Dynamical modelling of galaxies

Eugene Vasiliev

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Plan

- Overview of dynamical modelling problem
- Theoretical and observational approaches
- The Schwarzschild orbit superposition method
- Fundamental restrictions on the parameter determination
- Other modelling approaches

Overview of dynamical modelling

A galaxy in dynamical equilibrium satisfies the Poisson equation

$$\nabla^2\Phi(\vec{\boldsymbol{r}}) = 4\pi \sum_c \rho_c(\vec{\boldsymbol{r}}) \ ,$$

and the collisionless Boltzmann equations for each component c:

$$\frac{\partial f_c(\boldsymbol{r},\boldsymbol{v})}{\partial t} + \boldsymbol{v}\,\frac{\partial f_c}{\partial \boldsymbol{r}} - \frac{\partial \Phi}{\partial \boldsymbol{r}}\,\frac{\partial f_c}{\partial \boldsymbol{v}} = 0.$$

The aim of dynamical modelling is to find the distribution function $f_c(\boldsymbol{r},\boldsymbol{v})$, or to provide useful constraints on the potential Φ and f (e.g. its moments – $\rho(\boldsymbol{r}) \equiv \int f(\boldsymbol{r},\boldsymbol{v})\,d\boldsymbol{v}$, $\quad \rho\,\sigma^2 \equiv \int f(\boldsymbol{r},\boldsymbol{v}) v^2\,d\boldsymbol{v}$, etc.)

Two “flavours” of dynamical modelling – theoretical and observational

Dynamical models: theoretical input

Jeans theorem : In a steady state, the distribution function may only depend on integrals of motion (in the given potential).

Thus we may have, for instance, in a spherical system, the energy E and angular momentum L as integrals of motion, and the d.f. is $f(E, L)$.

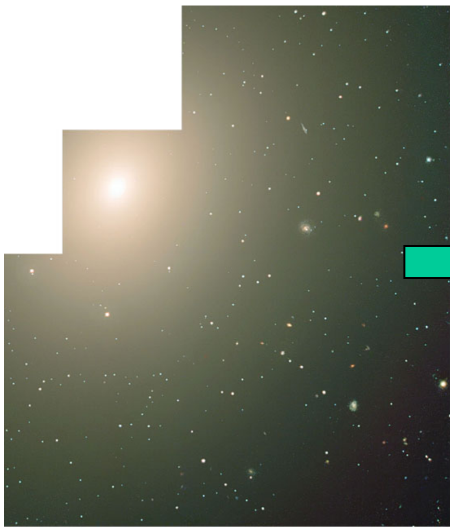
For simple cases, it is possible to find f if we specify ρ and Φ.
(e.g. Eddington inversion formula for $f(E)$ or its generalizations for $f(E,L)$.

In general case, we may not know the integrals of motion explicitly, they may not exist for every orbit, and there is no general way of finding f.

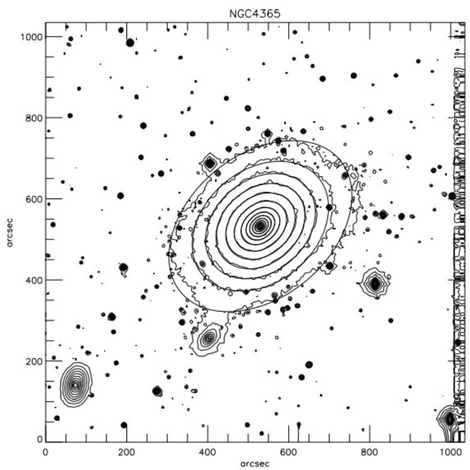
A popular approach is to represent a system as an N-body model (sample the distribution function by discrete particles).
It is guaranteed to be self-consistent (potential satisfies the Poisson eqn), and the model may be reasonably close to a steady state (not evolving).

