Evolution of binary supermassive black holes: the final parsec problem

Eugene Vasiliev
Lebedev Physical Institute

Fabio Antonini
Canadian Institute for Theoretical Astrophysics

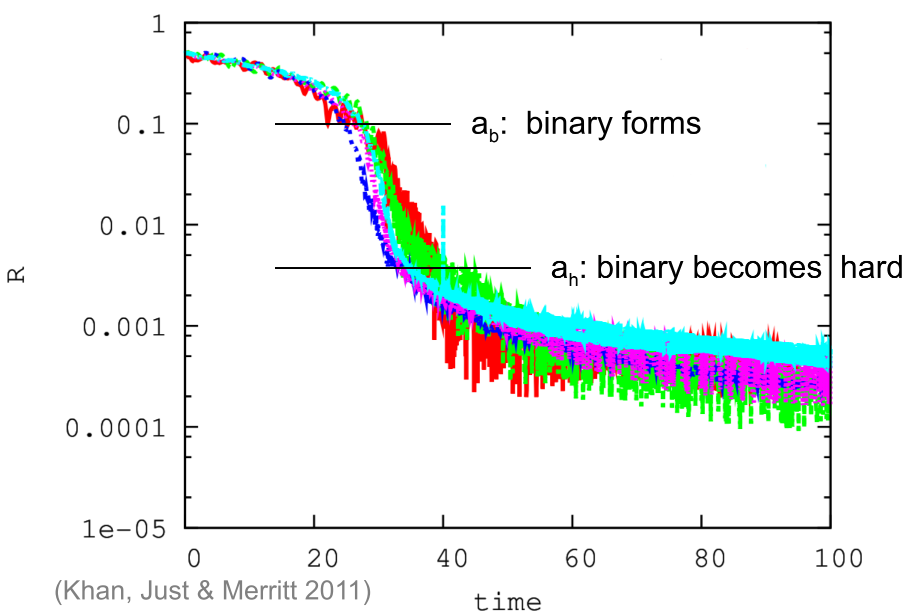
David Merritt
Rochester Institute of Technology

Plan of the talk

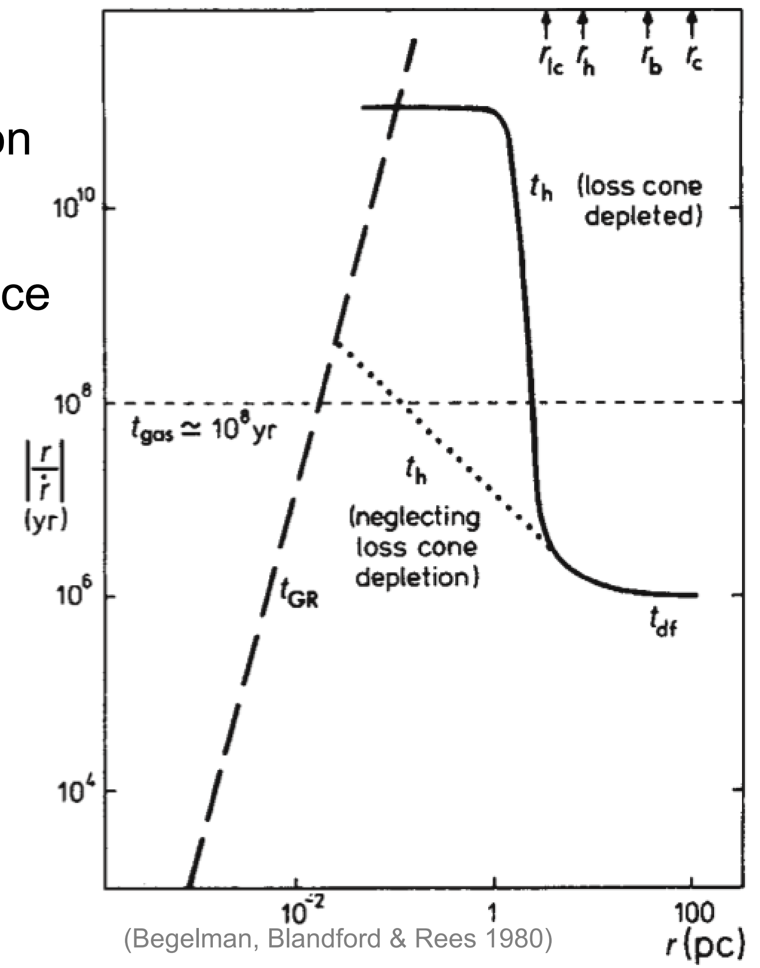
- Evolution of supermassive black hole binaries
- Definition of the "final parsec problem"
- Possible solution in non-spherical galactic nuclei
- Is it a viable solution or a numerical artifact?

Evolution of supermassive black hole binaries

- Merger of two galaxies creates a common galactic nucleus
- Dynamical friction brings two black holes to a distance a_b where they form a bound binary
- The binary shrinks ("hardens") down to a separation at which gravitational radiation becomes effective
- GW emission finally drives the binary to coalescence



(Khan, Just & Merritt 2011)



(Begelman, Blandford & Rees 1980)

Evolution of supermassive black hole binaries

- Dynamical friction timescale:

$$t_{\rm DF} \sim 10^6 \text{ yr} \left(\frac{r}{100 \text{ pc}}\right)^2 \left(\frac{\sigma}{200 \text{ km/s}}\right) \left(\frac{m_2}{10^8\ M_\odot}\right)^{-1} \left(\frac{\ln \Lambda}{15}\right)^{-1}$$

