Chaos and secular evolution of triaxial elliptical galaxies

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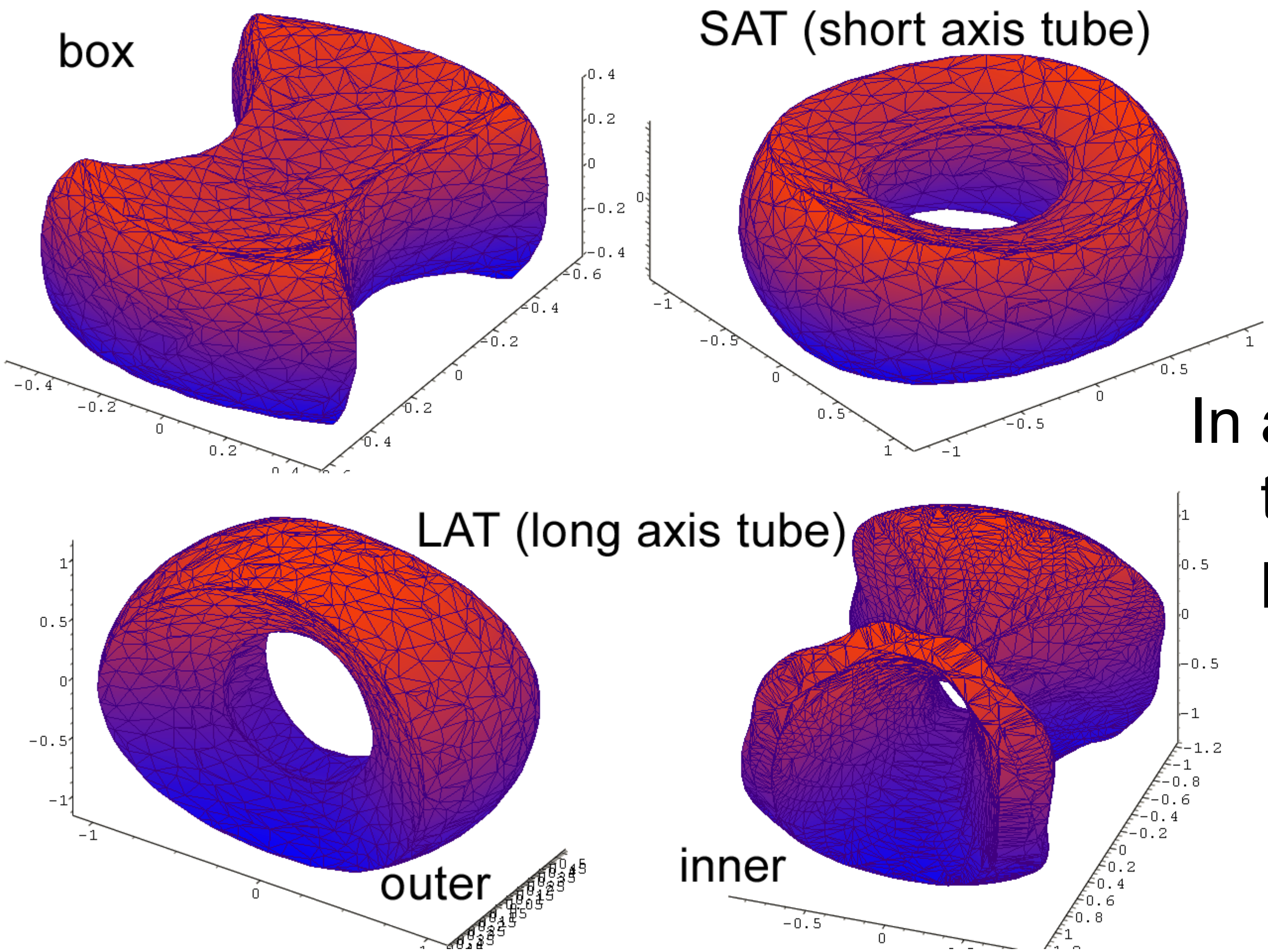
Plan of the talk