Dynamical modelling: observations

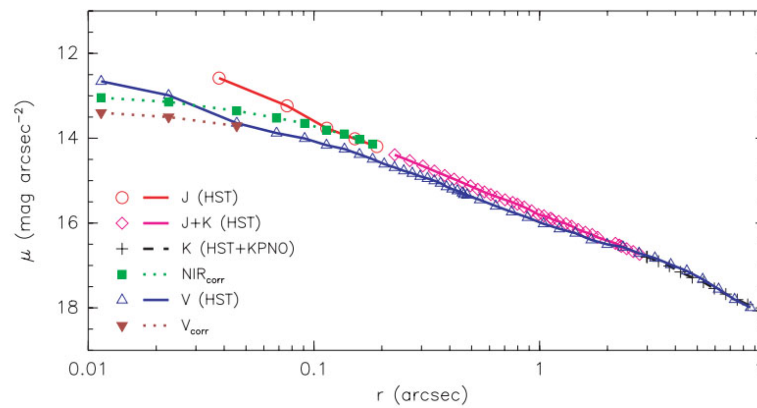
Photometric data



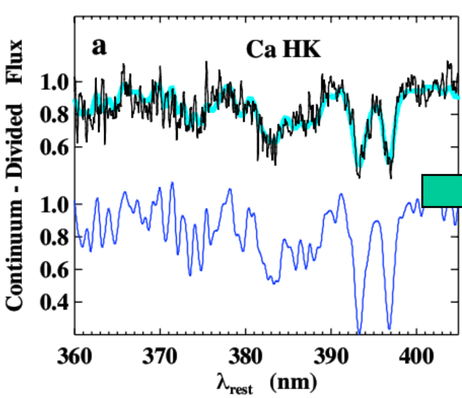
surface brightness map

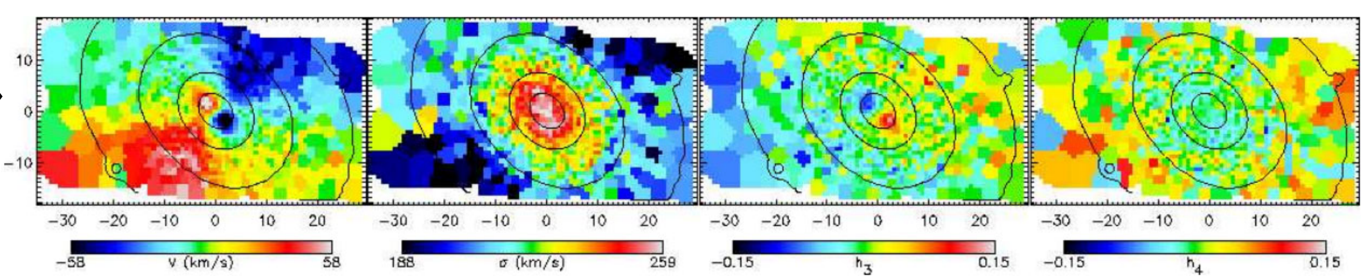


luminosity profile (+flattening)



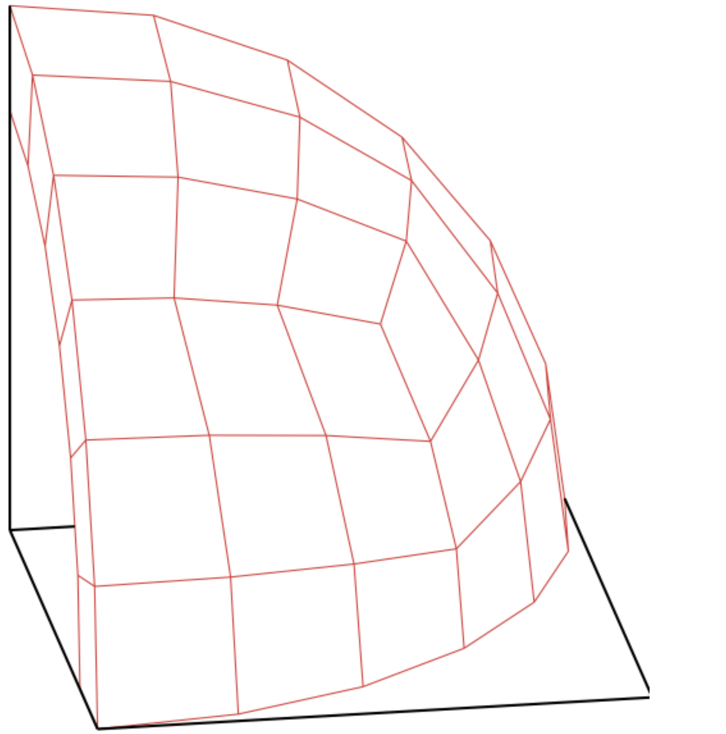
integral-field spectroscopy   =>   kinematic map (mean velocity, dispersion and higher moments, or full line-of-sight velocity distribution from fitting absorption line profiles)





A self-consistent dynamical model using Schwarzschild's method

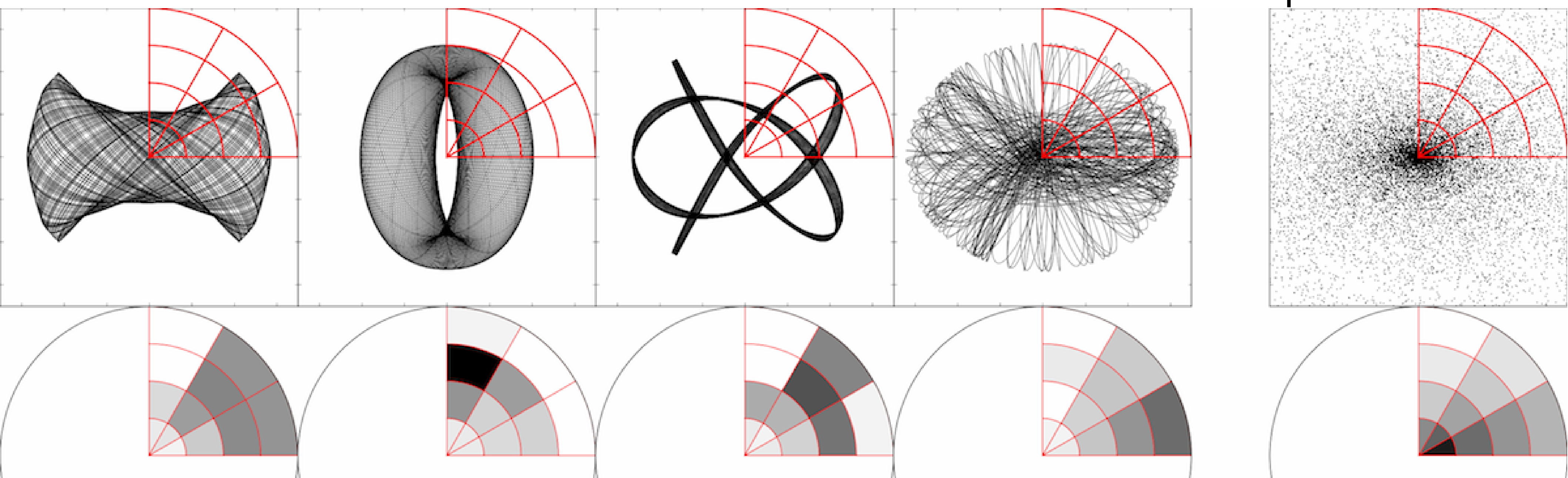
- take an arbitrary density profile $\rho(r)$ and potential $\Phi(r)$ (not necessarily self-consistent);
- discretize the space into a 3d grid; compute the mass in each grid cell;
- numerically compute a large number of orbits in the given potential, and record their spatial shape on the grid;
- assign orbit weights in such a way as to reproduce the required (discretized) density profile, and possibly additional (e.g. kinematic) constraints.