- A binary is called hard if its orbital velocity exceeds that of the field stars, or the separation is less than a_h:

$$a_h = \frac{G\mu}{\sigma^2} \approx 2.7\text{pc}(1+q)^{-1} \left(\frac{m_2}{10^8\ M_\odot}\right) \left(\frac{\sigma}{200\text{km/s}}\right)^{-2}, \qquad \mu \equiv \frac{m_1 m_2}{m_1 + m_2},\ q \equiv \frac{m_2}{m_1}$$

- The timescale for coalescence due to GW emission is (Peters 1964)

$$t_{\rm GW} = \frac{5}{256\,F(e)} \frac{c^5}{G^3} \frac{a^4}{\mu(m_1+m_2)^2} \approx 7 \times 10^8 \text{yr} \frac{q^3}{(1+q)^6} \left(\frac{m_1+m_2}{10^8\ M_\odot}\right)^{-0.6} \left(\frac{a}{10^{-2} a_h}\right)^4$$

$$F(e) \equiv (1-e^2)^{7/2} \left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right)$$

Gravitational slingshot and binary hardening

A star passing at a distance $\lesssim 3a$ from the binary will experience a complex 3-body interaction which results in ejection of the star with velocity $v_{\rm ej} \sim \sqrt{\frac{m_1 m_2}{(m_1+m_2)^2}}\, v_{\rm bin}$.

These stars carry away energy and angular momentum from the binary, so that its separation decreases:

$$\frac{d}{dt}\left(\frac{1}{a}\right) \approx 16 \frac{G\,\rho}{\sigma}$$

Thus, if density of field stars ρ remains constant, the binary hardens with a constant rate.

However, the reservoir of low angular momentum stars which can be ejected is finite and may be depleted quickly, so that the binary stalls at a radius $a_{\rm stall} \sim (0.1 - 0.4) a_h$.

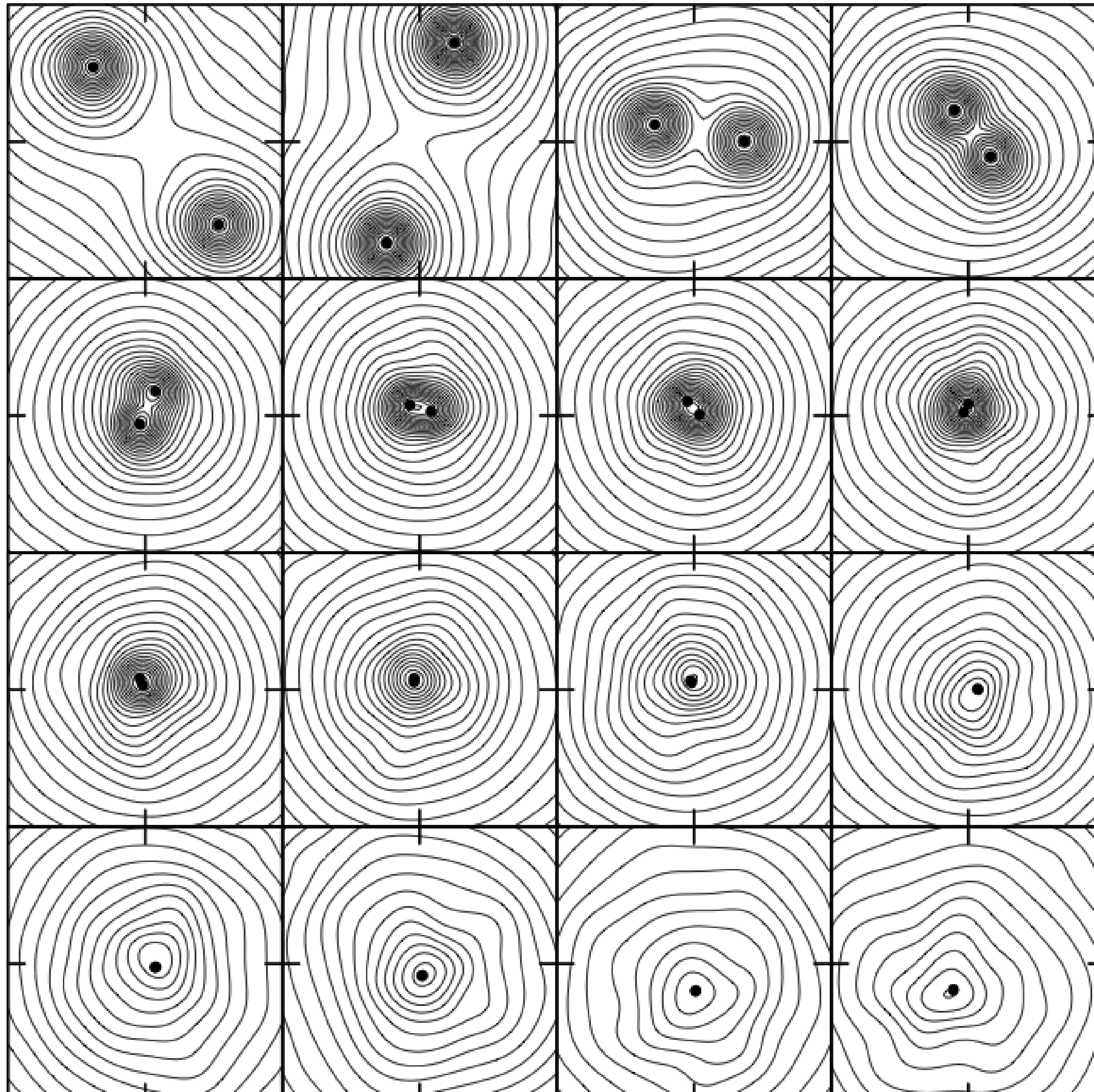
Evolution of density profile in the merger

