Evolution of binary supermassive black holes and the final parsec problem

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IAU312, Beijing, August 2014

Plan of the talk

Why binary supermassive black holes?

Evolutionary stages of binary black holes

Loss cone theory in spherical and non-spherical systems

The novel Monte Carlo simulation method

Results and conclusions

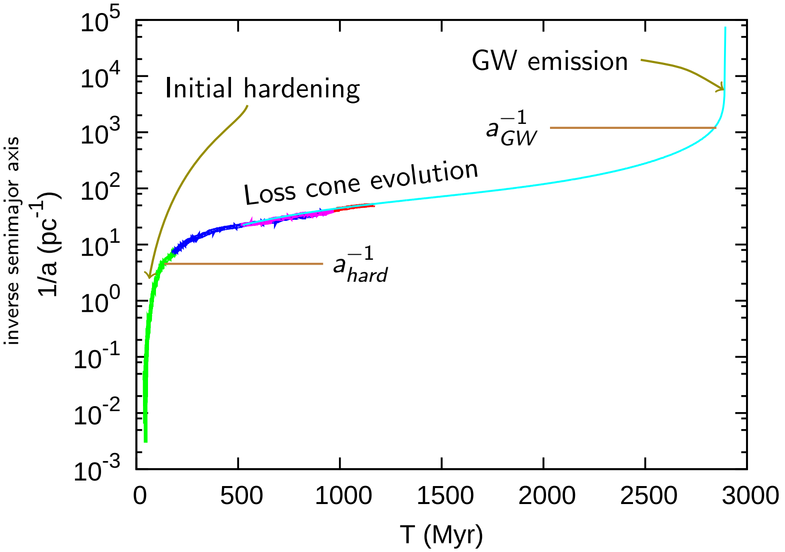
Origin of binary supermassive black holes (SBH)

- Most galaxies are believed to host central massive black holes
- In the hierarchical merger paradigm, galaxies in the Universe have typically 1–3 major and multiple minor mergers in their lifetime
- Every such merger brings two central black holes from parent galaxies together to form a binary system
- We dont see much evidence for widespread binary SBH (to say the least) – therefore they need to merge rather efficiently
- Merger is a natural way of producing huge black holes from smaller seeds

Evolutionary track of binary SBH

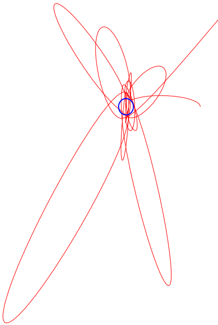
- Merger of two galaxies creates a common nucleus; dynamical friction rapidly brings two black holes together to form a binary (distance: $a \sim 10$ pc)
- Three-body interaction of binary with stars of galactic nucleus ejects most stars from the vicinity of the binary by the slingshot effect; a "mass deficit" is created and the binary becomes "hard" ($a \sim 1$ pc)
- The binary further shrinks by scattering off stars that continue to flow into the "loss cone", due to two-body relaxation or other factors
- As the separation reaches $\sim 10^{-2}$ pc, gravitational wave emission becomes the dominant mechanism that carries away the energy
- Reaching a few Schwarzschild radii ($\sim 10^{-5}$ pc), the binary finally merges

Evolutionary stages and timescales



[from Khan+ 2012]

Gravitational slingshot and binary hardening



A star passing at a distance $\lesssim 3a$ from the binary will experience a complex 3-body interaction which results in ejection of the star with velocity $v_{\mathrm{ej}} \sim \sqrt{\frac{m_1 m_2}{(m_1+m_2)^2}}\, v_{\mathrm{bin}}$.
These stars carry away energy and angular momentum from the binary, so that its semimajor axis a decreases:

$$\frac{d}{dt}\left(\frac{1}{a}\right) \approx 16\frac{G\,\rho}{\sigma} \;\equiv\; H_{\mathrm{full}}$$ [Quinlan 1996]

Thus, if density of field stars ρ remains constant, the binary hardens with a constant rate.
However, the reservoir of low angular momentum stars which can be ejected is finite and may be depleted quickly, so that the binary stalls at a radius $a_{\mathrm{stall}} \sim (0.1-0.4)a_h$.