Orbit superposition

Orbits in the model

Target density profile



Discretized orbit density

(fraction of time t_{ic} that i-th orbit spends in c-th cell)

Discretized model density

(mass in grid cells – m_c)

For each c-th cell we require $\Sigma\, w_i\, t_{ic} = m_c$, where w_i is orbit weight

Linear optimization problem

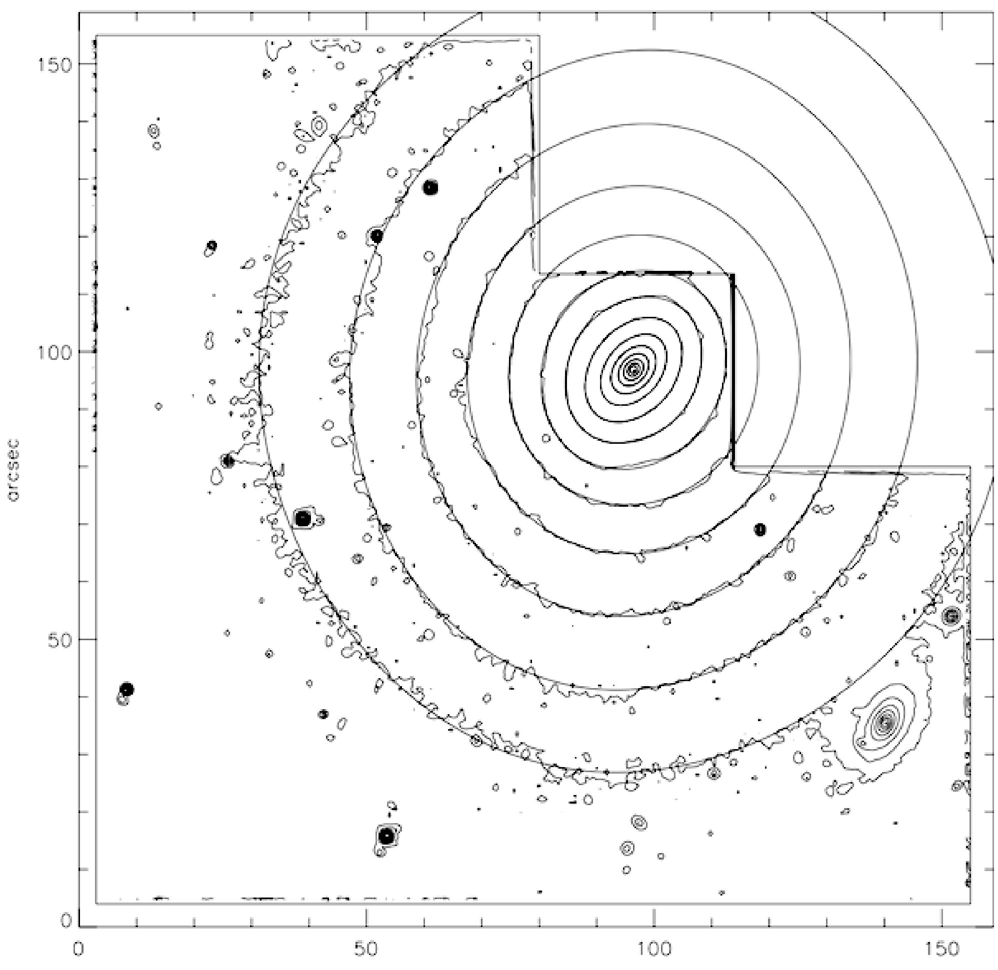
Solve the matrix equation for orbit weights w_o under the condition that $w_o \geq 0$

$$N_{\text{cell}} \left\{ \overbrace{\begin{vmatrix} \\ t_{ic} \\ \\ \end{vmatrix}}^{N_{\text{orb}}} \right. \times \begin{vmatrix} \vdots \\ w_o \\ \vdots \end{vmatrix} = \begin{vmatrix} \vdots \\ m_c \\ \vdots \end{vmatrix}$$

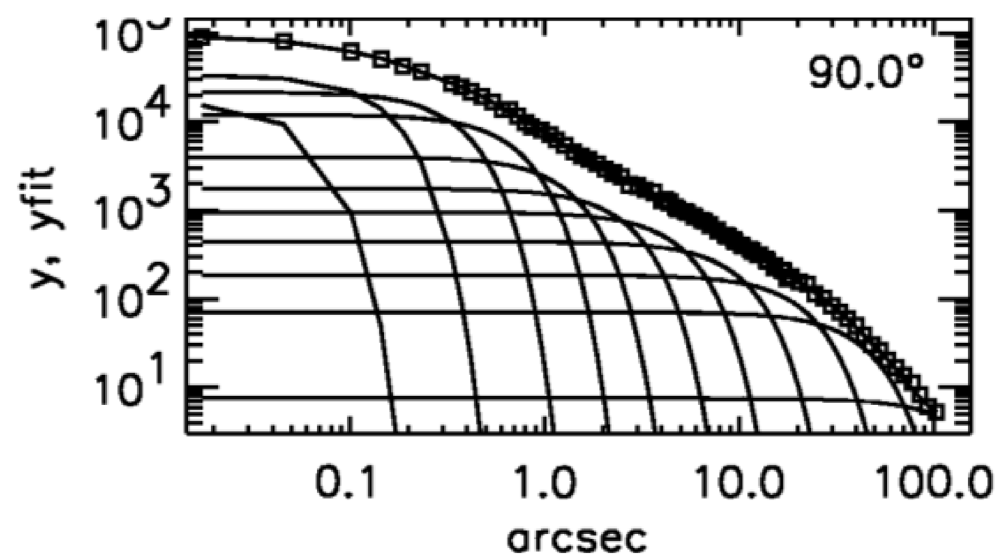
Typically $N_{orb} \gg N_{cell}$, so the solution to the above system, if exists, is highly non-unique. The number of orbits with non-zero weights may be as small as N_{cell}, and moreover, orbit weights may fluctuate wildly (which is considered unphysical). To make the model smoother, some regularization is typically applied (in which case the problem becomes non-linear, for instance, quadratic in w_o).