Dynamical friction

Bound pair

Ejection of stars via gravitational slingshot

(Milosavljevic & Merritt 2001)



“Mass deficit” in observations of galactic nuclei

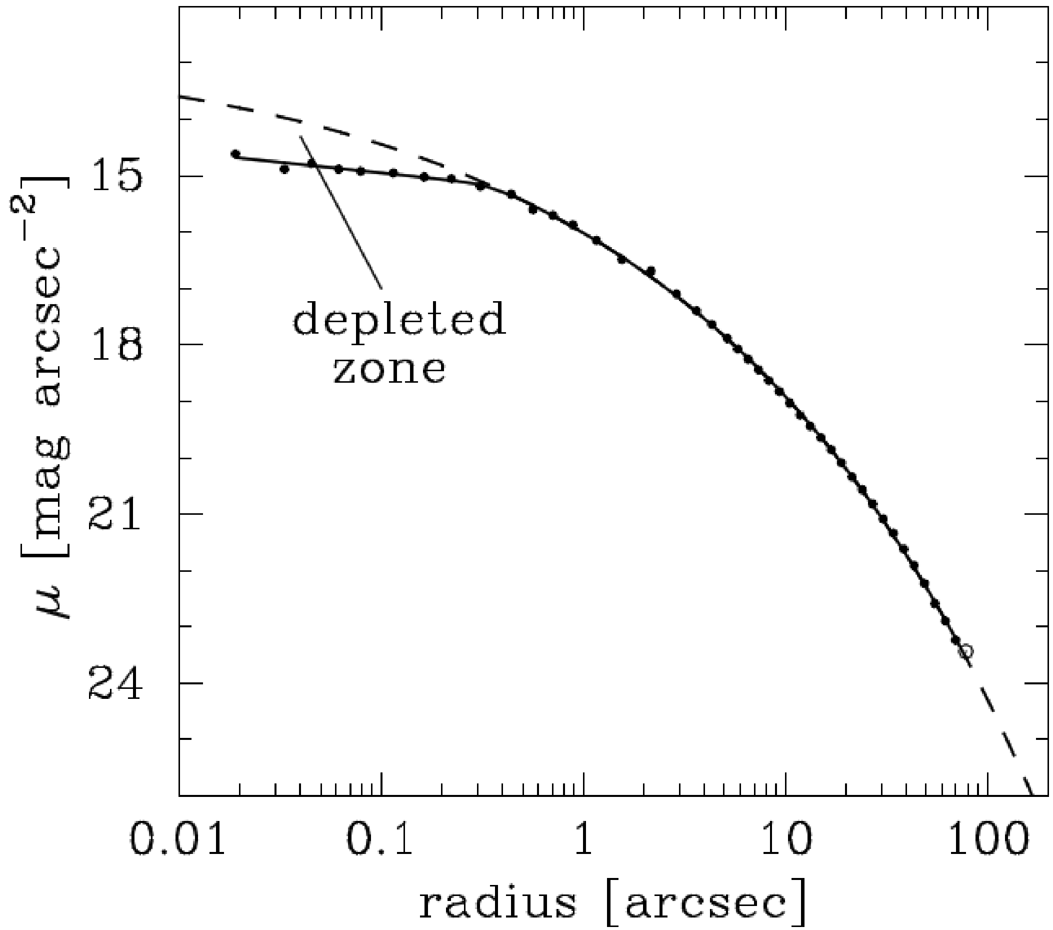


Figure 11: *Observed surface brightness profile of NGC 3348. The dashed line is the best-fitting Sersic model to the large-radius data. Solid line is the fit of an alternative model, the “core-Sersic” model, which fits both the inner and outer data well. The mass deficit is illustrated by the area designated “depleted zone” and the corresponding mass is roughly* $3 \times 10^8 M_\odot$ [Graham 2004]

Loss cone dynamics

The region of phase space with $L^2 < L_{\rm LC}^2 \equiv 2G(m_1+m_2)\,3a$ is called loss cone.
In the absence of other processes, the loss cone is repopulated on a timescale