Loss cone theory

The region of phase space with angular momentum
$L^2 < L_{\rm LC}^2 \equiv 2G(m_1 + m_2)\, a$ is called the loss cone.
Gravitational slingshot eliminates stars from the loss cone in one orbital period $T_{\rm orb}$. The crucial parameter for the evolution is the timescale for repopulation of the loss cone.
In the absence of other processes, the repopulation time is

$$T_{\rm rep} \sim T_{\rm rel}\, \frac{L_{\rm LC}^2}{L_{\rm circ}^2}, \text{ where } T_{\rm rel} = \frac{0.34\, \sigma^3}{G^2\, m_\star\, \rho_\star\, \ln\Lambda} \text{ is the relaxation time.}$$

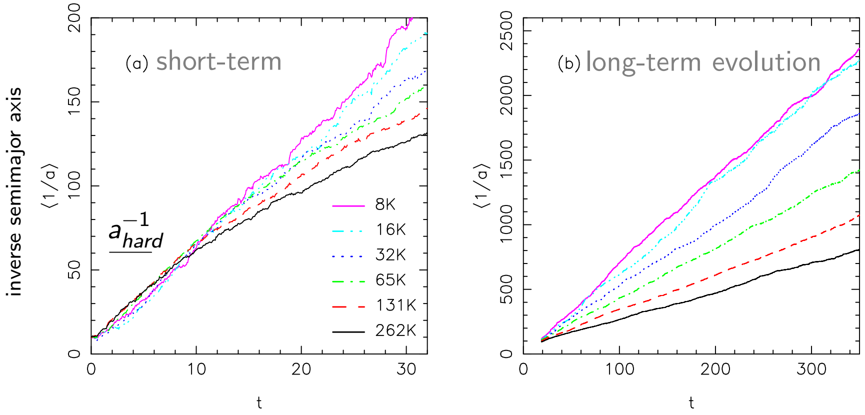
If $T_{\rm rep} \lesssim T_{\rm orb}$, the loss cone is full (refilled faster than orbital period). In real galaxies, however, the opposite regime applies – the empty loss cone. In this case the hardening rate

$$H \equiv \frac{d}{dt}(a^{-1}) \simeq \frac{T_{\rm orb}}{T_{\rm rep}} H_{\rm full}.$$

Relaxation is too slow for an efficient repopulation of the loss cone: in the absense of other processes the binary would not merge in a Hubble time.
This is the "final parsec problem" [Milosavljević&Merritt 2003]

N-scaling in the empty loss cone regime

Hardening rate $H \equiv \frac{d}{dt}(a^{-1}) \propto T_{rel}^{-1} \propto N^{-1}$



[from Merritt+ 2007]

Possible ways to enhance the loss cone repopulation

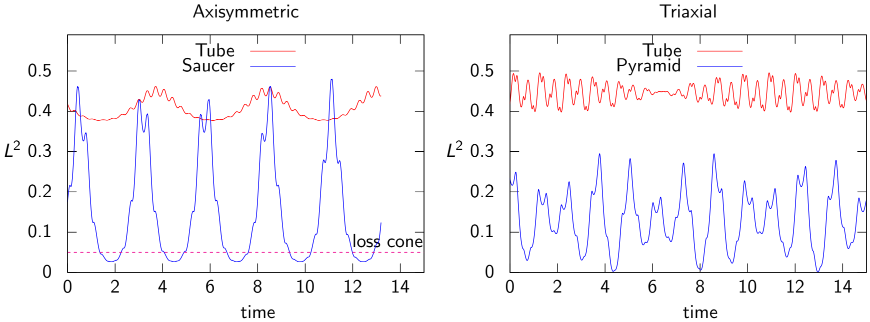
- Brownian motion of the binary (enables interaction with larger number of stars) [Milosavljević&Merritt 2001; Chatterjee+ 2003]
- Non-stationary solution for the loss cone repopulation rate [Milosavljević&Merritt 2003]
- Secondary slingshot (stars may interact with binary several times) [MM03]
- Gas physics – under special circumstances [Lodato+ 2009, Roškar+ 2014]
- Perturbations to the stellar distribution arising from transient events (such as infall of large molecular clouds, additional minor mergers and massive black holes, ...)
- Effects of non-sphericity on the orbits of stars in the nucleus [Berczik+ 2006; Preto+ 2011; Khan+ 2011,2012,2013; Vasiliev+ 2014]

Loss cone in non-spherical stellar systems

Angular momentum L of any star is not conserved, but experiences oscillations due to torques from non-spherical distribution of stars.

