

# Interaction Effects in Disordered Conductors

**Andrei D. Zaikin**

Institute for Nanotechnology  
Forschungszentrum Karlsruhe  
Karlsruhe Institute for Technology

Tamm Dept. of Theoretical Physics  
Lebedev Physics Institute, Moscow

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**Dima Golubev**



**Artem Galaktionov**



# Outline

- Introduction and motivation
- Short coherent conductors: Weak Coulomb blockade
- Short coherent conductors: interaction correction and shot noise
- Interactions and higher current cumulants
- Transport and interactions in quantum dots and arrays: universal model for ANY diffusive conductor
- Weak localization and decoherence at  $T=0$

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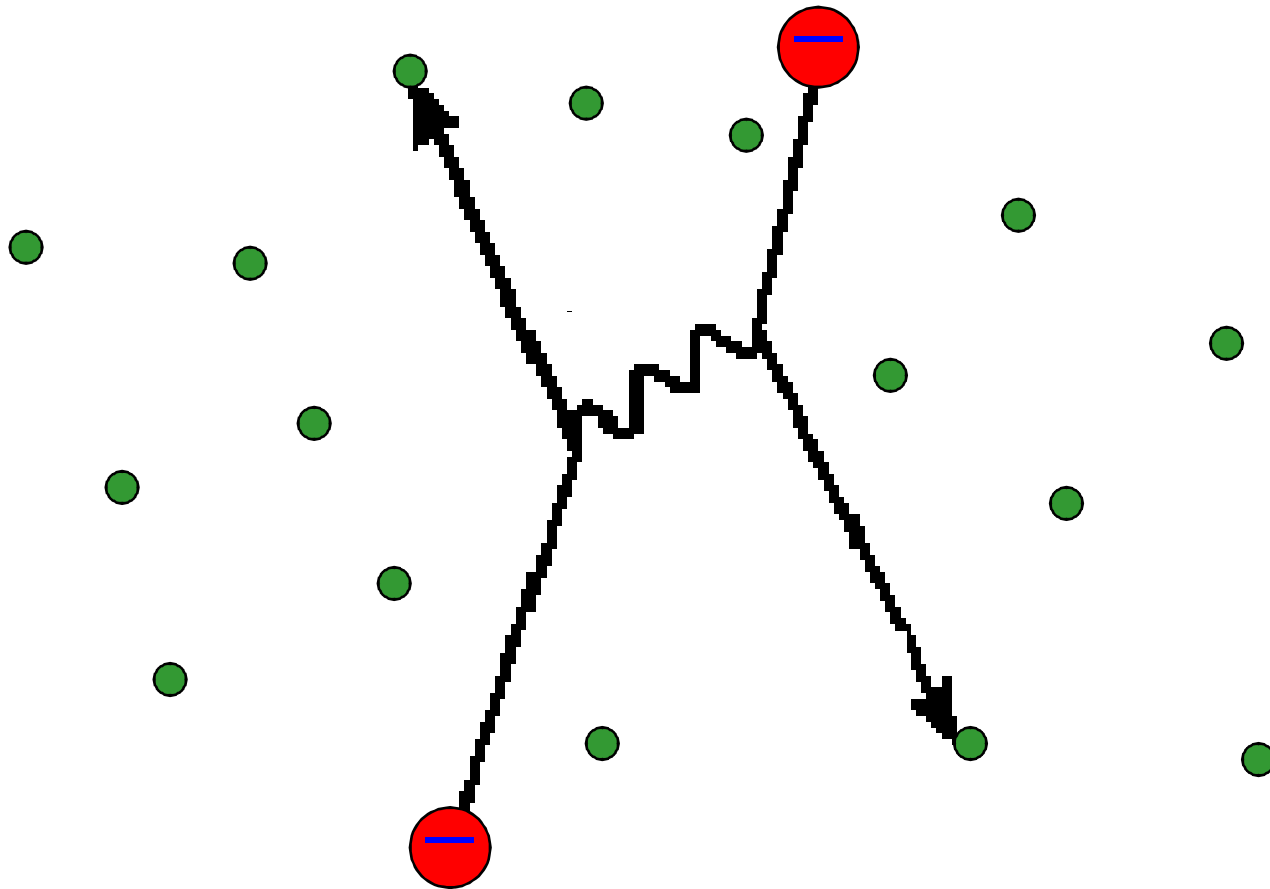
# Coulomb interaction + disorder



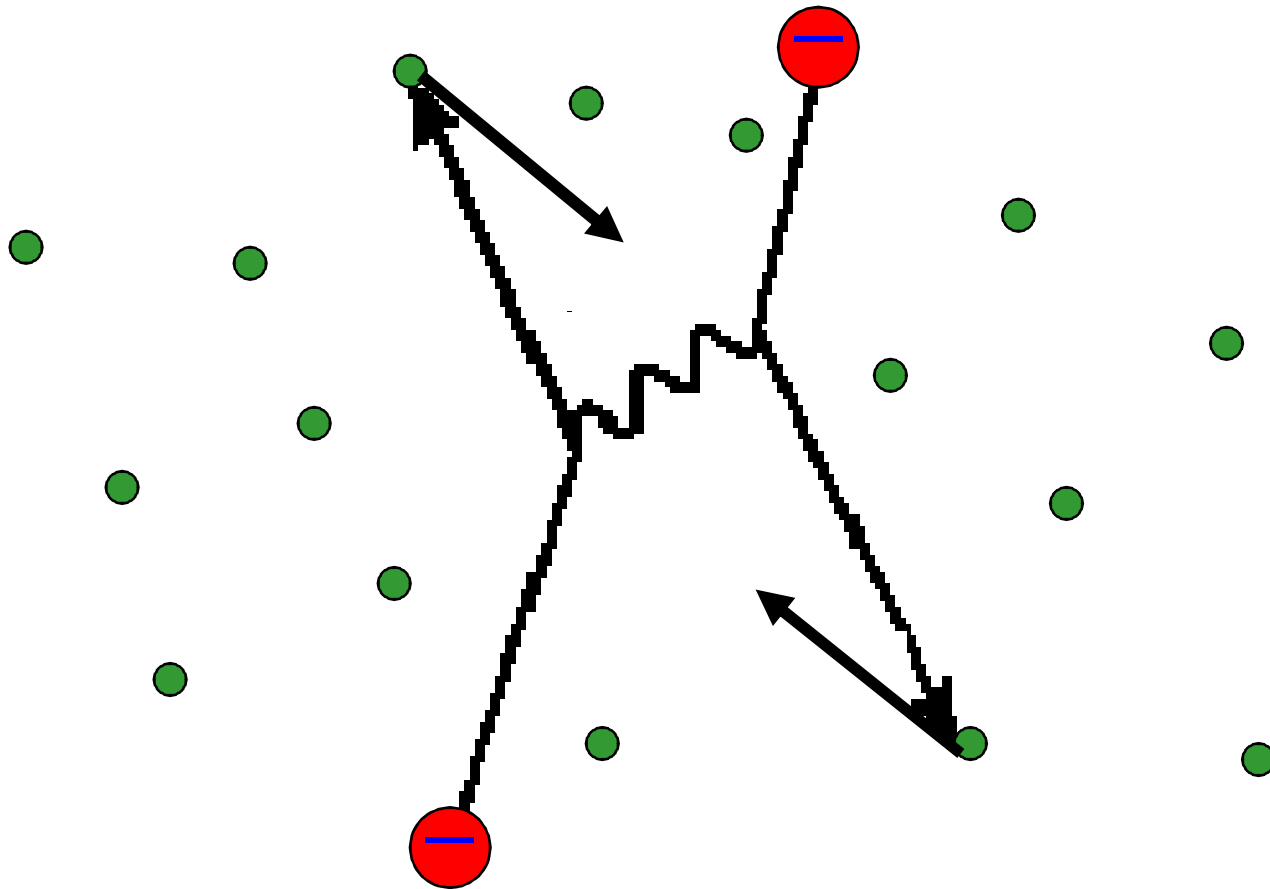
# Coulomb interaction + disorder



# Coulomb interaction + disorder

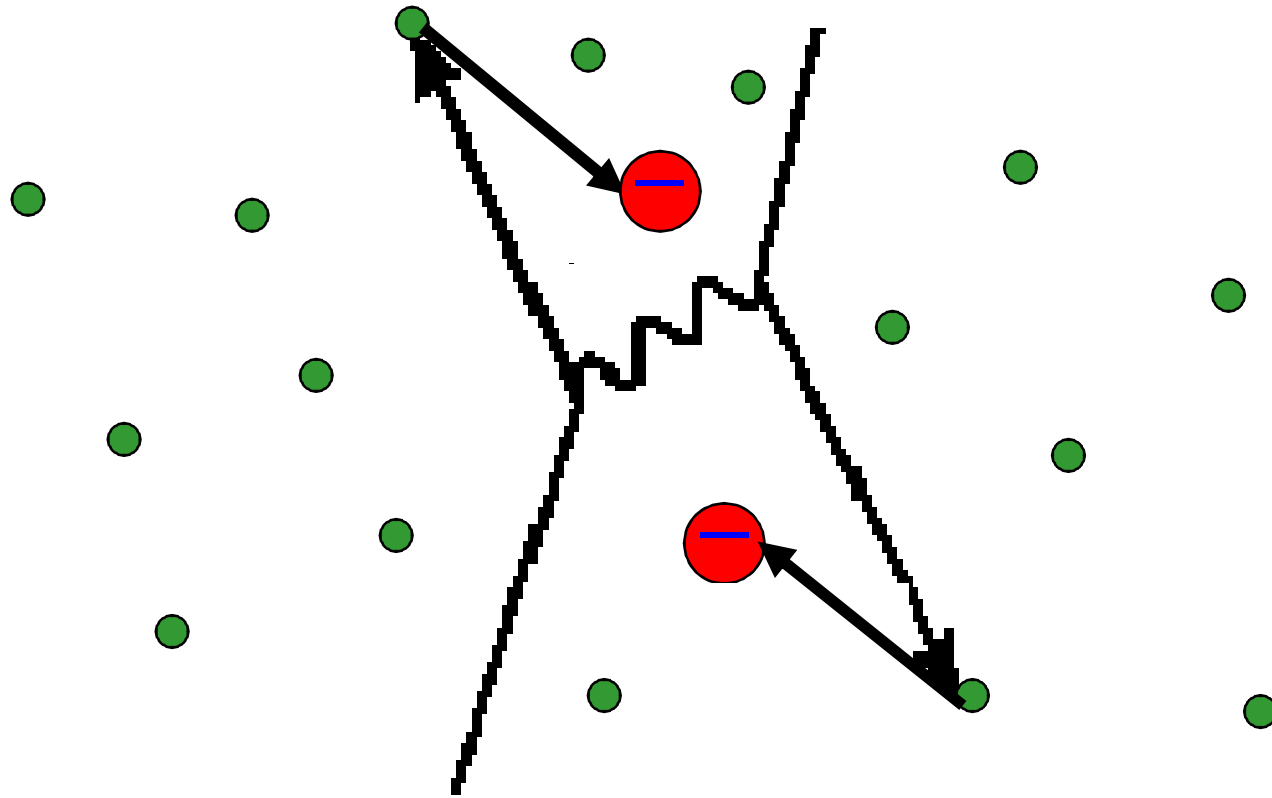


# Coulomb interaction + disorder





# Coulomb interaction + disorder







$$G(T) = G_{\text{Drude}} - \delta G_{\text{int}} - \delta G_{\text{WL}}$$

**Interaction  
correction**

**Weak  
localization  
correction**



$$G(T) = G_{\text{Drude}} - \delta G_{\text{int}} - \delta G_{\text{WL}}$$

- $\frac{\delta G_{\text{int}}}{G_{\text{Drude}}} \sim \frac{L_{\text{T}}}{L_{\text{loc}}}, \quad L_{\text{T}} \sim \sqrt{\frac{D}{T}}, \quad L_{\text{loc}} \sim Nl$
- $\frac{\delta G_{\text{WL}}}{G_{\text{Drude}}} \sim \frac{L_{\varphi}}{L_{\text{loc}}}, \quad L_{\varphi} \sim L_{\text{loc}}^{1/3} L_{\text{T}}^{2/3},$



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- $\frac{\delta G_{\text{WL}}}{G_{\text{Drude}}} \sim \frac{L_{\varphi}}{L_{\text{loc}}}, \quad L_{\varphi} \sim L_{\text{loc}}^{1/3} L_{\text{T}}^{2/3},$

**Crossover to Anderson localization at:**

$$L_{\text{T}} \sim L_{\varphi} \sim L_{\text{loc}}$$

## Intrinsic Decoherence in Mesoscopic Systems

P. Mohanty, E. M. Q. Jariwala, and R. A. Webb

*Center for Superconductivity Research, Department of Physics, University of Maryland, College Park, Maryland 20742*

(Received 17 December 1996)

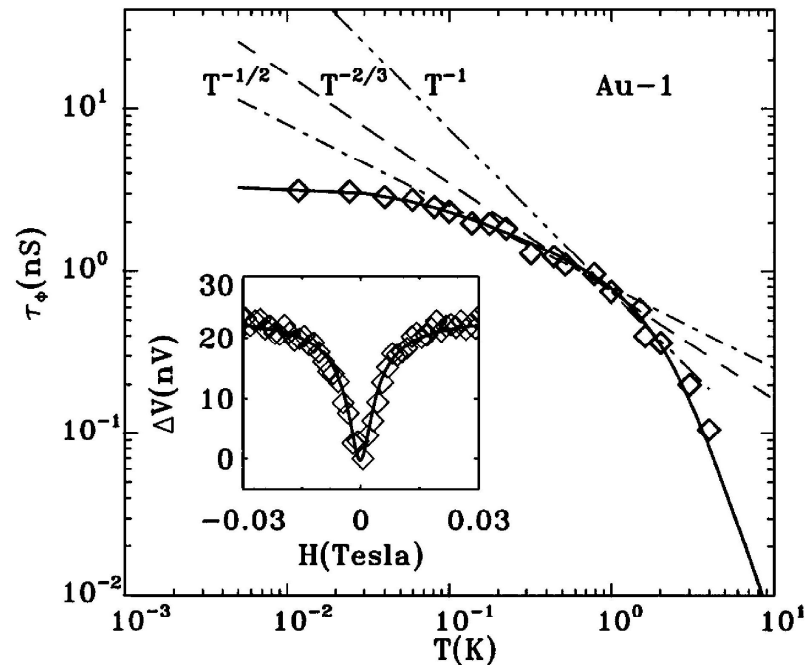


FIG. 1. Temperature dependence of  $\tau_\phi$  for sample Au-1. The broken lines are the functional forms expected from previous theories. The solid line is a fit to Eq. (1) with phonons. The inset shows the typical weak localization data taken with 2 nA at 11 mK with a fit to the standard 1D theory.

