Sum Rules for Spin-dependent Parton Distributions, Gravitational Formfactors and Equivalence Principle

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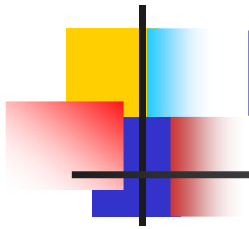
Oleg Teryaev

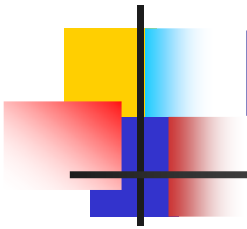
JINR, Dubna



Main Topics

- QCD factorization and two sources of Sum Rules
- Bjorken Sum Rule at low Q
- Momentum Sum Rules and Equivalence Principle
- Extension of Equivalence Principle and AdS/QCD

QCD factorization and Sum Rules



- Hard subprocess + soft parton distribution
- New processes (spin-dependent, exclusive) -> (zoology of) new distributions
- Moments -> hadron matrix elements of quark&gluon local operators
- (talk of C. Llewellyn-Smith)
- Fixed by (anomalous non-) conservation or known from other experiments -> two sources of Sum Rules

Bjorken sum rule

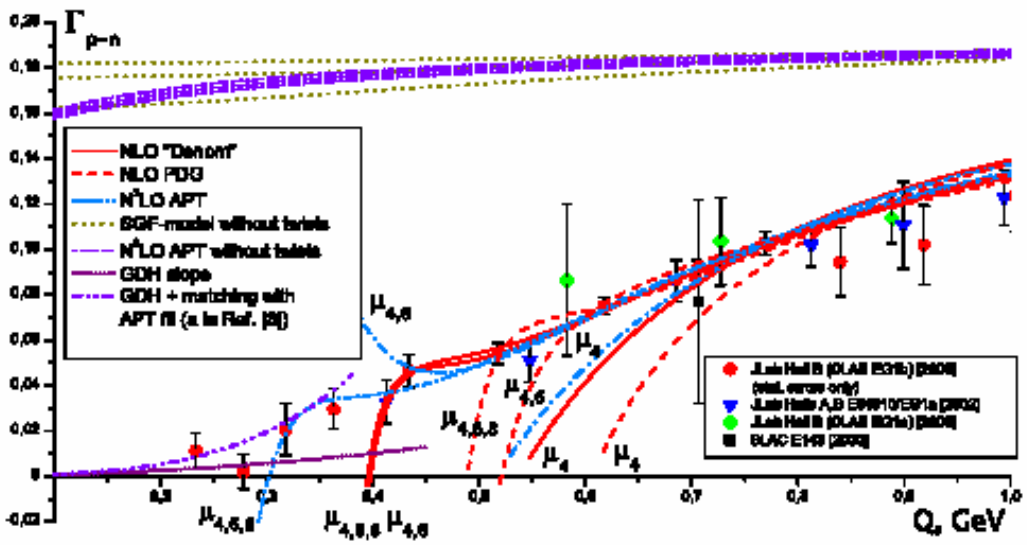
- Spin-dependent structure functions
- Matrix element of axial current – known from beta decay + Isospin invariance (2nd type of SR)
- High order perturbative corrections are calculated
- Very accurate Jlab data are now available

$$\Gamma_1^{p-n}(Q^2) = \int_0^1 dx (g_1^p(x,Q^2) - g_1^n(x,Q^2))$$

$$\Gamma_{1,PT}^{p-n}(Q^2) = \frac{g_A}{6}\left[1 - \frac{\alpha_s}{\pi} - 3.558\left(\frac{\alpha_s}{\pi}\right)^2 - 20.215\left(\frac{\alpha_s}{\pi}\right)^3 - O(\alpha_s^4)\right] + \sum_{i=2}^{\infty}\frac{\mu_{2i}}{Q^{2i-2}}$$

Higher twists from Bjorken Sum Rule (Pasechnik, Shirkov, OT)

- Accurate data + IR stable coupling -> move pQCD frontier down to low Q region!