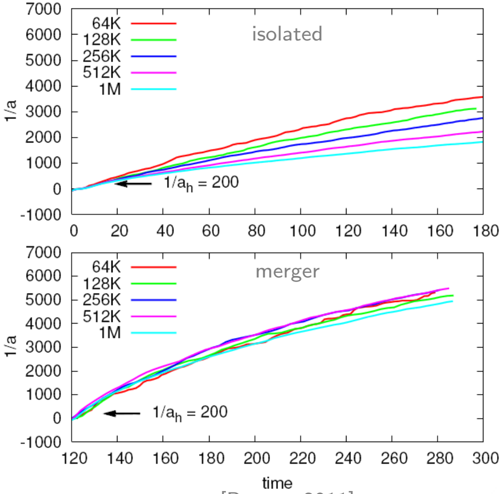
Therefore, much larger number of stars can attain low values of L and enter the loss cone at some point in their (collisionless) evolution, regardless of two-body relaxation.

This has led to a conclusion that the loss cone should remain full in axisymmetric and especially triaxial systems.

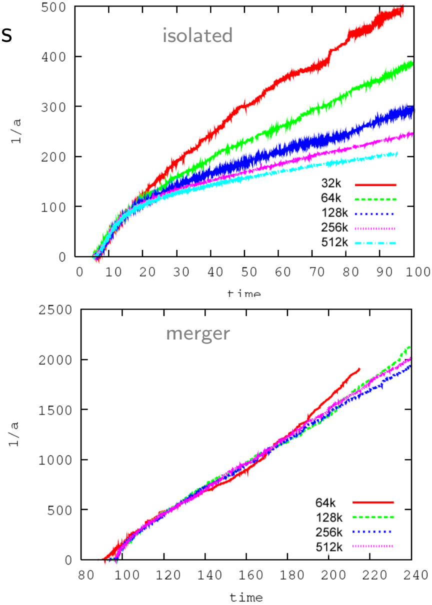


Merger simulations hint for a full loss cone

Hardening rates in merger simulations were found to be N-independent



[Preto+ 2011]



[Khan+ 2011]

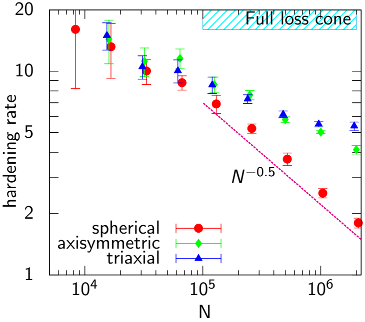
Evolution of isolated systems in different geometries

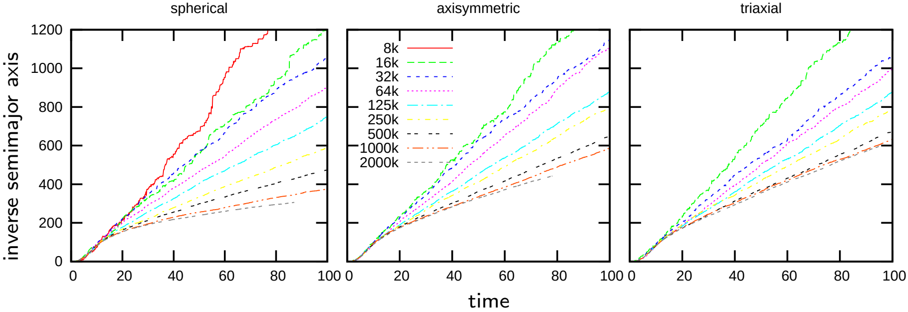
We have performed simulations of binary black hole evolution in three sets of models: spherical, axisymmetric and triaxial.

In all three cases the hardening rate appears to drop with N in the range $10^5 \lesssim N \lesssim 10^6$, but it drops slower in non-spherical cases.