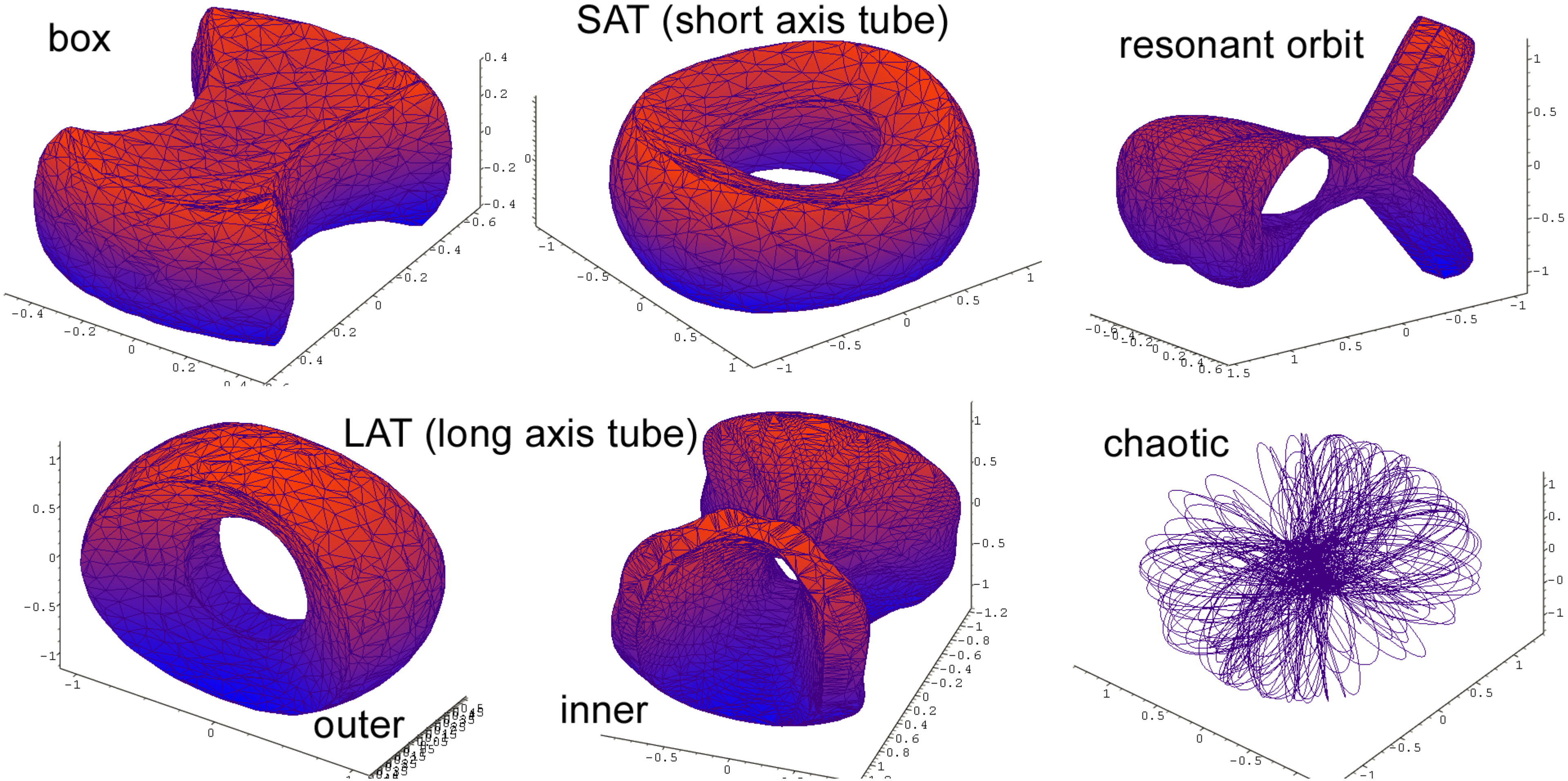
- Types of orbits in a triaxial potential
- Resonant and sticky chaotic orbits
- Construction of equilibrium triaxial galaxy models by Schwarzschild method
- Role of chaos in the change of galaxy shape

Types of orbits in triaxial potentials



In an integrable potential these are the only possible orbit types

Types of orbits in triaxial potentials

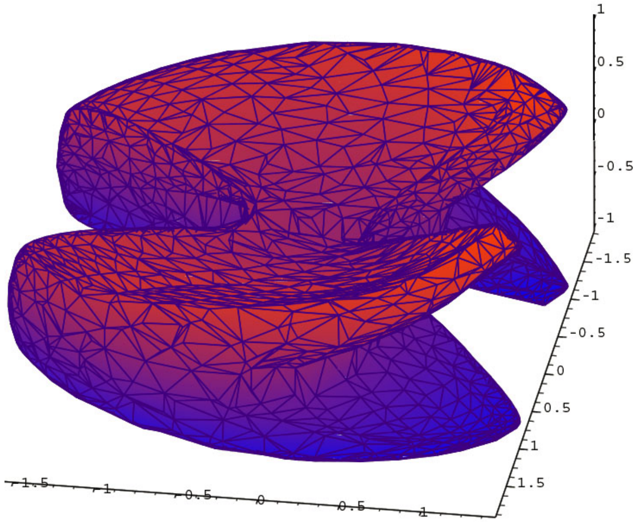


Resonant and thin orbits

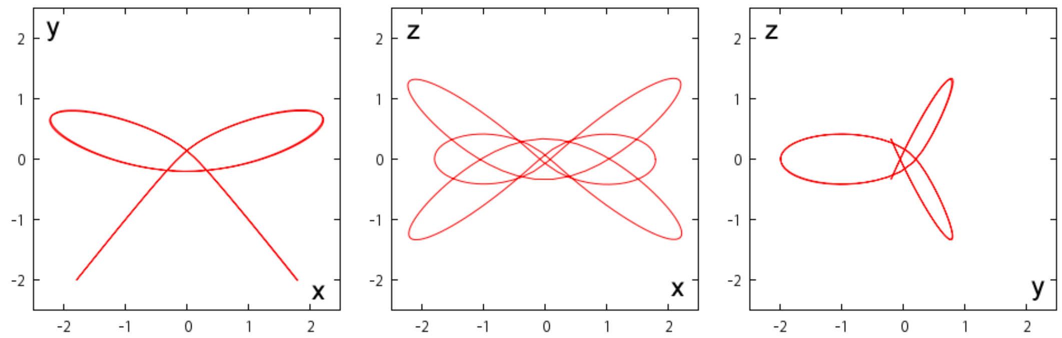
$\omega_x, \omega_y, \omega_z$ – leading frequencies of a regular orbit

Resonant or thin orbit: $n_1\omega_1 + n_2\omega_2 + n_3\omega_3 = 0.$

(2,1,-2) thin orbit



projections of a 3:4:5 resonant orbit



These orbits have associated regions in phase space,
and boundary layers are occupied by `sticky' chaotic orbits