- HT – small indeed

PHYSICAL REVIEW D **78,** 071902(R) (2008)

TABLE I. Combined fit results for the HT terms in APT and conventional PT in PDG and denominator forms.

Method	$Q^2_{\min}$, GeV^2	μ_4/M^2	μ_6/M^4	μ_8/M^6
NLO PDG	0.50	−0.043(2)	0	0
$\Lambda = 380$ MeV	0.30	−0.074(4)	0.025(2)	0
	0.27	−0.049(5)	−0.007(5)	0.009(1)
NLO denom	0.47	−0.046(2)	0	0
$\Lambda = 340$ MeV	0.17	−0.066(2)	0.013(4)	0
	0.17	−0.061(4)	0.009(3)	0.0005(3)
N^{2}LO APT	0.47	−0.054(1)	0	0
$\Lambda = 380$ MeV	0.17	−0.065(2)	0.0081(5)	0
	0.10	−0.069(2)	0.0114(9)	−0.0006(1)

Momentum/angular momentum sum rules – of both types

- Follow from momentum/angular momentum conservation – fixed matrix elements of Energy-Momentum Tensors

$$\sum_{a=q,g} H^a(0) = \sum_{a=q,g} \int_{-1}^{1} \mathrm{d}x\, x\, f^a(x) = 1$$

$$P^a = \int_{-1}^{1} \mathrm{d}x\, x\, f^a(x)$$

- Matrix elements of EMT –coupling to Gravity
- Fixed by Equivalence Principle
- Especially interesting for (non-forward) Generalized Parton Distribution

$$J^a = \tfrac{1}{2}(H^a(0) + E^a(0))$$

1-st moments - EM, 2-nd - Gravitational Formfactors

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle = \bar{u}(p')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}p^{\nu)} + B_{q,g}(\Delta^2)P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M]u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment : $\mu_G = J$ (g=2)

$$P_{q,g} = A_{q,g}(0) \qquad A_q(0) + A_g(0) = 1$$

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right] \qquad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe ainteraction with both classical and TeV gravity

Electromagnetism vs Gravity

- Interaction – field vs metric deviation

$$M = \langle P'|J^{\mu}_{q}|P\rangle A_{\mu}(q) \qquad M = \frac{1}{2}\sum_{q,G}\langle P'|T^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu}(q)$$

- Static limit

$$\langle P|J^{\mu}_{q}|P\rangle = 2e_{q}P^{\mu} \qquad \sum_{q,G}\langle P|T^{\mu\nu}_{i}|P\rangle = 2P^{\mu}P^{\nu}$$

$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P|J^{\mu}_{q}|P\rangle A_{\mu} = 2e_{q}M\,\phi(q) \qquad M_0 = \frac{1}{2}\sum_{q,G}\langle P|T^{\mu\nu}_{i}|P\rangle h_{\mu\nu} = 2M\cdot M\,\phi(q)$$

- Mass as charge – equivalence principle

Equivalence principle

- Newtonian – “Falling elevator” – well known and checked
- Post-Newtonian – gravity action on SPIN – known since 1962 (Kobzarev and Okun’) – not checked on purpose but in fact checked in atomic spins experiments at % level (Silenko,OT’07)
- Anomalous gravitomagnetic moment iz ZERO or
- Classical and QUANTUM rotators behave in the SAME way

Gravitomagnetism

- Gravitomagnetic field – action on spin – ½ from $M = \frac{1}{2}\sum_{q,G}\langle P'|T^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu}(q)$

$\vec{H}_J = \frac{1}{2} rot\vec{g};\ \vec{g}_i \equiv g_{0i}$ spin dragging twice smaller than EM

- Lorentz force – similar to EM case: factor ½ cancelled with 2 from $h_{00} = 2\phi(x)$ Larmor frequency same as EM $\vec{H}_L = rot\vec{g}$
- Orbital and Spin momenta dragging – the same - Equivalence principle $\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L$