Moreover, this rate is several times lower than the rate that would be expected in the full loss cone regime.

[Vasiliev, Antonini & Merritt 2014]





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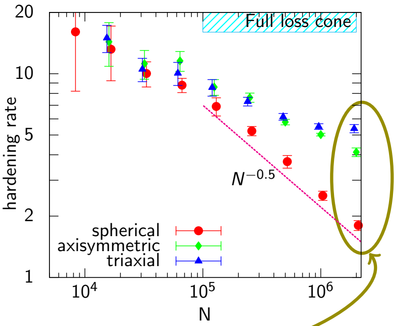
[Vasiliev, Antonini & Merritt 2014]



- Is there a convergence in the limit $N \to \infty$?
- If yes, why the limiting hardening rate seems to be much smaller than the full loss cone value?
- Does it stay constant with time, after all?
- Why the results of merger simulations are different?

Problems with direct N-body simulations

- We model galaxies with $N_\star \sim 10^{10-12}$, but our simulations are feasible only for $N \sim 10^6$;
- therefore, one needs to extrapolate our findings to much higher N;
- but as we have seen, the scaling is non-trivial;
- the contribution of collisional relaxation to loss cone repopulation scales as N^{-1}, but the collisionless processes are independent of N;
- we cannot afford having much larger N even with the best hardware and improved algorithms (but see talk by Y.Meiron).
- need a simulation method in which we may adjust the relaxation rate independently of particle number;
- Fokker-Planck and fluid models are impractical for complex geometry ⇒ need to use a particle-based Monte Carlo method.

[GRAPE cluster in RIT]

Monte Carlo method for collisional stellar systems

Name	Reference	relaxation treatment	timestep	1:1[1]	BH[2]	remarks
Princeton	Spitzer&Hart(1971), Spitzer&Thuan(1972)	local dif.coefs. in velocity, Maxwellian background $f(r,v)$	$\propto T_{dyn}$	−	−	
Cornell	Marchant&Shapiro (1980)	dif.coef. in E, L, self-consistent background $f(E)$	indiv., T_{dyn}	−	+	particle cloning
Hénon	Hénon(1971)	local pairwise interaction, self-consistent bkgr. $f(r, v_{\parallel}, v_{\perp})$	$\propto T_{rel}$	−	−	
	Stodołkiewicz(1982) Stodołkiewicz(1986)	Hénon's	block, $T_{rel}(r)$	−	−	mass spectrum, disc shocks binaries, stellar evolution
Mocca	Giersz(1998)	same	same	+	−	3-body scattering (analyt.)
	Hypki&Giersz(2013)	same	same	+	−	single/binary stellar evol., few-body scattering (num.)
Cmc	Joshi+(2000)	same	$\propto T_{rel}(r=0)$	+	−	partially parallelized
	Umbreit+(2012), Pattabiraman+(2013)		(shared)	+	+	fewbody interaction, single/binary stellar evol., GPU
Me(ssy)[2]	Freitag&Benz(2002)	same	indiv.$\propto T_{rel}$	−	+	cloning, SPH physical collis.
Raga	this study (Vasiliev 2014)	local dif.coef. in velocity, self-consistent background $f(E)$	indiv.$\propto T_{dyn}$	−	+	arbitrary geometry

[1] One-to-one correspondence between particles and stars in the system

[2] Massive black hole in the center, loss-cone effects

The novel Monte Carlo method for arbitrary geometry

- **Gravitational potential:**
  particles move in a smooth potential with arbitrary geometry (with a well-defined center), represented by a basis-set expansion in spherical harmonics (SCF or MEX, see also talk by Y.Meiron).
- **Orbit integration:**
  variable timestep Runge-Kutta; orbits are computed in parallel, independently from each other, during each update interval.
- **Two-body relaxation:**
  local (position-dependent) velocity diffusion coefficients, computed under an approximation of a spherical isotropic distribution function (DF) of background stars.
- **Potential and DF update:**
  update interval $\gg$ dynamical time $\Rightarrow$ temporal smoothing;
  use many sampling points per particle during each update interval
  $\Rightarrow$ reduce discreteness fluctuations, suppress artificial relaxation.