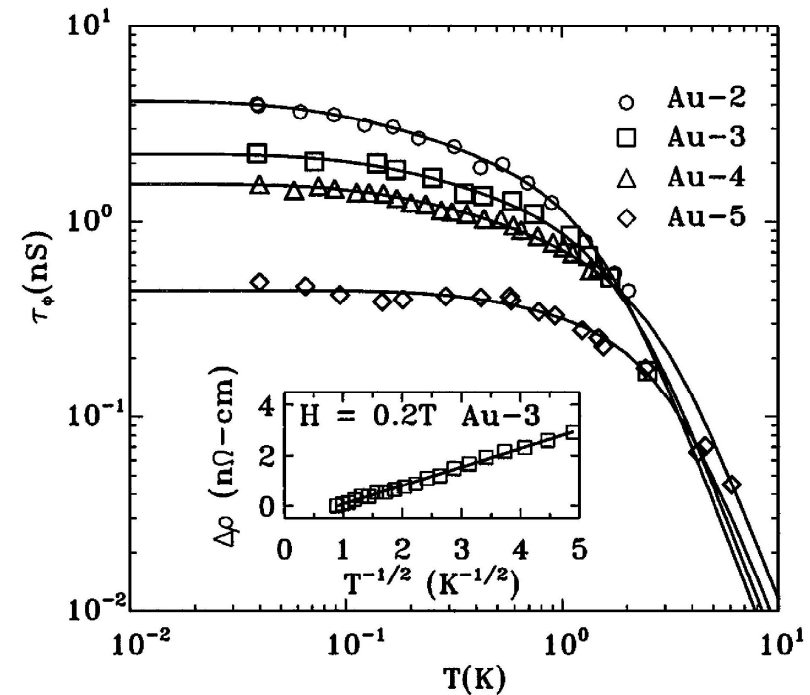


FIG. 2. Temperature dependence of  $\tau_\phi$  for four Au wires. Solid lines are fits to Eq. (1) with phonons. The inset is the EE contribution to  $\Delta\rho$  with the theoretical prediction.



$$G(T) = G_{\text{Drude}} - \delta G_{\text{int}} - \delta G_{\text{WL}}$$

- $\frac{\delta G_{\text{int}}}{G_{\text{Drude}}} \sim \frac{L_{\text{T}}}{L_{\text{loc}}}, \quad L_{\text{T}} \sim \sqrt{\frac{\bar{D}}{T}}, \quad L_{\text{loc}} \sim Nl$

## No Anderson localization!

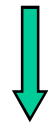
- **low T** :  $L_{\varphi} \sim \sqrt{Nl}, \quad \frac{\delta G_{\text{WL}}}{G_{\text{Drude}}} \sim \frac{1}{\sqrt{N}} \ll 1$

Disorder + Interactions:

Universal theory

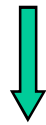


Disorder + Interactions:



Scattering  
matrix  
approach

# Disorder + Interactions:



Scattering  
matrix  
approach



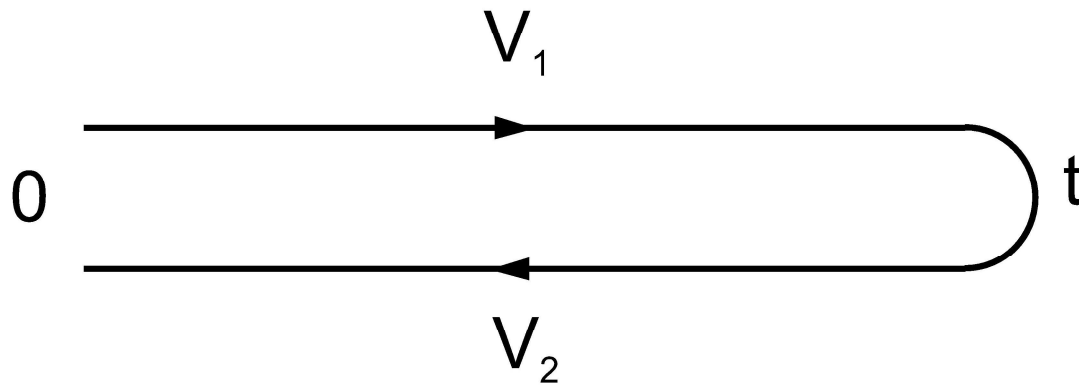
Hubbard-Stratonovich  
+  
Keldysh

## Hamiltonian

$$\mathbf{H} = \int d\mathbf{r} \psi_{\sigma}^{\dagger}(\mathbf{r}) h(\mathbf{r}) \psi_{\sigma}(\mathbf{r}) + \mathbf{H}_{int},$$

$$h(\mathbf{r}) = -\frac{\nabla^2}{2m} - \mu + U(\mathbf{r}),$$

$$\mathbf{H}_{int} = \frac{1}{2} \int d\mathbf{r} \int d\mathbf{r}' \psi_{\sigma}^{\dagger}(\mathbf{r}) \psi_{\sigma'}^{\dagger}(\mathbf{r}') \frac{e^2}{|\mathbf{r} - \mathbf{r}'|} \psi_{\sigma'}(\mathbf{r}') \psi_{\sigma}(\mathbf{r})$$



$$\hat{G} = \frac{\int \mathcal{D}V_1 \mathcal{D}V_2 G_V \exp(iS)}{\int \mathcal{D}V_1 \mathcal{D}V_2 \exp(iS)}$$

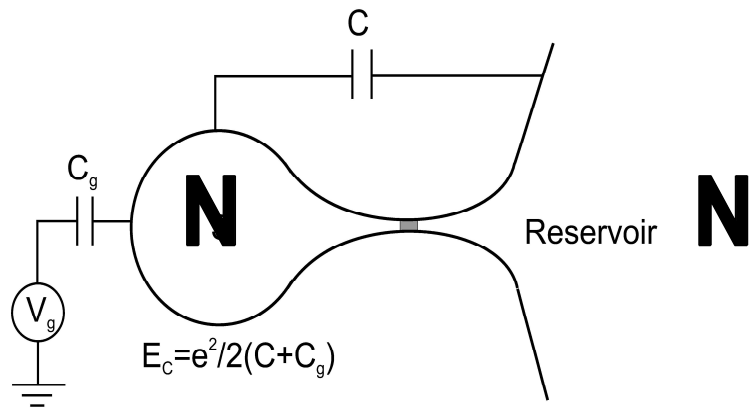
$$\begin{aligned} & \begin{pmatrix} i\frac{\partial}{\partial t'} + h(\mathbf{r}') + eV_1(t', \mathbf{r}') & 0 \\ 0 & i\frac{\partial}{\partial t'} + h(\mathbf{r}') + eV_2(t', \mathbf{r}') \end{pmatrix} \hat{G}_V(t't'', \mathbf{r}'\mathbf{r}'') \\ & \qquad \qquad \qquad = \sigma_z \delta(t' - t'') \delta(\mathbf{r}' - \mathbf{r}''). \end{aligned}$$

### Effective action

$$iS = 2\text{Tr} \ln \hat{G}_V^{-1} + i \int_0^t dt' \int d\mathbf{r} \frac{(\nabla V_1)^2 - (\nabla V_2)^2}{8\pi}.$$

# Outline

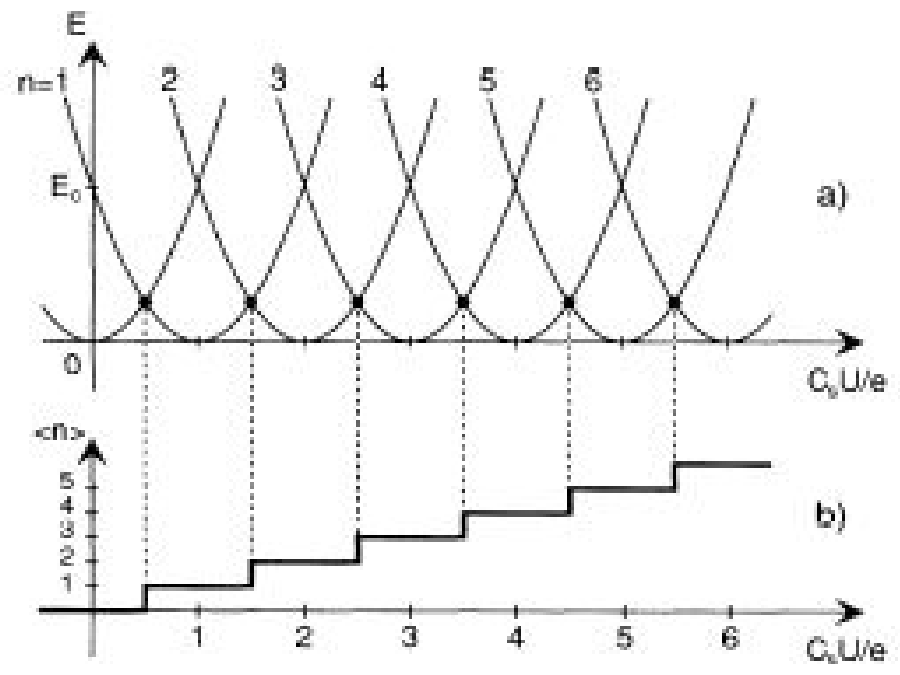
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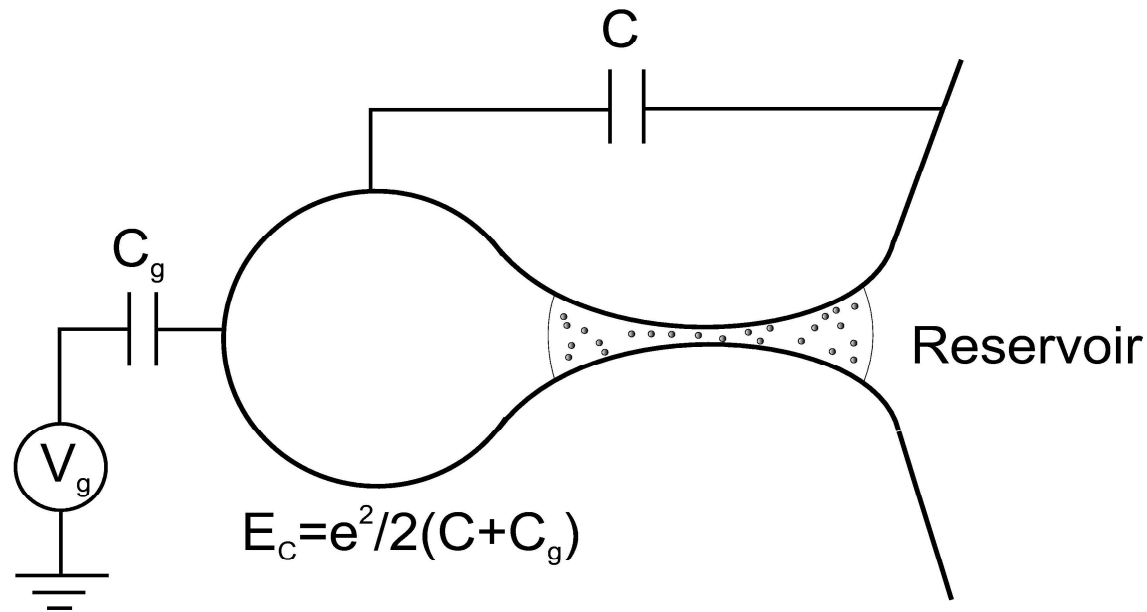
$$E = (Q_g - ne)^2 / 2(C + C_g)$$

$$Q_g = C_g V_g$$

$$n_g = Q_g / e$$



# Coherent scatterer: basic model



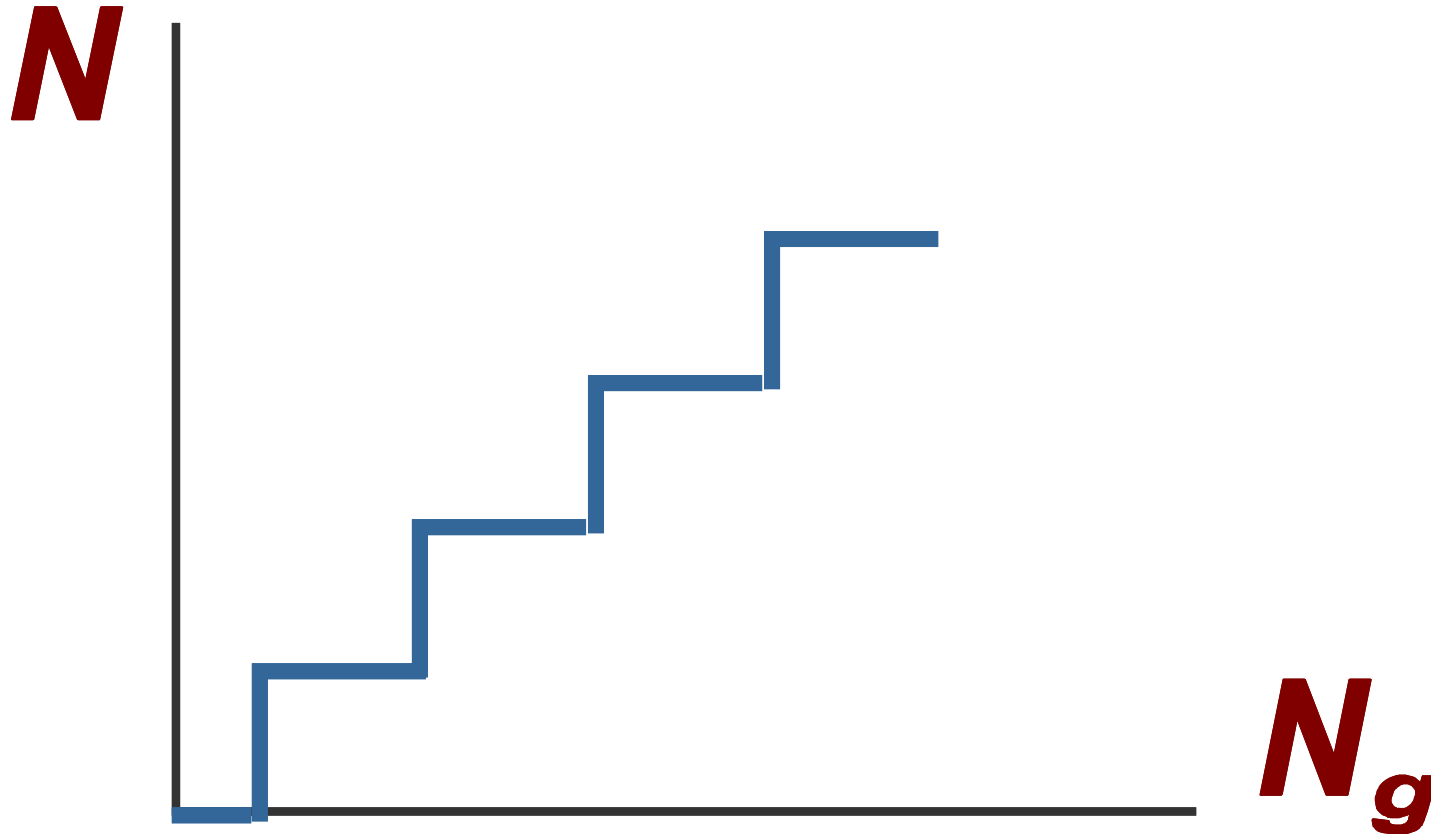
$$\frac{1}{R} = \frac{e^2}{\pi \hbar} \text{tr}[\hat{t}^\dagger + \hat{t}] = \frac{e^2}{\pi \hbar} \sum_n T_n$$

$$g = \frac{2\pi \hbar}{e^2 R} = 2 \sum_n T_n$$

# Tunnel junction, weak tunneling

$$\text{all } T_n \ll 1 \quad g \ll 1$$

Averin-Likharev'86 ("Orthodox" theory)



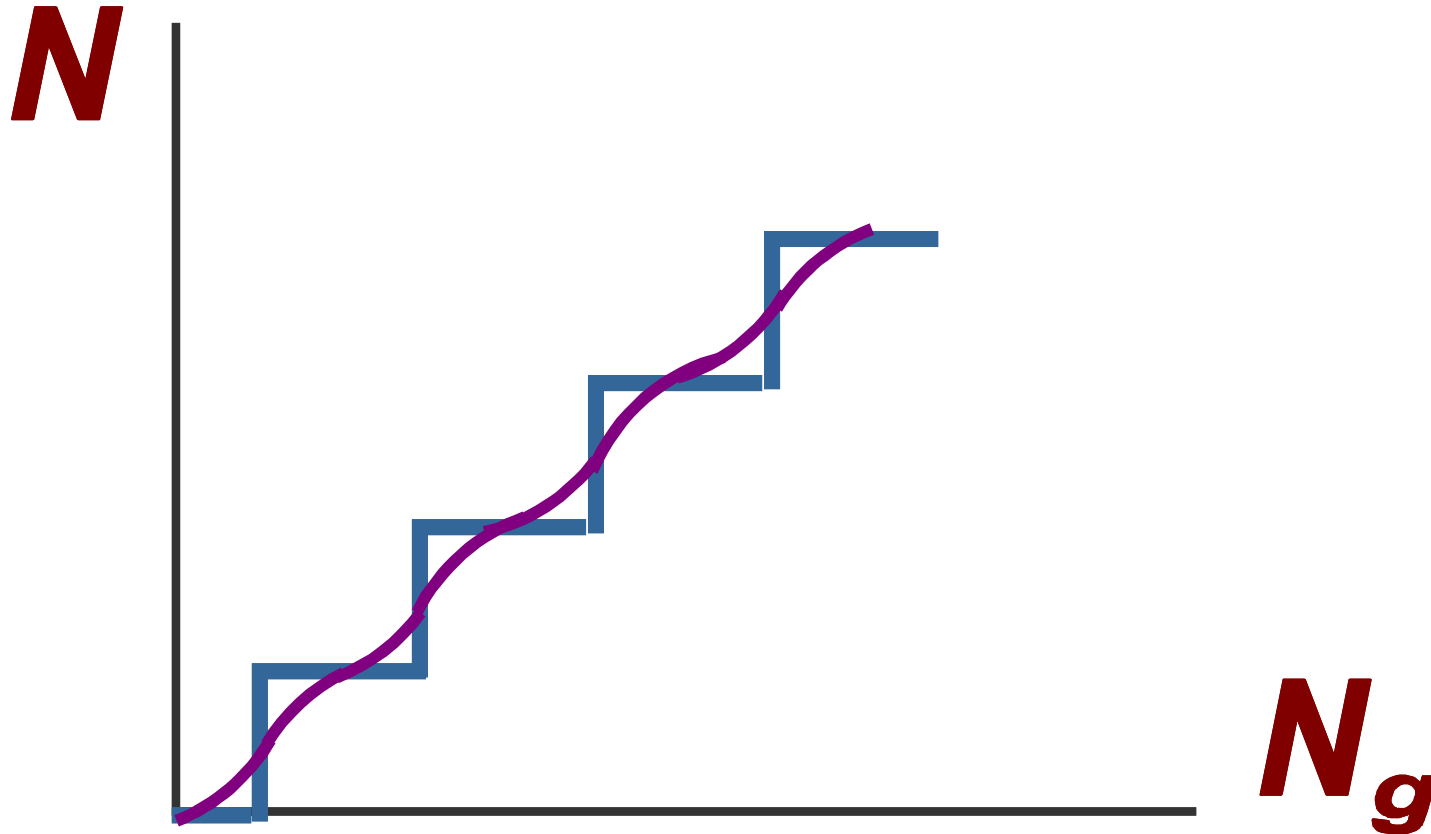


# Tunnel junction, strong tunneling

all  $T_n \ll 1$  but  $g \gg 1$

Panyukov, A.D.Z.'88'91 (**Instanton technique**)

$$\tilde{E}_c \propto E_c \exp(-g/2)$$



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## One channel, arbitrary $T_1$

