New variants of Schwarzschild modelling

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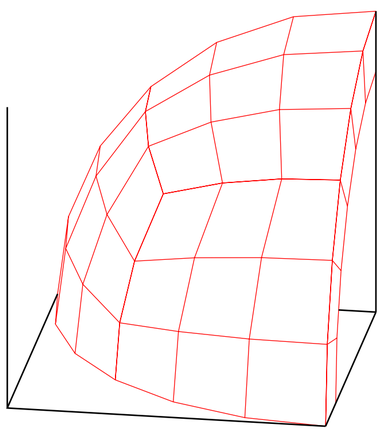
I propose new approaches for creating equilibrium models of stellar systems by Schwarzschild modelling (*SM*) method. As in the classical *SM*, a library of orbits is created for a given potential model, and their weights are assigned by solving an optimization problem so that the weighted sum of orbits reproduces the density corresponding to this potential. Details differ as to how evaluate the contribution of an orbit to the net potential.

Schwarzschild modelling overview

The core idea of Schwarzschild (1979) modelling is to find a superposition of N_o orbits with weights w_o such that N_c constraints are satisfied, by solving a matrix equation

$$\sum_{o=1}^{N_o} A_{oc}\, w_o = B_c\ ,\quad c = 1..N_c\ ,\quad w_o \geq 0 \qquad (1)$$

In the classical *SM*, the configuration space is partitioned into N_c cells and B_c in the above equation is the mass within a cell, while A_{oc} is the fraction of time each orbit spends in each cell.



Equation (1), with possible additional constraints (e.g. kinematic), may be solved by method of linear or quadratic programming (in the latter case, $\sum_o w_o^2$ is minimized to provide more smooth distribution of orbit weights).

Possible modifications

The meaning of variables in (1) needs not be related to mass in cells of spatial grid. For example, the potential and density of a model may be represented as some finite series of basis functions with certain coefficients A_{nlm}:

$$\begin{aligned}\Phi(r,\theta,\phi) &= \sum_{n=0}^{n_{max}} \sum_{l=0}^{l_{max}} \sum_{m=-l}^{l} A_{nlm}\, \Phi_{nlm}(r,\theta,\phi)\\ \rho(r,\theta,\phi) &= \sum_{n=0}^{n_{max}} \sum_{l=0}^{l_{max}} \sum_{m=-l}^{l} A_{nlm}\, \rho_{nlm}(r,\theta,\phi)\end{aligned} \qquad (2)$$

Typically, the basis functions are factorized as $\Phi_{nlm} = \Phi_{nl}(r)\, Y_l^m(\theta,\phi)$, where the angular part of basis functions is represented in spherical harmonic functions. For the radial basis, several choices exist, most widely used is the Hernquist-Ostriker(1992) basis set, for which the zeroth-order term corresponds to Hernquist profile, $\rho(r) = r^{-1}(1+r)^{-3}$.

Then, the variables in Eq. 1 may be taken to be the coefficients of expansion A_{nlm} for density of the underlying mass model and of each individual orbit in *SM*. In some sense, this is analogous to the self-consistent field (SCF) method of Hernquist & Ostriker (1992), with the difference that the coefficients are built up by summing not over individual particles, but over entire orbits.

Another option for representing potential is a direct decomposition of $\Phi(r,\theta,\phi)$ into sum of spherical harmonics with coefficients being smooth functions of radius:

$$\Phi(r,\theta,\phi) = \sum_{l=0}^{l_{max}} \sum_{m=-l}^{l} A_{lm}(r)\, Y_l^m(\theta,\phi) \qquad (3)$$

In practice, A_{lm} may be evaluated at a finite number of grid points in radius, and spline-interpolated between them. *SM* variables then are the coefficients $A_{lm,k;o}(r_k)$ of expansion of potential created by o-th orbit at radius r_k.

Both new modifications share the advantage of being 'non-localized' in angular dependence of density, i.e. they avoid sharp changes at the boundaries of grid cells which may arise in traditional *SM*. A disadvantage is that coefficients may vary by orders of magnitude, which requires applying weight functions in the optimization solver; moreover, the first method works well only if a density model is well represented by a small number of terms in basis-set expansion. The second method is completely flexible in radial dependence and yields good results for galactic models that are not too highly flattened.

A new software for orbit analysis and Schwarzschild modelling

The two new methods, along with the traditional grid-based *SM*, are implemented in a new software named *SMILE*, which is intended to be a flexible tool for studying theoretical galactic models and analyzing simulations. The key features of *SMILE* are:

- A number of standard potential models (e.g. triaxial Dehnen profile), and several general-purpose potential approximations: Zhao (1996) basis-set expansion (which is more general than H-O basis set), spline-based spherical harmonic expansion, and a fixed-background N-body potential based on tree-code, with possible density-adaptive softening. A central point mass (SMBH) may be also included.
- Orbit analysis methods: classification of orbit shapes (boxes, tubes, resonant orbits, chaotic orbits, centrophilic/centrophobic orbits); frequency analysis, detection of chaotic orbits by means of Lyapunov exponent computation and frequency diffusion rate. May analyse orbits from external simulations as well as from internal orbit integrator.
- Tools for studying gross dynamics of a potential model: Poincaré surfaces of section, frequency map plots.
- Schwarzschild modelling based on grid, basis-set expansion, or spherical-harmonic expansion at radial grid points. Generation of initial conditions for orbits randomly rather than from a predefined start-space, which is important for stability of N-body models created from *SM*, as shown by Vasiliev & Athanassoula (2012).
- An interactive graphical interface and scripting support.
- Written in C++, portable to Linux, Mac and Windows. Modular structure allows to use parts of the code in external programs, and easily include additional features.