Modelling of observational data – photometry

Photometry => approximation by a suitable smooth surface brightness profile => deprojection (what to assume for the inclination angle?) => 3d density profile (assuming constant M/L? not necessarily..)



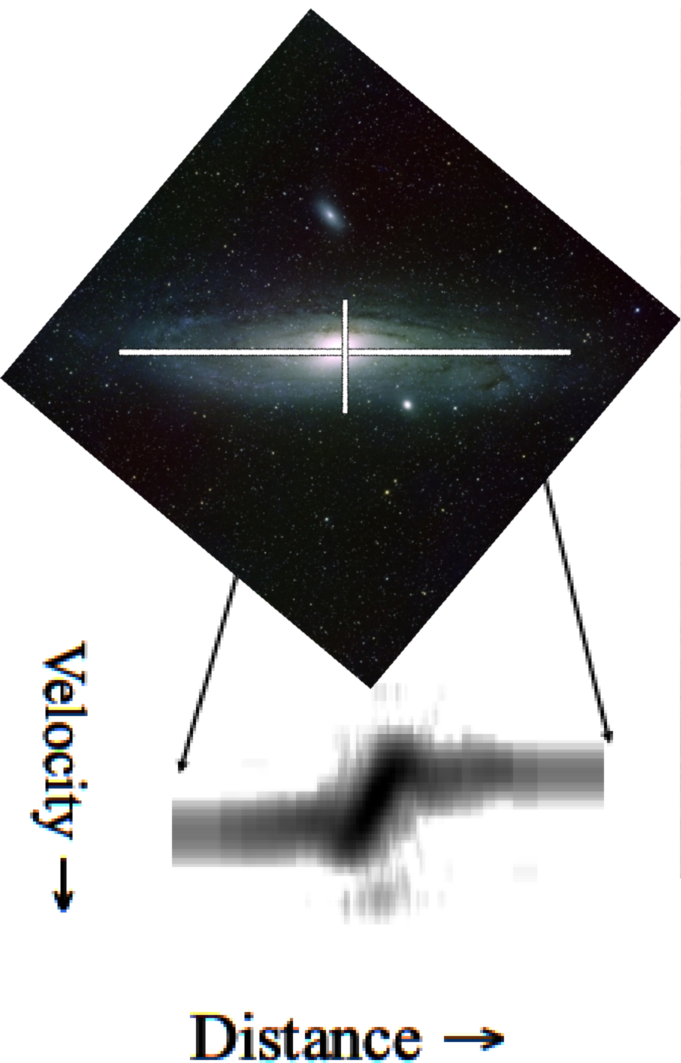
Usually approximate the density profile with a Multi-Gaussian Expansion



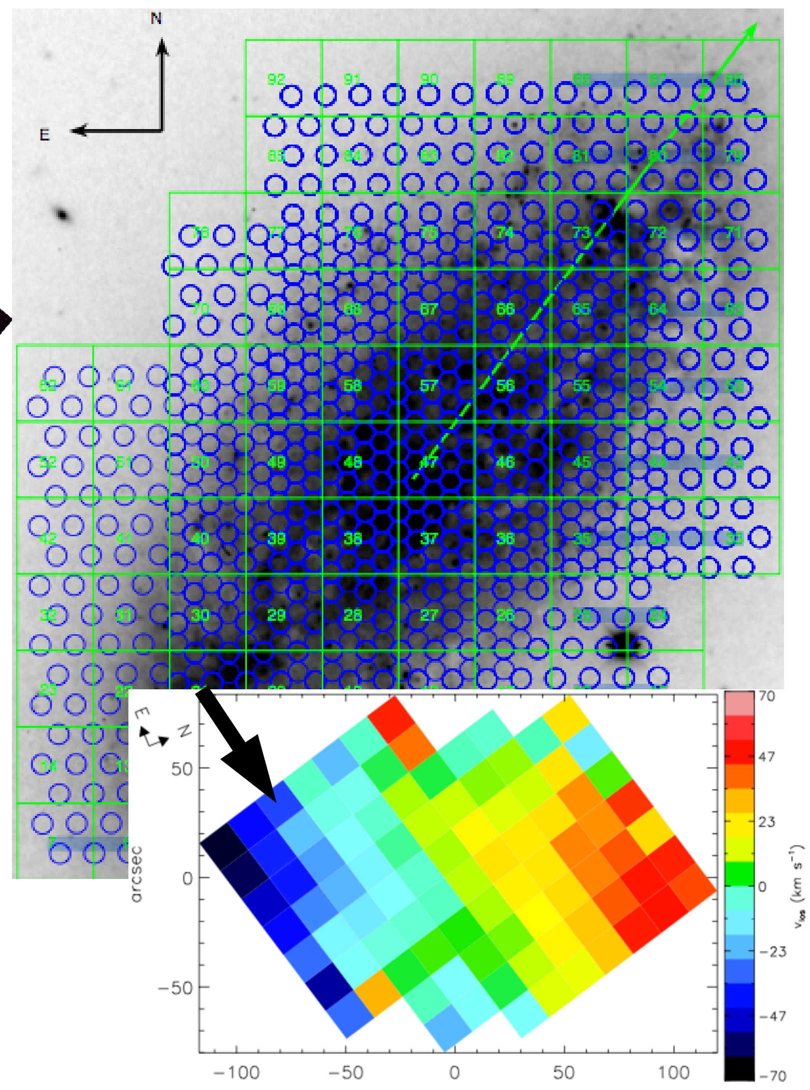
Alternatives: basis-set expansion,
Fourier decomposition for spiral galaxies, etc..