Matveev'95 (bosonization)

$$\tilde{E}_c = 0 \quad \text{for } T_1 = 1$$

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## One channel, arbitrary $T_1$

Matveev'95 (bosonization)

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## Arbitrary scatterer, $g \gg 1$

Nazarov'99 (effective action, instantons)

$$\tilde{E}_c \propto E_c \prod_n [1 - T_n] = E_c \exp(-ag)$$

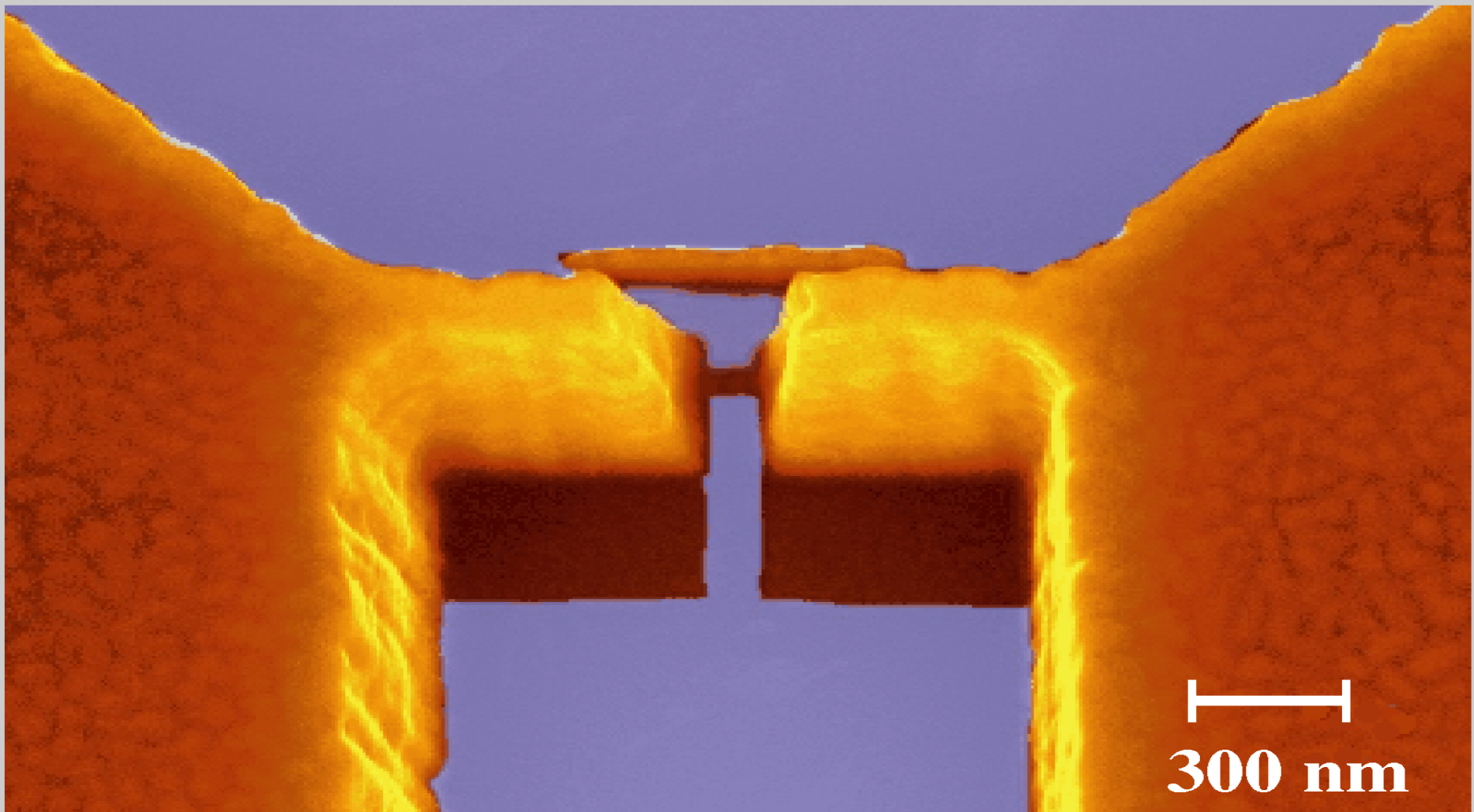
diffusive conductor:  $a = \pi^2/8$

# Outline

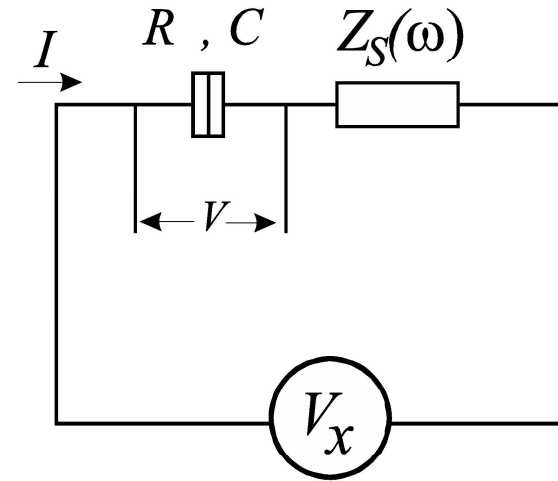
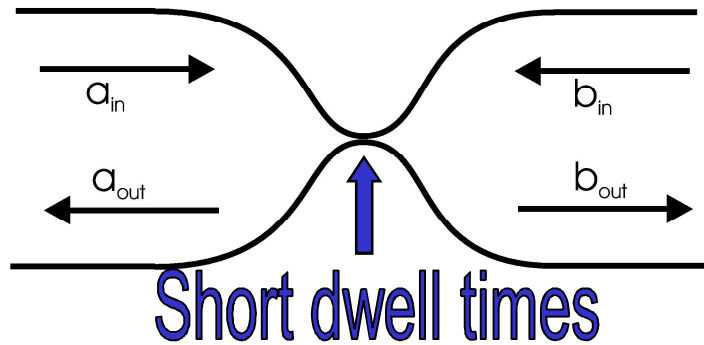
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# Diffusive metallic bridges

*H. Weber et al., PRB'01*



# Scatterer+Environment

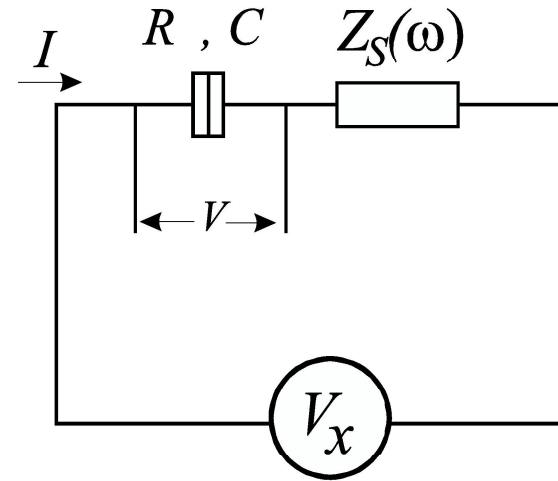
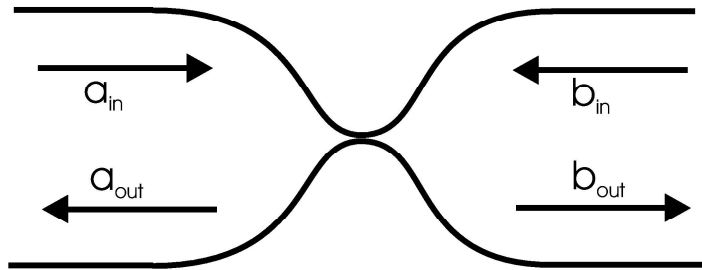


Golubev,  
A.D.Z.'01:

$$R \frac{dI}{dV} = 1 - \beta f(V, T)$$

$$\beta = \frac{\sum_n T_n (1 - T_n)}{\sum_n T_n}$$

# Scatterer+Environment



Golubev,  
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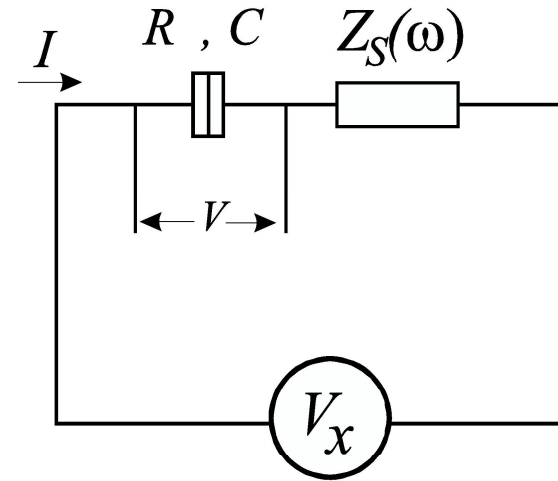
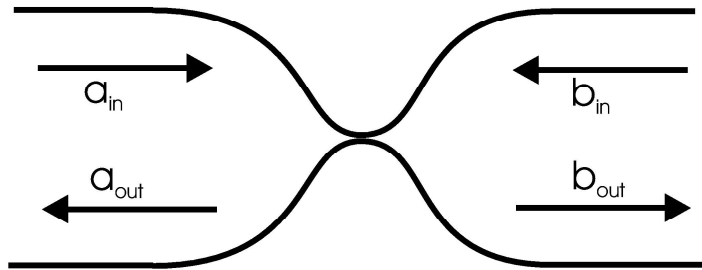
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Universal  
function

$$\beta = \frac{\sum_n T_n (1 - T_n)}{\sum_n T_n}$$

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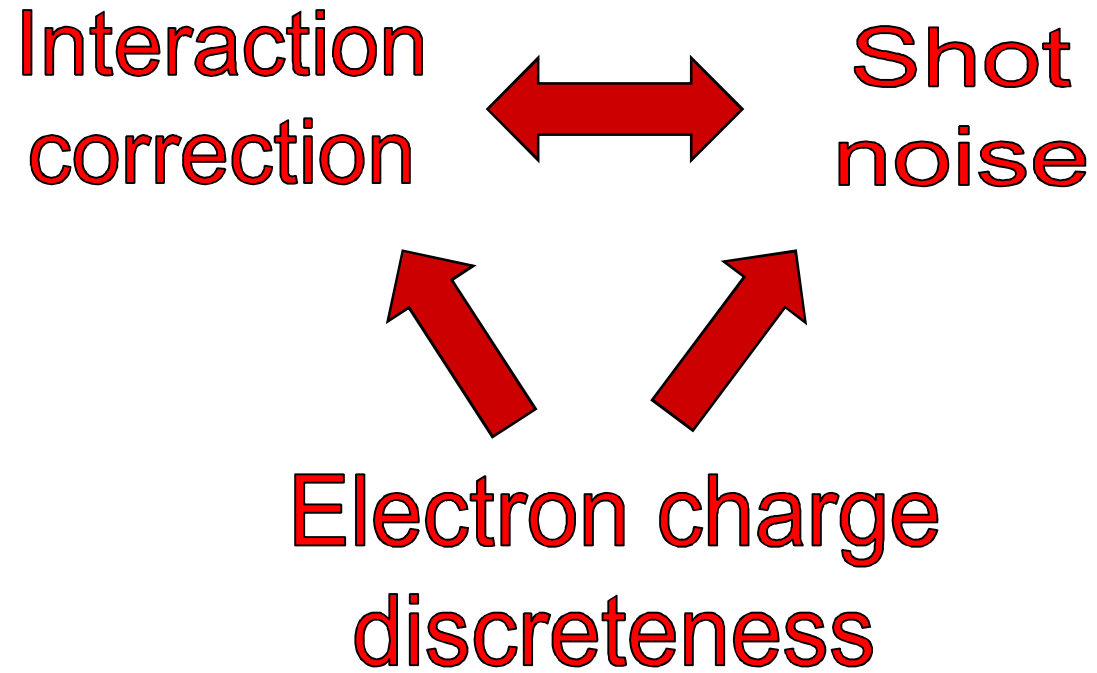
Shot  
noise