Available at `http://td.lpi.ru/~eugvas/smile/`.

References

Hernquist L., Ostriker J., 1992, ApJ, 386, 375

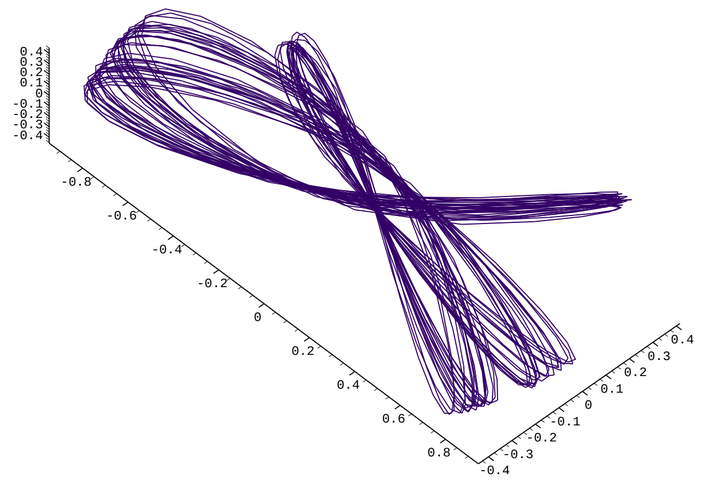
Schwarzschild M., 1979, ApJ, 232, 236

Valluri M., Merritt D., 1998, ApJ, 506, 686

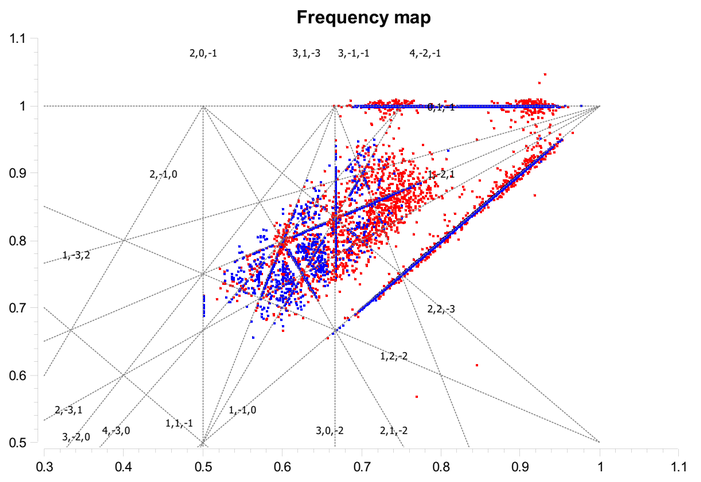
Vasiliev E., Athanassoula E., 2012, MNRAS, 419, 3268

Zhao H.-S., 1996a, MNRAS, 278, 488

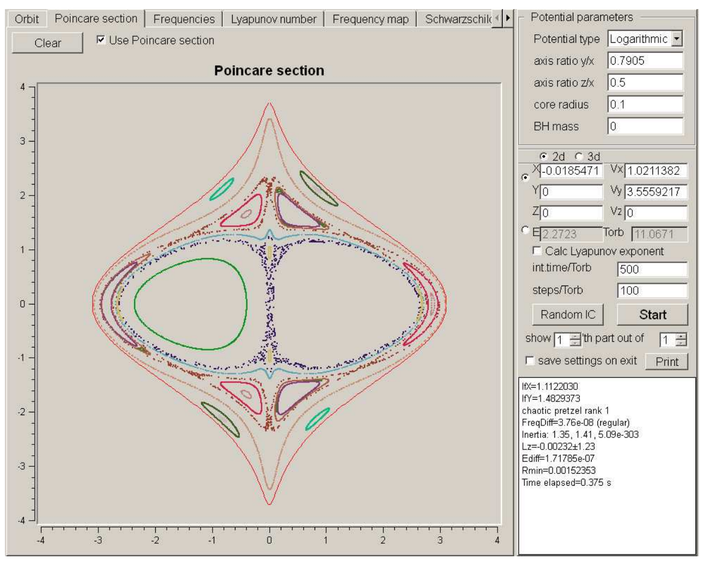
Screenshots



3d rendering of a 3:4:5 resonant orbit.



Frequency map plot: each point corresponds to an orbit with the ratio of leading frequencies ω_y/ω_x and ω_z/ω_x as coordinates. The plot shows 10^4 points from a triaxial Hernquist model, which has a significant population of resonant orbits, grouped along several lines (e.g Valluri & Merritt, 1998). Color denotes the regular(blue) or chaotic(red) character of an orbit.



Poincaré surface of section for a 2d logarithmic potential