

Equilibrium models in practical applications

Eugene Vasiliev

Institute of Astronomy, Cambridge

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Outline

- ▶ Galactic globular clusters
- ▶ Nuclear star cluster
- ▶ Bar/bulge
- ▶ Disc, Solar neighbourhood
- ▶ Stellar and dark matter halo
- ▶ Overall structure and mass distribution
- ▶ Nearby galaxies

Characteristic scales:	length	velocity	time	mass
globular clusters	few pc	few km/s	10^6 yr	$10^5 M_{\odot}$
Galactic disc	few kpc	tens km/s	10^8 yr	$10^9 M_{\odot}$
Galactic dark halo	tens kpc	100 km/s	10^9 yr	$10^{12} M_{\odot}$
dwarf galaxies	~ 1 kpc	3 – 15 km/s	10^8 yr	$10^{7\pm 1} M_{\odot}$

The Galaxy in Context: Structural, Kinematic, and Integrated Properties

Joss Bland-Hawthorn¹ and Ortwin Gerhard²

¹Sydney Institute for Astronomy, School of Physics, University of Sydney, New South Wales 2006, Australia; email: jbh@physics.usyd.edu.au

²Max Planck Institute for Extraterrestrial Physics, 85741 Garching, Germany; email: gerhard@mpe.mpg.de

Annu. Rev. Astron. Astrophys., 2016, 54, 529

Globular clusters

The simplest ever (but still useful) model: Plummer sphere

$$\Phi(r) = -\frac{GM}{\sqrt{r^2 + a^2}},$$

$$\rho(r) = \frac{3M}{4\pi a^3 [1 + (r/a)^2]^{5/2}},$$

$$f(E) = \frac{3a^2 (-2E)^{7/2}}{7\pi^3 G^5 M^4}$$

Only two free parameters (mass M and scale radius a) –
not enough to describe variations in the internal structure!



NGC 104 (47 Tuc)



NGC 5904 (M 5)



NGC 7078 (M 15)



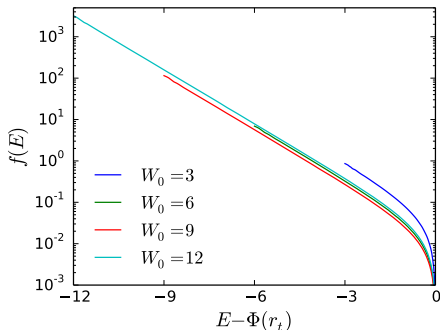
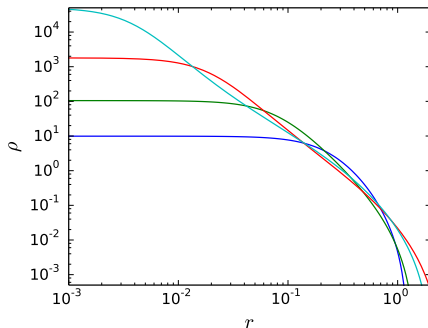
NGC 7099 (M 30)

Globular clusters

The next level of complexity: King (lowered isothermal) models

$$f(E) = A \exp\left(-\frac{E - \Phi(r_t)}{\sigma^2}\right)$$

Density ρ and potential Φ are found numerically by solving a 1d ODE. Density has a central core of radius r_c and drops to zero at the “tidal radius” r_t ; the degree of central concentration is set by the parameters $C \equiv \log_{10}(r_t/r_c)$ or $W_0 \equiv [\Phi(r_t) - \Phi(0)]/\sigma^2$.



Globular clusters

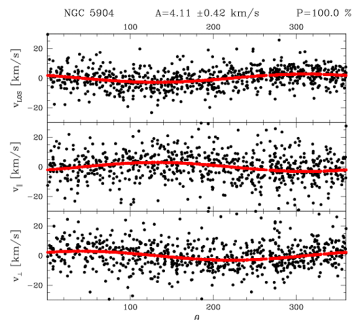
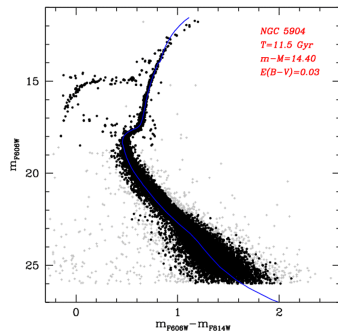
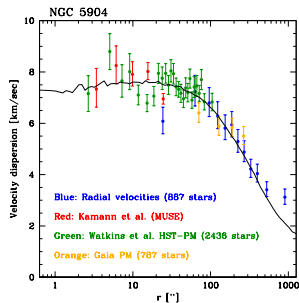
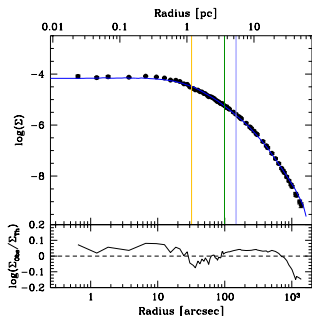
Further complexities:

- ▶ more flexible tidal cutoff [Wooley 1954; Wilson 1975; Gomez-Leyton&Velazquez 2014]
- ▶ velocity anisotropy [Michie 1963; Gieles&Zocchi 2015]
- ▶ rotation and flattening [Bertin&Varri 2008; Bianchini+ 2013]
- ▶ stellar mass spectrum, mass segregation [many studies]
- ▶ potential escapers and extratidal stars [Claydon+ 2019]
- ▶ central IMBH [Lützgendorf+ 2011+; Lanzoni+ 2013; Baumgardt 2017; Zocchi+ 2018]
- ▶ stellar binaries
- ▶ multiple populations
- ▶ different evolutionary states (N -body or Monte Carlo models)
- ▶ ...