Equivalence principle for moving particles

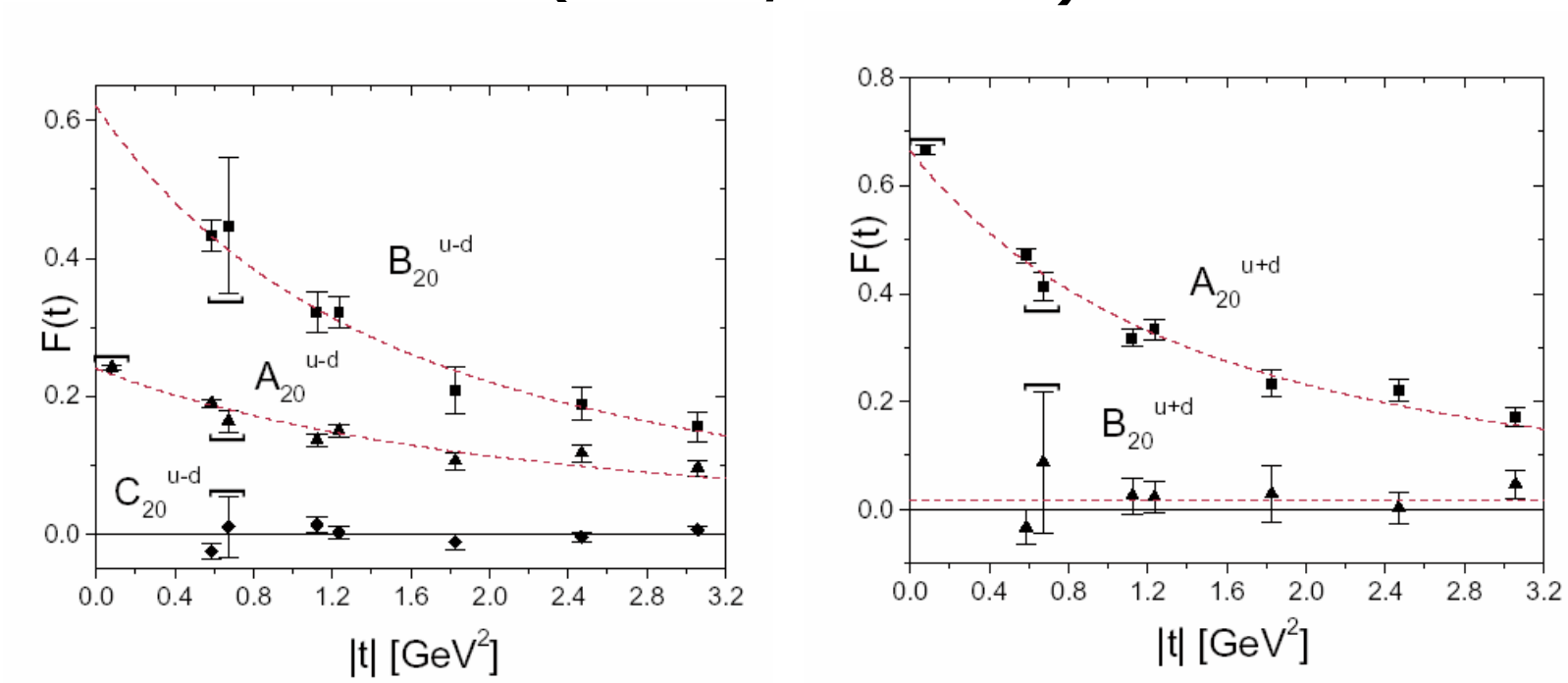
- Compare gravity and acceleration: gravity provides EXTRA space components of metrics $h_{zz} = h_{xx} = h_{yy} = h_{00}$
- Matrix elements DIFFER

$$\mathcal{M}_g = (\epsilon^2 + p^2)h_{00}(q), \qquad \mathcal{M}_a = \epsilon^2 h_{00}(q)$$

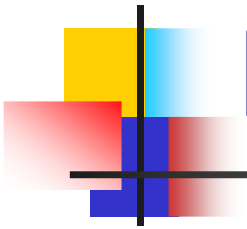
- Ratio of accelerations: $R = \frac{\epsilon^2 + p^2}{\epsilon^2}$ - confirmed by explicit solution of Dirac equation (Silenko, O.T.)