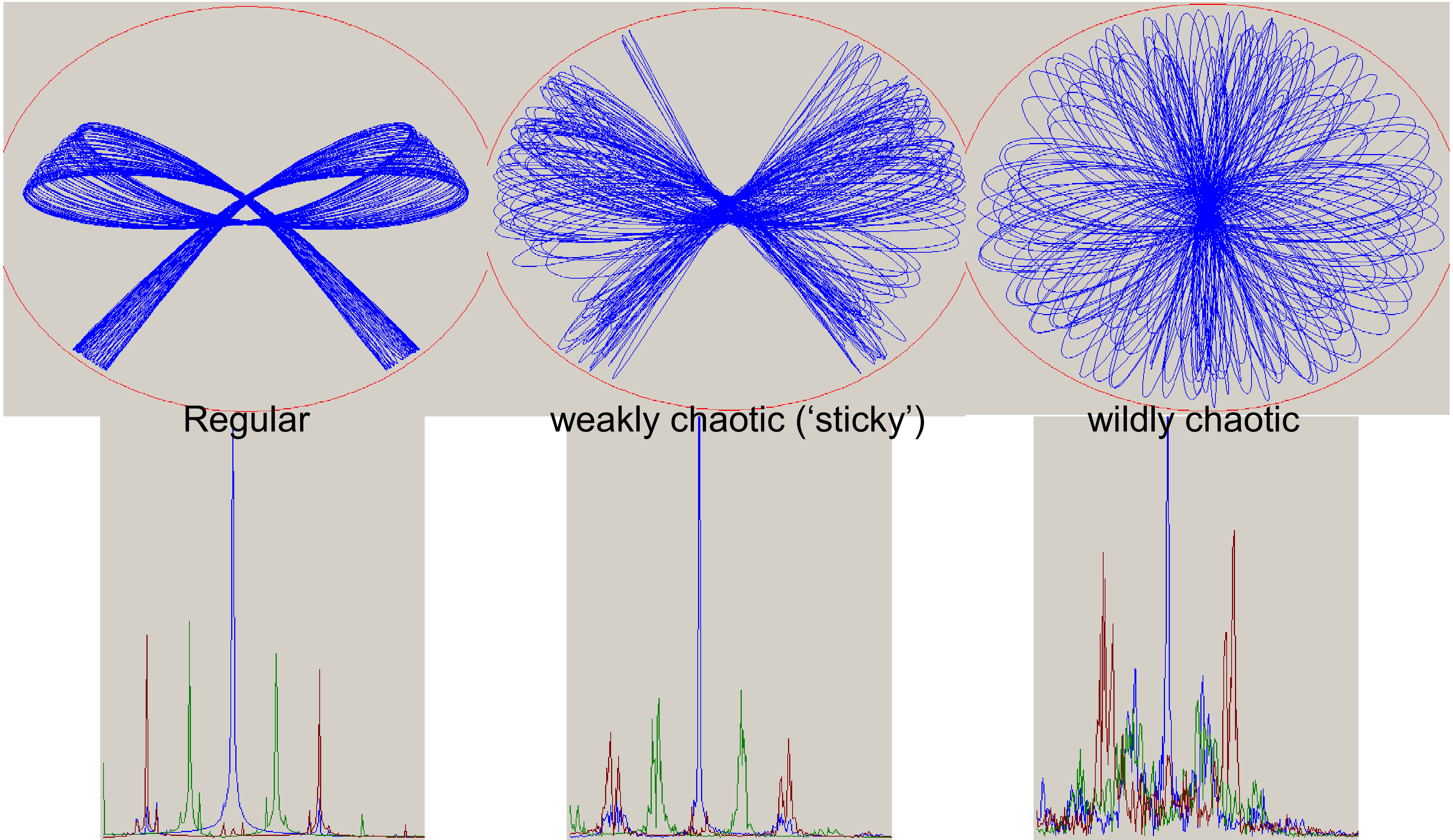
Definition of chaos

In a system with N degrees of freedom
a regular orbit has N integrals of motion,
a chaotic one has less than N.

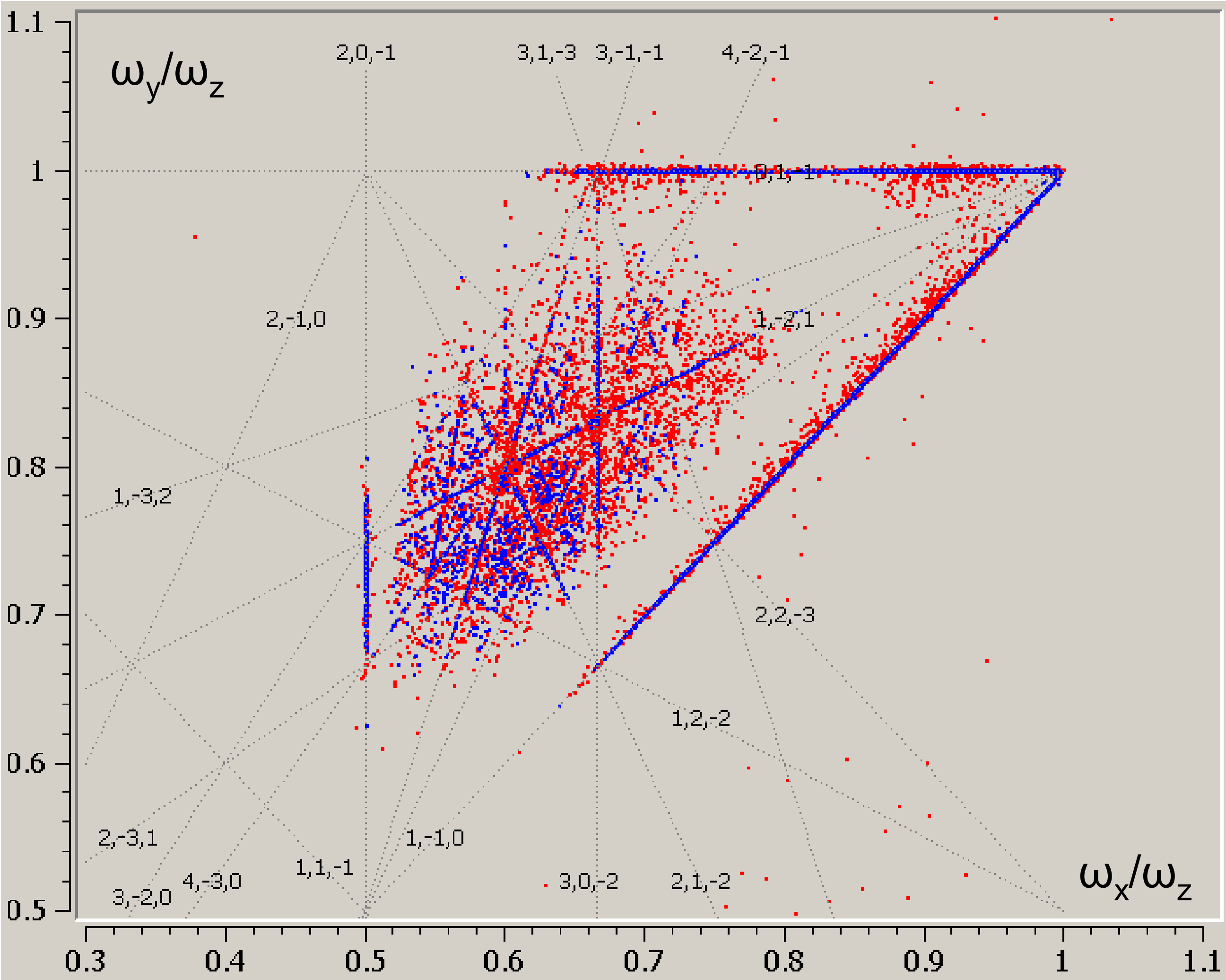
But the integrals are rarely known in explicit form!