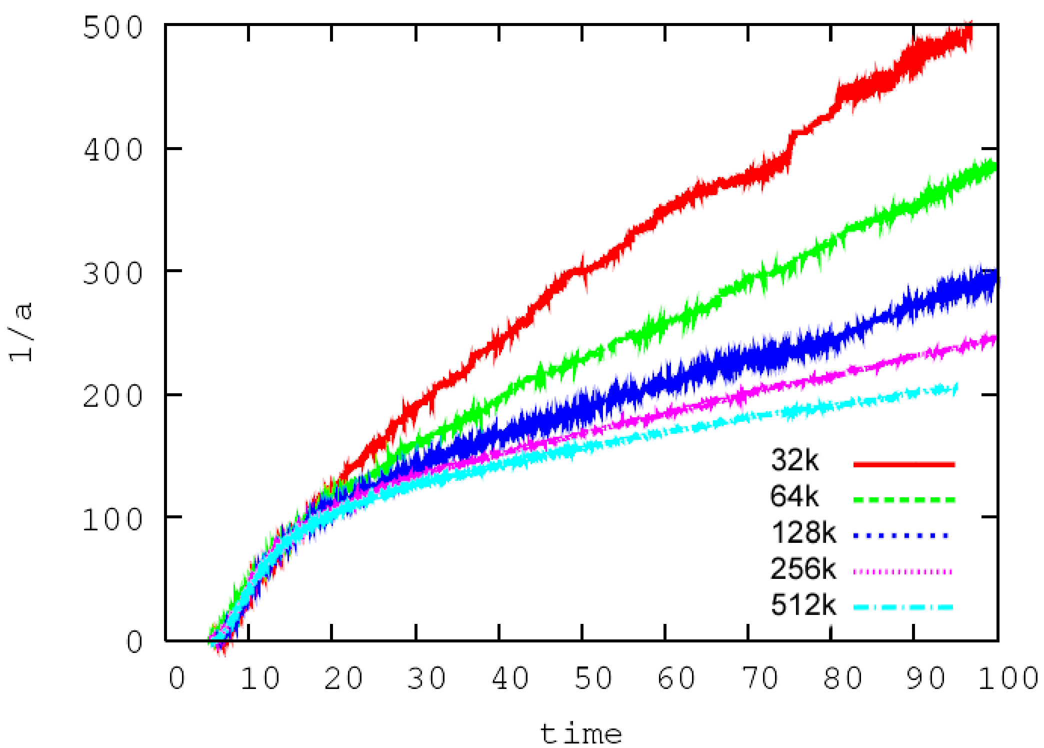
$$T_{\rm rep} \sim T_{\rm rel}\,\frac{L_{\rm LC}^2}{L_{\rm circ}^2}\ , \text{ where } T_{\rm rel} = \frac{0.34\,\sigma 3}{G^2\, m_\star\, \rho_\star\, \ln\Lambda} \text{ is the relaxation time.}$$

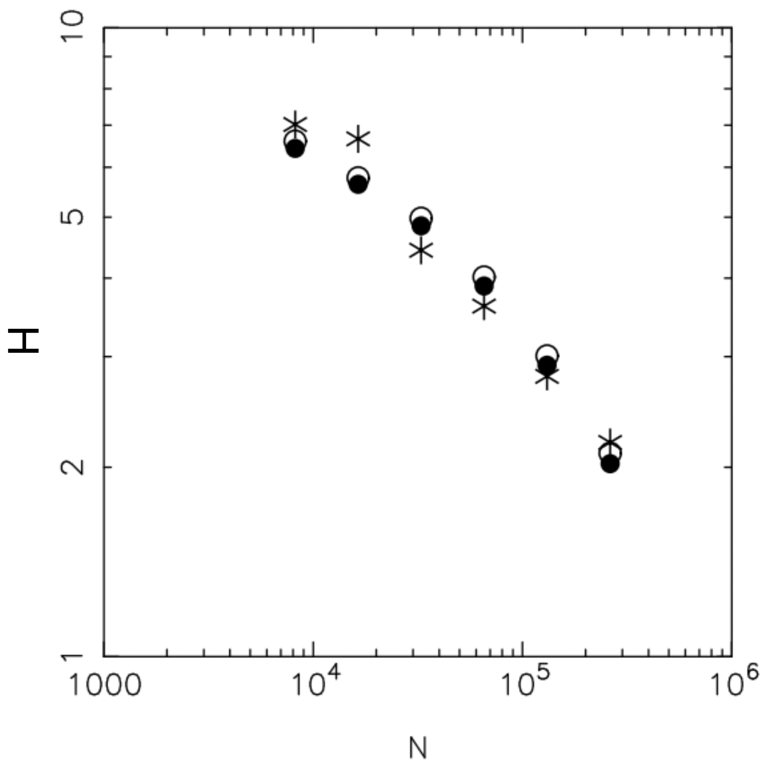
If $T_{\rm rep} \lesssim T_{\rm orb}$, the loss cone is essentially full – it is refilled faster than orbital period.
In real galaxies, however, the opposite regime applies – the empty loss cone.
In this case the hardening rate $H \equiv \dfrac{d}{dt}(a^{-1}) \simeq \dfrac{T_{\rm orb}}{T_{\rm rep}} H_{\rm full}$.

In the N-body simulations, the full loss cone regime is manifested by independence of hardening rate on the number of particles, while the empty loss cone regime should have $H \propto m_\star \propto 1/N$.

N-dependence of hardening rate in simulations

For simulations of a spherical galaxy with large enough N,
the hardening rate appears to drop with N, as predicted by two-body relaxation theory.





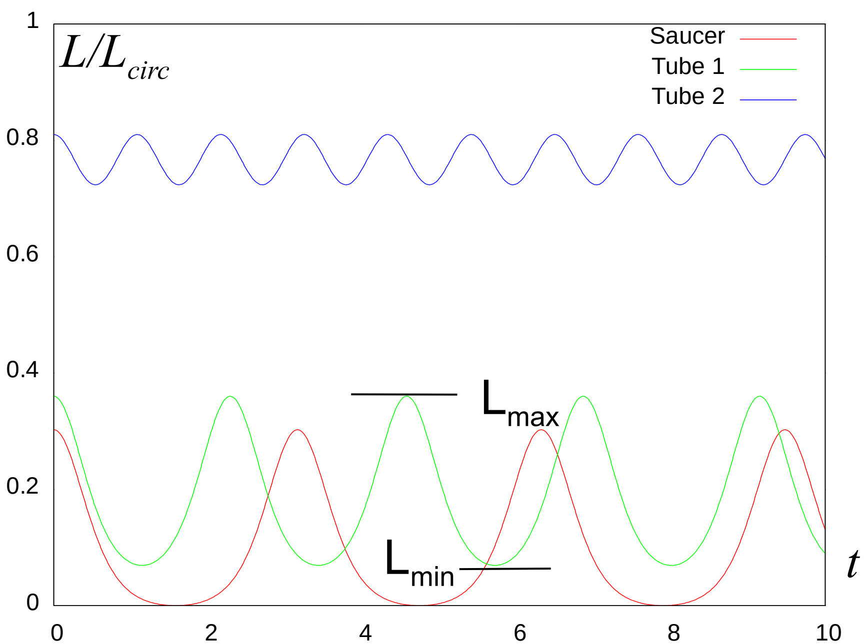
(Merritt et al.2007)