Generalization of Equivalence principle

- Various arguments: AGM ≈ 0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)



Extended Equivalence Principle=Exact EquiPartition



- In pQCD – violated
- Reason – in the case of EEP- no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- Supported by smallness of E (isoscalar AMM)

Vector mesons and EEP

- J=1/2 -> J=1. QCD SR calculation of Rho's AMM gives g close to 2.

- Maybe because of similarity of moments
- g-2=<E(x)>; B=<xE(x)>
- Directly for charged Rho (combinations like p+n for nucleons unnecessary!). Not reduced to non-extended EP: Gluons momentum fraction sizable. Direct calculation of AGM are in progress.

EEP and AdS/QCD

- Recent development – calculation of Rho formfactors in Holographic QCD (Grigoryan, Radyushkin)
- Provides g=2 identically!
- Experimental test at time –like region possible

Another (new!) manifestation of post-Newtonian (E)EP for spin 1 hadrons

- Tensor polarization - coupling of EMT to spin in forward matrix elements - inclusive processes
- Second moments of tensor distributions should sum to zero

$$A_T = \frac{\sigma_+ + \sigma_- - 2\sigma_0}{3\bar{\sigma}}$$

$$\langle P,S|\bar{\psi}(0)\gamma^\nu D^{\nu_1}...D^{\nu_n}\psi(0)|P,S\rangle_{\mu^2} = i^{-n}M^2 S^{\nu\nu_1}P^{\nu_2}...P\nu_n\int_0^1 C_q^T(x)x^n dx$$

$$\sum_q \langle P,S|T_i^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^\mu P^\nu(1-\delta(\mu^2)) + 2M^2 S^{\mu\nu}\delta_1(\mu^2)$$

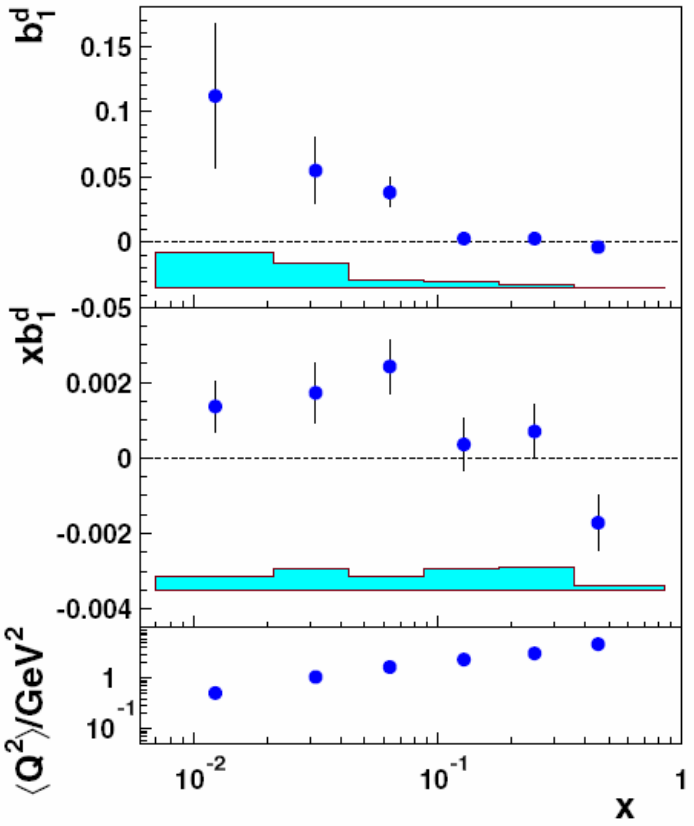
$$\langle P,S|T_g^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^\mu P^\nu\delta(\mu^2) - 2M^2 S^{\mu\nu}\delta_1(\mu^2)$$

$$\sum_q \int_0^1 C_i^T(x)xdx = \delta_1(\mu^2)$$ =0 for EEP

HERMES – data on tensor spin structure function

PRL **95**, 242001 (2005)