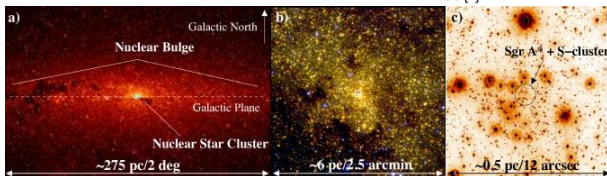
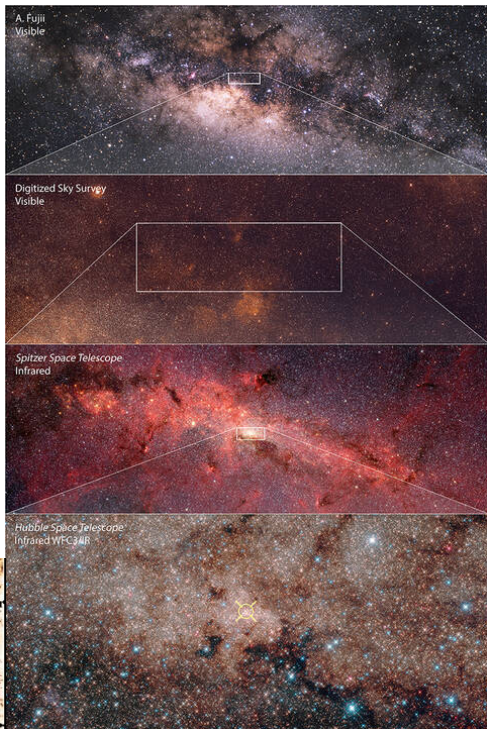
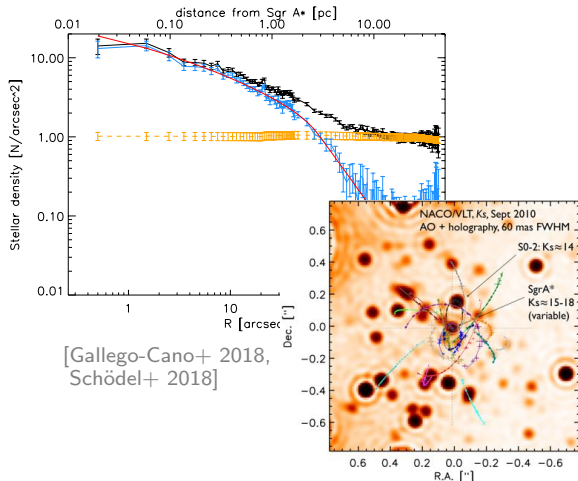
Globular clusters

Example of N -body models fitted to observations for a globular cluster NGC 5904 [Baumgardt+ 2019].

Data include: surface brightness profile, line-of-sight velocities and proper motions, CMD, stellar mass function, ...

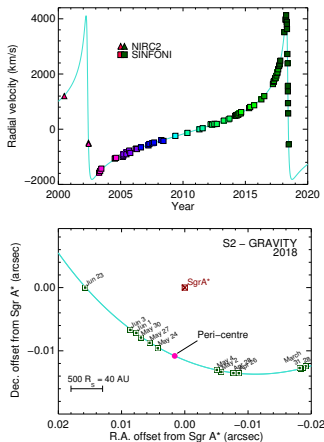
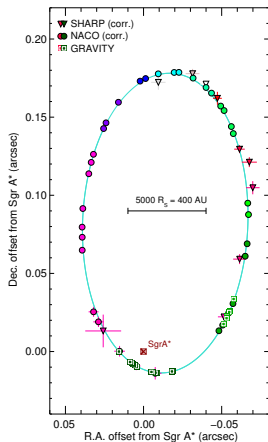


Nuclear star cluster

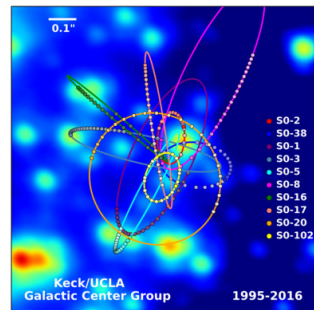


Nuclear star cluster

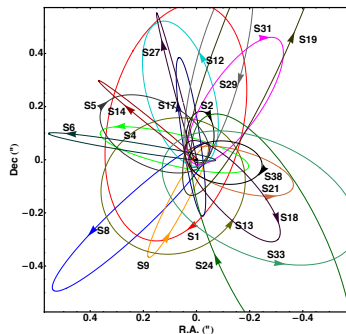
Individual stellar orbits around the central SMBH Sgr A* are traced over $\gtrsim 20$ years, with some stars completing more than orbit. The mass of the SMBH is $\sim 4.15 \times 10^6 M_{\odot}$ measured with $\lesssim 0.5\%$ precision.



[GRAVITY collaboration 2018]

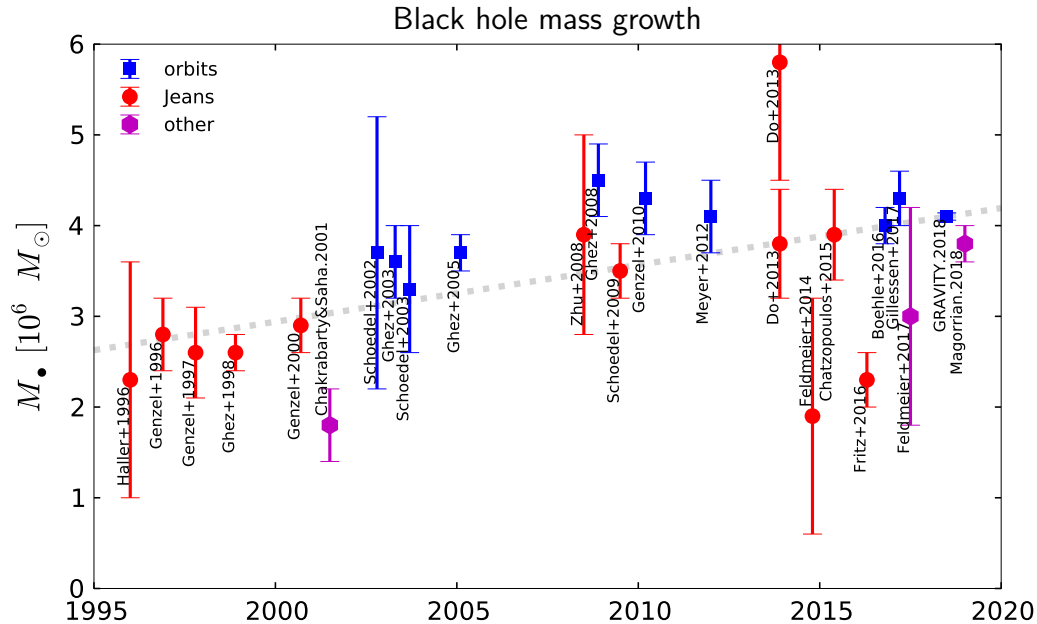


[Ghez+ 2003+]