Modelling of observational data – kinematics

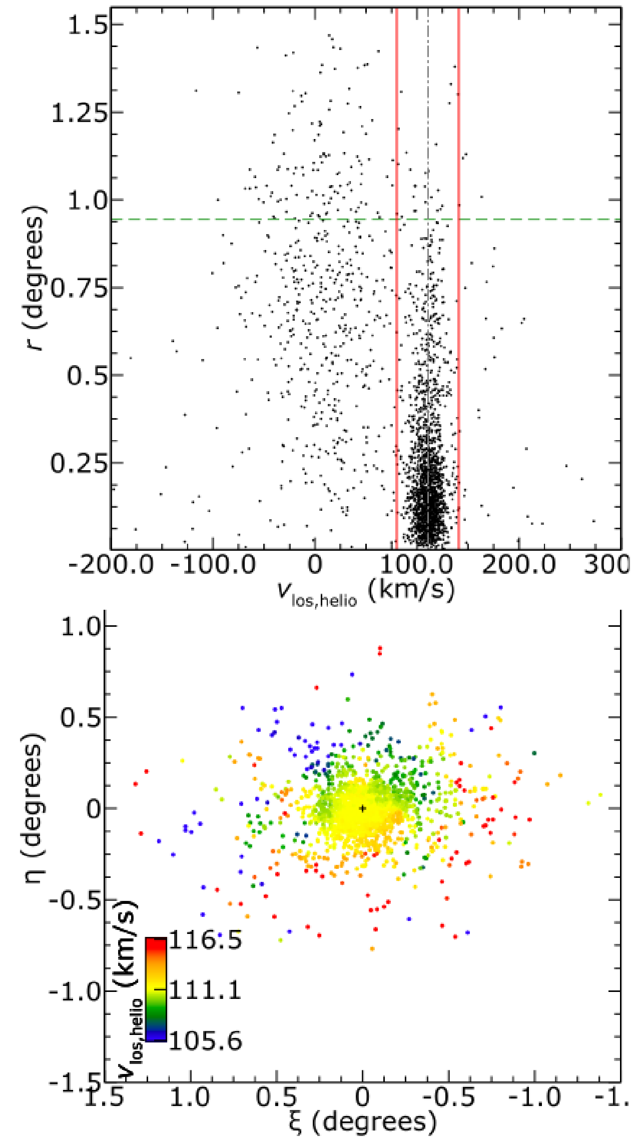
Long-slit spectroscopy



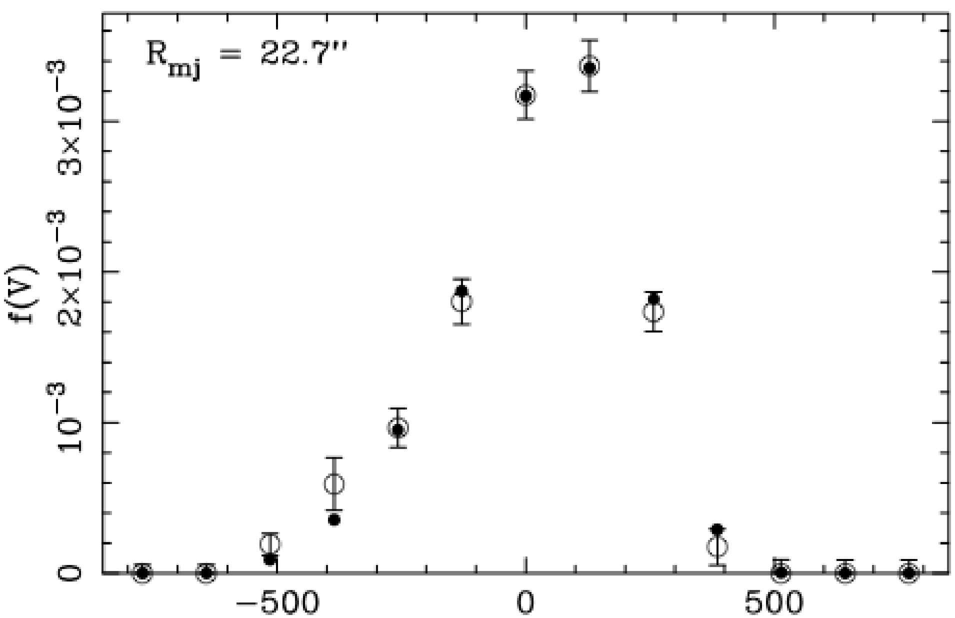
Integral-field spectroscopy

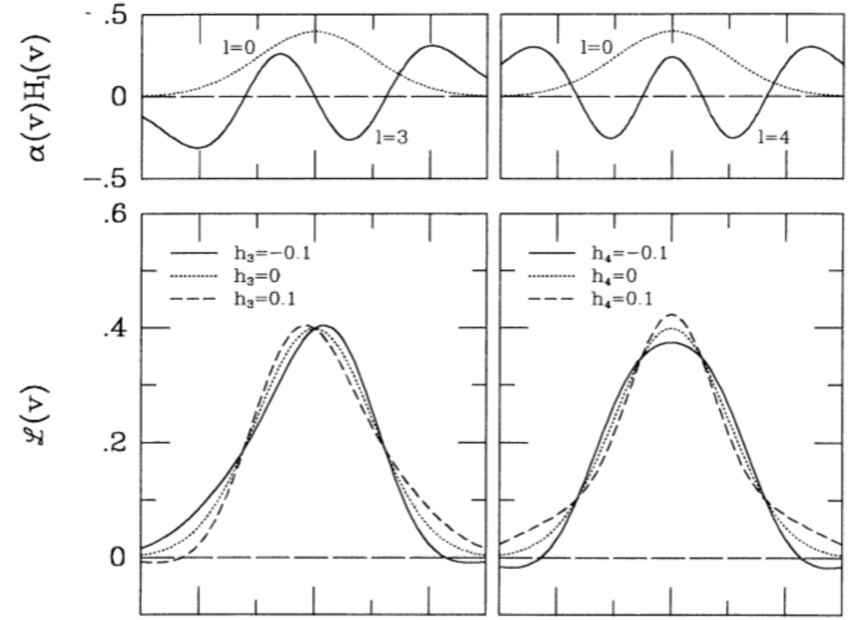


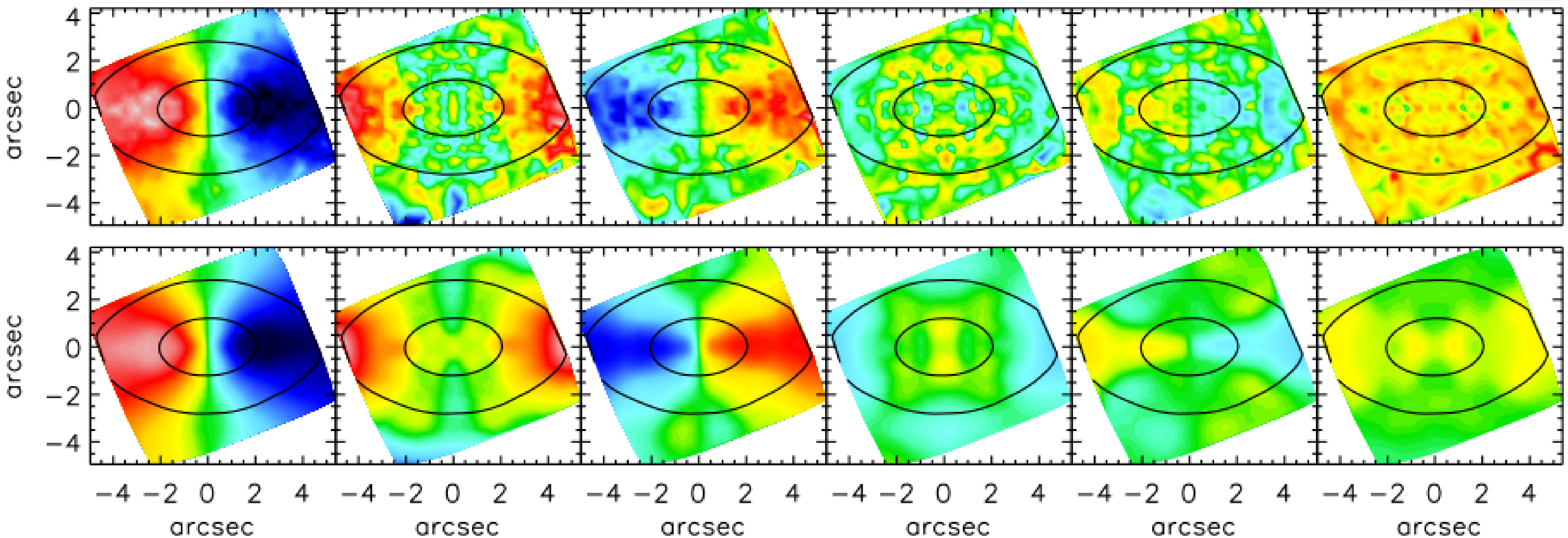
Individual star velocities



Kinematics: LOSVD, Gauss-Hermite moments,...







Schwarzschild modelling for observations

- Take some guess for the total gravitational potential $\Phi(\boldsymbol{r})$;
- Compute a large number of orbits (10^3–10^5), record density and kinematic information, including PSF and other instrumental effects;