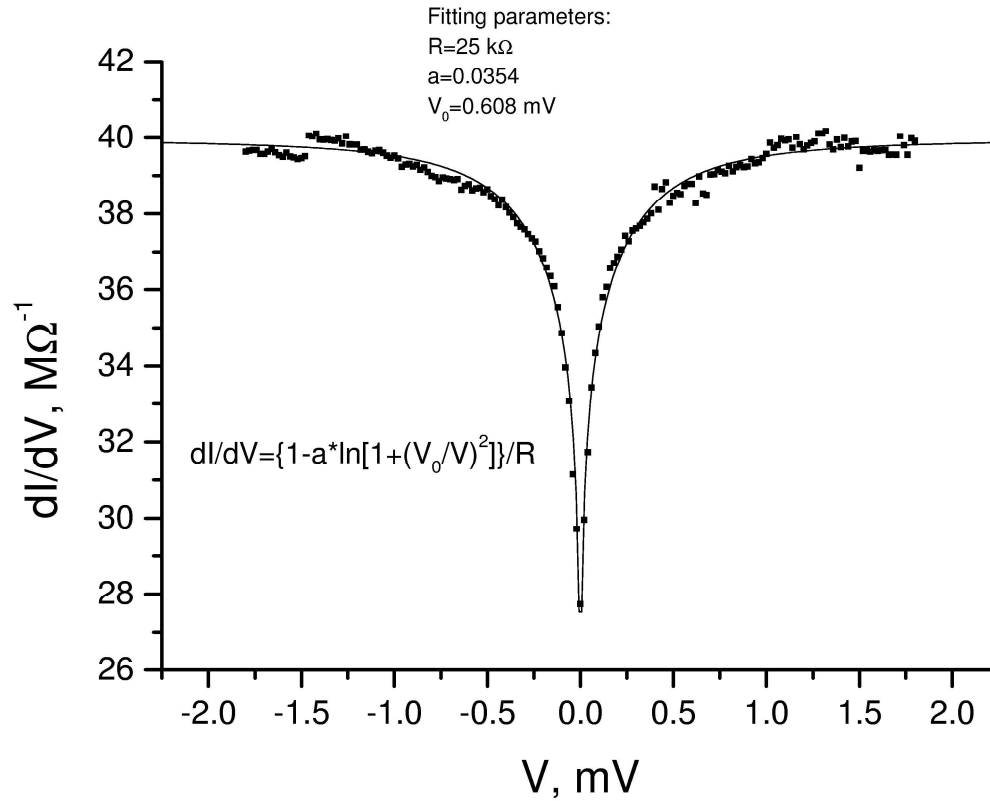
Interaction  
correction



Shot  
noise



## Experiment: Krupenin et al., APL'02 (Granular metals)

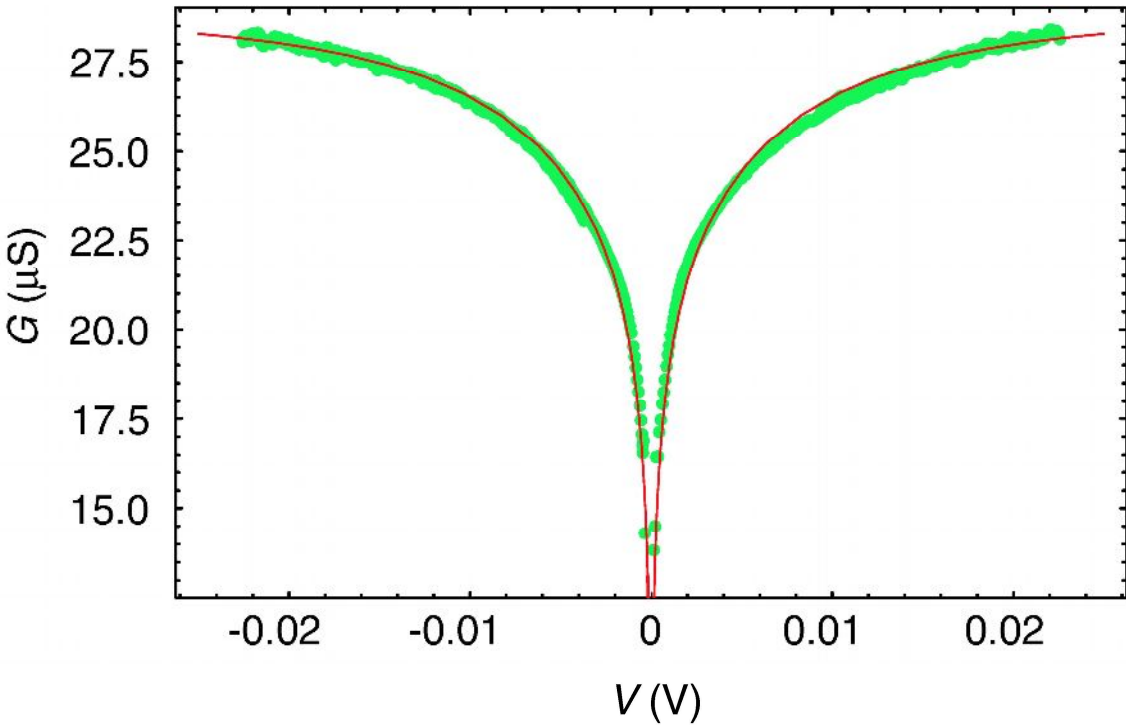


***I – V curve at low T:***

$$R \frac{dI}{dV} = 1 - \frac{\beta}{g} \ln \left( 1 + \frac{1}{(eVRC)^2} \right)$$

# Multiwalled Carbon Nanotubes

Paalanen et al. (LTL, Helsinki)



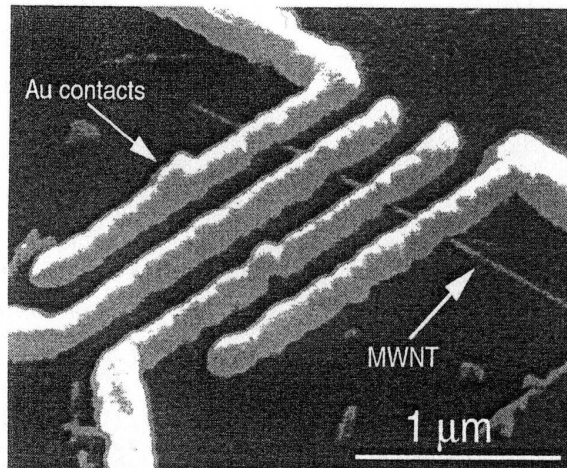
## Interference and Interaction in multi-wall carbon nanotubes

C. Schönenberger<sup>1</sup>, A. Bachtold<sup>1</sup>, C. Strunk<sup>1</sup>, J.-P. Salvetat<sup>2</sup>, L. Forró<sup>2</sup>

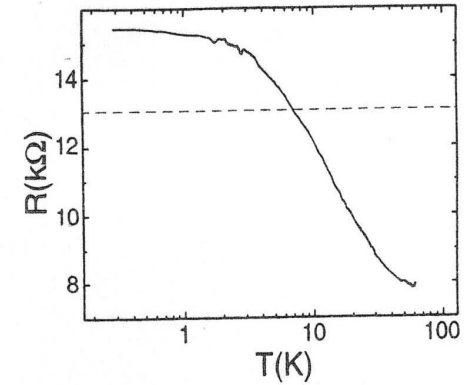
<sup>1</sup> Institut für Physik, Universität Basel, Klingelbergstr. 82, CH-4056 Basel, Switzerland

<sup>2</sup> Institut de Génie Atomique, École Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland

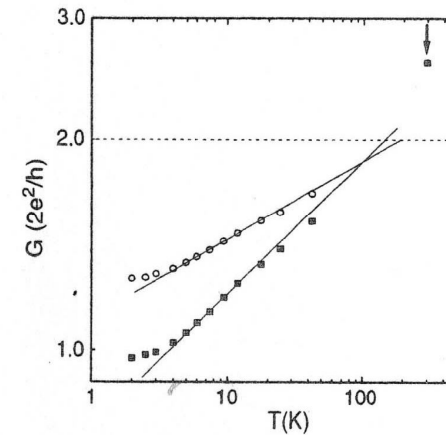
Received: 17 May 1999/Accepted: 18 May 1999/Published online: 4 August 1999



**Fig. 1.** Scanning electron microscopy image of a single multi-wall nanotube (MWNT) electrically contacted by four Au fingers from above. The separation between the contacts is 350 nm center to center

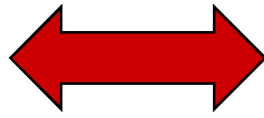


**Fig. 2.** Typical temperature-dependent electrical resistance  $R(T)$  of a single MWNT measured in a four-probe configuration, i.e. the current is passed through the outer contacts and voltage is measured over the inner ones. The dashed line corresponds to the resistance quantum  $h/2e^2$



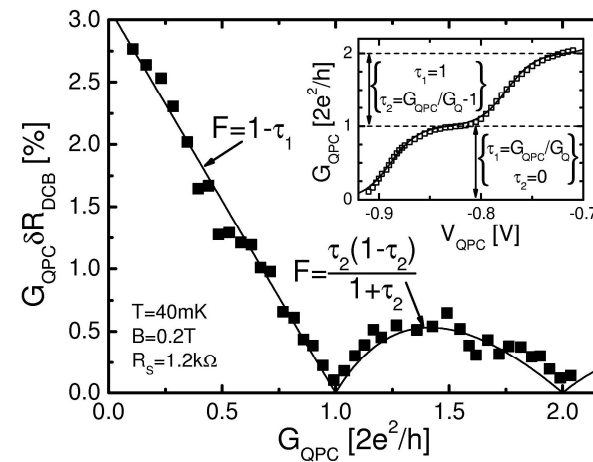
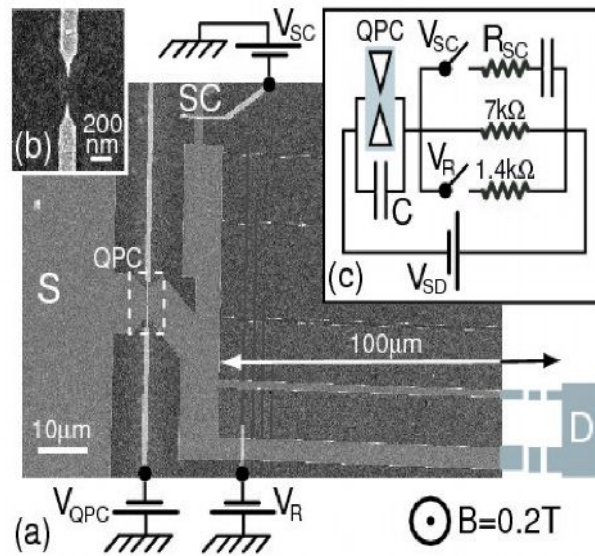
**Fig. 6.** Conductance  $G$  at zero magnetic field as a function of temperature  $T$  for the measurement shown in Fig. 5. Filled squares correspond to  $G$  at  $B = 0$  and open circles to  $G - \delta G_{WL}$  with  $\delta G_{WL}$  the contribution to the conductance from weak localization. The dashed horizontal line is the conductance expected for an ideal metallic carbon nanotube. The arrow points to the measured room temperature value

Interaction  
correction



Shot noise  
(Fano factor)

Experiments on break junctions: Altimiras et al. PRL'07

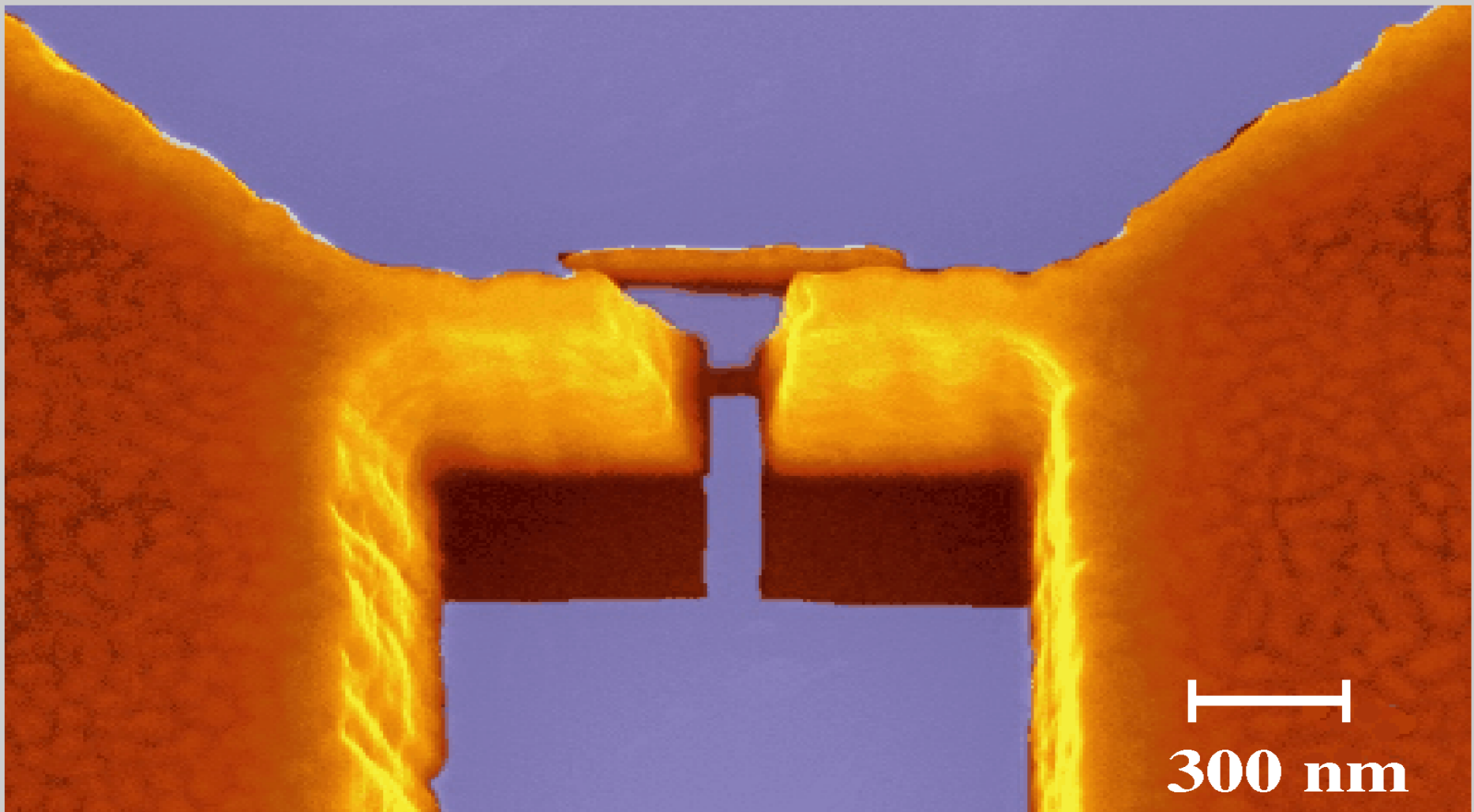


**Zero bias conductance:**

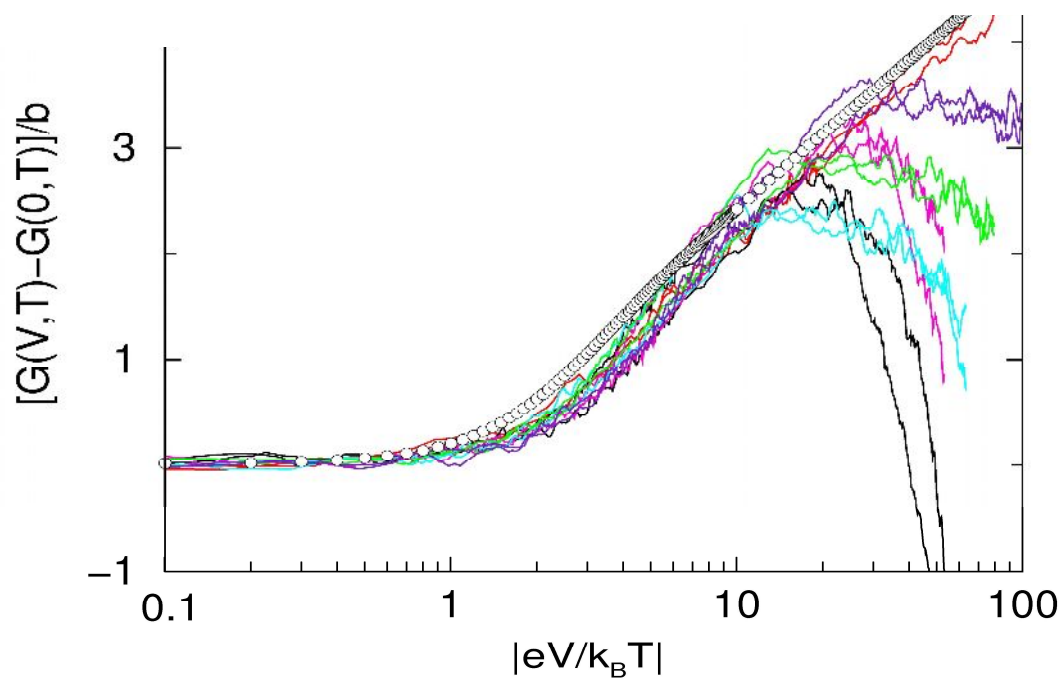
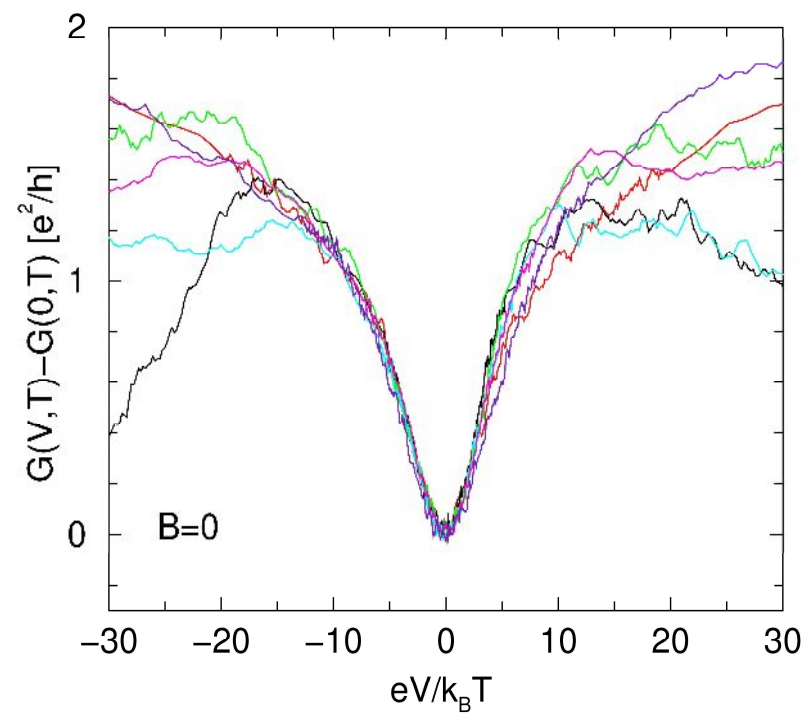
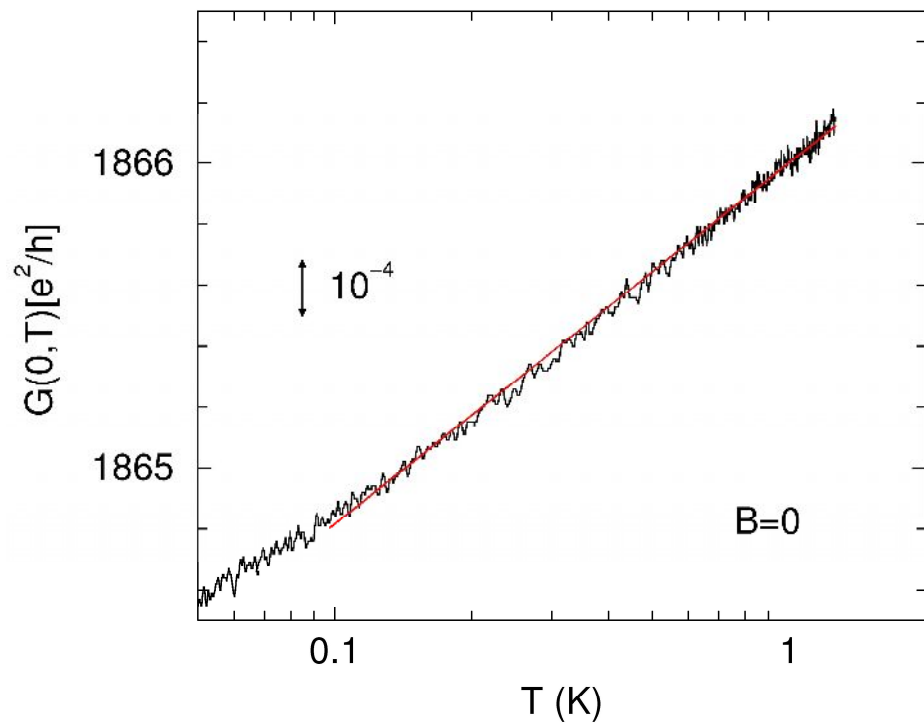
$$RG(T) \simeq 1 - \frac{2\beta}{g} \ln \left( \frac{gE_C}{T} \right)$$