The treatment of two-body relaxation

Local (position-dependent) velocity diffusion coefficients:

$$v\langle\Delta v_\parallel\rangle = -\left(1+\frac{m}{m_\star}\right) I_{1/2}\,,$$
$$\langle\Delta v_\parallel^2\rangle = \tfrac{2}{3}\left(I_0 + I_{3/2}\right),$$
$$\langle\Delta v_\perp^2\rangle = \tfrac{2}{3}\left(2I_0 + 3I_{1/2} - I_{3/2}\right),$$

here m and $m_\star$ are masses of the test and field stars, and

$$I_0 \equiv \Gamma \int_E^0 dE'\, \boxed{f(E')},$$
$$I_{n/2} \equiv \Gamma \int_{\Phi(r)}^{E} dE'\, f(E') \left(\frac{E' - \boxed{\Phi(r)}}{E-\Phi(r)}\right)^{n/2},$$
$$\Gamma \equiv 16\pi^2 G^2 m_\star \ln\Lambda = 16\pi^2 G^2 M_{\rm tot} \times \boxed{(N_\star^{-1}\ln\Lambda)}.$$

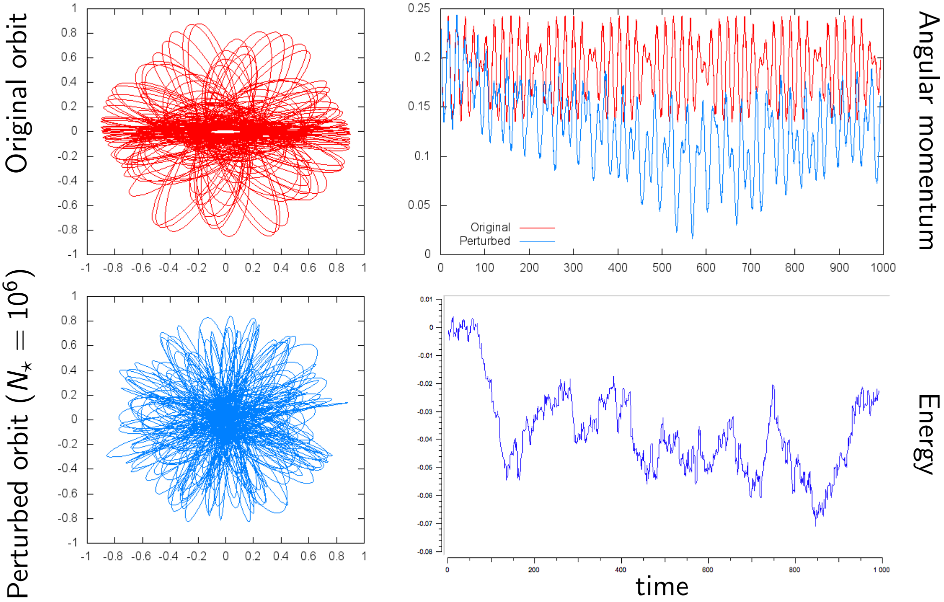
distribution function of stars
(isotropic approximations)
gravitational potential
scalable amplitude of perturbation

After each timestep, the perturbations to the velocity are computed as

$$\Delta v_\parallel = \langle\Delta v_\parallel\rangle\Delta t + \zeta_1\sqrt{\langle\Delta v_\parallel^2\rangle\Delta t}\,,$$
$$\Delta v_\perp = \zeta_2\sqrt{\langle\Delta v_\parallel^2\rangle\Delta t}\,,$$

where ζ_1, ζ_2 are two independent normally distributed random numbers.