[ESO/VLT: Gillessen+ 2009]

Nuclear star cluster



Nuclear star cluster

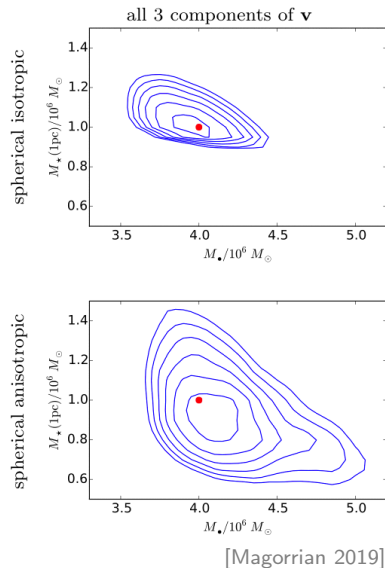
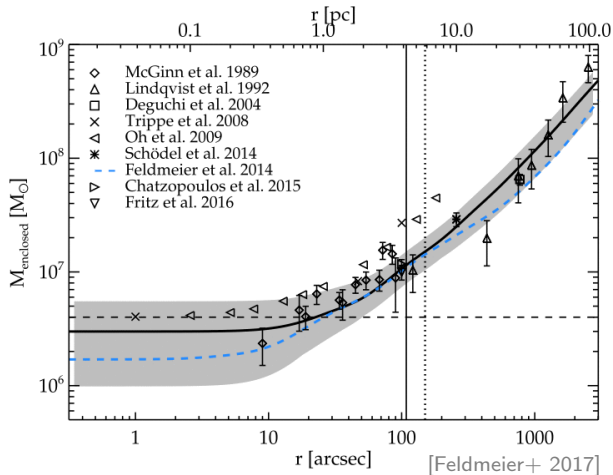
Summary of recent stellar-dynamical models

Reference	data	method	$M_{\bullet}/10^6 M_{\odot}$
Schödel, Merritt & Eckart 2009	6000 PM $R < 0.8$ pc	sph.isotr.Jeans sph.aniso.Jeans	$3.6^{+0.2}_{-0.4}$ $3.5^{+0.15}_{-0.35}$
Do+ 2013	PM (Yelda+2013) 265 v_{los} (Keck/OSIRIS)	sph.iso.Jeans sph.aniso.Jeans	$3.77^{+0.62}_{-0.52}$ $5.76^{+1.76}_{-1.26}$
Feldmeier+ 2014	\bar{v}_{los} , σ_{los} integrated light (ISAAC) $R < 4$ pc	axi.aniso.Jeans	$1.7^{+1.4}_{-1.1}$
Fritz+ 2016	10000 PM $R < 1.4$ pc 2500 v_{los} (VLT/SINFONI)	sph.isotr.Jeans same+ $M/L=\text{const}$	2.26 ± 0.26 4.35 ± 0.12
Chatzopoulos+ 2015	same data	axi.isotr.Jeans	3.9 ± 0.5
Feldmeier+ 2017	LOSVD (v , σ , h_3 , h_4) from F+14	triax.Schwarzschild	$3.0^{+1.1}_{-1.3}$
Magorrian 2018	PM+ v_{los} from F+16 PM from S+09	sph.Schwarzschild	3.76 ± 0.22

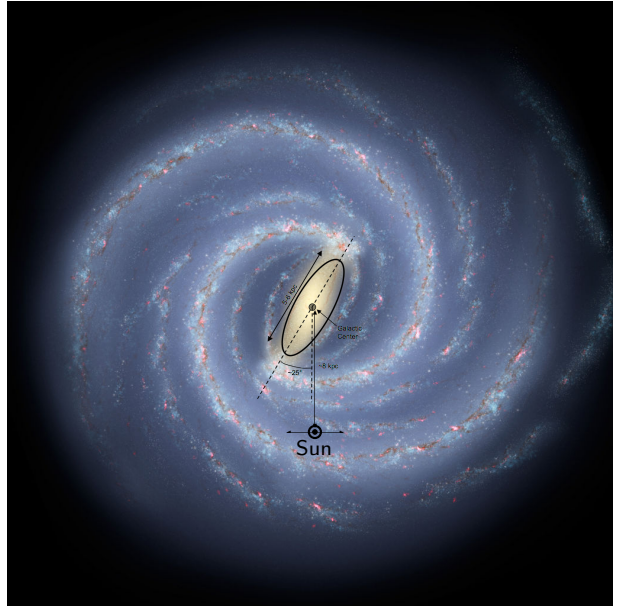
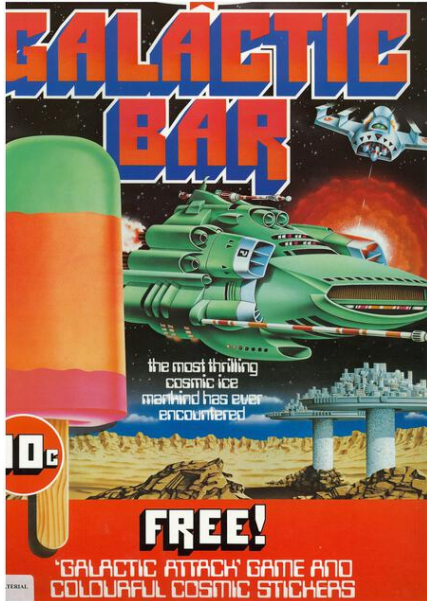
Nuclear star cluster

Ingredients: star counts, diffuse light profile,
individual V_{los} and proper motion measurements

Outputs: M_{SMBH} , stellar M/L , enclosed mass
profile, velocity anisotropy, ...