# Diffusive metallic bridges

*H. Weber et al., PRB'01*







**Zero bias conductance:**

$$RG(T) \simeq 1 - \frac{2\beta}{g} \ln \left( \frac{gE_C}{T} \right)$$

$$+ \frac{4}{g^2} \ln^2 \left( \frac{gE_C}{T} \right) \left[ \frac{\beta(1-\beta)}{2} - \gamma \right] + \dots$$

$$\gamma = \frac{\sum_n T_n^2 (1 - T_n)}{\sum_n T_n}$$

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# Interaction correction to higher cumulants and full counting statistics

**Shot noise:**  $S_2 = \langle I I \rangle_+$

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$$\delta S_2^{\text{int}} \sim (2\gamma - \beta)/g$$

# Interaction correction to higher cumulants and full counting statistics

**Shot noise:**  $S_2 = \langle I I \rangle_+$

$$\delta S_2^{\text{int}} \sim (2\gamma - \beta)/g \sim S_3$$

$$\tilde{S}(t, t') = \tilde{S}^{\text{ni}}(t, t') + \delta\tilde{S}(t, t')$$

## Nyquist noise...

$$\delta\tilde{S}_\omega = -\frac{4\beta T}{R_q} \ln \frac{gE_C}{T}, \quad \text{if } |\omega|, |eV| \ll T \ll gE_C,$$

$$\delta\tilde{S}_\omega = -\frac{2\beta|\omega|}{R_q} \ln \frac{gE_C}{|\omega|}, \quad \text{if } T, |eV| \ll |\omega| \ll gE_C$$

... always suppressed

$$\tilde{S}(t, t') = \tilde{S}^{\text{ni}}(t, t') + \delta\tilde{S}(t, t')$$

## Shot noise...

$$\delta\tilde{S}_\omega = -\frac{2(\beta - 2\gamma)|eV|}{R_q} \ln \frac{gE_C}{|eV|},$$

if  $T, |\omega| \ll |eV| \ll gE_C$

... enhanced for  $\beta < 2\gamma$



# Shot noise

Large voltages:

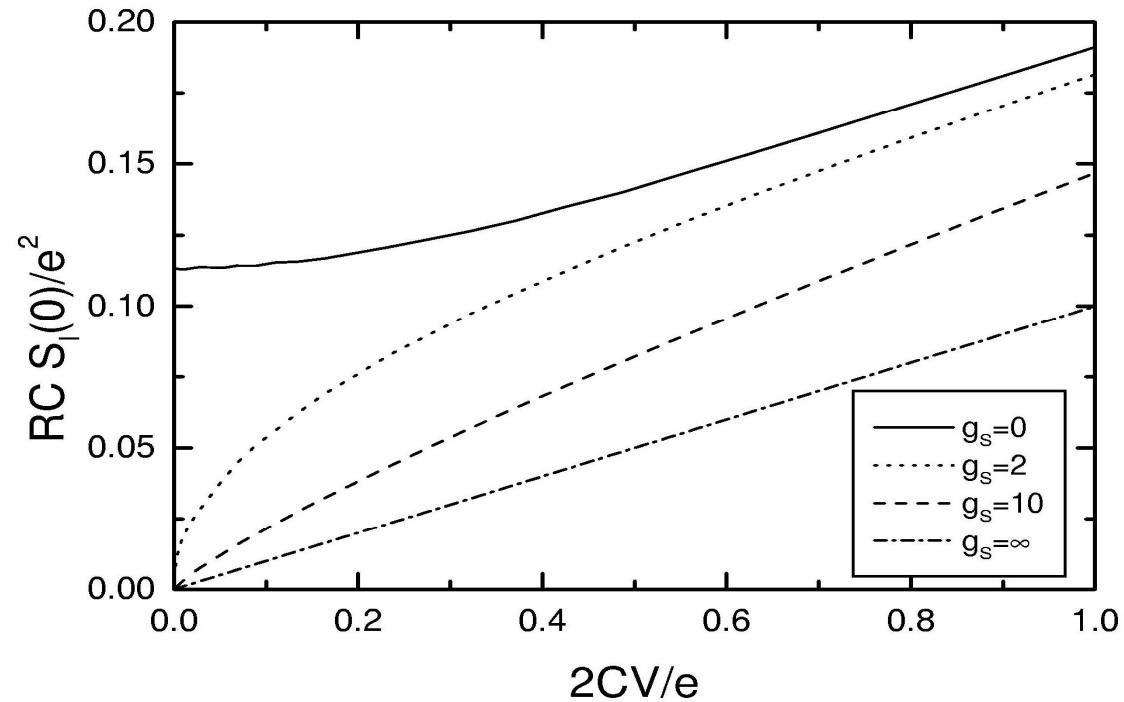
$$S_I(0) = \frac{2e^3\mathcal{R}}{\pi} \left( V + \frac{e}{2C} \right),$$

Small voltages:

$$S_I(0) = \frac{2e^2\mathcal{R}(R + R_S)e^{\frac{2\gamma_0}{g+g_s}}}{\pi\Gamma\left(2 - \frac{2}{g+g_s}\right)RR_S C} \left( \frac{e|V|RR_S C}{R + R_S} \right)^{1 - \frac{2}{g+g_s}}$$

# Shot noise

Golubev, Galaktionov, A.D.Z.'05



Interaction-induced excess noise

# Renormalization group

Kinderman, Nazarov'03  
Golubev, A.D.Z.'04  
Bagrets, Nazarov'05

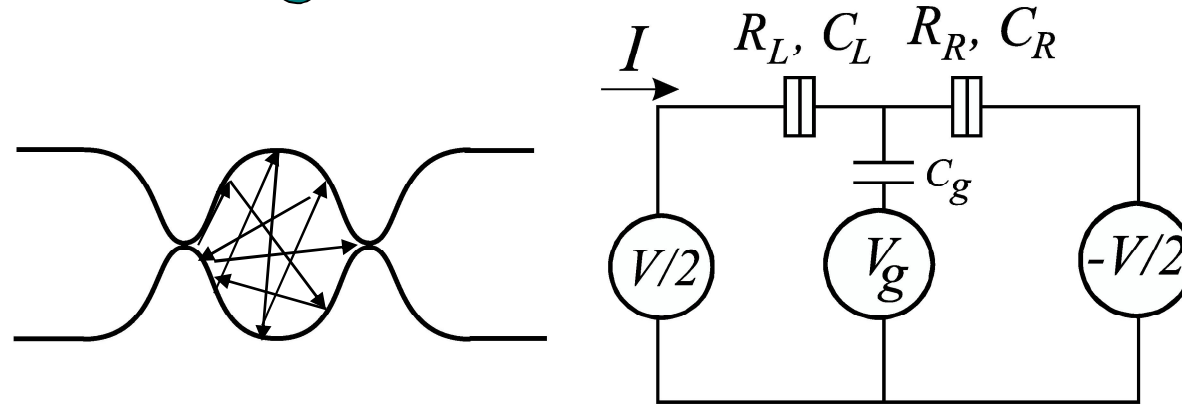
$$\frac{dT_n}{d\ln V} = \frac{2T_n(1-T_n)}{g}$$

$$g \gg 1$$

# Outline

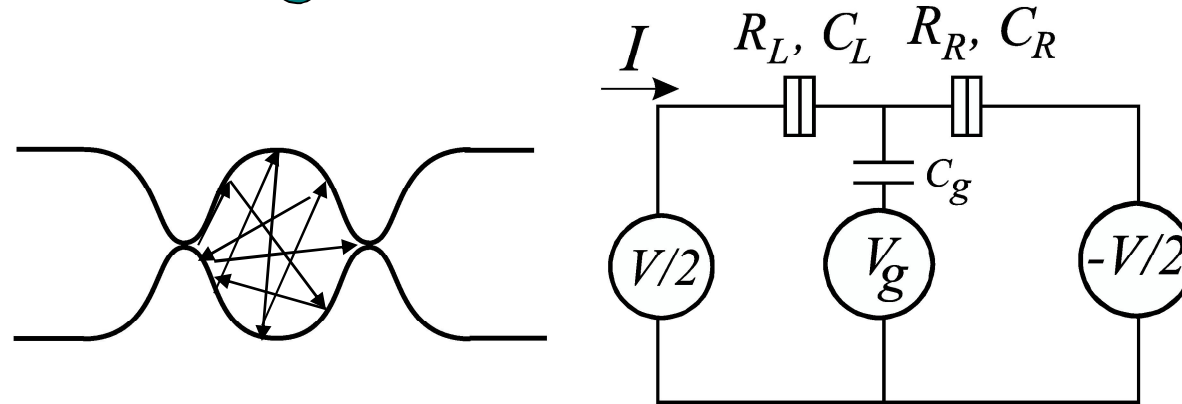
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# Long dwell times...



$$RG(T) \simeq 1 - \frac{2(\beta_L g_R + \beta_R g_L)}{(g_L + g_R)^2} \ln \left( \frac{g E_C}{T} \right)$$

# Long dwell times...



$$RG(T) \simeq 1 - \frac{2(\beta_L g_R + \beta_R g_L)}{(g_L + g_R)^2} \ln \left( \frac{g E_C}{T} \right)$$

**Finite  $\tau_D$  ?**

## Non-zero external impedance

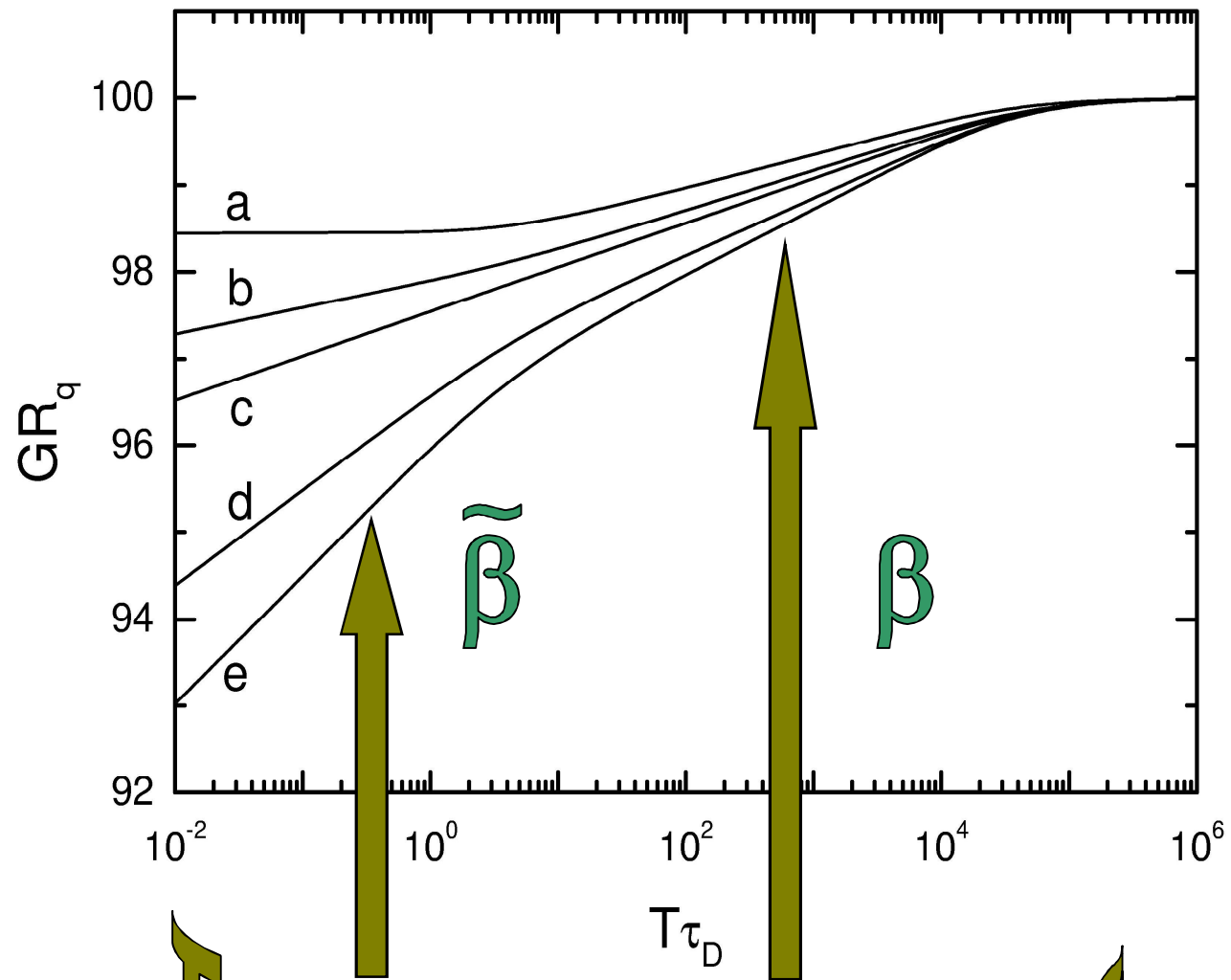
- $1/\tau_D \ll T \ll gE_C$  :

$$G = G_0 + \frac{\beta\chi}{R_q} \ln(T/gE_C), \quad 1 \leq \chi \leq 2,$$

- $T \ll 1/\tau_D$  :

$$G \simeq \tilde{G}_0 + \frac{2\tilde{\beta}}{R_q} \frac{1}{1 + 4R/R_S} \ln(T\tau_D),$$