- Isoscalar target – proportional to the sum of u and d quarks – combination required by EEP
- Second moments – compatible to zero better than the first one (collective glue << sea)



What about vector mesons – sum rules (A. Oganesian, **Phys.Atom.Nucl.71:1439-1444,2008**)

- Very different for longitudinal and transverse rho
- Reason – smallness of tensor polarization dependent part? – oscillates

$$A_T = \frac{\sigma_+ + \sigma_- - 2\sigma_0}{3\bar{\sigma}}$$

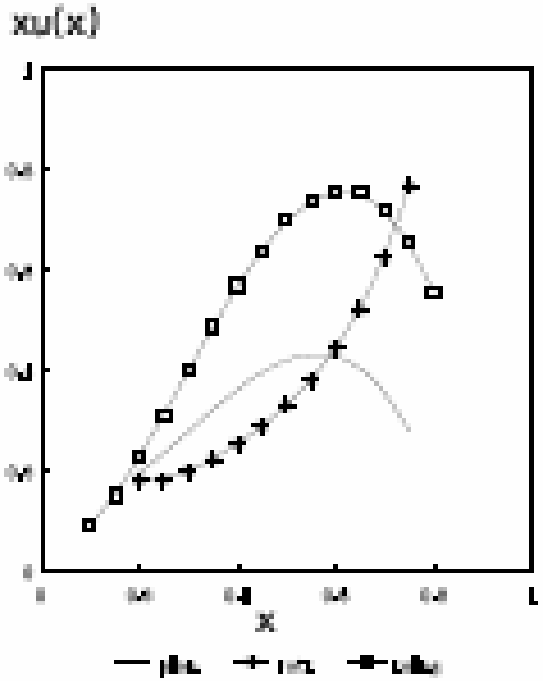


Figure 5: Quark distribution function for pion, transversally and longitudinally polarized ρ - solid line, line with asters, and line with squares correspondingly

Another relation of Gravitational FF and NP QCD (first reported at 1992: **hep-ph/9303228**)

- BELINFANTE (relocalization) invariance :

decreasing in coordinate –

$$M^{\mu,\nu\rho} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} J^5_{S\sigma} + x^\nu T^{\mu\rho} - x^\rho T^{\mu\nu}$$

smoothness in momentum space

$$M^{\mu,\nu\rho} = x^\nu T^{\mu\rho}_B - x^\rho T^{\mu\nu}_B$$

- Leads to absence of massless pole in singlet channel – U_A(1)

$$\epsilon_{\mu\nu\rho\alpha} M^{\mu,\nu\rho} = 0.$$

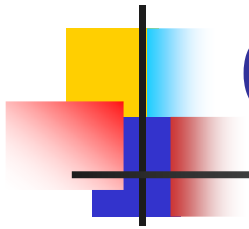
- Delicate effect of NP QCD

$$(g_{\rho\nu}g_{\alpha\mu} - g_{\rho\mu}g_{\alpha\nu})\partial^\rho (J^\alpha_{5S} x^\nu) = 0$$

- Equipartition – deeply related to relocalization invariance by QCD evolution

$$q^2 \frac{\partial}{\partial q^\alpha}\langle P|J^\alpha_{5S}|P+q\rangle = (q^\beta \frac{\partial}{\partial q^\beta} - 1) q_\gamma \langle P|J^\gamma_{5S}|P+q\rangle$$

$$\langle P,S|J^5_\mu(0)|P+q,S\rangle = 2MS_\mu G_1 + q_\mu (Sq) G_2,$$
$$q^2 G_2|_0 = 0$$



CONCLUSIONS

- New processes – new parton distributions – new sum rules from 2 sources
- Bjorken SR – very accurate description at very low Q
- Momentum sum rules – both sources
- Relations to post-Newtonian EP
- Spin-1 hadrons – also in inclusive processes
- A number of evidences for validity of EP for quarks and gluons separately