For time-independent potential – energy;
~~For spherically symmetric – angular momentum;~~
~~For axially-symmetric – *z* component of angular momentum~~
For triaxial - ……

No well-defined transition to chaos!



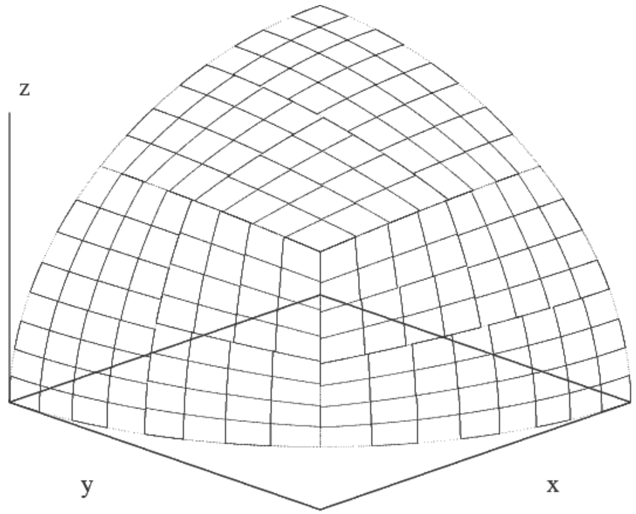
Frequency map as an instrument of studying the structure of phase space



Construction of equilibrium models
by Schwarzschild's orbit superposition method

- Take a specified density profile $\rho(\mathbf{r})$ / potential $\Psi(\mathbf{r})$
- Divide space into N_c cells with masses m_c
- Integrate N_o orbits in given potential ($N_o \gg N_c$)
  and calculate the fraction of time t_{oc}
  that *o*-th orbit spends in *c*-th cell
- Solve optimization problem:
  find orbit weights $w_o \geq 0$ so that

$$\sum_{i=1}^{N_o} w_o t_{oc} = m_c \quad c = 1..N_c$$



Features of Schwarzschild method

- Non-uniqueness of solution (if it exists at all)
- No guarantee of stability
  (only self-consistent equilibrium is assured)
- If we include chaotic orbits which may change their shape in time, then the solution may turn out to be non-stationary

To address these issues:

- Study the influence of chaotic orbits on the evolution of model shape
- Test the stability by evolving N-body representation of Schwarzschild model

Triaxial models considered

We consider two variants of triaxial Dehnen model, with density profile

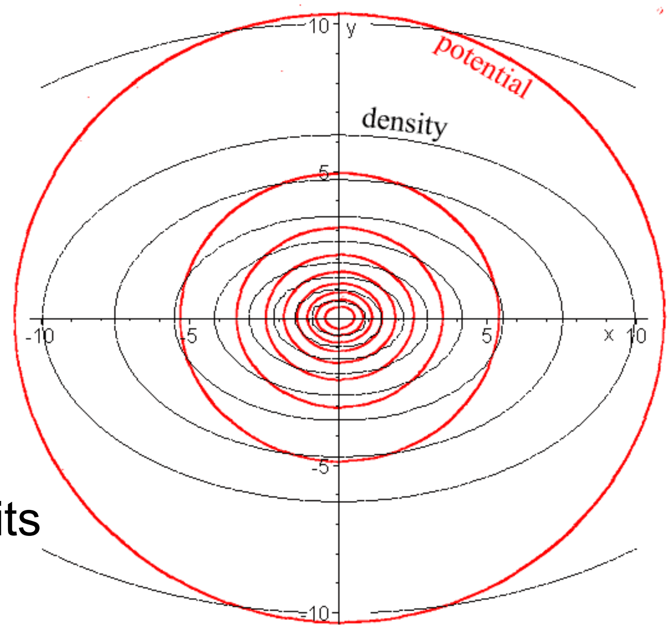
$$\rho(r) = \frac{(3-\gamma)M}{4\pi abc} \frac{1}{m^{\gamma}(1+m)^{4-\gamma}} \tag{1}$$

$m = [(x/a)^2 + (y/b)^2 + (z/c)^2]^{1/2}$ is the elliptic radius. We adopt dimensionless units in which $M = 1$, $a = 1$, $G = 1$ (which also fixes the time unit), and choose the axial ratios $b/a = \sqrt{5/8}$, $c/a = 1/2$, which corresponds to triaxiality parameter $T = (a^2 - b^2)/(a^2 - c^2)$ equal to $1/2$ (maximal triaxiality). For the cusp slope γ we choose two values: $\boxed{\gamma = 1}$ (weak cusp) and $\boxed{\gamma = 2}$ (strong cusp).

