'}(y)\, P_{a'} + (dz + dy^{m'} A_{m'}(y))\, P_7 + (dz - A(y))\, T_d + \Omega(y)\, M_{SU(3)}$

$T_d = 1/2\,(T_1 - T_2)$ is $U_d(1)$ generator, $P_7 = 1/2\,(T_1 + T_2)$ – translation along S^1 fiber

Hopf fibration of $OSp(8|4)/SO(7)\times SO(1,3)$

- $K_{11,32}$ - D=11 superspace with the bosonic subspace $AdS_4\times S^7$ and 32 fermionic directions

$$K_{11,32} = \mathcal{M}_{10,32} \times S^1 \quad \text{(locally)}$$

base fiber

- $\mathcal{M}_{10,32}$ - D=10 superspace with the bosonic subspace $AdS_4\times CP^3$ and 32 fermionic directions (it is not a coset space)

$\mathcal{M}_{10,32}$ *is the superspace we are looking for!*

1st step: Hopf fibration over $K_{10,24} = OSp(6|4)/U(3)xSO(1,3)$

- $K_{11,24} = K_{10,24} \times S^1$ (locally) $\supset AdS_4 \text{ x } S^7$

$$K_{11,24} = \frac{OSp(6|4) \times U(1)}{SO(1,3)\times SU(3)\times U_d(1)} \qquad OSp(6|4)\times U(1) \subset OSp(8|4)$$

$$K_{11,24}\,(x,y,\vartheta,z) = K_{10,24}\,(x,y,\vartheta)\, e^{zT_1} = e^{x^a P_a}\, e^{y^{a'} P_{a'}}\, e^{\vartheta\, Q_{24}}\, e^{zT_1}$$

Cartan forms:

$$K^{-1}dK = E^a(x,y,\vartheta)\, P_a + E^{a'}(x,y,\vartheta)\, P_{a'} + E^{\alpha\alpha'}(x,y,\vartheta)\, Q_{\alpha\alpha'}$$
$$+ (dz + A(x,y,\vartheta))\, P_7 + (dz - A)\, T_d + \Omega\,(x,y,\vartheta)\, M_{SU(3)}$$

2nd step: adding 8 fermionic directions υ_8

$$K_{11,32}\,(x,y,\vartheta,z,\upsilon)=K_{11,24}\,(x,y,\vartheta,z)\;e^{\upsilon Q_8}\;=\;K_{10,24}\,(x,y,\vartheta)\;e^{zT_1}\,e^{\upsilon Q_8}$$

Q_8 are 8 susy generators which extend OSp(6|4) to OSp(8|4)

OSp(6|4) superalgebra:

$$\{Q_{24},Q_{24}\}=M_{SO(2,3)}+M_{SU(4)}$$

OSp(2|4) superalgebra:

$$\{Q_8,Q_8\}=M_{SO(2,3)}+T_1$$

Extension to OSp(8|4): $\{Q_{24},Q_8\}=\mathrm{M}_\perp\subset\; SO(8)/SU(4)\times U(1)$

Cartan forms of $OSp(8|4)/SO(7)\times SO(1,3)$:

$$K^{-1}dK\;=\;[E^a(x,y,\vartheta,\upsilon)+dz\;V^a(\upsilon)]\;P_a\;+\;E^{a'}(x,y,\vartheta,\upsilon)\;P_{a'}$$

$$+[dz\;\Phi(\upsilon)+A(x,y,\vartheta,\upsilon)]\;P_7\;+\;E^{24}(x,y,\vartheta,\upsilon)\;Q_{24}+\;E^8(x,y,\vartheta,\upsilon)\;Q_8$$

+ connection terms

3rd step: Lorentz rotation in $AdS_4 \times S^1$ (fiber) tangent space

$$K^{-1}dK = (E^a(x,y,\vartheta,\upsilon) + dz\, V^a(\upsilon))\, P_a + E^{a'}(x,y,\vartheta,\upsilon)\, P_{a'}$$

$$+ (dz\, \Phi(\upsilon) + A(x,y,\vartheta,\upsilon))\, P_7 + E^{24}(x,y,\vartheta,\upsilon)\, Q_{24} + E^{8}(x,y,\vartheta,\upsilon)\, Q_8$$

+ connection terms

Lorentz transformation of the supervielbeins:

$\mathcal{M}_{10,32}$:

$$\mathcal{E}^a(x,y,\vartheta,\upsilon) = (E^b(x,y,\vartheta,\upsilon) + dz\, V^b(\upsilon))\, \Lambda_b{}^a(\upsilon)$$

$$+ (dz\, \Phi(\upsilon) + A(x,y,\vartheta,\upsilon))\, \Lambda_7{}^a(\upsilon)$$

$$\mathcal{E}^{a'}(x,y,\vartheta,\upsilon) = E^{a'}(x,y,\vartheta,\upsilon)$$

$\mathcal{E}^{24}(x,y,\vartheta,\upsilon)$, $\mathcal{E}^{8}(x,y,\vartheta,\upsilon)$ – rotated 32 fermionic vielbeins

S^1:

$$dz\,\Psi(\upsilon) + \mathcal{A}(x,y,\vartheta,\upsilon) = (dz\, \Phi(\upsilon) + A(x,y,\vartheta,\upsilon))\, \Lambda_7{}^7(\upsilon)$$

$$+ (E^b(x,y,\vartheta,\upsilon) + dz\, V^b(\upsilon))\, \Lambda_b{}^7(\upsilon)$$

Superstring action in $AdS_4 \times CP^3$ superspace

$$S = -\int d^2\xi \sqrt{-\det g_{ij}} - \int B_2$$

$g_{ij} = \mathcal{E}_i^A \mathcal{E}_j^B \eta_{AB}$ – induced worldsheet metric

NS-NS field B_2 is obtained by dimensional reduction of A_3 in D=11

Kappa-symmetry gauge fixing (*P.A.Grassi, L.Wulff and D.S. arXiv:0903.5407*)

(Killing spinor, supersolvable, or superconformal gauge, 1998)

$$ds^2_{AdS_4} = \frac{1}{r^2}(dx^m dx^n \eta_{mn} + dr dr), \qquad x^m \ (m = 0,1,2)$$ - parametrize 3d slice of AdS_4

$$\Theta = \frac{1}{2}(1 + \Gamma^0\Gamma^1\Gamma^2)\Theta$$ - “chirality” condition on the AdS boundary

Guage fixed superstring action in $AdS_4 \times CP^3$

(upon a T-dualization)

AdS$_4$ CP^3

$$S = -\frac{1}{2}\int d^2\xi\, \eta^{ij}\left[\frac{1}{r^2}(\partial_i x^m \partial_j x_m + \partial_i r \partial_j r) + e_i^{a'}(y)\, e_j^{a'}(y)\right]\left(1 - 6(\upsilon\upsilon)^2\right)$$

$$-\int \left(\frac{i}{r} dx^m \Theta\Gamma_m D\Theta + \frac{i}{r} dr\, \Theta\gamma\,\Gamma_{11} D\Theta + ie^{a'}(y)\,\Theta\Gamma_{a'}\Gamma_{11} D\Theta\right)$$

$$+\int\left(\frac{i}{r^2} dx^m dx^n \upsilon\gamma\,\Gamma_{mn}\Gamma_{11}\upsilon + \frac{2i}{r} dr\, e^{a'}(y)\vartheta\Gamma_{a'}\Gamma_{11}\upsilon + ie^{a'}(y) e^{b'}(y)\,\Theta\gamma\Gamma_{a'b'}\Gamma_{11}\upsilon\right)$$

$$\Theta_{32} = (\theta_{24},\ \upsilon_8), \qquad \gamma = \Gamma^0\Gamma^1\Gamma^2\,\Gamma^3$$

<u>The action contains fermionic terms up to a quartic order only</u>

Conclusion

- The complete $AdS_4 \times CP^3$ superspace with 32 fermionic directions has been constructed. Its superisometry is OSp(6|4)

- The explicit form of the Type IIA superstring and D-branes in $AdS_4 \times CP^3$ superspace have been derived

- The simplest possible gauge-fixed form of the superstring action has been obtained

- These results can be use for studying various problems of String Theory in $AdS_4 \times CP^3$, in particular, to perform higher-loop computations (involving fermionic modes) for testing AdS_4 / CFT_3 correspondence: integrability, Bethe ansatz, S-matrix etc. in the dual planar $\mathcal{N}=6$ CS-matter theory