Possible ways to enhance the loss cone repopulation

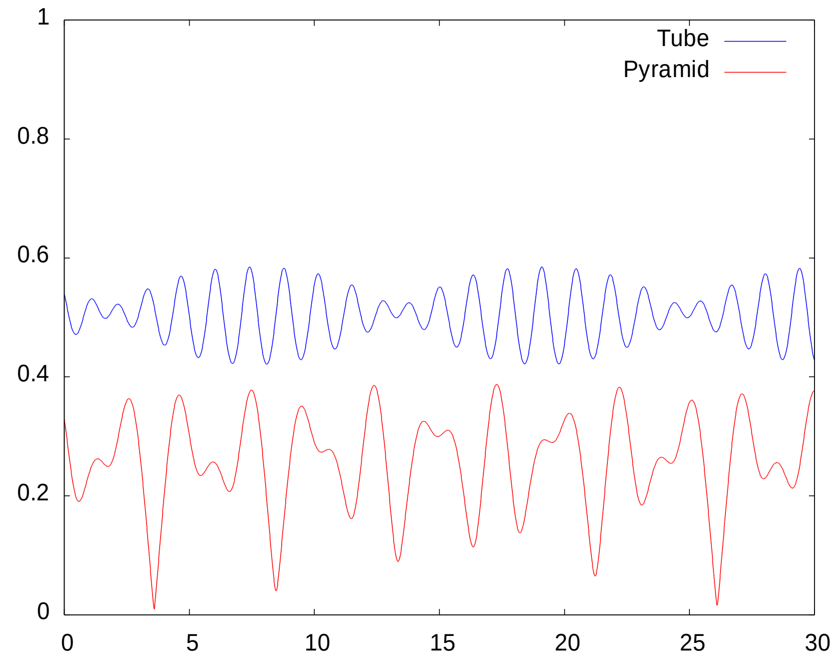
- Brownian motion of the binary (enables interaction with larger number of stars) [Milosavljevic&Merritt 2001]
- Non-stationary solution for the loss cone repopulation rate [Milosavljevic&Merritt 2003]
- Secondary slingshot (stars may interact with binary several times) [MM'03]
- Gas physics – under special circumstances [Lodato+ 2009]
- Perturbations to the stellar distribution arising from transient events (such as infall of large molecular clouds, additional minor mergers, …)
- Effects of non-sphericity on the orbits of stars in the nucleus

Evolution of angular momentum of an orbit in a non-spherical nuclear star cluster

Total angular momentum squared, L^2, is not conserved but experiences regular oscillations due to torques from non-spherical stellar distribution.
Therefore, much larger number of stars can attain low values of angular momentum at some point in their regular precession
Especially in a triaxial nucleus, the fraction of centrophilic orbits may be large enough to sustain full loss cone regime for the entire evolution of the binary