An example of orbit in a triaxial potential

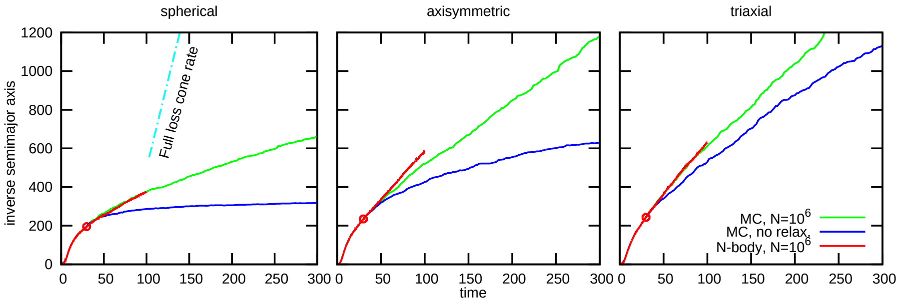


Application to the final-parsec problem

- Follow the merger and initial hardening by a conventional N-body code;
- after the formation of hard binary, switch to Monte Carlo method:
- during each episode, evolve particles in a time-dependent potential of binary MBH moving on a Keplerian orbit with fixed parameters;
- at the end of episode, record the changes of energy and angular momentum of each particle during each close encounter with the binary, sum them up and adjust the orbit of the binary using conservation laws [e.g. Sesana+ 2007, Meiron&Laor 2012];
- this automatically accounts for depletion of the loss cone, secondary slingshot, and change of shape of the gravitational potential; does not account for brownian motion;
- may also include two-body relaxation in addition to non-spherical torques $\Rightarrow$ naturally interpolate between $N = 10^6$ and $N = \infty$.

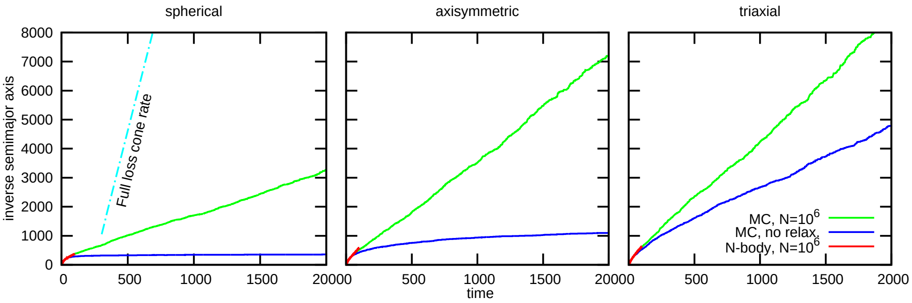
Preliminary results of Monte Carlo simulations

- Monte Carlo simulations are in qualitative agreement with direct N-body simulations, although somewhat underestimate hardening rate.
- Hardening rate does not stay constant, but decreases with time; it is never even close to the full loss cone rate.



Preliminary results of Monte Carlo simulations

- Monte Carlo simulations are in qualitative agreement with direct N-body simulations, although somewhat underestimate hardening rate.
- Hardening rate does not stay constant, but decreases with time; it is never even close to the full loss cone rate.
- It's not clear if the "final parsec" gap can be overcome in the purely collisionless axisymmetric case [in contrast with Khan+ 2013].
- In the triaxial case there is little "benefit" from relaxation.



Summary

- The final parsec problem in the binary MBH evolution is connected to the efficiency of repopulation of the loss cone.
- This repopulation occurs faster in non-spherical geometry.
- In simulations of isolated systems, the loss cone never stays "full", even in triaxial geometry.
- It is difficult to disentangle collisional and collisionless effects (suppress 2-body relaxation) in conventional N-body simulations.
- A novel Monte Carlo method with arbitrary geometry and adjustable relaxation rate is proposed.
- Preliminary results suggest that the final parsec problem may be overcome only in triaxial systems (axisymmetry is not enough).
- Remaining problem: apparently different situation (higher hardening rate) in merger simulations.

THANK YOU!

Bonus: feeding rates of single MBH

A similar simulation method has been applied to the problem of feeding single MBH by star captures [Vasiliev 2014, CQG in press].

Here again conventional N-body simulations cannot follow the correct proportion between collisional and collisionless effects (however, one may simultaneously scale N and capture radius while keeping this proportion).

Black holes with $M_\bullet \gtrsim 10^7\, M_\odot$ are deep in the empty loss cone regime, and the inclusion of non-spherical torques greatly increases the capture rate.

Possible future improvements:
– inclusion of stellar mass spectrum and stellar evolution;
– application for the capture rates by binary MBH.
[Chen+ 2009, Wegg&Bode 2011]