$$\tilde{G}_0 \simeq G_0 - \frac{\beta\chi}{2R_q} \ln \frac{\tau_D}{RC}$$



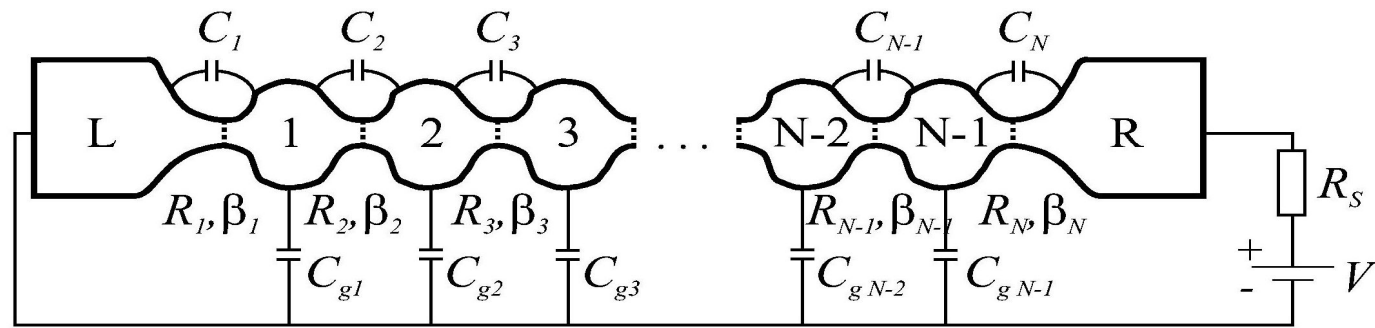
Two different logs

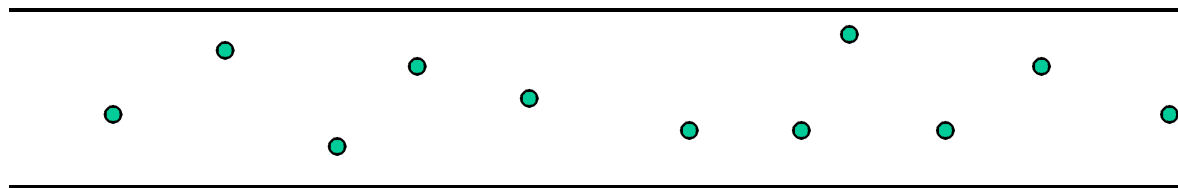
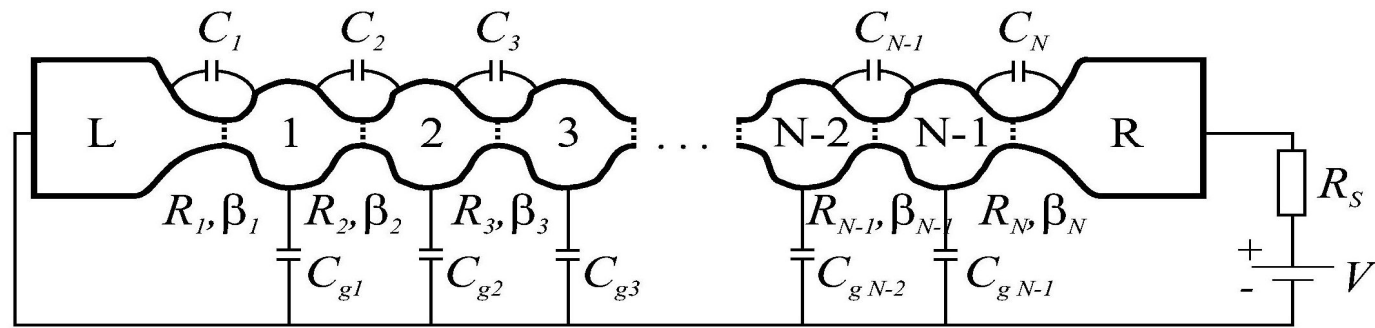




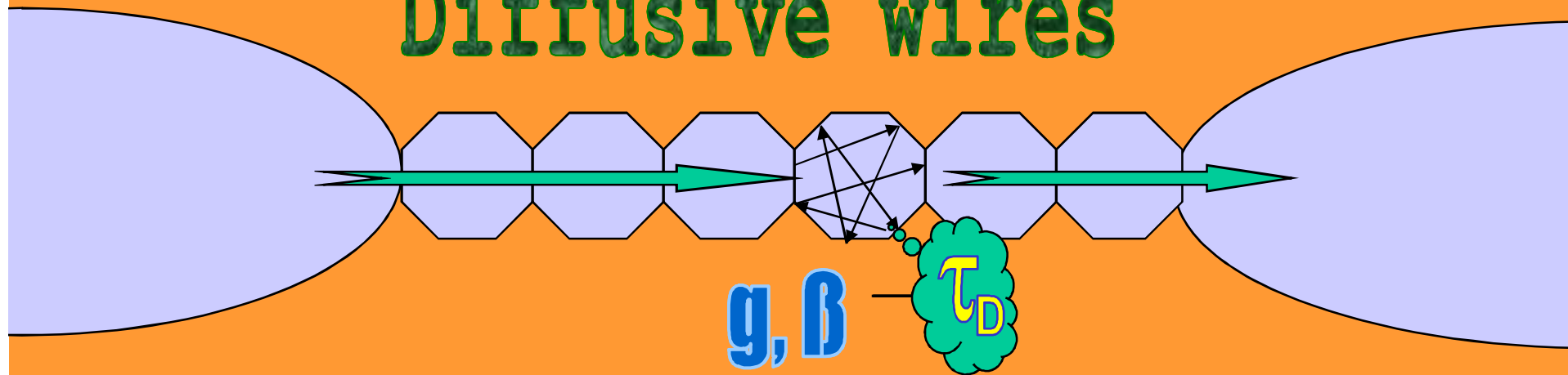
- Diffusive metallic nanobridges (Weber et al.)
- Break junctions (Saclay)
- Metallic constrictions (Natelson et al.)
- Carbon nanotubes (numerous)
- Single tunnel junctions and granular arrays (numerous)
- Diffusive metallic wires (Mohanty-Webb)

**Both logs have been observed**

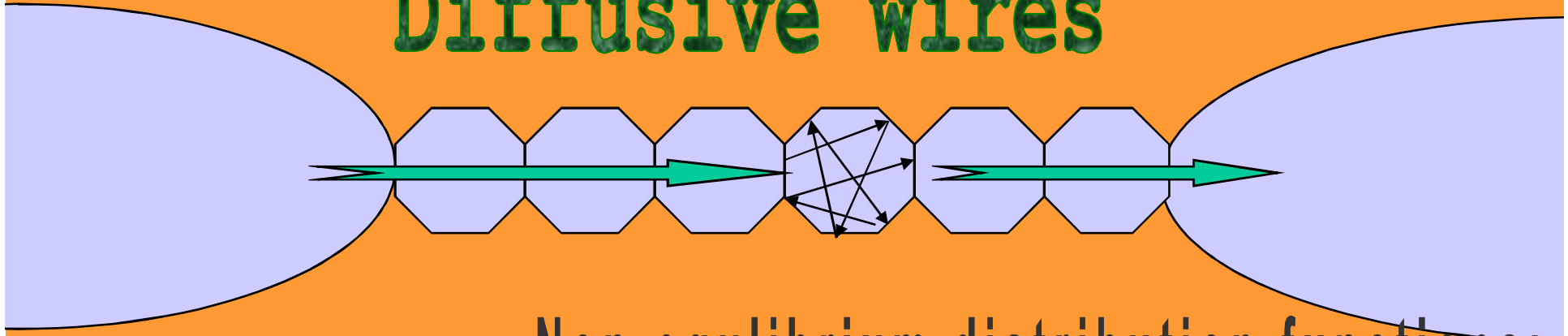




# Diffusive wires



# Diffusive wires



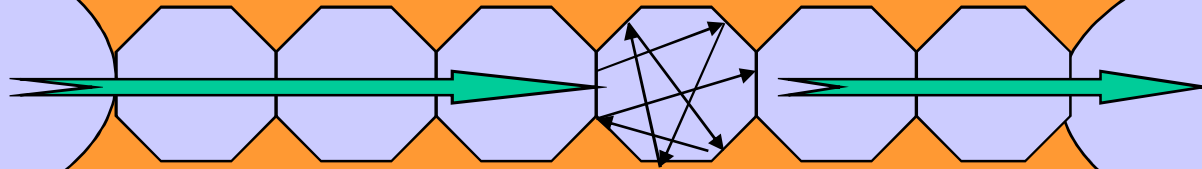
Non-equilibrium distribution functions:  
shot noise modified

*General  
strategy:*

Kinetic equation

Balance of charges and voltages

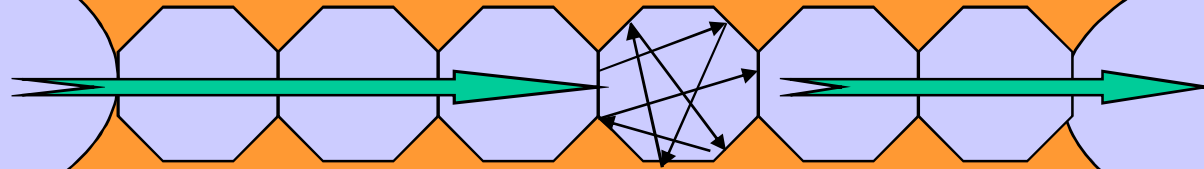
## Diffusive wires



## Shot Noise:

$$\tilde{\beta} = 1/3 + \frac{\sum_n (1/g_n^3)(\beta_n - 1/3)}{(\sum_n 1/g_n)^3}$$

# Diffusive wires



*Noise and interaction correction*

$$IR_{\Sigma} = V + \sum_n R_n \langle \xi_n \rangle$$

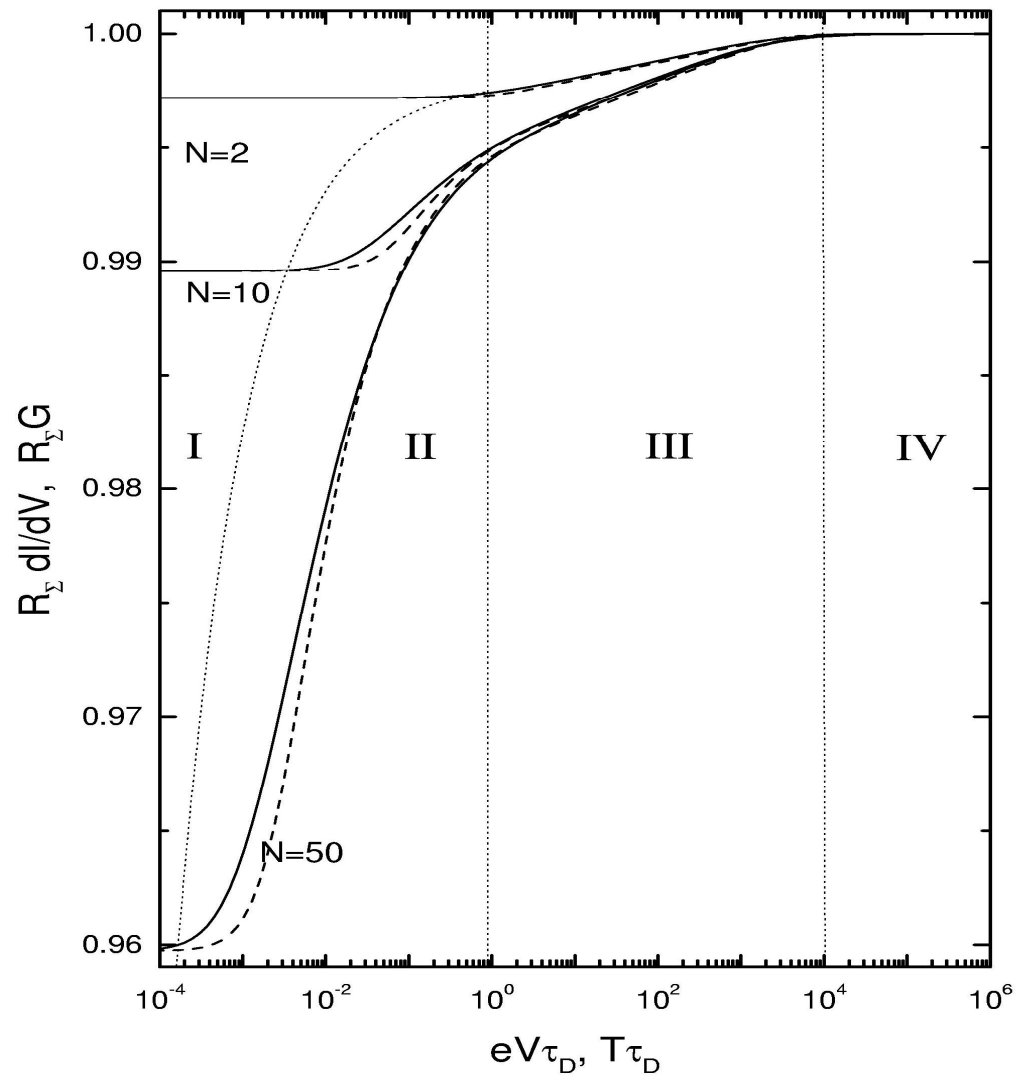
# Perturbation theory: complete expression

$$I = V/R_{\Sigma} + \delta I,$$

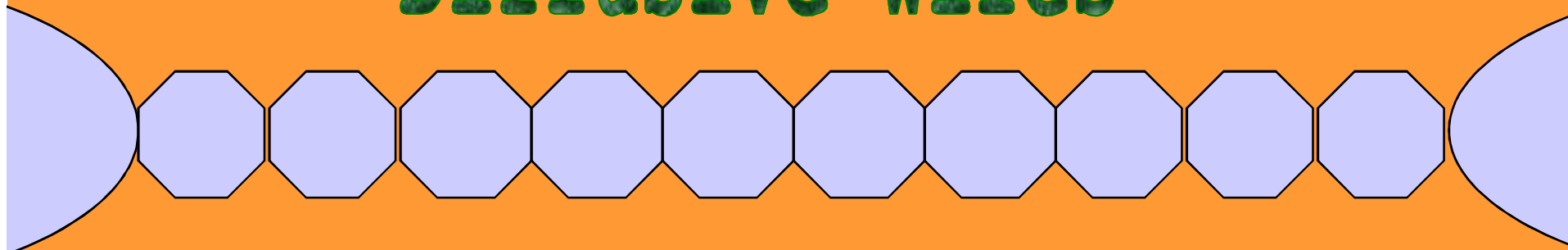
$$\begin{aligned} \delta I = & -\frac{2Te}{N^2} \sum_{q=1}^{N-1} \left[ \beta + \frac{\cos \frac{\pi q}{N}}{4 \sin^2 \frac{\pi q}{2N}} \right] \left[ W \left( \frac{2 \sin^2 \frac{\pi q}{2N}}{\pi T R (4C \sin^2 \frac{\pi q}{2N} + C_g)} + \frac{\sin^2 \frac{\pi q}{2N}}{\pi T \tau_D} + \frac{ieV}{2\pi T} \right) - W \left( \frac{\sin^2 \frac{\pi q}{2N}}{\pi T \tau_D} + \frac{ieV}{2\pi T} \right) \right] \\ & - \frac{4Te}{N^4} \sum_{p,q=1}^{N-1} \frac{(1 - (-1)^{p+q}) \sin^2 \frac{\pi q}{N} \sin^2 \frac{\pi p}{N}}{\left( \cos \frac{\pi q}{N} - \cos \frac{\pi p}{N} \right)^3 \left( 2 \sin^2 \frac{\pi p}{2N} + \frac{R}{2\tau_D} (4C \sin^2 \frac{\pi p}{2N} + C_g) \left( \cos \frac{\pi q}{N} - \cos \frac{\pi p}{N} \right) \right)} \\ & \times \left[ W \left( \frac{2 \sin^2 \frac{\pi p}{2N}}{\pi T R (4C \sin^2 \frac{\pi p}{2N} + C_g)} + \frac{\sin^2 \frac{\pi p}{2N}}{\pi T \tau_D} + \frac{ieV}{2\pi T} \right) - W \left( \frac{\sin^2 \frac{\pi p}{2N}}{\pi T \tau_D} + \frac{ieV}{2\pi T} \right) \right]. \end{aligned}$$