Axisymmetric system

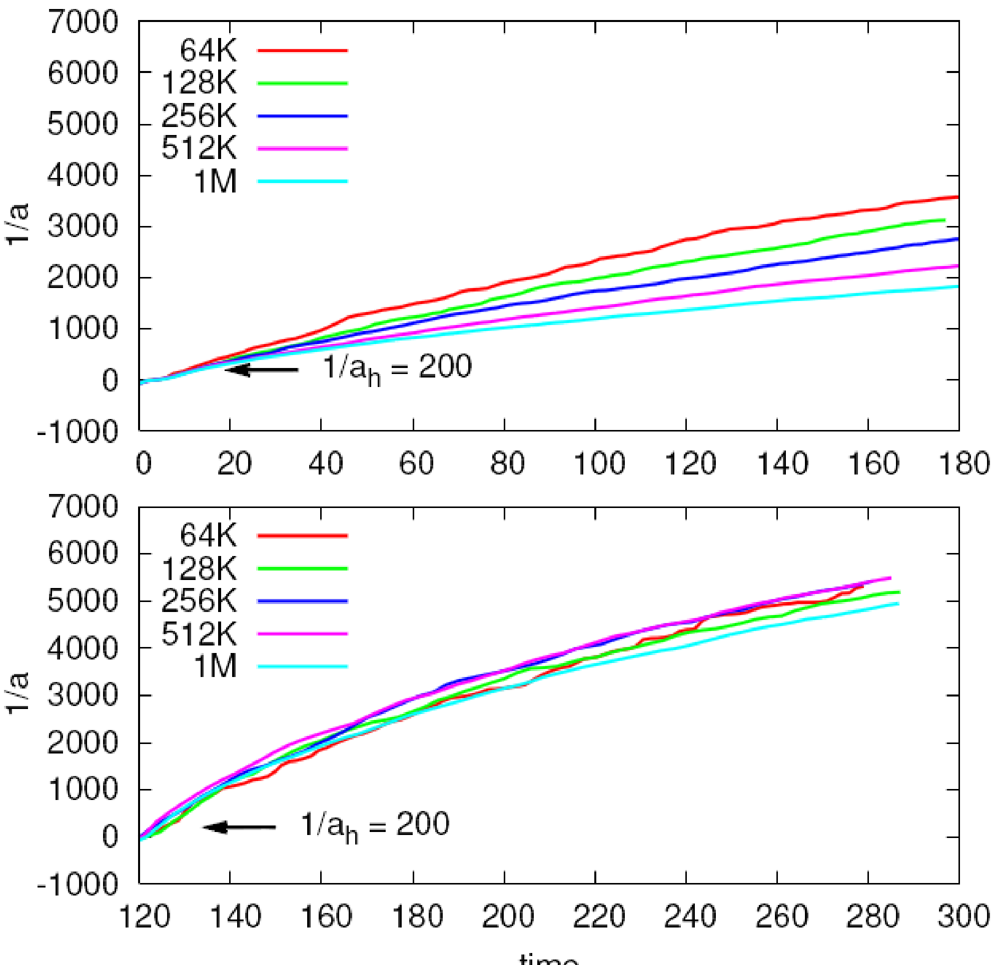


triaxial system

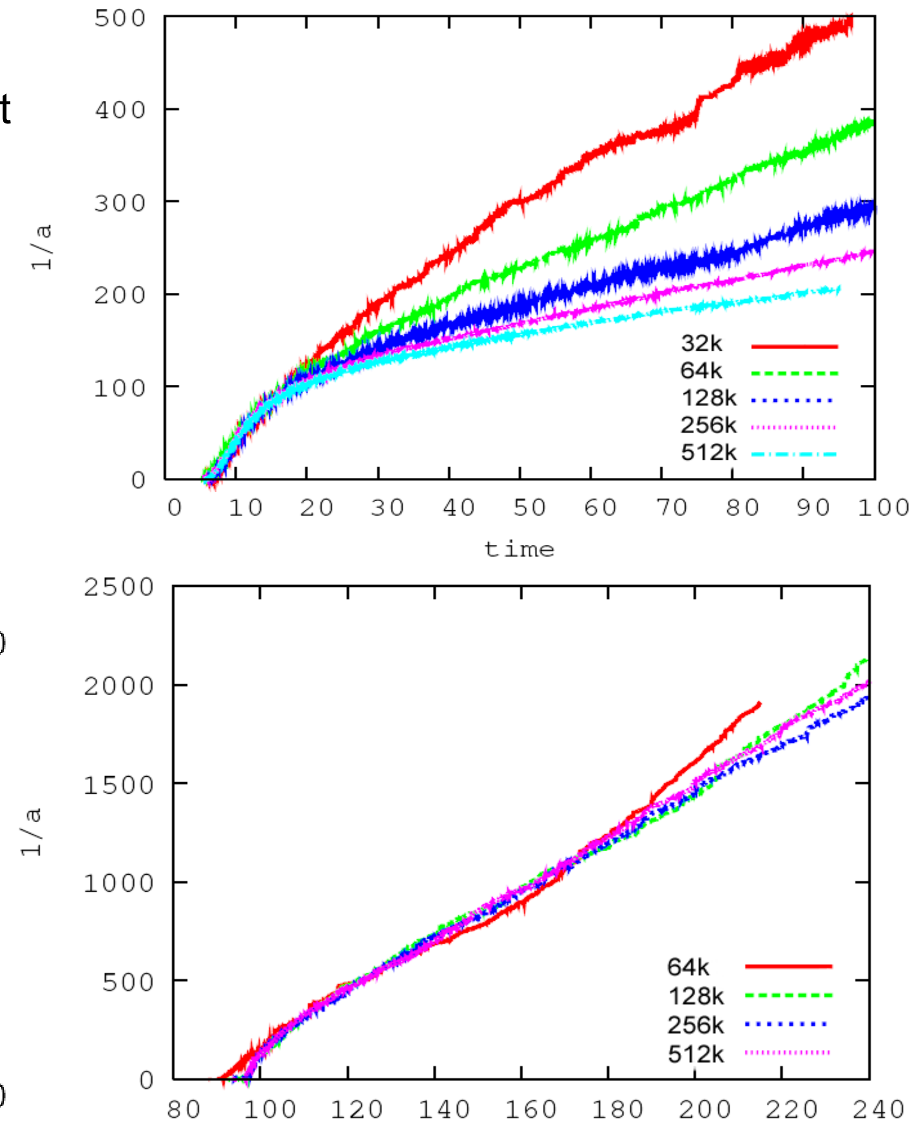
Hardening rates in non-spherical simulations

...were recently found to be N-independent

for merger simulations with non-spherical remnant



(Preto et al.2011)



(Khan et al.2011)

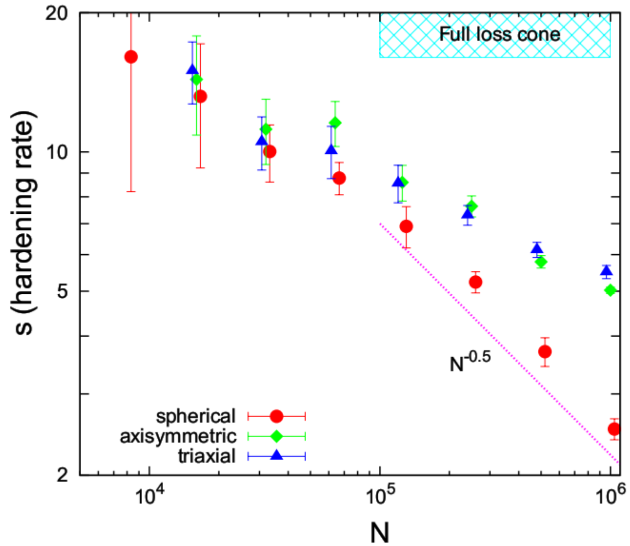
Is the final-parsec problem solved?