- Solve for orbit weights w_o while minimizing the deviation χ^2 between predicted and observed kinematic constraints Q and adding some regularization λ:

$$\chi^2 = \sum_{i=1}^{N_{\rm obs}} \left(\frac{Q_{i,\rm mod} - Q_{i,\rm obs}}{\Delta Q_{i,\rm obs}} \right)^2 + \mathcal{F}_{\rm additional} \, .$$

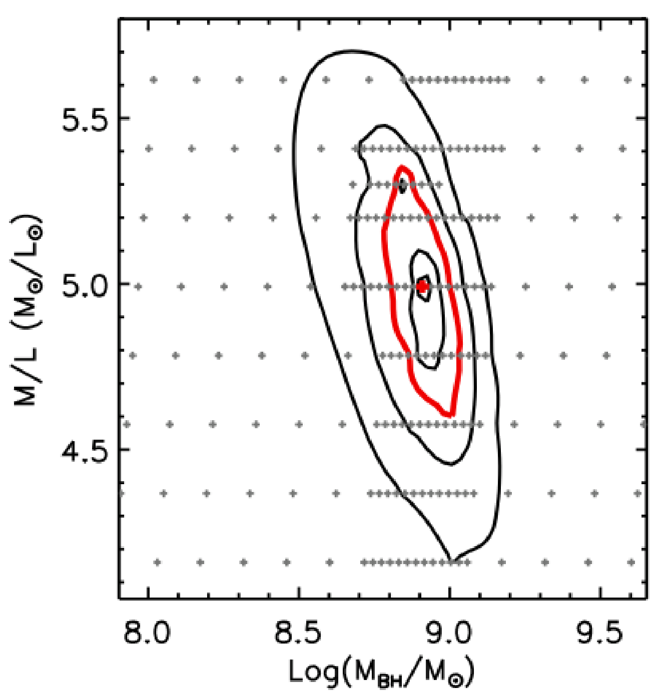
$$\text{Maximum-entropy approach: } \mathcal{F}_{\rm additional} = -\lambda S \equiv \lambda \frac{1}{M_{\rm total}} \sum_{o=1}^{N_o} w_o \ln(w_o/\tilde{w}_o) \, ,$$