Chaotic diffusion and the change of model shape

Chaotic orbits may change the overall shape of density profile because if an orbit initially being sticky may eventually escape and become more wildly chaotic, and therefore tend to fill more uniformly the equipotential surface (which is rounder than equidensity surface).

The opposite process tends to be suppressed because initial (self-consistent) distribution of orbits did not contain much wildly chaotic ones.



To see whether this process may be important on a Hubble timescale, we look at the shape evolution of chaotic orbits in a *fixed* potential

Then we look how do they behave in a live *N*-body simulation

Chaotic diffusion and the change of model shape

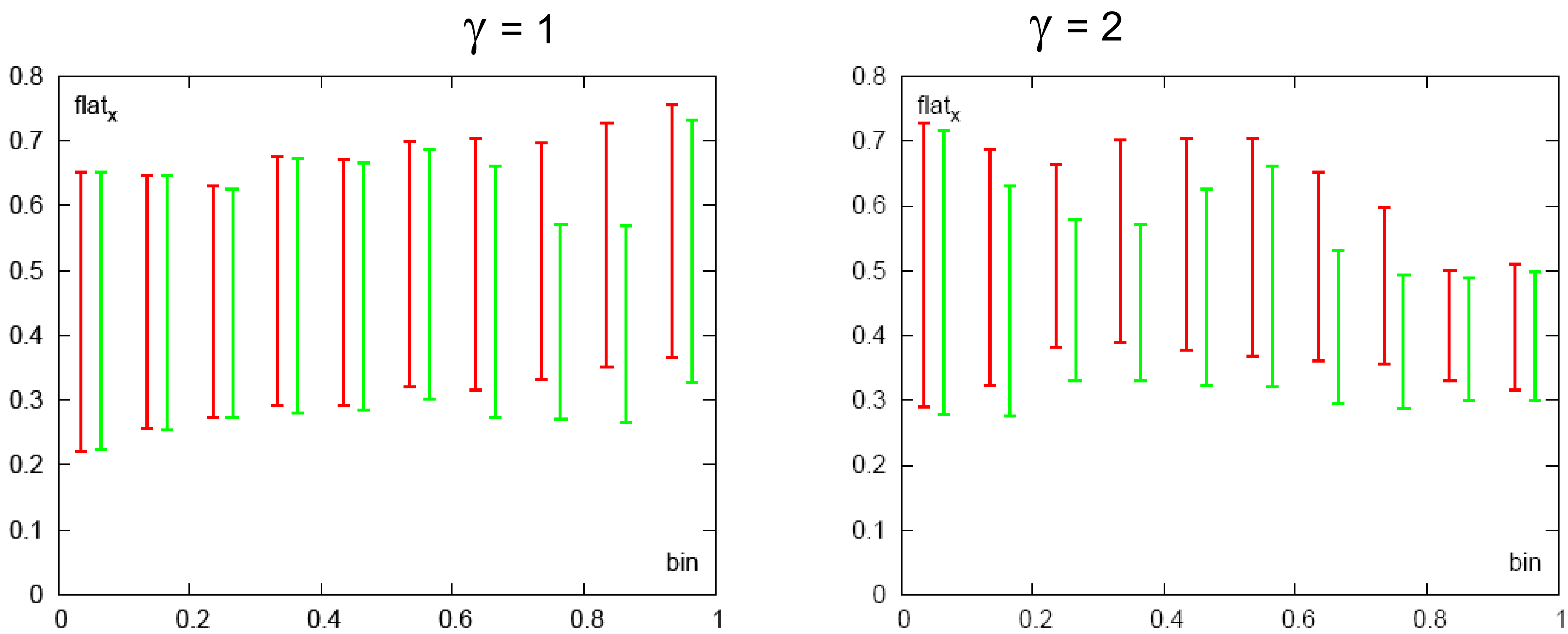
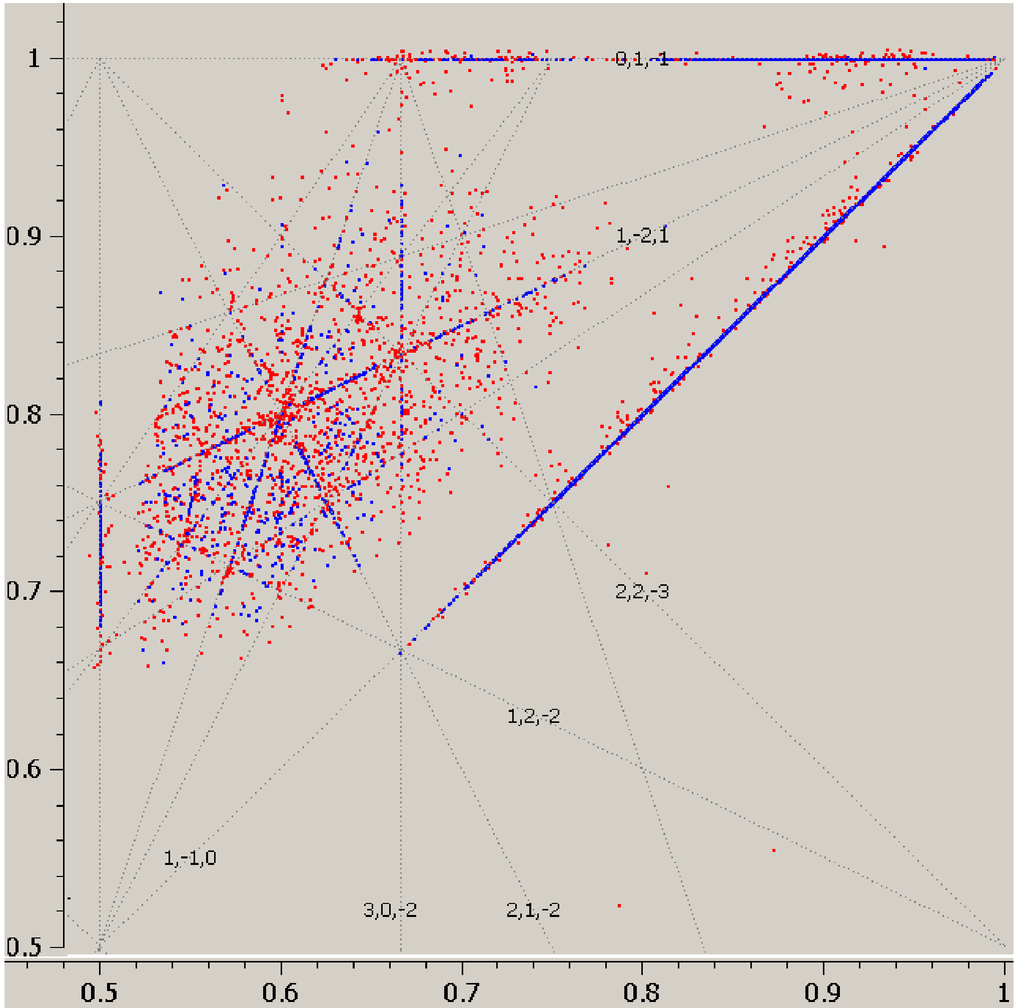
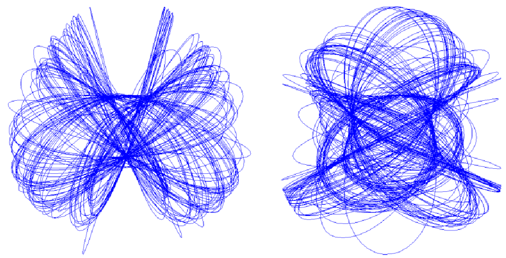