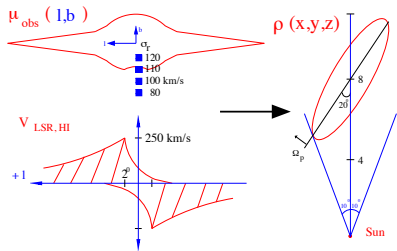


Galactic bulge and bar



[credit: R.Hurt / NASA]

Galactic bulge and bar



side-on view

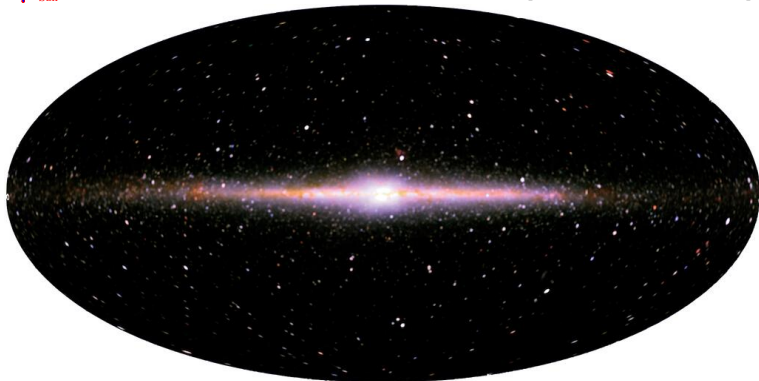
nearly end-on view

[credit: E.Athanassoula]

[from Zhao 1998]

First models of the Galactic bar using the Schwarzschild method:

Zhao 1996; Häfner+ 2000,
or N -body sims: FuX 1997

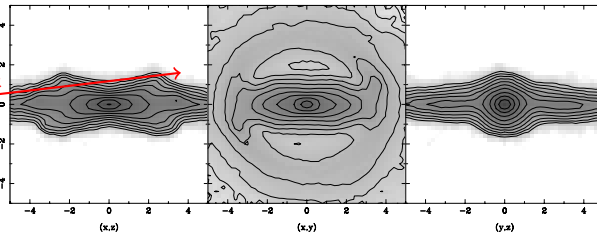


[COBE satellite / NASA]

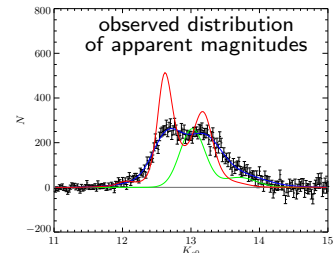
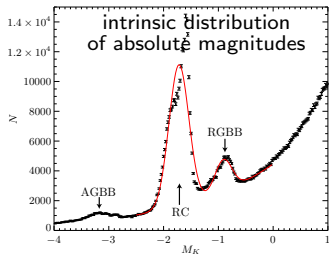
Galactic bulge and bar

X-shape and split red clump

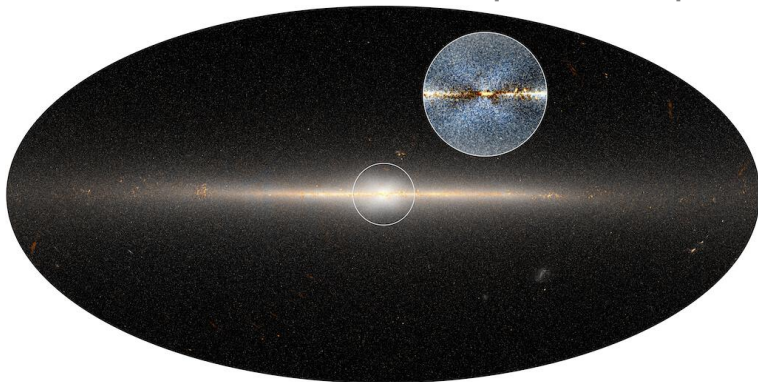
line of sight



[Athanassoula 2007]



[Wegg&Gerhard 2013]



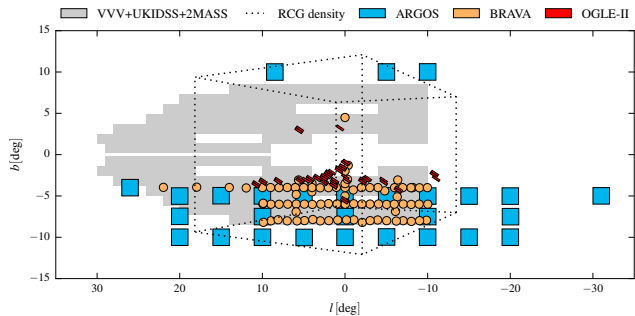
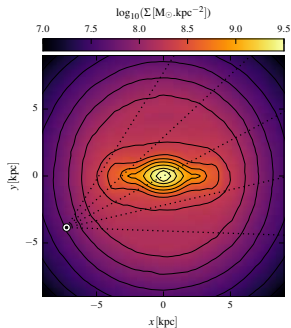
[WISE satellite / Ness&Lang 2016]

Galactic bulge and bar

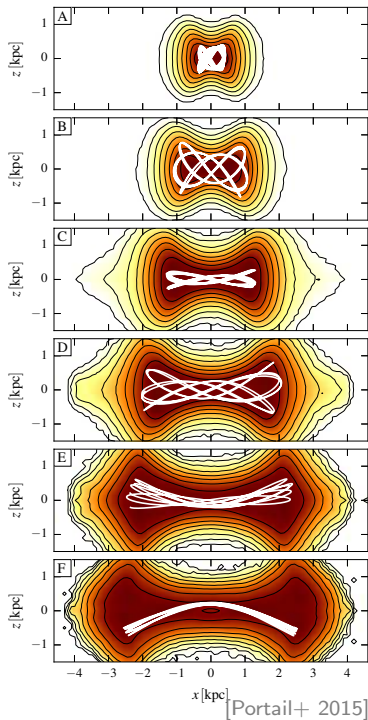
Made-to-measure models of the inner Galaxy by Ortwin Gerhard's group at MPE:

Inputs: 3d tomography of RC stars; V_{los} from various surveys; microlensing depth

Outputs: orbital structure, 3d morphology, mass, pattern speed, predictions of proper motions



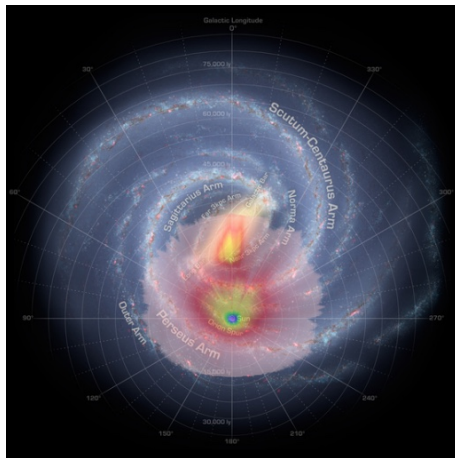
[Portail+ 2017]



[Portail+ 2015]

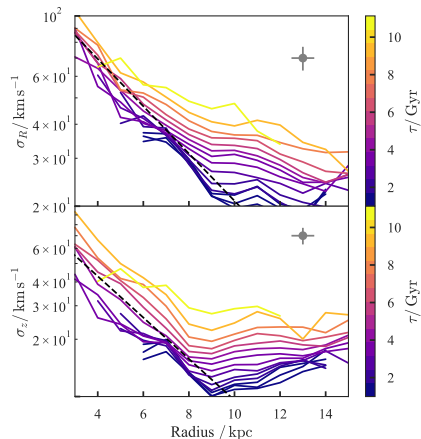
Galactic disc

Solar neighbourhood (within a few kpc) is where we have the most detailed information about stellar distribution in the 6d phase-space, chemistry, ages, etc.



Gaia 6d sample [credit: ESA]

Stellar velocities depend on the age and other properties, which are rarely explored in models



[Sanders&Das 2018] – GaiaDR2

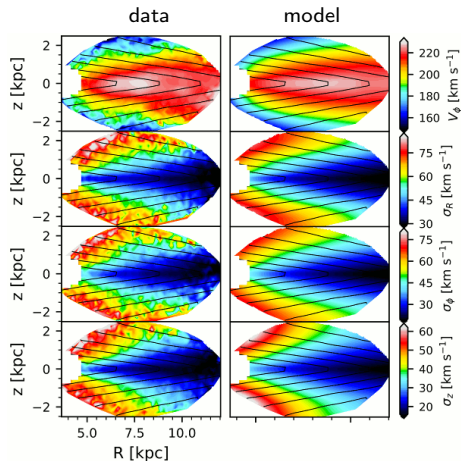
Galactic disc

Data: velocity dispersions in three directions

across the disc; parametrized density profile

Method: Jeans Anisotropic model (JAM)

Outputs: potential, dark halo profile and shape



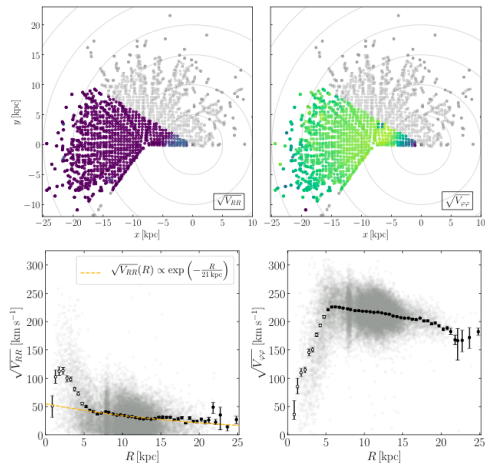
[Nitschai+ 2020]

Data: velocity dispersions in the disc

plane; parametrized density profile

Method: 1d Jeans equation for $\Phi(R)$

Outputs: circular-velocity curve



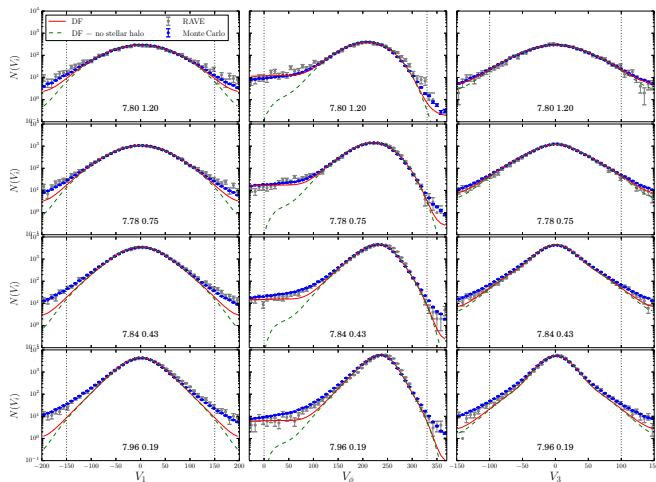
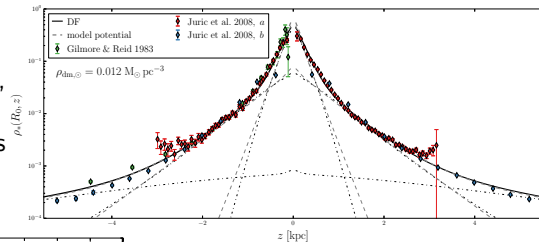
[Eilers+ 2019]

Galactic disc

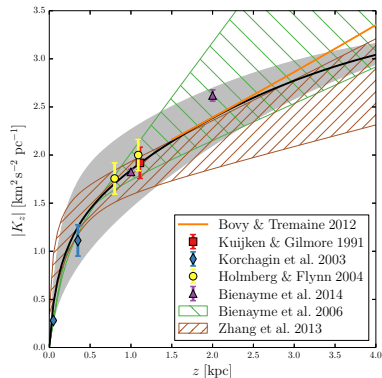
Data: velocity distributions at various locations,
vertical density profile

