# Chaotic mixing and the secular evolution of triaxial cuspy galaxy models built with Schwarzschild's method

E. Vasiliev (Lebedev Physical Institute, Moscow, Russia & Rochester Institute of Technology, Rochester, NY, USA)
E. Athanassoula (Laboratoire d'Astrophysique de Marseille, Marseille, France)

We explore the stability and the long-term secular evolution of non-rotating, triaxial spheroidal galactic models, using both $N$-body simulations and integration in fixed potentials. More specifically we consider Dehnen models with inner density cusp $\rho \propto r^{-\gamma}$, built with the Schwarzschild method. We show that short-term stability depends on the degree of velocity anisotropy (radially anisotropic models are subject to rapid development of radial-orbit instability). Long-term stability, on the other hand, depends mainly on the properties of the potential, and in particular, on whether it admits a substantial fraction of strongly chaotic orbits. We show that in the case of a weak density cusp ($\gamma = 1$ Dehnen model) the $N$-body model is remarkably stable, while the strong-cusp ($\gamma = 2$) model exhibits substantial evolution of shape away from triaxiality, which we attribute to the effect of chaotic diffusion of orbits. The different behaviour in these two cases originates from the different phase space structure of the potential; in the weak-cusp case there exist numerous resonant orbit families that impede the chaotic duffusion. Moreover, we found that varying the fraction of chaotic orbits in Schwarzschild model has no effect on the shape evolution of $N$-body model, which we attribute to impossibility to preserve the chaotic properties of an orbit in the $N$-body simulation.

## Introduction

We study the evolution of triaxial spheroidal galaxies represented by Dehnen density profile

$$\rho(r) = \frac{(3-\gamma)M}{4\pi abc}\frac{1}{m^\gamma(1+m)^{4-\gamma}},$$

where $m = [(x/a)^2+(y/b)^2+(z/c)^2]^{1/2}$ is the elliptic radius, for two cases: $\gamma = 1$ (weak-cusp model) and $\gamma = 2$ (strong-cusp), with $b/a = 0.79$ and $c/a = 0.5$. The models were created with Schwarzschild method implemented in $SMILE$ software [1]. We explore the stability of these models with $N$-body simulations, using $N \geq 10^6$ particles evolved with gyrfalcON code [2], on a timescale corresponding to Hubble time. We find that on a short timescale the models develop radial-orbit instability if the central velocity anisotropy coefficient $\beta = 1-\frac{\sigma_t^2}{2\sigma_r^2}$ exceeds $0.3-0.4$, in agreement with [3]. In the opposite case the models appear to be quite stable for many dynamical times. However, the strong-cusp model exhibits substantial evolution of shape towards more axisymmetric, see Fig. 1.

## Resonances and chaotic diffusion

The triaxial Dehnen models contain a large fraction ($\sim 30-40\%$) of chaotic orbits: it is impossible to create a self-consistent model using only the regular orbits. There is gradual transition from regular through weakly chaotic to strongly chaotic orbits (example is given in Fig. 2). One of chaos indicators is frequency diffusion rate $\delta\omega$, measuring the difference in leading frequencies of motion between the first and the second halves of integration time. As seen from Fig. 3 (right panel), there is a smooth distribution of orbits by $\delta\omega$, but for the strong-cusp case there are more strongly chaotic orbits [4]. More importantly, in the weak-cusp case there exist numerous non-tube resonant orbit families, for which the leading frequencies of oscillation in three coordinates satisfy one or more resonant relation $n_1\omega_x+n_2\omega_y+n_3\omega_z = 0$, with integer $n_i$. They are visible as concentration of points along lines on the frequency map plot (Fig. 3, left); by contrast, the frequency map for strong-cusp model (Fig. 3, middle) does not have significant resonances, apart from $1:2$ $x-z$ banana orbit.

The resonant orbits are important because they both represent "building blocks" with various geometry, and inhibit chaotic diffusion in phase space [5]. While chaotic orbits exist in both weak- and strong-cusp models, in the latter case they become on average rounder in the course of evolution, more uniformly filling the equipotential surface; while in the former case their diffusion is mostly limited to the vicinity of resonant orbit families, which have more well-defined nontrivial shape.