$$W(x) = \text{Im}[x\Psi(1+x)]$$





# Diffusive wires



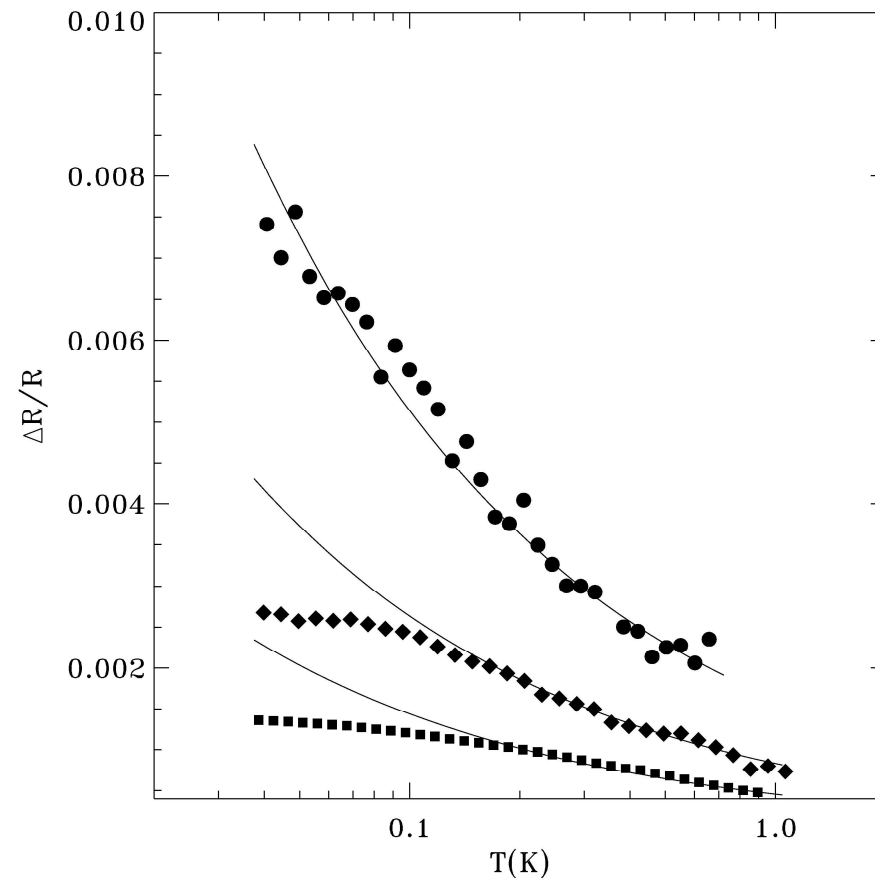
$L_T$

$L_T \sim (D/T)^{1/2}$

$$\delta G \sim \beta(L_T)/g(L_T) \sim T^{-1/2}$$

# Quasi-1D Disordered Wires

## Mohanty-Webb'98



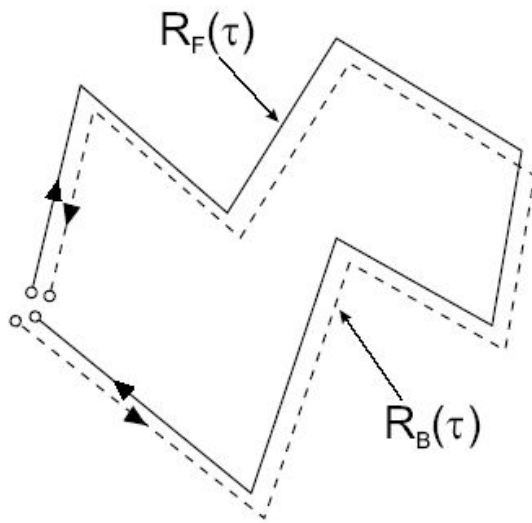
# Outline

- Introduction and motivation
- Short coherent conductors: Weak Coulomb blockade
- Short coherent conductors: interaction correction and shot noise
- Interactions and higher current cumulants
- Transport and interactions in quantum dots and arrays: universal model for ANY diffusive conductor
- **Weak localization and decoherence at  $T=0$**

# Interference (weak localization) correction to conductance:

$$\sigma_{\alpha\beta}(r, r') = -\frac{e^2}{4\pi m^2} \int_{-\infty}^t dt_0 \int dt' (\nabla_{r_1}^\alpha - \nabla_{r_2}^\alpha)_{r_1=r_2=r} (\nabla_{r'_1}^\beta - \nabla_{r'_2}^\beta)_{r'_1=r'_2=r'} \langle G_R(t, t'; r_1, r'_2) G_A(t_0, t; r'_1, r_2) \rangle_{\text{imp}}$$

$$G_R(t, t'; r_1, r'_2) \sim e^{iS_0[t, t'; R_F(\tau)]}, \quad G_A(t_0, t; r'_1, r_2) \sim e^{-iS_0[t, t_0; R_B(\tau)]}$$



time reversed paths

$$R_B(\tau) = R_F(t + t_0 - \tau), \quad t' = t_0 \Rightarrow \\ S_0[t, t'; R_F(\tau)] = S_0[t, t_0; R_B(\tau)]$$

weak localization correction

$$\delta\sigma_{\alpha\beta}^{\text{WL}}(r, r') = -\frac{2e^2 D(r)}{\pi} \delta_{\alpha\beta} \delta(r - r') \int_{-\infty}^t dt_0 C(t, t_0; r, r)$$

# Decoherence:

Electron-electron interaction  $\longrightarrow$  fluctuating field

$$G_R(t, t'; r_1, r'_2) \sim e^{iS_0[t, t'; R_F(\tau)]} e^{-i \int_{t_0}^t d\tau eV(\tau, R_F(\tau))}, \quad G_A(t_0, t; r'_1, r_2) \sim e^{-iS_0[t, t_0; R_B(\tau)]} e^{i \int_{t_0}^t d\tau eV(\tau, R_F(t+t_0-\tau))}$$

Hence

$$\delta\sigma_{\alpha\beta}^{\text{WL}}(r, r') \approx -\frac{2e^2 D(r)}{\pi} \delta_{\alpha\beta} \delta(r - r') \int_{-\infty}^t dt_0 C(t, t_0; r, r) e^{-F(t-t_0, r)}$$

where

$$e^{-F(t-t_0, r)} = \left\langle e^{-i \int_{t_0}^t d\tau [eV(\tau, R_F(\tau)) - eV(\tau, R_F(t+t_0-\tau))]} \right\rangle_{V, \text{ paths}} \approx e^{-(t-t_0)/\tau_\varphi}$$

## Saturation of electron dephasing time is observed in:

- Quantum dots (0d)
- Quasi-1d metallic wires
- Quasi-1d semiconductors
- Carbon nanotubes
- 2d metallic films
- 2DEGs
- Graphene
- Bulk metals (3d)
- ...

## Low-Temperature Saturation of the Dephasing Time and Effects of Microwave Radiation on Open Quantum Dots

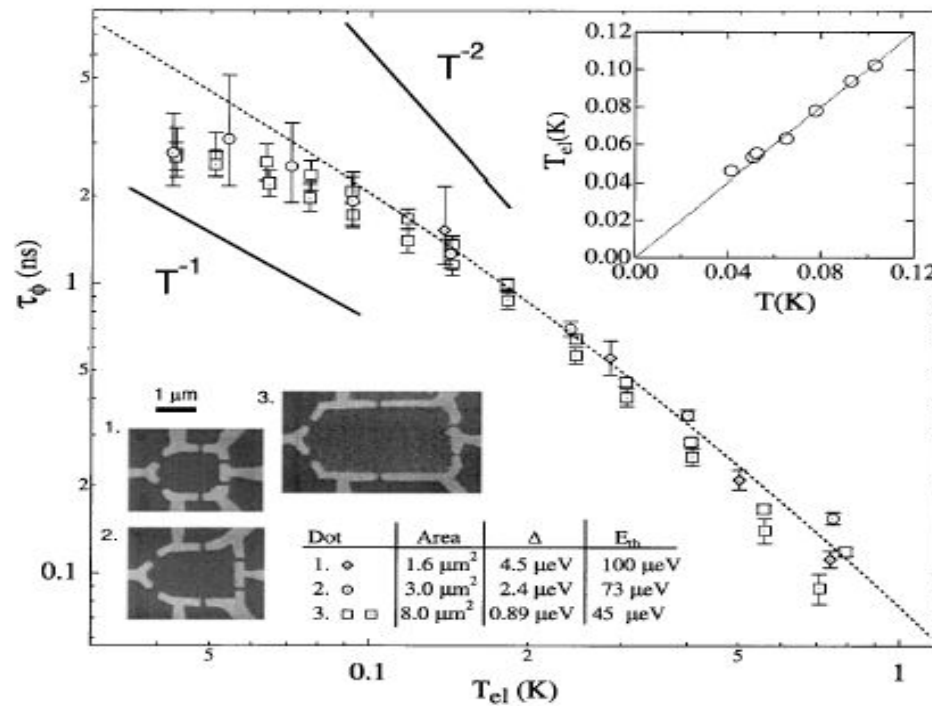
A. G. Huibers, J. A. Folk, S. R. Patel, and C. M. Marcus

*Department of Physics, Stanford University, Stanford, California 94305*

C. I. Duruöz and J. S. Harris, Jr.

*Department of Electrical Engineering, Stanford University, Stanford, California 94305*

(Received 20 April 1999)

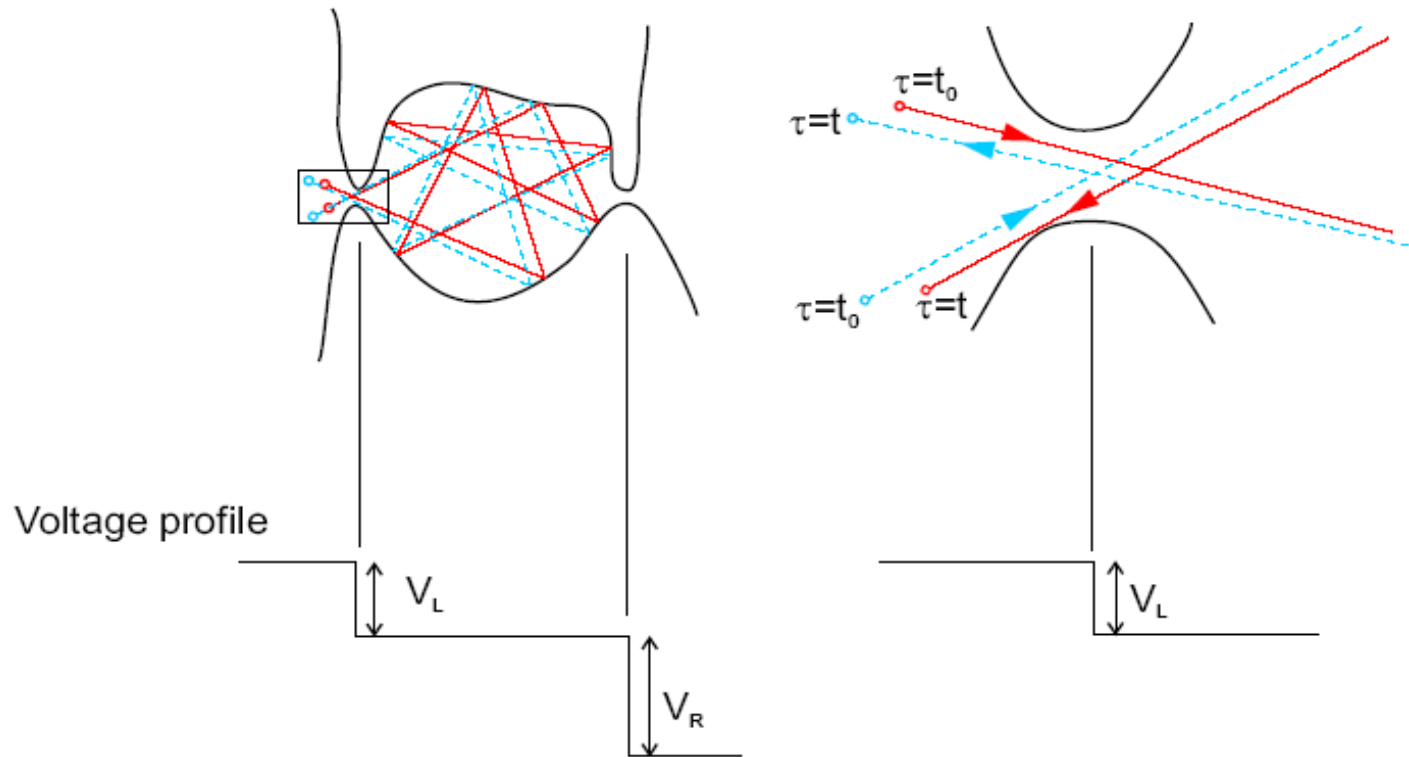


High T:  
power law

Low T:  
saturation



# Two scatterers: no dephasing at ANY T



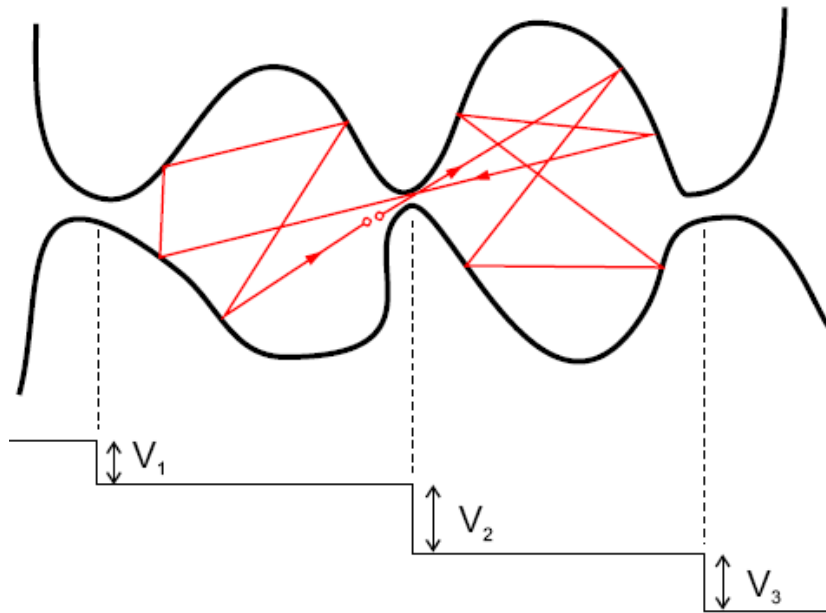
Since  $R_F(\tau)$  and  $R_B(\tau) = R_F(t + t_0 - \tau)$  cross the left barrier at the same time,

$$eV(\tau, R_F(\tau)) - eV(\tau, R_F(t + t_0 - \tau)) = 0$$

and

$$\left\langle e^{-i \int_{t_0}^t d\tau [eV(\tau, R_F(\tau)) - eV(\tau, R_F(t + t_0 - \tau))]} \right\rangle_{V, \text{ paths}} = 1 \Rightarrow \underline{\text{no dephasing}}$$

# Three barriers:



Forward path crosses the central barrier twice:  
at  $\tau = t_0$  and at  $\tau = s$ .