Method: action-based self-consistent DF models

Outputs: potential, local dark matter density



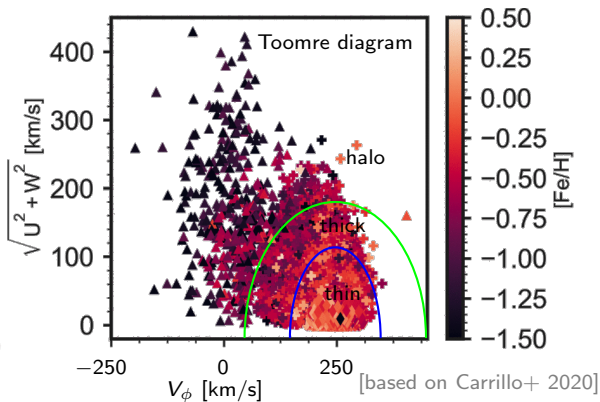
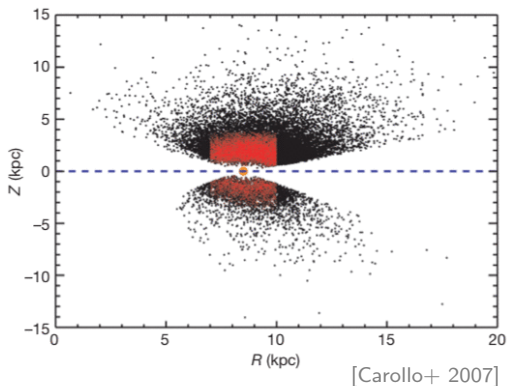
[Binney+ 2014; Piffl+ 2014]



Galactic stellar halo

How to select stars belonging to the halo (as opposed to disc):

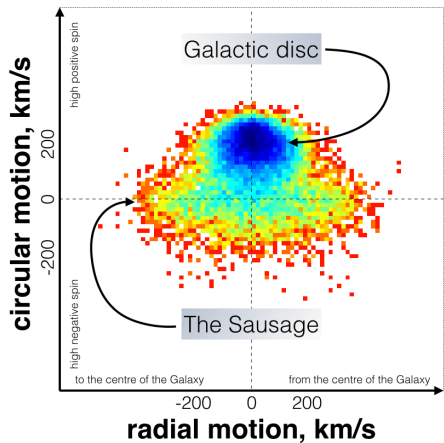
- ▶ Spatial selection: high above/below disc plane (\gtrsim a few kpc) or at large distances (\gtrsim 20 kpc from the Galactic centre)
- ▶ Kinematics (local): select stars with high velocity relative to the local standard of rest (LSR) – only $\sim 1\%$ of stars in the Solar neighbourhood
- ▶ Kinematics (orbit): select stars with high eccentricity and inclination
- ▶ Chemistry: select only relatively metal-poor stars (e.g., $[\text{Fe}/\text{H}] < -1$)



Galactic stellar halo

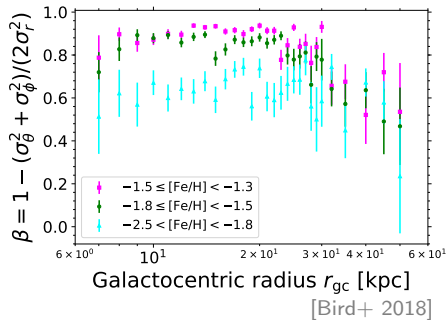
Recent discovery of a strong radial bias in velocity dispersion of relatively metal-rich halo stars – likely a remnant of an ancient merger (8 – 10 Gyr ago).

Motions of 7,000,000 Gaia stars

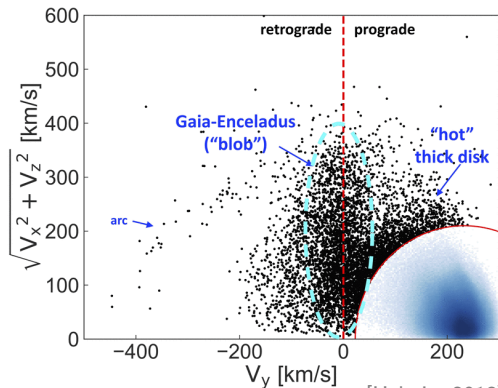


[Belokurov+ 2018]

turn 90°



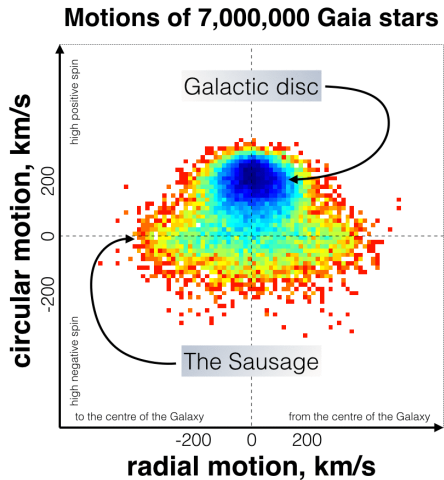
[Bird+ 2018]



[Helmi+ 2018]

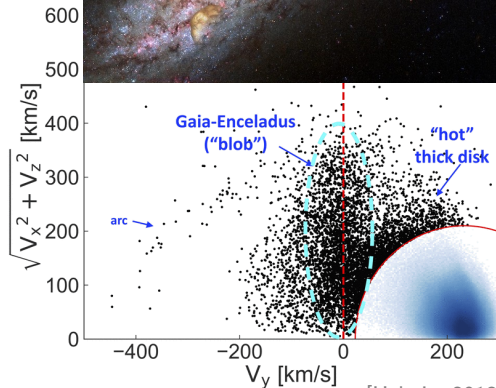
Galactic stellar halo

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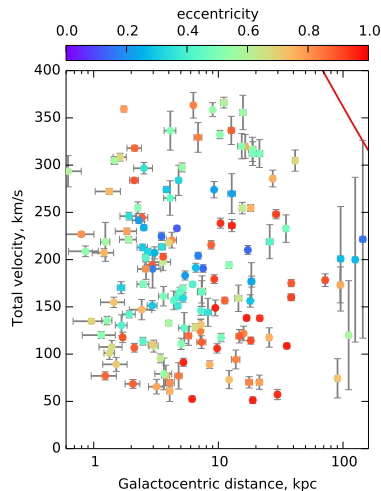