We also found that the variation of the chaotic orbit fraction in the Schwarzschild model does not change the rate of shape evolution of $N$-body model. This is explained by the impossibility to preserve chaotic properties of a given orbit in the $N$-body simulation, even if the orbit shape is well preserved. The evolution is therefore governed by gross features of the potential, not by a particular arrangement of orbits. Another demonstration of this is that models built with Schwarzschild's method evolve similarly to those created with iterative method for constructing dynamical equilibrium models [6]. It is also important to note that a similar amount of evolution is observed in fixed-potential integration, when particles are evolved in a static potential and do not interact with each other (Fig. 1, red).

## References

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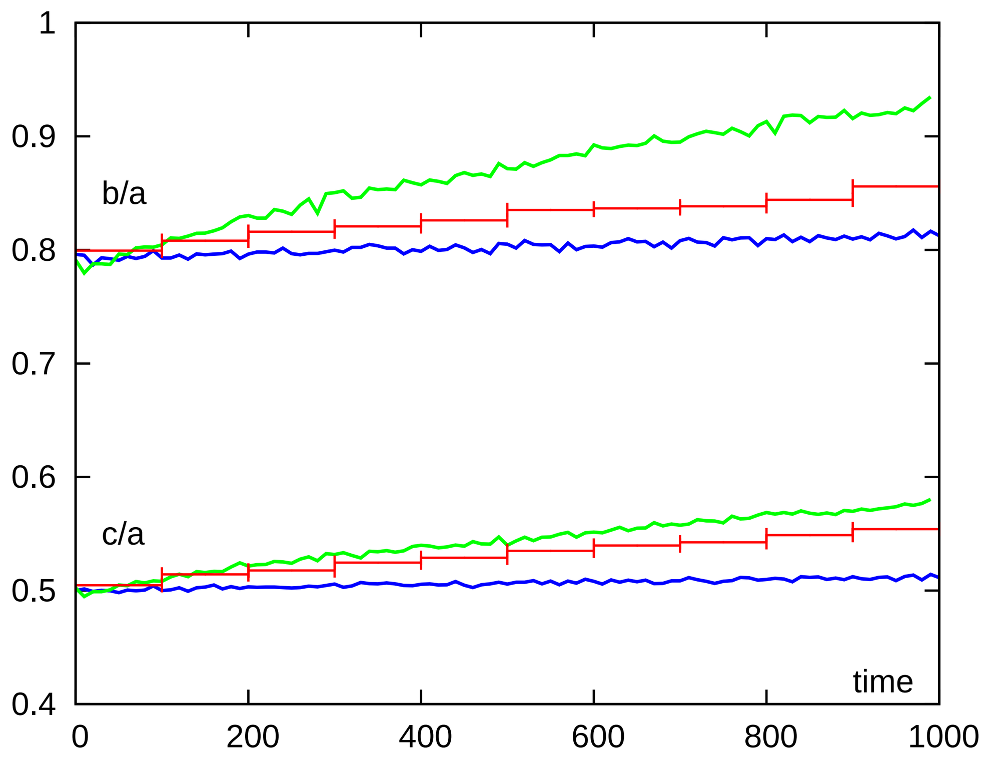


Fig. 1: Evolution of axis ratios $b/a$ and $c/a$ (starting from 0.79 and 0.5, correspondingly). Blue line – $\gamma = 1$ $N$-body model with $N = 10^6$, which shows very little evolution; green line – $\gamma = 2$ $N$-body model with $N = 10^6$, which changes shape substantially to become more axisymmetric; red line – integration in fixed potential for $\gamma = 2$ model, which shows comparable amount of evolution exclusively due to chaotic diffusion. $T = 1000$ corresponds roughly to Hubble time; dynamical time at half-mass radius is $\mathcal{O}(10)$.