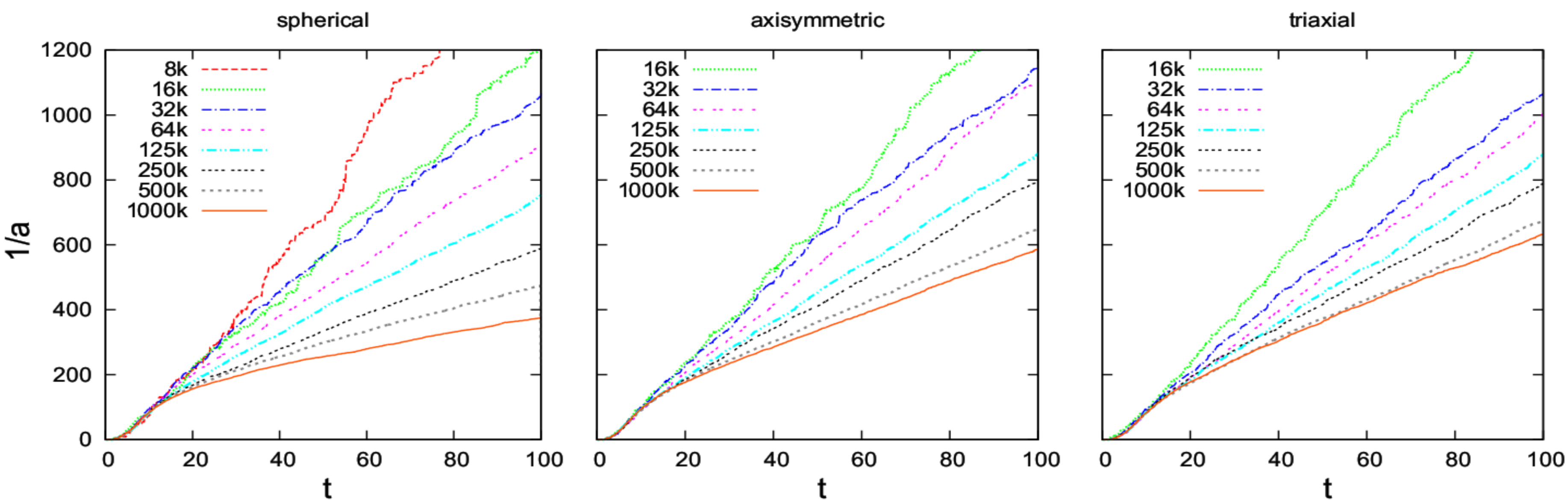
(by assuming non-spherical shape of galactic nucleus)

We have performed simulations of binary black hole evolution in three sets of models: spherical, axisymmetric and triaxial.

In all three cases the hardening rate appears to drop with **N**, although it is factor of 2-3 larger for non-spherical models for the largest N in our runs.

Moreover, this rate is several times **lower** than the rate that would be expected in the full loss cone regime.

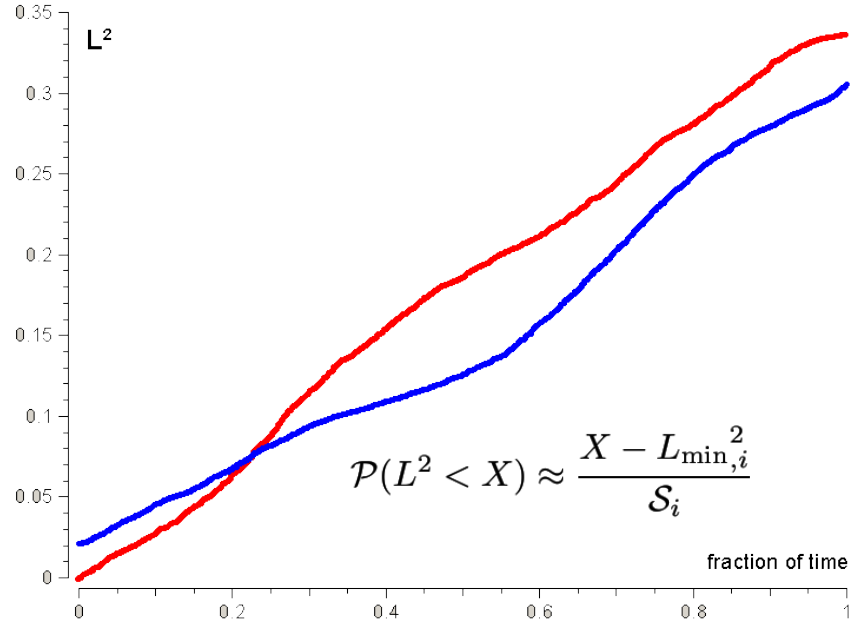




What happens? Insights from orbit analysis

The key property of an orbit in a non-spherical potential is the fraction of time that it spends in the low angular momentum region.

We may estimate the rate at which stars come close enough to origin to interact with the binary, by studying the orbit population of a model – or – N-body simulation.