Other kinematic tracers in the outer Galaxy

objects with 6d phase-space information after Gaia DR2

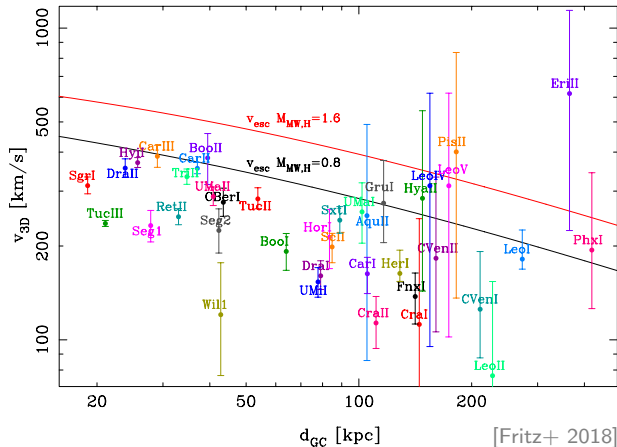
~ 150 globular clusters

[Helmi+ 2018; Vasiliev 2019; Baumgardt+ 2019]



a few dozen satellite galaxies

[Simon 2018; Pace&Li 2018; Massari&Helmi 2018; Fritz+ 2018; McConnachie&Venn 2020]

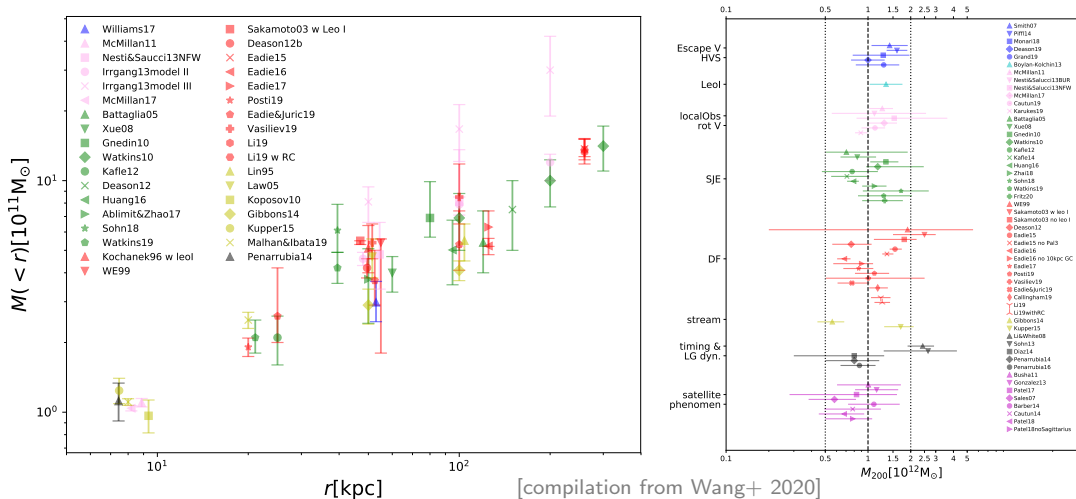


Mass modelling of the Milky Way dark halo

Data: stellar velocities in the outer halo (3d position, 1d velocity – v_{los} only); star clusters and satellites (6d phase space); local high-velocity stars; stellar streams

Methods: DF-based models; Jeans eqn; escape speed; stream fitting; N -body sims

Outputs: enclosed mass profile / circular-velocity curve; halo shape

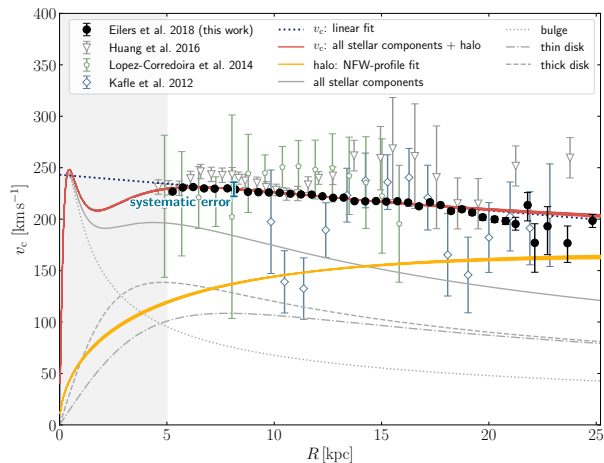


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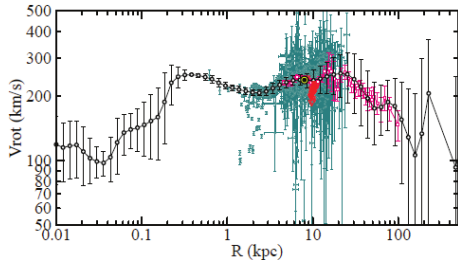
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[Eilers+ 2019] – Gaia DR2

$$V_{\text{circ}}(R) \equiv \sqrt{R \frac{\partial \Phi}{\partial R}} \approx \sqrt{\frac{G M(< R)}{R}}$$