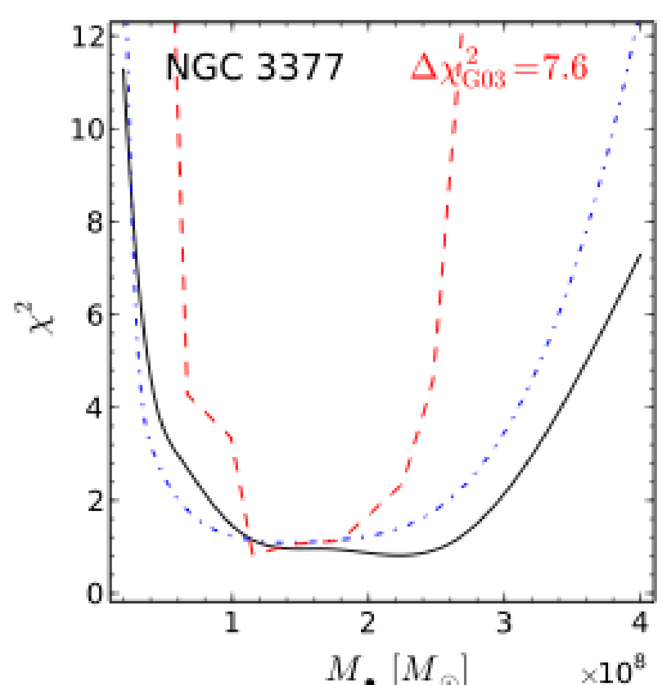
$$\text{or quadratic regularization: } \mathcal{F}_{\rm additional} = \frac{\lambda}{N_o} \sum_{o=1}^{N_o} (w_o/\tilde{w}_o)^2 \, .$$

Solution obtained by linear or quadratic programming, or non-negative least squares (NNLS)

Search through parameter space

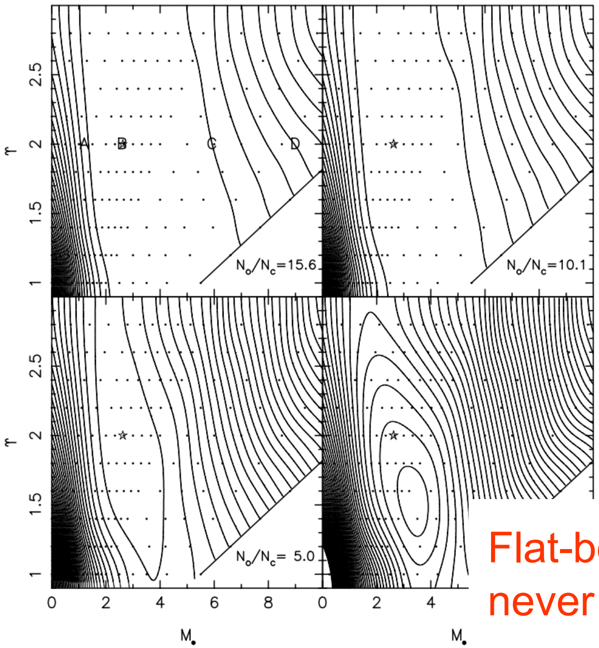
- Take some guess for the total gravitational potential and other model parameters;
- Construct an orbit superposition model that fits the observed kinematics and photometry; evaluate the goodness-of-fit $\chi 2$;
- Repeat with different parameters (M/L, M_{BH}, inclination, …) find best-fitting model and confidence intervals.
- Marginalize over unknown params (e.g. inclination)
- If possible, determine total potential (including dark matter halo) nonparametrically

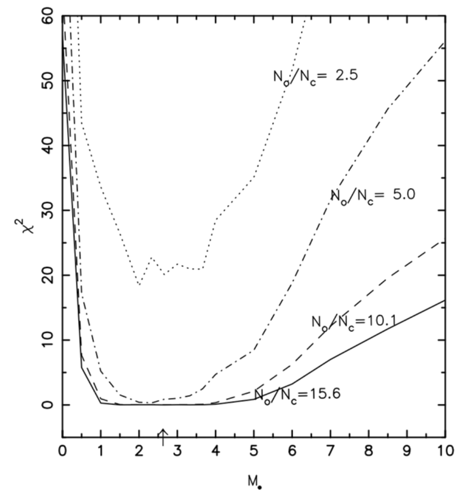




A fundamental indeterminacy problem

- The distribution function of stars generally is a function of three variables (integrals of motion); the gravitational potential, in a general case, is another unknown function of 3 coordinates.
- Observations typically may provide at most 3-dimensional data cube (1d LOSVD at each point in a 2d image) [exception: GAIA, etc]
- We cannot infer 2 unknown functions in a unique way from observations!
- Therefore, parameters are intrinsically degenerate
- If the confidence range for determined parameters is too narrow, it most likely means that the model was not general/flexible enough.





Flat-bottomed χ^2 plots are almost never seen in published papers!

Implementations of Schwarzschild method

Observation-oriented:

Axisymmetric:

- The “Nukers” group (Gebhardt, Richstone, Kormendy, et al...)
- The “Leiden” code (van der Marel, Cretton, Rix, Cappellari, …)
- The “Rutgers” code (Valluri, Merritt, Emsellem)

Triaxial:

- van den Bosch, van de Ven & de Zeeuw
- Zhao, Wang, Mao (for Milky Way)