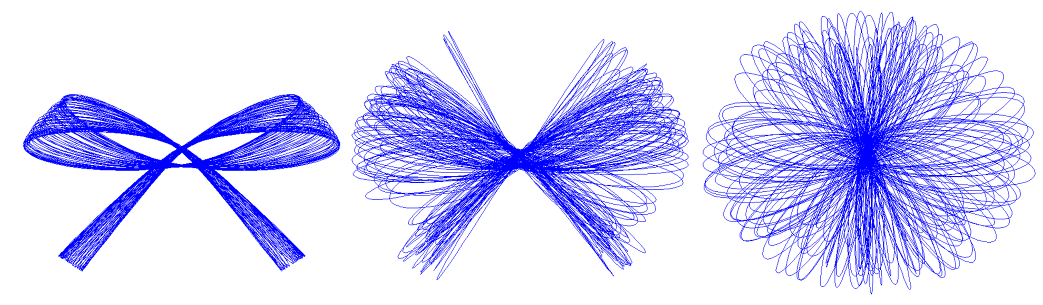

Fig. 2: Example of a regular resonant orbit (left, frequency diffusion rate $\delta\omega \sim 10^{-4}$), a weakly chaotic orbit (center, $\delta\omega \sim 10^{-2.5}$), and a strongly chaotic one (right, $\delta\omega \sim 10^{-1}$).

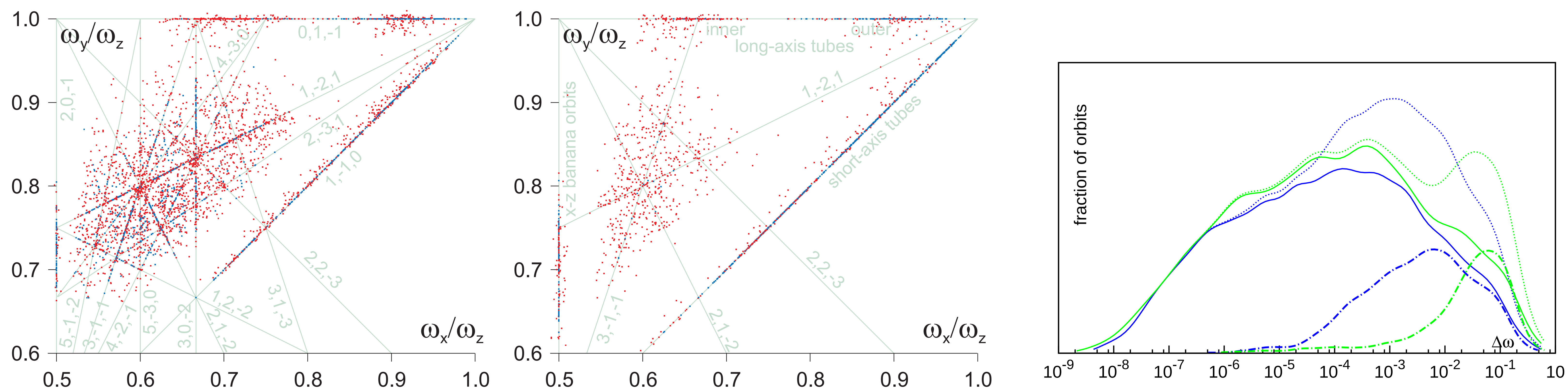


Fig. 3: Frequency map plots for $\gamma = 1$ (left) and $\gamma = 2$ (center) models: each point represents an orbit with given ratio of leading frequencies in three coordinates ($\omega_y/\omega_x$ and $\omega_z/\omega_x$). Blue dots represent regular orbits and red – chaotic ones, based on the value of their Lyapunov exponent. There is remarkable difference between the two models: in the former case there exist substantial population of orbits in resonant families, which show up as points grouping along certain lines on the plot. In the latter case most orbits which do not belong to $1:1$ tube families and $1:2$ $x-z$ banana family, are chaotic.
Right panel displays the distribution of frequency diffusion rates (blue – for $\gamma = 1$ model, green – for $\gamma = 2$ model). Dotted lines – all orbits, solid lines – tube-like orbits only, dot-dashed lines – all other non-tube orbits, including resonant orbit families and chaotic orbits. It demonstrates that the latter population is indeed different in the two models, in the $\gamma = 2$ case being notably composed of strongly chaotic orbits with $\delta\omega \gtrsim 10^{-2}$.