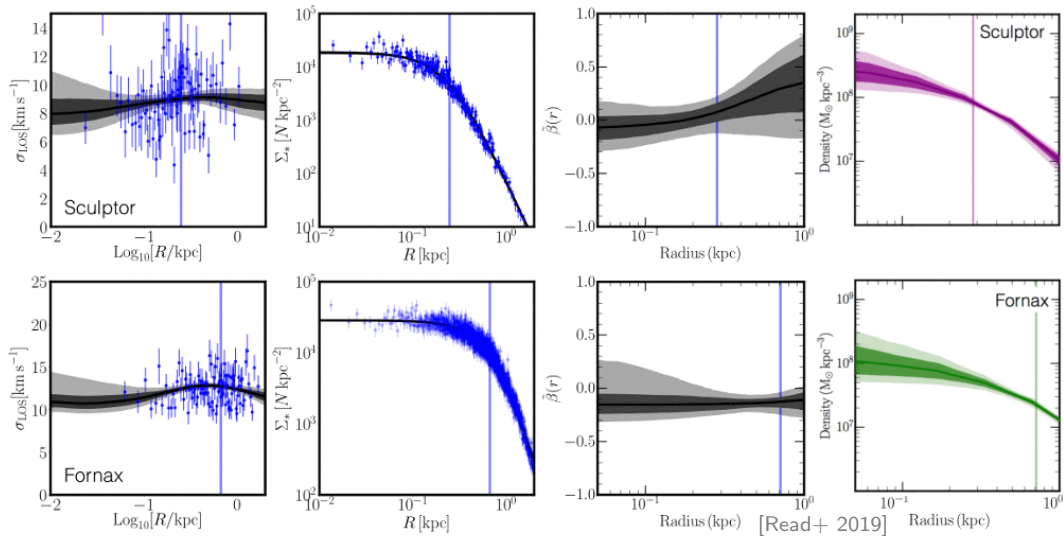
[Sofue 2020] – compilation

Dwarf galaxies – Milky Way satellites

Data: surface density, v_{los} for $\mathcal{O}(10^2 - 10^3)$ stars; sometimes – metallicity estimates

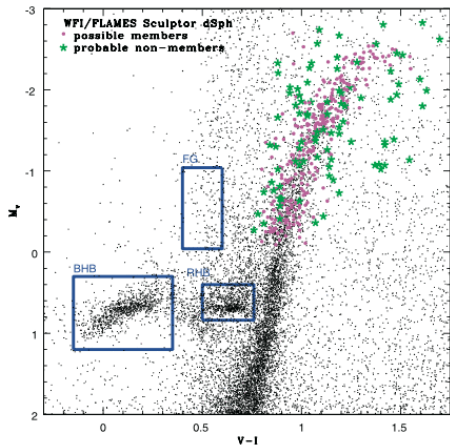
Methods: spherical and axisymmetric Jeans, Schwarzschild and DF-based models of all sorts

Outputs: dark matter profiles – core vs. cusp?

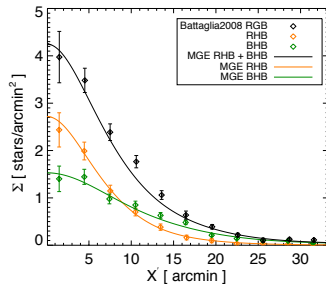


Dwarf galaxies – Milky Way satellites

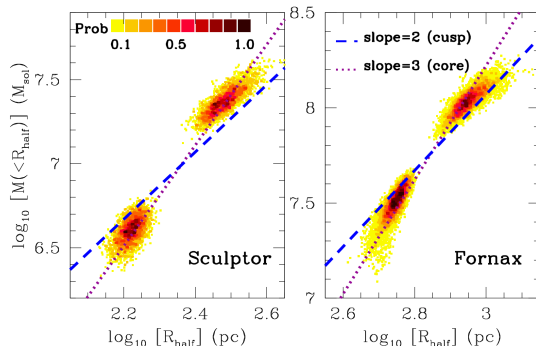
Two-population modelling: chemically distinct stellar populations with different kinematics, living in the same potential
 \Rightarrow probe the dark matter profile at different spatial scales



[Tolstoy+ 2004]

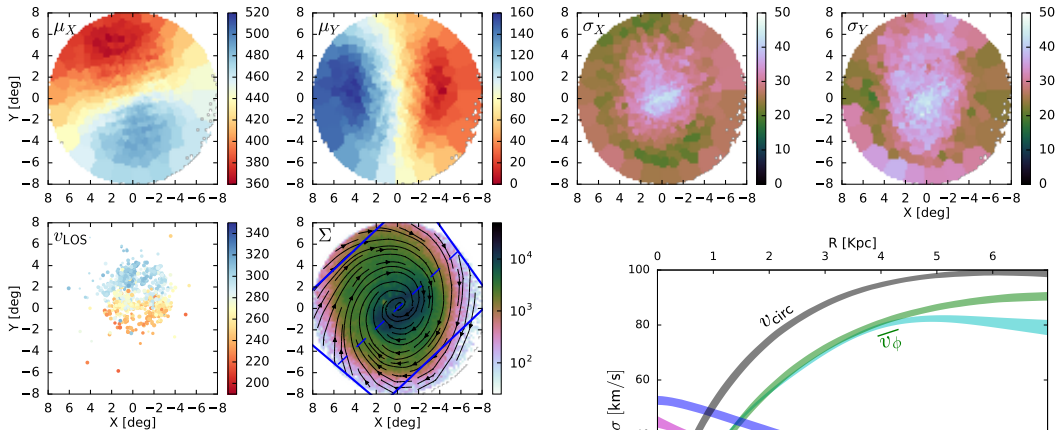


[Zhu+ 2016]

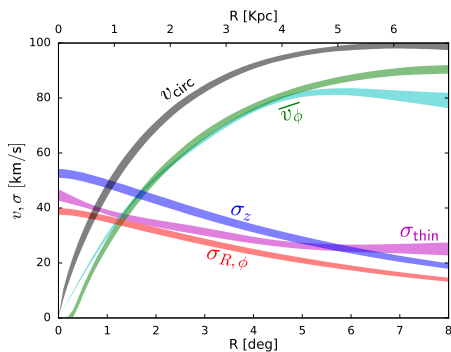


[Walker&Peñarrubia 2011]

Dwarf galaxies – Milky Way satellites



With Gaia DR2, it became possible to measure the internal 3d kinematics of the most massive and closest satellites – LMC, SMC and Sgr dSph (and in combination with HST, even for distant galaxies – Sculptor, Draco [Massari+ 2018, 2019])



Jeans model of the LMC [Vasiliev 2018]