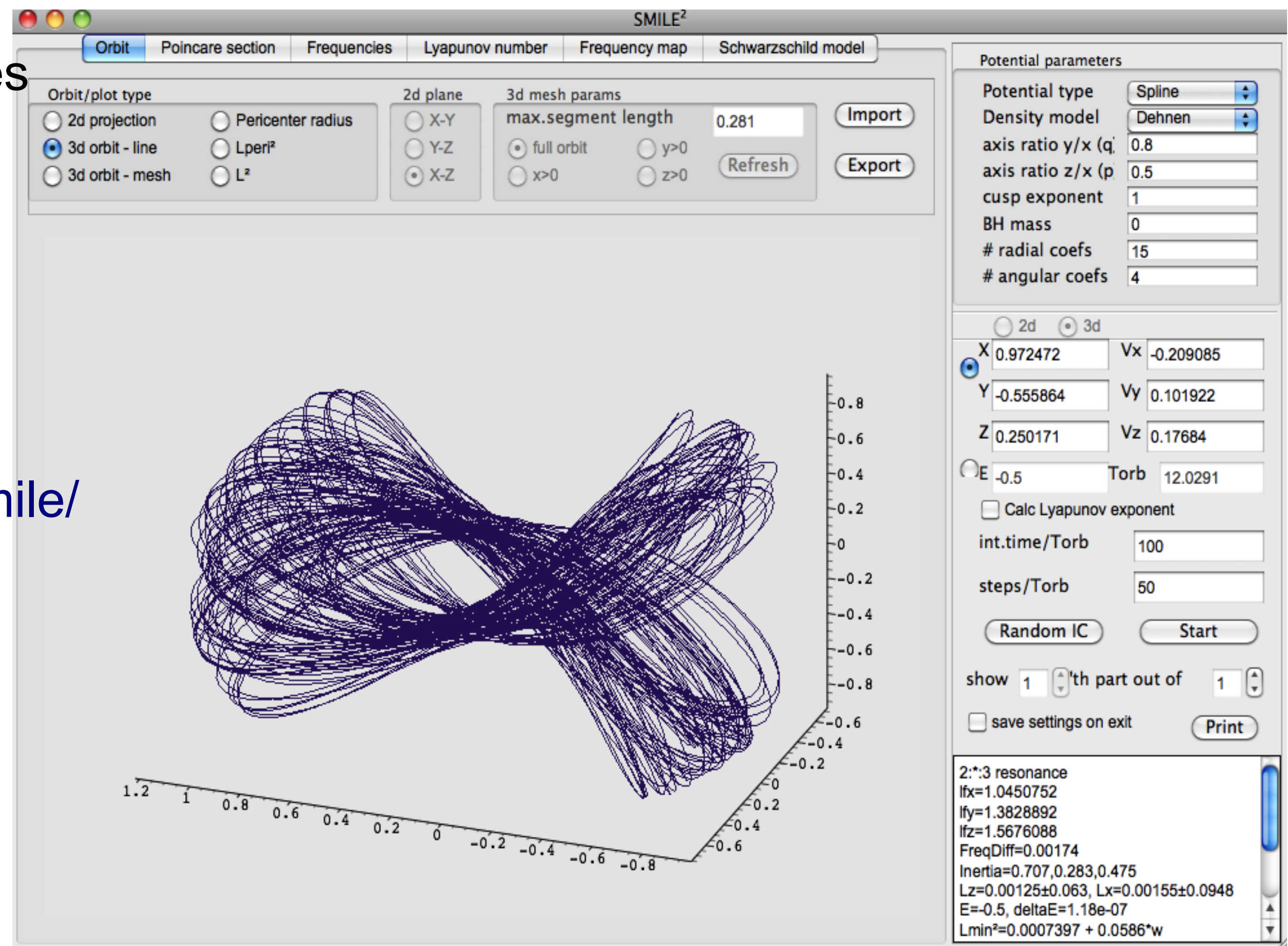
Theory-oriented:

- Schwarzschild(1979+)
- Pfenniger(1984)
- Merritt&Fridman(1996)
- Siopis&Kandrup(2000)
- Vasiliev(2013)

A bit of advertisement

SMILE orbit analysis and Schwarzschild modelling software

- Explore properties of orbits in arbitrary non-spherical potential;
- Various chaos detection tools and phase space visualization
- Create Schwarzschild models for triaxial galaxies (elliptical and disky)
- Educational and practical applications
- GUI interface
- Publically available at http://td.lpi.ru/~eugvas/smile/
- So far a “Theorist's tool”, but extension to observational modelling is planned



Other dynamical modelling methods

Based on Jeans equations:

- Jeans Anisotropic Models (Cappellari+)
  + easy to understand and apply, fast, efficient exploration of parameter space
  – only first two velocity moments; limited flexibility (axisymmetry; fixed orientation of velocity ellipsoid); existence of positive distribution function not guaranteed
- MAMPOSSt (Mamon+) – spherical, DF-based, flexible anisotropy, fast

Based on N-particle models:

- Made-to-measure (M2M) (Syer&Tremaine; Gerhard, de Lorenzi, Morganti; Dehnen; Long, Mao; Hunt, Kawata):
  particle evolution in a self-adapting potential; changing particle masses to adapt to observations; similar to Schwarzschild method but without an orbit library
- Iterative method (Rodionov, Athanassoula):
  adaptation of velocity field to dynamical self-consistency and observations
- GALIC (Yurin, Springel): like Schw. without orbit library, iteratively adjust velocities

Other approaches:

- Torus modelling (Binney, McMillan)
- Near-equilibrium flattened models (Kuijken, Dubinski; Dehnen, Binney; Contopoulos; ...)

Conclusions

- Dynamical modelling requires the knowledge of both density distribution and kinematics; usually the assumption of stationary state is also necessary
- The problem of finding the unknown potential from the tracer population of visible matter with unknown distribution is indeterminate; some assumptions are usually made to make any progress
- Various dynamical modelling methods offer a spectrum of opportunities: usually the more sophisticated and flexible ones that have least number of assumptions are also most expensive, while the simpler ones may suffer from model restrictions
- Confidence intervals on model parameters are often determined by hard to control systematic restrictions rather than the data itself;
more flexible methods may generally give a wider range of allowed parameters, which reflects true physical indeterminacy

Happy modelling!