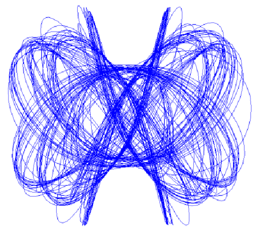
Figure 1. Shape change for chaotic orbits during $T = 1000\,T_{dyn}$. Each pair of error bars shows the spread in distribution of x-axis flattening $(I_{xx}/(I_{xx}+I_{yy}+I_{zz})$, where I_{ij} are the inertia tensor components), for initial (left, red) and final (right, green) $100\,T_{dyn}$ intervals of time, averaged over the ensemble of chaotic $(\Delta\omega > 10^{-3})$ orbits in a given energy bin. The horizontal axis corresponds to 10 bins each of which contains 10% of the total mass, with the innermost particles in the left bin). Top panel: $\gamma = 1$, bottom: $\gamma = 2$ models. The decrease in the average value in each pair means that orbits become rounder (y- and z-axis components increase at the expense of x-axis component). It is evident that in the weak-cusp case only the chaotic orbits in the outer shells do change shape systematically to become rounder, while in the strong-cusp case this tendency exists for most of the radial shells.

Phase space of weak-cusp model



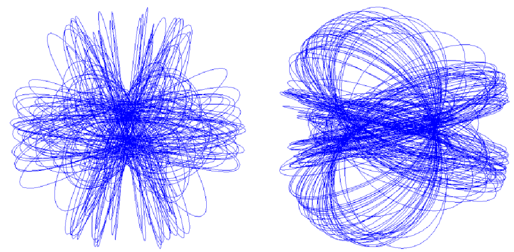
Many resonant orbit families; most chaotic orbits are associated with resonances and are `sticky'

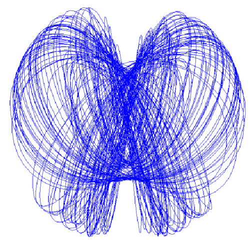




Initial segment:

(4,-1,-2) thin orbit

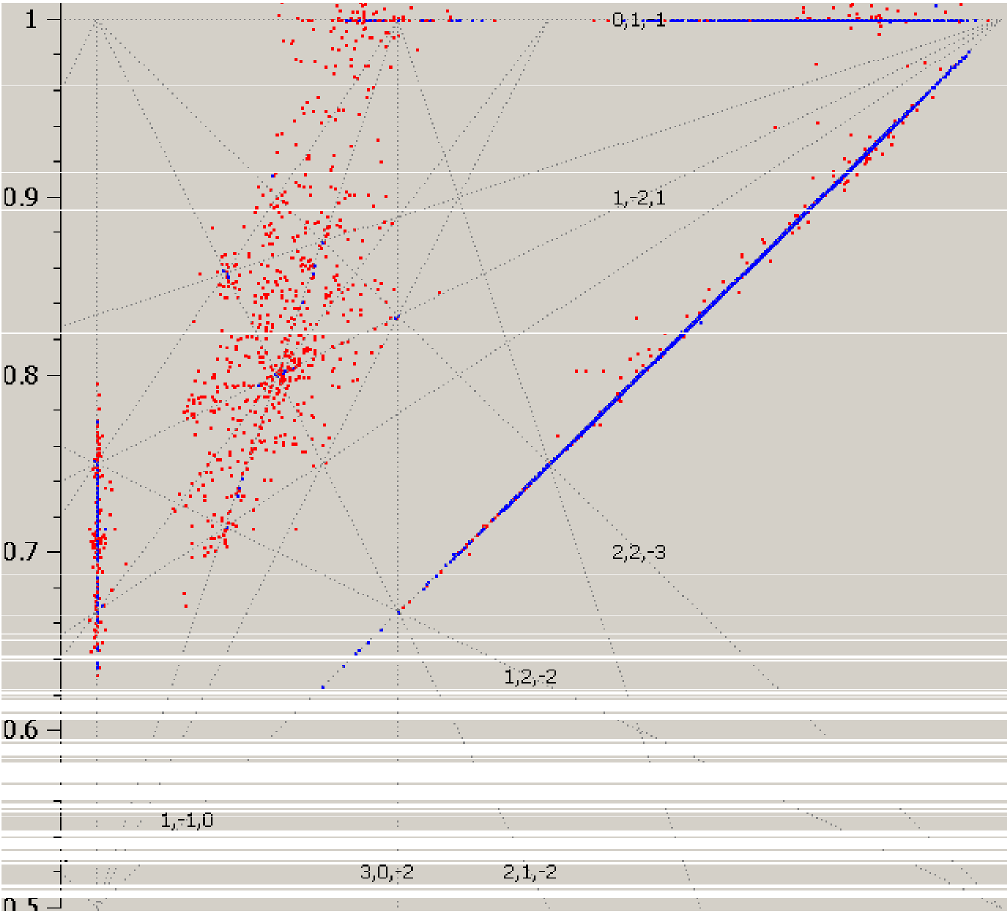


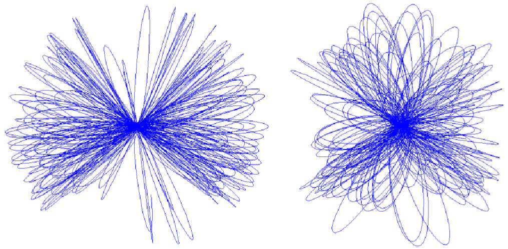


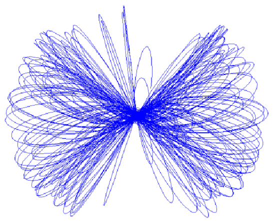
Final segment:

6:7:8 resonance

Phase space of strong-cusp model

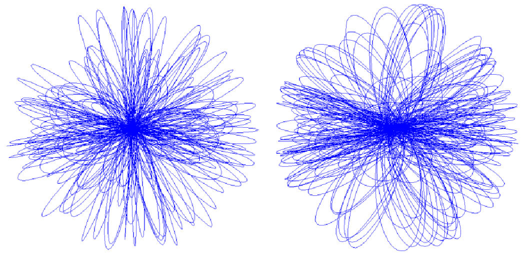


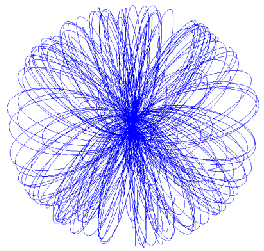




Initial segment:

flattened in x direction





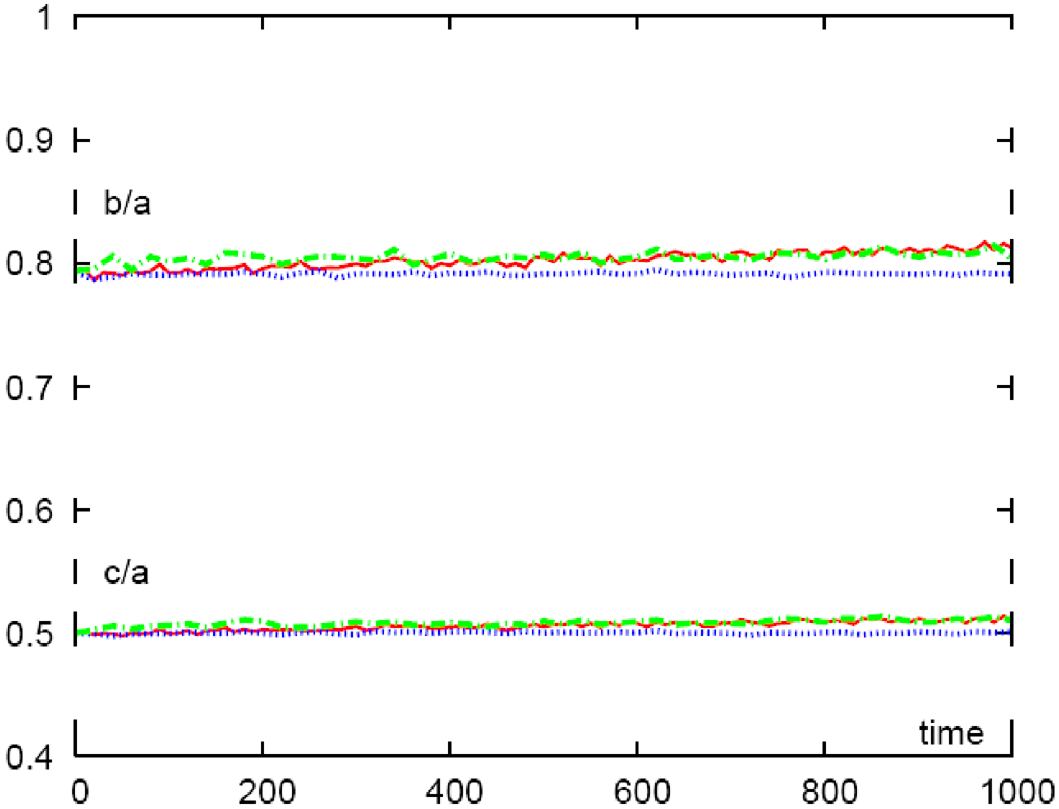
Final segment:

almost round

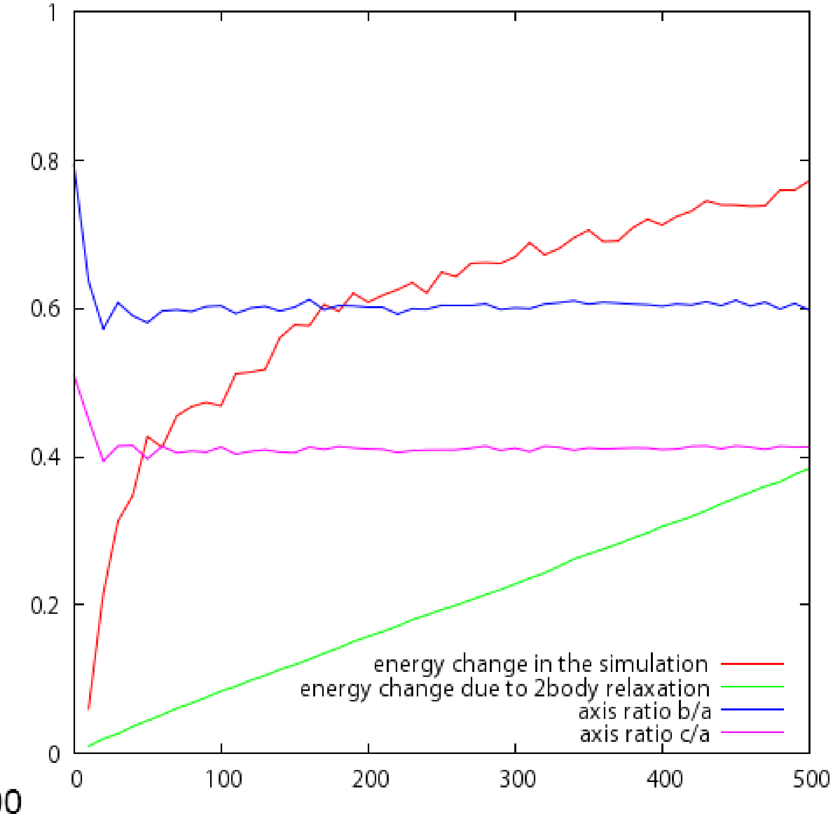
Almost all non-tube orbits are chaotic, often `wildly chaotic'

Stability of weak-cusp model (γ=1)

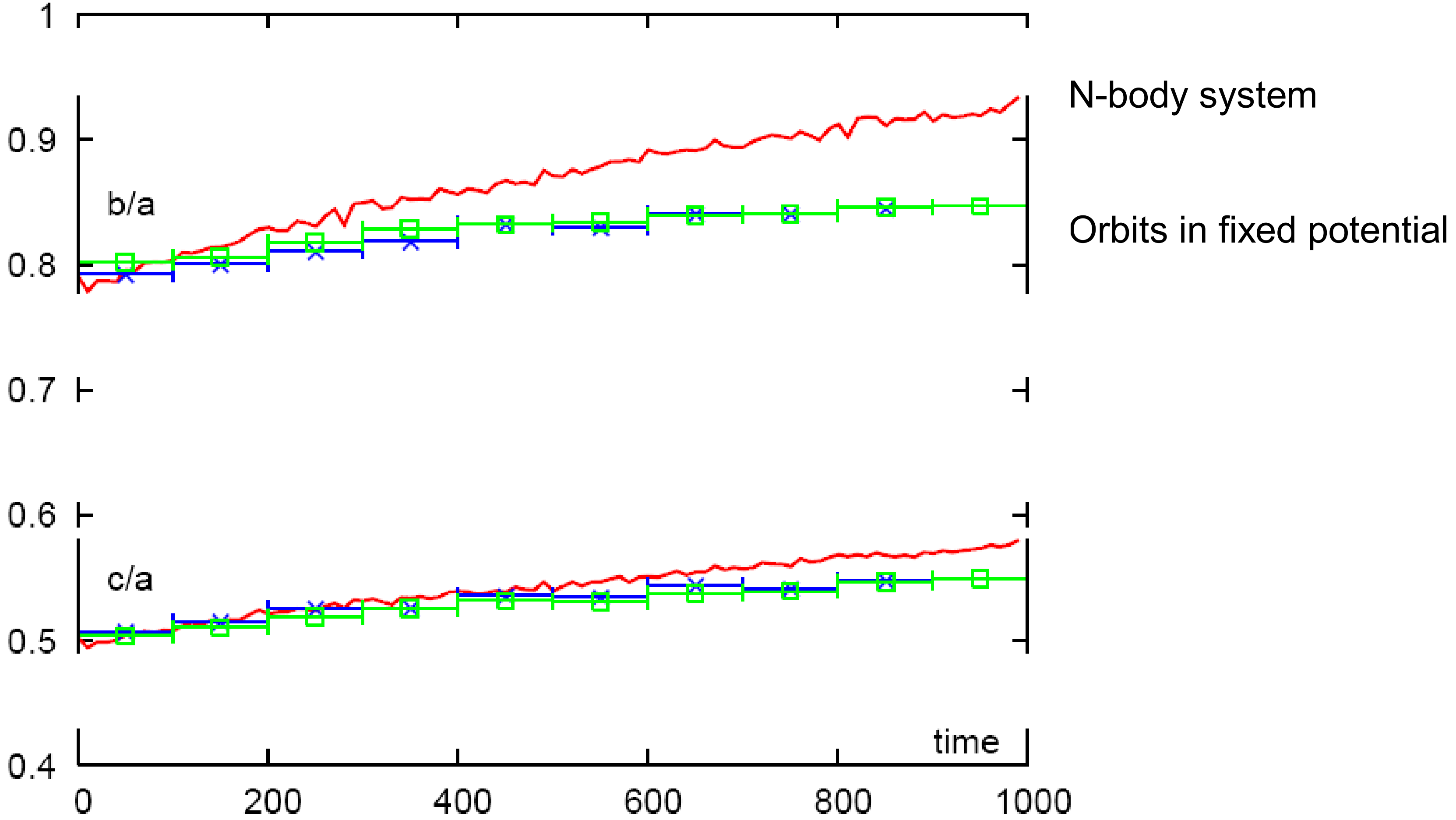
Typical stable model



Radially anisotropic model is subject to rapid onset of radial-orbit instability



Evolution of strong-cusp model (γ=2)



Conclusions

- Long-term (secular) evolution of galaxy shape may be caused by chaotic diffusion of orbits.
- It drives galaxies towards more spherical, or axisymmetric shape.
- Its rate and outcome depend on the phase space structure of potential:
  for the weak-cusp model it has complicated network of resonant orbits, which slow down chaotic diffusion;
  for the strong-cusp case there are no major resonances, and most chaotic orbits eventually become rounder.
- N-body evolution of model shape agrees well with the results of chaotic diffusion in fixed potential.