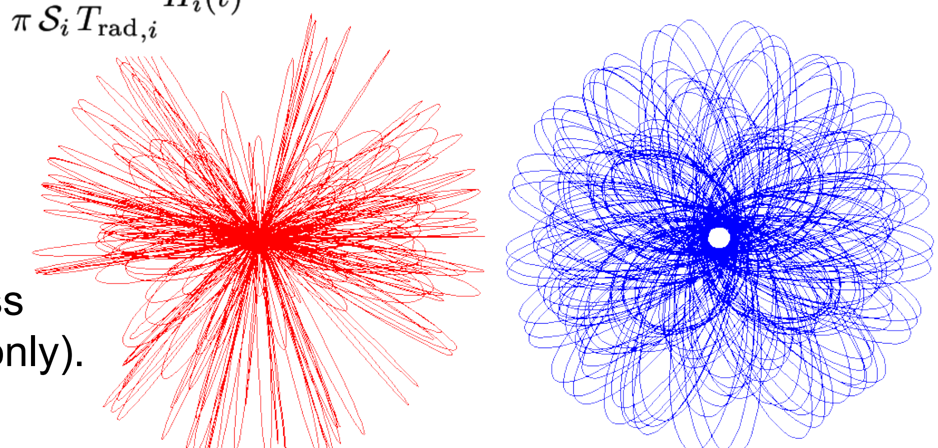


The coupled evolution of binary separation a and the weights of orbits m_i in the model is described by a system of coupled ODE.

$$s(t) \equiv \frac{d}{dt}\left(\frac{1}{a}\right) = \sum_{i=1}^{N} \frac{2m_i(t)}{M_{\rm bin}\, a(t)\, T_{{\rm rad},i}} \int_0^1 d\mathcal{P}\, C(\chi(\mathcal{P})) = \sum_{i=1}^{N} \frac{G\, m_i(t)}{\pi\, \mathcal{S}_i\, T_{{\rm rad},i}} H_i(t)$$

$$\frac{dm_i}{dt} = -m_i \frac{G M_{\rm bin}\, a(t)\, J H_i(t)}{\pi \mathcal{S}_i\, T_{{\rm rad},i}}$$

By solving this system we obtain the collisionless rate of evolution (due to non-spherical torques only).



centrophilic   and   centrophobic orbits

Collisionless evolution vs. N-body simulations

The models for collision**less** evolution predict the hardening rate which is comparable – to within factor of 2 – to the rate found in the highest-N simulations.

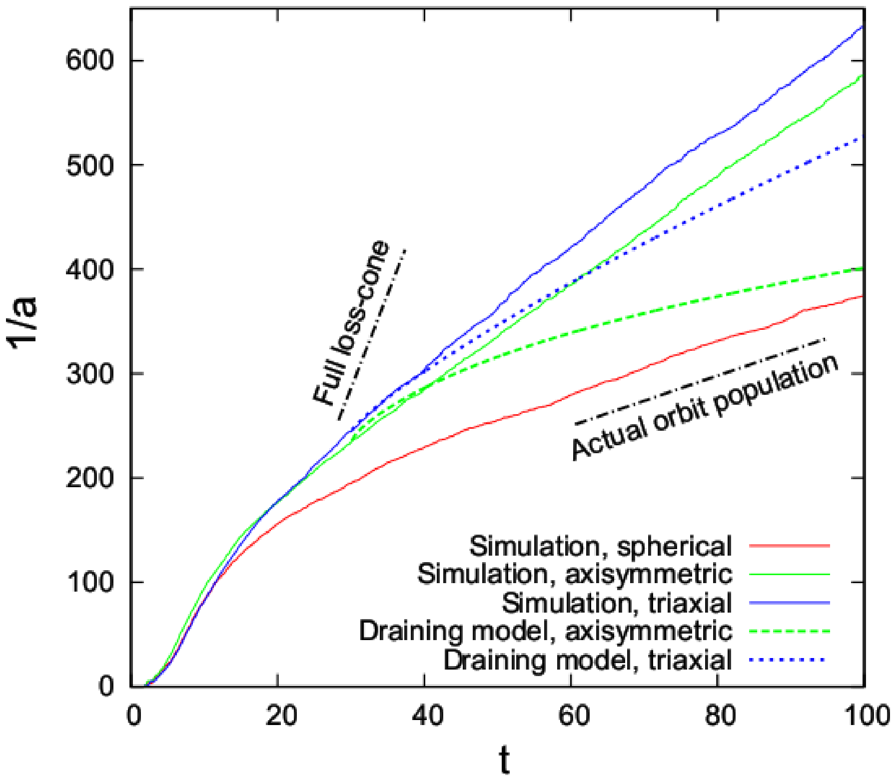
In other words, the contribution from collision**al** effects (i.e. two-body relaxation) is still non-negligible in the N-body simulations.

We cannot simply extrapolate their results to $N \to \infty$.

To address the problem, one has to either

1) increase N, or
2) beat down 2-body relaxation in other way.

(option 2 is under development now...)



The question remains, why in the merger simulations we see almost no N-dependence.

It could be due to properties of the remnants not accounted for in the steady-state models, such as clumpiness or net rotation.

Analysis of orbital population of merger remnants is underway..

Conclusions

- Formation of binary supermassive black holes results in their coalescence in a reasonable time only if there is a continuous supply of low angular momentum stars which can interact with the binary and make it shrink.
- The standard loss cone theory for a spherical galaxy predicts that this reservoir is quickly depleted and very slowly repopulated – this is the final parsec problem.
- In the case of realistic, non-spherical merger remnants, this problem is believed to be alleviated because of existence of large amount of stars on centrophilic orbits, which can overwhelm the loss cone depletion.
- Our detailed analysis indicates that the non-spherical torques alone still cannot keep the loss cone full.
- Two-body relaxation still plays substantial role in the simulations, which it shouldn't in a real galaxy.
  It is not yet clear whether we may simulate the binary evolution reliably.