Then the backward, time reversed path, crosses it  
at  $\tau = t + t_0 - s$  and at  $\tau = t$

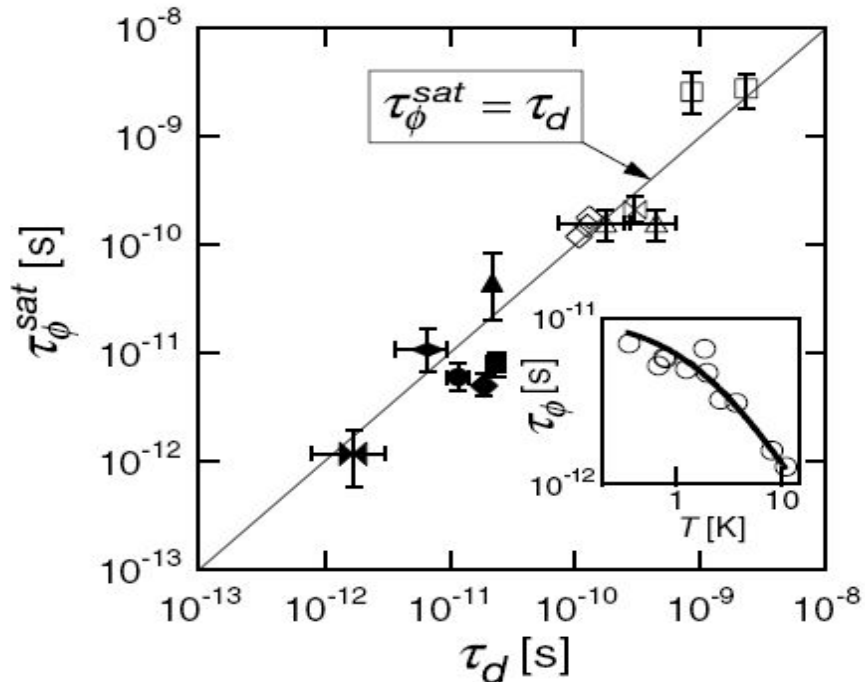
$$\varphi_2(\tau) = \int_{t_i}^{\tau} d\tau' eV_2(\tau')$$

$$G_R \propto e^{iS_0[R_F]} e^{-i\varphi_2(t_0) + i\varphi_2(s)}, \quad G_A \propto e^{-iS_0[R_B]} e^{-i\varphi_2(t+t_0-s) + i\varphi_2(t)}$$

$$\left\langle e^{-i\varphi_2(t_0) + i\varphi_2(s)} e^{-i\varphi_2(t+t_0-s) + i\varphi_2(t)} \right\rangle < 1 \Rightarrow \underline{\text{finite dephasing}}$$

# QUANTUM DOTS

Experiment:



Theory:

$$\tau_{\phi 0} = \frac{\tau_D}{(2E_C/\delta)^{4/g} - 1}$$

$$g \gg 1:$$

$$\tau_{\phi 0} \simeq \frac{g\tau_D}{4 \ln(2E_C/\delta)} = \frac{\pi}{\delta \ln(2E_C/\delta)}$$

- $1/\tau_{\phi} = 1/\tau_{\phi 0}^{\text{GZ}} + 1/\tau_{\phi}^{\text{AAK}} \quad (\text{T})$

- $\tau_{\phi 0}^{\text{GZ}} \sim D^3 / \ln D, \quad l < t, w$

- $\tau_{\phi}^{\text{AAK}} \sim D^{1/3} T^{-2/3} (tw)^{1/3}$

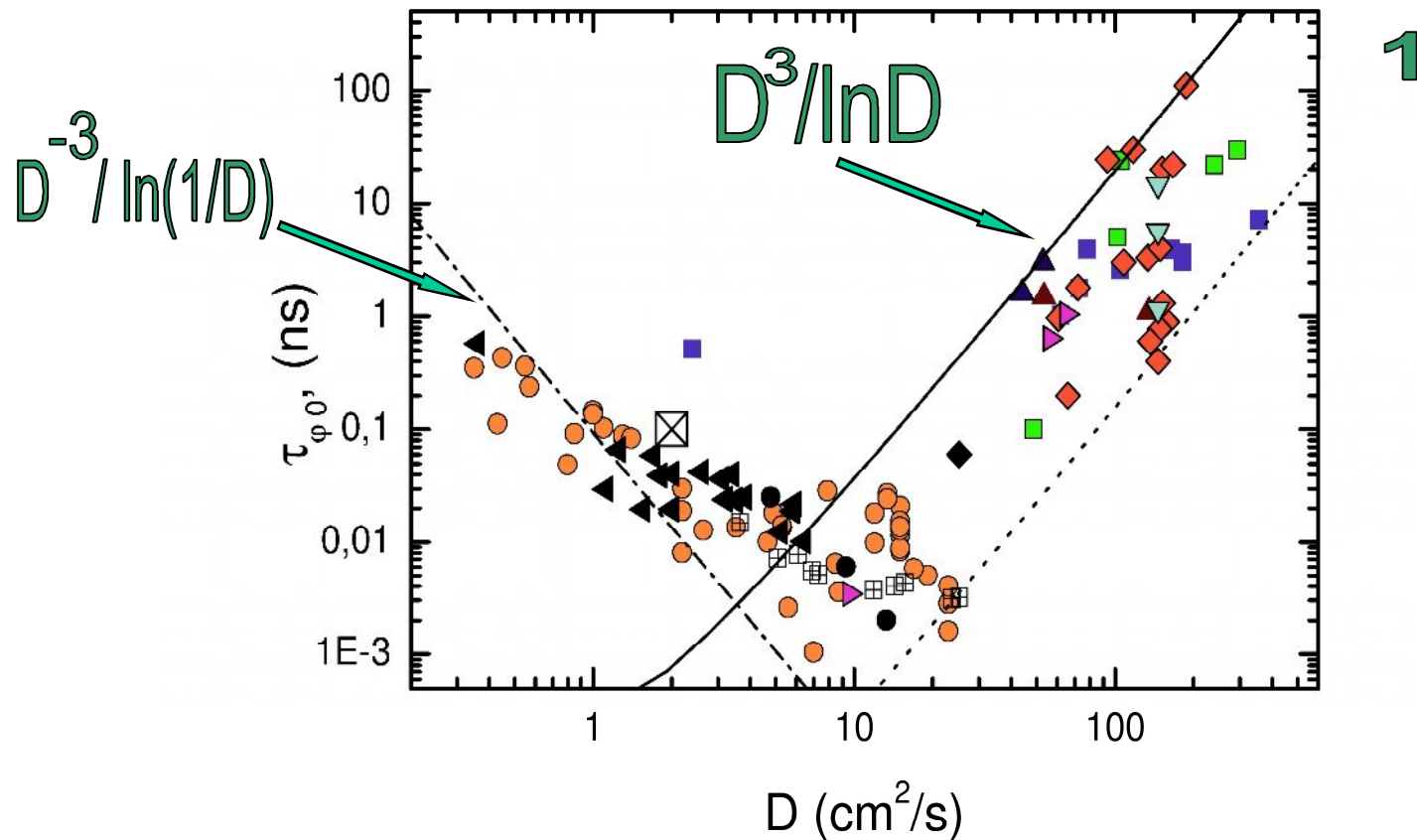
- $T_{\mathbf{q}} \sim (tw)^{1/2} D^{-4}$

# DIFFUSIVE CONDUCTORS

~ 130 samples

- |   |               |   |                  |
|---|---------------|---|------------------|
| ▲ | Gershenson'93 | ■ | Mohanty et al'97 |
| ■ | Grenoble      | ▲ | Mohanty Webb'03  |
| ● | Lin'07        | ● | Bird'03          |
| ◆ | Saclay-MSU    | ▼ | Birge'06         |
| ⊠ | Sahnoune'92   | ◀ | Lin'01           |
| ▶ | Natelson'05   | ⊞ | Lin'06           |
| ◆ | Alomare'05    |   |                  |

$$\gamma = 0.2$$



**Thank you!**

**The End**

$$\tilde{S}(t, t') = \tilde{S}^{\text{ni}}(t, t') + \delta\tilde{S}(t, t')$$

# Shot noise...

$$\delta\tilde{S}_\omega = -\frac{2(\beta - 2\gamma)|eV|}{R_q} \ln \frac{gE_C}{|eV|},$$

if  $T, |\omega| \ll |eV| \ll gE_C$



### Quantum Transport in a Multiwalled Carbon Nanotube

L. Langer, V. Bayot, E. Grivei, and J.-P. Issi

*Unité de Physico-Chimie et de Physique des Matériaux, Université Catholique de Louvain, Place Croix du Sud 1, B-1348 Louvain-la-Neuve, Belgium*

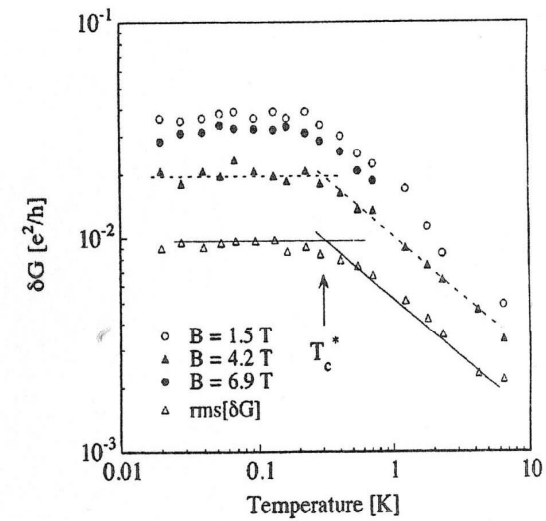
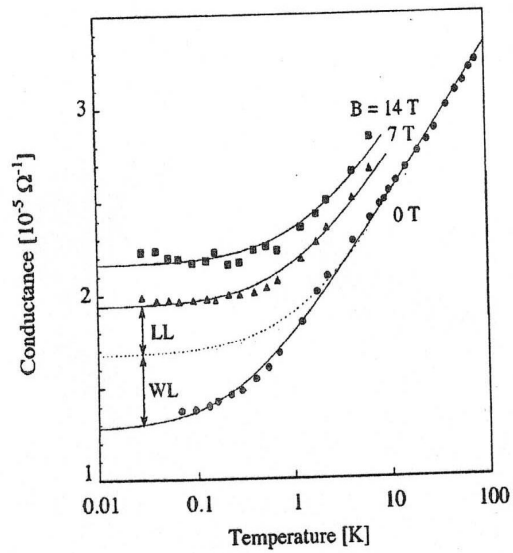
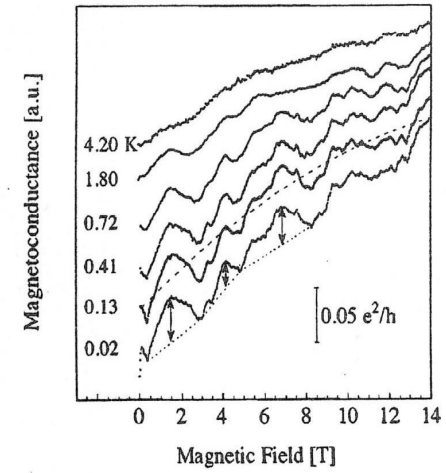
J. P. Heremans and C. H. Olk

*Physics Department, General Motors Research, Warren, Michigan 48090*

L. Stockman,\* C. Van Haesendonck, and Y. Bruynseraede

*Laboratorium voor Vaste-Stoffysika en Magnetisme, Katholieke Universiteit Leuven, Celestijnenlaan 200 D, B-3001 Leuven, Belgium*

(Received 2 August 1995)

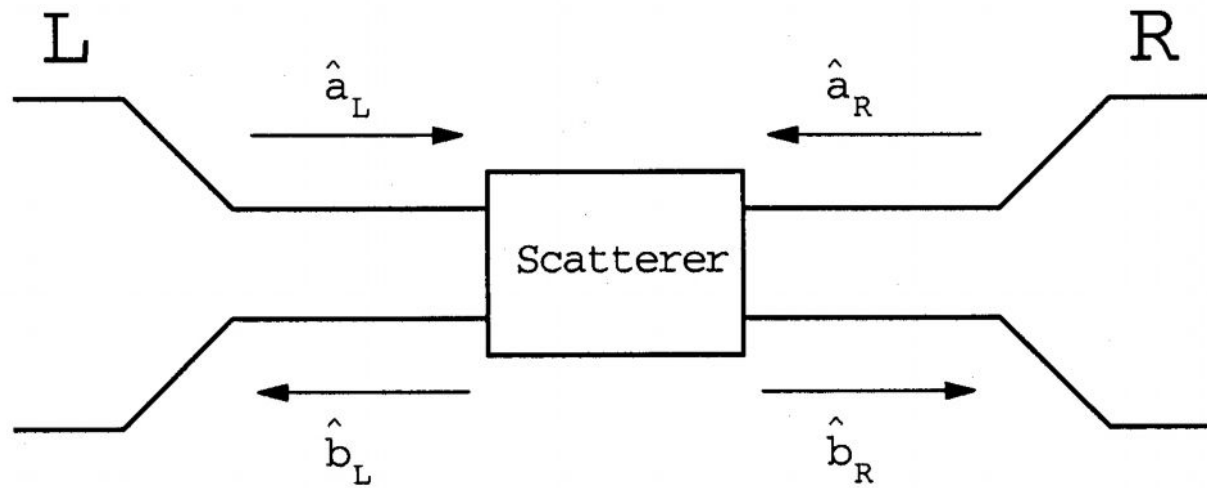


- $1/\tau_{\phi} = 1/\tau_{\phi 0}^{\text{GZ}} + 1/\tau_{\phi}^{\text{AAK}} \text{ (T)}$

- $1/\tau_{\phi} = 1/\tau_{\phi 0}^{\text{GZ}} + 1/\tau_{\phi}^{\text{AAK}} (T)$

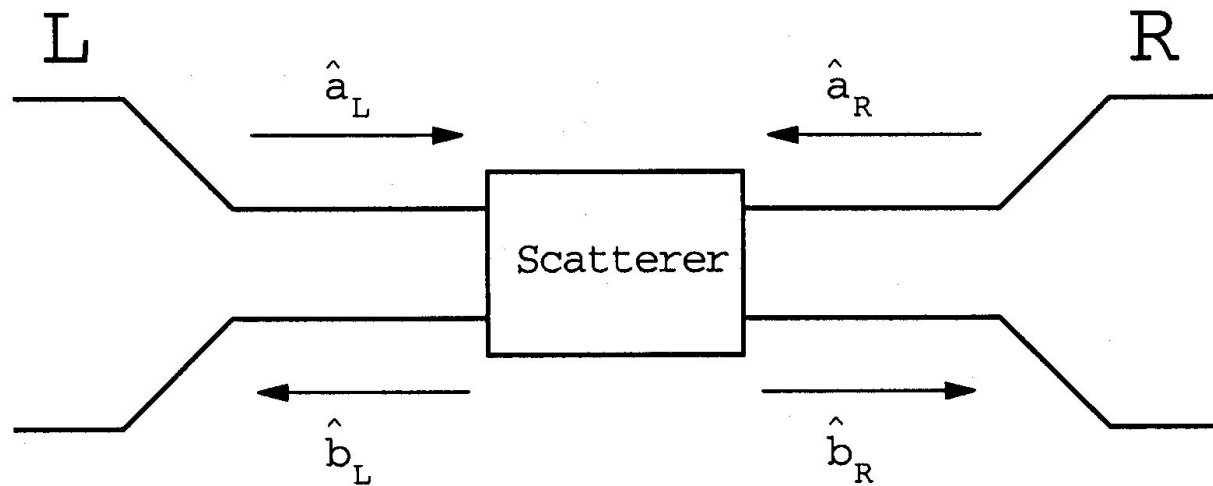
- $\tau_{\phi 0}^{\text{GZ}} \sim D^3, \quad l < t, w$

- $\tau_{\phi}^{\text{AAK}} \sim D^{1/3} T^{-2/3} (tw)^{1/3}$



$$-\frac{\nabla^2}{2m}\psi(\mathbf{r}) + W(\mathbf{r})\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

$$\psi(\mathbf{r}) = \sum_n c_n \Phi_n(\mathbf{r}_\perp) \chi_n(x).$$



$$-\frac{1}{2m} \frac{d^2}{dx^2} \chi_n(x) = (E - E_n) \chi_n(x).$$

$$-\frac{\nabla_{\perp}^2}{2m} \Phi_n(\mathbf{r}_{\perp}) + W(\mathbf{r}_{\perp}) \Phi_n(\mathbf{r}_{\perp}) = E_n \Phi_n(\mathbf{r}_{\perp}),$$

## Scattering matrix:

$$\begin{pmatrix} b_{L1} \\ \dots \\ b_{LN_L} \\ b_{R1} \\ \dots \\ b_{RN_R} \end{pmatrix} = \hat{S}(\xi) \begin{pmatrix} a_{L1} \\ \dots \\ a_{LN_L} \\ a_{R1} \\ \dots \\ a_{RN_R} \end{pmatrix}$$

$$(N_L + N_R) \times (N_L + N_R)$$

$$\hat{S}(\xi) = \begin{pmatrix} \hat{r}(\xi) & \hat{t}'(\xi) \\ \hat{t}(\xi) & \hat{r}'(\xi) \end{pmatrix}$$



$$G(T) = G_{\text{Drude}} - \delta G_{\text{int}} - \delta G_{\text{WL}}$$

- $\frac{\delta G_{\text{int}}}{G_{\text{Drude}}} \sim \frac{L_{\text{T}}}{L_{\text{loc}}}, \quad L_{\text{T}} \sim \sqrt{\frac{D}{T}}, \quad L_{\text{loc}} \sim Nl$